Depinning and flow of a vortex line in an uniaxial random medium

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We study numerically and analytically the dynamics of a single directed elastic string driven through a 3dimensional disordered medium. In the quasistatic limit the string is super-rough in the direction of the driving force, with roughness exponent $\zeta_{\parallel}=1.25\pm0.01$, dynamic exponent $z_{\parallel}=1.43\pm0.01$, correlation-length exponent $\nu=1.33\pm0.02$, depinning exponent $\beta=0.24\pm0.01$, and avalanche-size exponent $\tau_{\parallel}=1.09\pm0.03$. In the transverse direction we find $\zeta_{\perp}=0.5\pm0.01, z_{\perp}=2.27\pm0.05,$ and $\tau_{\perp}=1.17\pm0.06.$ Our results show that transverse fluctuations do not alter the critical exponents in the driving direction, as predicted by the planar approximation (PA) proposed in 1996 by Ertas and Kardar (EK). We check the PA for the measured force-force correlator, comparing to the functional renormalization group and numerical simulations. Both Random-Bond (RB) and Random-Field (RF) disorder yield a single universality class, indistinguishable from the one of an elastic string in a two-dimensional random medium. While relations $z_{\perp}=z_{\parallel}+1/\nu$ and $\nu=1/(2-\zeta_{\parallel})$ of EK are satisfied, the transversal movement is that of a Brownian, with a clock set locally by the forward movement. This implies $\zeta_{\perp}=(2-d)/2$, distinct from EK. Finally at small driving velocities the distribution of local parallel displacements has a negative skewness, while in the transverse direction it is a Gaussian. For large scales, the system can be described by anisotropic effective temperatures defined from generalized fluctuationdissipation relations. In the fast-flow regime the local displacement distributions become Gaussian in both directions and the effective temperatures vanish as $T_{\tt eff}^{\perp}\sim 1/v$ and $T_{\tt eff}^{\parallel}\sim 1/v^3$ for RB disorder and as $T_{\rm eff}^{\perp} \approx T_{\rm eff}^{\parallel} \sim 1/v$ for RF disorder.

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I. INTRODUCTION

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References

Many driven systems display a depinning transition from a static to a sliding state, at a finite value of an applied force or stress. Examples are field-driven domain walls in ferromagnetic [1–3] or ferroelectric materials [4, 5], cracks under stress in heterogeneous materials [6–8], contact lines of liquids on a rough substrate [9, 10], imbibition of fluids in porous and fractured media [11], reaction fronts in porous media [12], solid-solid friction [13], sheared amorphous solids or yield-stress fluids [14], dislocation arrays in sheared crystals [15], current-driven vortex lattices in superconductors [16–22], skyrmion lattices in ferromagnets [23], or earthquakes models [24, 25]. Among these systems, the family of *directed elastic manifolds in random media*, where interactions between constituents are purely elastic, and topological defects absent, have become the framework of choice

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for understanding quantitatively universal properties of the depinning transition [16, 26–28].

Directed elastic manifolds embedded in a space of dimension D have an internal dimension d and an N-dimensional displacement field, such that D=N+d [29]. We focus on the overdamped zero-temperature dynamics when the manifold is driven by a force f in one of the N directions in a quenched random potential. In such a case, the competition of elasticity and disorder yields a critical force f_c , such that for $f < f_c$ the steady-state velocity vanishes, v=0, while for $f>f_c$ the interfaces slides according to a velocity-field characteristics v(f)>0. While one can impose the force f and then measure the velocity v(f), in most experiments the driving velocity v is imposed, and the pinning force f measured. This is achieved by confining the elastic manifold within a quadratic potential, given by the demagnetization field in magnets, gravity in contact-line depinning, or the bulk elasticity in earthquakes.

The case (d, N) = (d, 1) of a directed elastic interface has been studied in detail [30–33]. It was found that the depinning transition at f_c is continuous, reversible, and occurs at a welldefined characteristic threshold force f_c [34], below which the interface remains pinned in a metastable state. At f_c the interface is marginally blocked and the instability is described by a localized soft-spot or eigenvector [35]. Just above the threshold, the mean velocity v is given by the depinning law $v \sim (f - f_c)^{\beta}$, with β a non-trivial critical exponent. A divergent correlation length $\xi \sim (f-f_c)^{-\nu}$ and a divergent correlation relation time $au \sim \xi^z$ characterize the jerky motion as we approach f_c from above. Below the length scale ξ the interface is self-affine, with the displacement field growing as $u \sim x^{\zeta}$. Similarly $v \sim \xi^{\zeta-z}$ and $\beta = \nu(z-\zeta)$. The critical exponents have been estimated analytically [36-38] and numerically [32, 39–43]. The different universality classes are distinguished by d, the range [44–48] or nature [49] of the elastic interactions, the anisotropic [50] or isotropic correlations of the pinning force [51, 52], and by the presence of non-linear terms in addition to the pinning force [50, 53–57]. Boundary [58] or AC-driven [59] elastic interfaces have been studied as well. If the so-called statistical tilt symmetry (STS) holds, each depinning universality class has exactly two independent exponents, ζ and z. At large velocities, the disorder mimics thermal fluctuations with a velocity-dependent effective temperature, vanishing in the infinite-velocity limit. The particular case (d, N) = (1, 1), which represents an elastic line in a random medium, is relevant for bulk magnets [3, 60], and magnetic domain walls in ultra-thin ferrimagnetic materials at very low temperatures [61]. Variants are systems with longranged elasticity, as contact lines of liquid menisci [9, 10] or planar crack propagation [8]. When STS is broken, an additional KPZ term is generated, as in reaction fronts in porous media [12].

The depinning transition of a one-dimensional directed elastic line with a two-dimensional scalar displacement field (d,N)=(1,2) in a D=d+N=3-dimensional medium is realized for an isolated flux line (FL) in a superconductor. The 2-dimensional displacement field has one component u_{\parallel} parallel to the driving direction, and a perpendicular component u_{\perp} , while the string is directed in the x di-

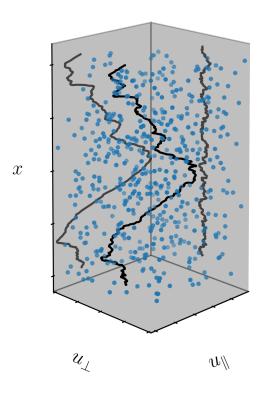


FIG. 1. Snapshot of an elastic line moving in an isotropic three-dimensional random environment. The projected profiles display the x-dependent displacements u_{\parallel} (parallel) and u_{\perp} (perpendicular) to the driving.

rection as shown in Fig 1. For an overdamped dynamics the study of such a system has been pioneered by Ertas and Kardar more than twenty five years ago [62]. Compared with the (d, N) = (1, 1) case of interface depinning considerable less progress has been made since then, with the exception of a precise study of the critical force in the case of isotropic disorder [63], relevant for superconductor applications [19, 64]. This is due to the difficulties the flux-line problem presents in addition to an elastic interface (and fewer applications). Indeed, even for harmonic elasticity and uncorrelated disorder one has to deal with a 2-component (vector) displacement field instead of the scalar displacement field of the interface. One of the important consequences of this difference is that Middleton's theorems [65], which lie at the heart of FRG calculations for depinning [36, 37, 66, 67], and which greatly accelerate numerical simulations [54, 68] are not valid for the FL, making a precise study of its large-scale properties difficult. In addition, renormalization-group calculations rely on an expansion in $\epsilon = 4 - d$, with d = 1 far away from the upper critical dimension. In Ref. 62 Ertas and Kardar postulated anisotropic scaling forms for the directions parallel and perpendicular to the driving, with exponents ζ_{\perp} , ζ_{\parallel} , z_{\perp} , z_{\parallel} , ν , β . The latter were calculated, both via FRG and via direct numerical simulations. They describe the anisotropic self-affine steady-state of the line below the depinning correlation length

$$\xi \sim (f - f_c)^{-\nu},$$

$$\langle [u_{\parallel}(x,t) - u_{\parallel}(0,0)]^2 \rangle = x^{2\zeta_{\parallel}} g_{\parallel}(t/x^{z_{\parallel}}),$$
 (1)

$$\langle [u_{\perp}(x,t) - u_{\perp}(0,0)]^2 \rangle = x^{2\zeta_{\perp}} g_{\perp}(t/x^{z_{\perp}}),$$
 (2)

with $\beta=\nu(z_{\parallel}-\zeta_{\parallel})$ and $g_{\parallel}(y)$ and $g_{\perp}(y)$ universal functions. The central idea of the 1996 Ref. 62 by Ertazs and Kardar (EK) is to propose a "planar approximation", postulating that the exponents corresponding to the direction parallel to the driving are equal to those for N=1, i.e. without a perpendicular direction. In a second step, exponents in the perpendicular direction are obtained, using the results in the parallel direction. EK then obtained the exponents $\zeta_{\parallel}=1$, and $\zeta_{\perp}=\zeta_{\parallel}-d/2=1/2$, using a FRG calculation at 1-loop order (followed by a numerical check).

They then claimed that this result holds to all orders in perturbation theory, using an argument advanced in 1993 for N=1 by Narayan and Fisher [31], and in agreement with numerical simulations at the time. In the meantime it has been established that for N=1 the exponent $\zeta>1$: The 2-loop FRG [36, 37] calculation of 2000 points out a mechanism absent from [31], which invalidates the argument for $\zeta\equiv(4-d)/3$ at depinning, while the latter remains valid in equilibrium. On the numerical side, it was shown that $\zeta>1$ [54, 69], though the 2-point function behaves as $(\mathcal{A}$ is a number)

$$\left\langle \left[u(x,t) - u(0,t) \right]^2 \right\rangle = \mathcal{A}|x|^2 L^{2\zeta - 2},$$
 (3)

which can easily be misinterpreted as $\zeta=1$. The most precise current value is $\zeta=1.25$ [43, 70], and probably exactly $\zeta=5/4$ [28, 71]. Assuming Ertasz and Kardar to keep $\zeta_{\parallel}|_{N>1}=\zeta|_{N=1}$, one may wonder whether they would keep the relation of $\zeta_{\perp}=\zeta_{\parallel}-d/2$, or replace it as well.

In this paper we revisit the problem of depinning and flow of an elastic string in three dimensions using numerical simulations and analytical arguments to address the above issues. Both for RB and RF disorder we confirm the validity of the anisotropic scaling forms and of the planar approximation for an extended set of exponents, together with the scaling relation $z_{\perp}=z_{\parallel}+1/\nu$ between dynamic exponents, as predicted in Ref. 62. Our results agree with the "improved" EK relation $\zeta_{\parallel}=\zeta_{\parallel N=1}=1.25$, while our perpendicular roughness is $\zeta_{\perp}=0.5\pm0.001$. We shall argue below that this is simply the thermal exponent of a d-dimensional elastic system,

$$\zeta_{\perp} = \frac{2-d}{2}.\tag{4}$$

We also calculate the force-force correlator, the avalanchesize and waiting-time distributions, as well as the joint distribution of the two components of the center-of-mass jumps in the quasistatic regime. We finally unveil a skewed distribution of local parallel displacements at low velocities, and show that the dynamical structure of the string at large scales has non-trivial features.

The large-velocity or fast-flow regime of elastic manifolds, usually considered trivial, present nevertheless interesting open issues. Using a perturbative analysis, Koshelev and

Vinokur [72] introduced the concept of shaking temperatures for moving vortex lattices by considering the effect of disorder as an effective thermal noise in the co-moving frame. Later on, for the same problem, it was numerically shown than an effective velocity-dependent temperature can be defined from generalized fluctuation-dissipation theorems [73]. A similar analysis can be performed for the fast-flow regime of elastic manifolds in general. For interfaces with harmonic elasticity in isotropic media, Functional Renormalization Group (FRG) calculations show that the large-scale structure is well described by the Edwards-Wilkinson equation with an effective temperature T which, at large velocities, vanishes as $T \sim 1/v$ [33]. In contrast, a perturbative analysis for a single vortex line finds that the shaking temperature is zero in the longitudinal direction and vanishes as 1/v in the transverse direction [16], contradicting the FRG prediction [33] for interfaces in combination with the planar approximation. Moreover, a single monomer driven in a two-dimensional disordered medium which may be thought of as the limiting case of very short correlation lengths along x, shows an effective temperature vanishing as $1/v^3$ in the parallel direction and 1/v in the transverse direction [74], hence adding a third different prediction for the same property. Understanding these aparent discrepancies is an open issue we address here.

We find that in the comoving frame the system can accurately be described by two effective temperatures, defined from a generalized fluctuation-dissipation relation. In particular, at large velocities v, for microscopic disorder of the random-bond (RB) type, this effective temperature vanishes as $T_{\tt eff}^{\perp} \sim 1/v$ in the transversal direction, and as $T_{\tt eff}^{\parallel} \sim 1/v^3$ in the driving direction, while for microscopic disorder of the random-field (RF) type we find $T_{\tt eff}^{\parallel} \sim T_{\tt eff}^{\perp} \sim 1/v$. Interestingly, applying the EK planar approximation in the fast-flow regime, the dependency on v of the effective temperatures in the RB case is incompatible with the longitudinal "shakingtemperature" obtained from a perturbative analysis [16], and the effective "Edwards-Wilkinson" temperature $\sim 1/v$ predicted in Ref. 33. Nevertheless, the latter prediction agrees with our result for microscopic RF disorder. Finally, for RB disorder we confirm that with increasing v the aspect ratio of the string width changes from being elongated in the driving direction, to being elongated in the perpendicular direction, as qualitatively predicted by Scheidl and Nattermann [16]. In contrast, the change of aspect ratio dissapears for RF disorder which crosses over from anisotropic fluctuations at intermediate velocities to isotropic ones at large velocities.

This paper is organized as follows: In Section II A we describe the model, the protocol and its numerical implementation. In section II B we define the observables of interest. Section III summarizes the theoretical results confirmed in our simulations, and proposes a new mechanism for the perpendicular direction. Due to its simplicity, we are able to predict many observables analytically. In section IV the main results are presented, followed by conclusions in section V.

II. MODEL AND OBSERVABLES

A. Model and implementations

We study a driven one-dimensional elastic line (string), directed along the x-direction, in a three-dimensional space at zero temperature. It is parameterized by the time-dependent 2-dimensional vector field $\mathbf{u}(x,t) = \{u_{\parallel}(x,t), u_{\perp}(x,t)\}$ which measures its continuum displacements along the parallel and perpendicular directions with respect to the driving direction (Fig. 1). The overdamped dynamics of the system is

$$\eta \partial_t \mathbf{u}(x,t) = c \partial_x^2 \mathbf{u}(x,t) + \mathbf{F}_p(\mathbf{u}(x,t),x) + \mathbf{f}(\mathbf{u}(\mathbf{x},\mathbf{t})).$$
 (5)

Here η and c are the friction and elastic constants, $\mathbf{f}(\mathbf{u})$ is the driving force and $\mathbf{F}_p(\mathbf{u},x)$ is a statistically isotropic random pinning force in the $\{u_\parallel,u_\perp\}$ plane. Eq. (5) is a minimal model for a single vortex line in a type-II superconductor induced by a magnetic field pointing in the x-direction [75, 76]. The left-hand side then represent Bardeen-Stephen friction. The first term on the right-hand side is a harmonic approximation for the elastic tension of a single vortex, the second the coupling to the defects in the material, and the third a force due to a uniform applied current or some other way to impose a uniform mean velocity. In this work, to impose a steady-state mean velocity along the u_\parallel direction $\mathbf{v}=\{v,0\}$ we use a moving parabolic trap,

$$\mathbf{f}[\mathbf{u}] = m^2 \left[\mathbf{w}(t) - \mathbf{u} \right]. \tag{6}$$

with $\mathbf{w}(t) = \{w,0\} = \{vt,0\}$ and $m \sim 1/L$, with L the size of the string. With this driving protocol, both the instantaneous center-of-mass velocity and the driving force fluctuate, as they do in most eperiments. We checked that the fixed-force ensemble (i.e. the one obtained by using a fixed force $\mathbf{f} = \{f,0\}$, and where the center-of-mass velocity fluctuates) yields equivalent results for large enough systems (see Appendix A). Moreover, driving with a confining potential allows us to measure the effective force-correlator, the central object of functional-renormalization-group calculations [28, 60, 77–80].

For RB disorder, we consider a pinning force $\mathbf{F}_p=\{F_p^\parallel,F_p^\perp\}=\{-\partial_{u_\parallel}V,-\partial_{u_\perp}V\}$ derived from a pinning potential

$$\overline{V(\mathbf{u}, x)V(\mathbf{u}', x')} = \delta(x - x')R(\mathbf{u} - \mathbf{u}'), \tag{7}$$

where $\overline{(...)}$ denotes average over disorder realizations and R(u) is assumed to be a short-ranged function.

For RF disorder we use

$$\overline{F_p^{\alpha}(\mathbf{u}, x)F_p^{\beta}(\mathbf{u}', x')} = \delta_{\alpha\beta}\Delta(|\mathbf{u} - \mathbf{u}'|)\delta(x - x'), \quad (8)$$

and $\Delta(u)$ a short-ranged correlated function. Note that this type of disorder is not conservative, thus may allow a momomer x to perform periodic orbits even in the presence of a finite viscosity. As an alternative, we could use Eq. (7), with $R(u) \sim |u|$. This could be done by generating Fourier modes for the disorder, multiply it with an appropriate kernel, and

then Fourier transforming. We will not do this since it is (i) prohibitively costly to implement, and (ii) not necessary close to the depinning transition. Indeed we show below that as long as m is sufficiently small, if a momomer position $\mathbf{u}(\mathbf{x})$ is updated, $u_{\parallel}(x)$ necessarily increases, forbidding periodic orbits. Both potentials represent isotropic disorder in the \mathbf{u} plane ("model A" in Ref. [62]).

We discretize the system along the x-direction, such that $\partial_x^2 \mathbf{u} \to \mathbf{u}(x+1) + \mathbf{u}(x-1) - 2\mathbf{u}(x)$, and implement different uncorrelated pinning potentials on the resulting layers $x = 1, \dots, L$, with periodic boundary conditions. We use dimensionless units with $\eta = c = 1$, and $R(0) = \Delta(0) = \mathcal{O}(1)$.

For computational convenience two different implementations for RB disorder were used, one for finite velocities, and the other for the quasi-static $v \to 0^+$ limit.

- (i) At finite velocities we directly solve Eq. (5) using standard finite-difference techniques, and consider for each monomer x the $V(\mathbf{u})$ to be the sum of Gaussian wells randomly distributed according to a Poissonian distribution.
- (ii) In the quasi-static limit the dynamics of the system becomes very slow, and the direct integration of Eq. (5) is computationally inefficient. We therefore use a cellular automaton. This algorithm is a generalization of the algorithm used in Ref. [81] for a particle dragged in a 2-dimensional disordered landscape. Here we generate a random pinningenergy landscape in a cubic lattice, described by the integer indices $\{x, n_{\parallel}, n_{\perp}\}$, using a uniform [0, 1) box distribution on each site, with uncorrelated energies at different sites. The 2-dimensional position of the monomers at each layer xis discretized on this grid, and thus given by two integers, $\mathbf{u}(x,t) = \{n_{\parallel}(x), n_{\perp}(x)\}.$ For a given configuration we compute the total energy of each monomer in all nine nearest neighboring positions, keeping the monomers of neighboring layers fixed. This energy for layer x is the sum of the random pinning energy, the quadratic potential well with curvature m^2 centered at $\{w,0\}$, and the elastic energy involving layers x + 1 and x - 1. In an iteration, each monomer moves to the neighboring site with the lowest energy. This elementary step is performed synchronously for all x and repeated M times for a fixed w. In the quasistatic limit, M is chosen ∞ , i.e. w is increased only after the string gets stuck. For v > 0, M > 1 is fixed (i.e. we let the string make M steps for each step δw in w). This protocol simulates a velocity that fluctuates around the mean velocity $v \sim \delta w/M$. We checked that this algorithm yields, in the low-velocity limit, the same universal results as the direct integration of the continuous displacements in the smooth pinning potential. Although most of our results are for RB disorder, we give some results, both for low and high velocities, for a RF type of disorder in order to detect possible dependencies on the microscopic disorder.

B. Observables of interest

We now define some observables of interest. The steadystate force-velocity characteristics f(v) is computed from the mean force in the steady state,

$$f(v) = m^2 [vt - u_{||}(t)].$$
 (9)

Here $u_{\parallel}(t)$ is the parallel component of the center-of-mass position

$$\mathbf{u}(t) = \langle \mathbf{u}(x,t) \rangle. \tag{10}$$

with $\langle (...) \rangle := L^{-1} \int \mathrm{d}x(...)$. The velocity-force characteristics, and in particular the critical force $f_c := \lim_{v \to 0} f(v)$, have a well-defined scaling behavior when $m \to 0$ [77]. For small v, we expect $v \sim [f(v) - f_c]^{\beta}$.

To characterize the geometry of the string we define the mean quadratic widths in both directions

$$W_{\alpha}^{2}(t) = \overline{\langle (u_{\alpha}(x,t) - u_{\alpha}(t))^{2} \rangle}, \tag{11}$$

with $\alpha = \{\parallel, \perp\}$. We also define the local displacement components with respect to the center of mass $\delta u_{\alpha}(x,t) := u_{\alpha}(x,t) - u_{\alpha}(t)$ and consider their distributions

$$P_{\alpha}^{u}(s) = \overline{\langle \delta(s - \delta u_{\alpha}(x, t)) \rangle}, \tag{12}$$

as well as the joint distribution

$$P^{u}(s_{\parallel}, s_{\perp}) = \overline{\langle \delta(s_{\parallel} - \delta u_{\parallel}(x, t)) \delta(s_{\perp} - \delta u_{\perp}(z, t)) \rangle}. \quad (13)$$

We define the anisotropic structure factor as

$$S_{\alpha}(q,t) = \overline{\left| \left\langle u_{\alpha}(x,t)e^{-iqx} \right\rangle \right|^{2}},\tag{14}$$

where $\alpha = \{\parallel, \perp\}$ and $q = 2\pi n/L$ with $n = 0, \ldots, L/2 + 1$. In the quasistatic $v \to 0^+$ regime, we consider the center of mass

$$\mathbf{u}(t) \to \mathbf{u}(w)$$
 (15)

as a function of w=vt instead of t. (Which object is referred to will be clear from the context.) For this we let the string relax completely before shifting $w\to w+\delta w$. To study avalanches, we compute the PDF

$$P_{\alpha}(S) = \overline{\langle \delta(S - Lu_{\alpha}(w + \delta w) + Lu_{\alpha}(w)) \rangle}.$$
 (16)

Discontinuous jumps of $u_{\alpha}(w)$ versus w are identified as shocks or avalanches. They occur at a discrete set of points, $w=w_i$, producing a finite set of jumps of sizes $S_i^{\alpha}=L(u_{\alpha}(w_i+\delta w)-u_{\alpha}(w_i))$, each one measuring the effective number of sites involved in the avalanche. The "waiting time" between consecutive jumps is defined as $\delta w_i=w_i-w_{i-1}$. Although δw_i is not a time, it yields the true waiting times $\delta w_i/v$ between successive avalanches in the $v\to 0^+$ limit. We are interested in its distribution

$$P^{w}(s) = \frac{1}{N} \sum_{i=1}^{N} \delta(s - \delta w_i)$$

$$\tag{17}$$

for a large sequence of $N \gg 1$ avalanches.

Finally, we consider the steady-state force-force correlation function in the driving direction, defined as [42, 77],

$$\Delta_{\alpha\beta}(w - w') = L^d \overline{f_{\alpha}(w) f_{\beta}(w')}^c, \tag{18}$$

where $f_{\parallel}(w) = m^2[w - u_{\parallel}(w)]$ is the (spatially averaged) parallel force for each metastable state at fixed w, and $f_{\perp}(w) =$

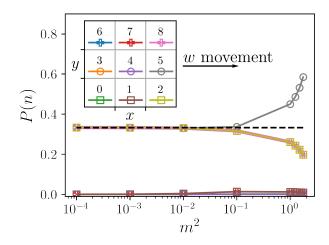


FIG. 2. Single monomer jump probability for the eight possible directions from position 4, in the cellular automata model.

 $-m^2u_\perp(w)$ is the perpendicular force. Due to symmetry, the cross correlator $\Delta_{\parallel\perp}(w)$ vanishes, and we are left with two (a priori) independent correlation functions. From the correlation theorem these functions can be computed using the Fourier transform as $\Delta=\mathscr{F}^{-1}\{|\hat{f}|^2\}$, with $f\xrightarrow{\mathscr{F}}\hat{f}$. This quantity is a central object of the theory and can be compared quantitatively with FRG calculations and experiments [28, 42, 60, 77–81].

III. ANALYTICAL RESULTS

A. Analytical results for the planar approximation

EK consider a range of models, of which we focus on the simplest one, their *model A*. Omitting some additional terms, and changing to our notations, the essence of EK's Eqs. (2.10a)-(2.10b) is

$$\eta \partial_t u_{\parallel}(x,t) = c \nabla_x^2 u_{\parallel}(x,t) + F + f_{\parallel} \big(x, \mathbf{u}(x,t) \big), \quad (19)$$

$$\eta \partial_t u_{\perp}(x,t) = c \nabla_x^2 u_{\perp}(x,t) + F + f_{\perp}(x,\mathbf{u}(x,t)).$$
 (20)

EK then say in Eq. (3.2) that Eq. (19) can be reduced to

$$\eta \partial_t u_\parallel(x,t) = c \nabla_x^2 u_\parallel(x,t) + F + f' \big(x, u_\parallel(x,t) \big). \tag{21} \label{eq:21}$$

EK argue that as long as $f_{\parallel}(x,\mathbf{u})$ is short-ranged correlated, so will be $f'(x,u_{\parallel})$. We agree with their analysis, and would like to justify it as follows: If we assume the pinning energies to be bounded, so will be the possible forces. The critical force f_c becomes large for $m \to 0$, thus (in the cellular automaton model) each monomer can choose among the three forward neighbors, while the three backward neighbors as well as the two sideway neighbors can be neglected. The locally chosen force (including the elastic forces) is the maximum force among the three possible choices, i.e. the maximally possible descent in energy. That this image is correct, and each monomer only moves forward, is shown in Fig. 2.

As the minimum of three (correlated) random variables, we expect it to have (roughly) the same statistics as one of them (say the middle one). Thus it is short-ranged, with a minimal history dependence. As an immediate consequence,

$$\zeta_{\parallel}\Big|_{N\geq 2} = \zeta\Big|_{N=1}.\tag{22}$$

For the line (d = 1) this yields [28, 70, 71]

$$\zeta_{\parallel}\Big|_{d=1} = \frac{5}{4}.\tag{23}$$

As for N=1, the time-integrated response function is protected by STS [31, 62]

$$\zeta_{\parallel} + \frac{1}{\nu} = 2.$$
 (24)

EK then argue that for the mean forces

$$\mathbf{F}(\mathbf{v}) = F(v)\hat{v} \qquad \hat{v} := \frac{\mathbf{v}}{v} \tag{25}$$

one has

$$\frac{\partial}{\partial v_{\parallel}} \mathbf{F}(\mathbf{v}) = \frac{\partial F(v)}{\partial v} \begin{pmatrix} 1\\ 0 \end{pmatrix}, \tag{26}$$

$$\frac{\partial}{\partial v_{\perp}} \mathbf{F}(\mathbf{v}) = \frac{F(v)}{v} \begin{pmatrix} 0\\1 \end{pmatrix}. \tag{27}$$

The only non-vanishing components are

$$\frac{\partial}{\partial v_{\parallel}} \mathbf{F}_{\parallel}(\mathbf{v}) = \frac{\mathrm{d}F(v)}{\mathrm{d}v},\tag{28}$$

$$\frac{\partial}{\partial v_{\perp}} \mathbf{F}_{\perp}(\mathbf{v}) = \frac{F(v)}{v}.$$
 (29)

Using that $v \sim (F - F_c)^{\beta}$, $\beta = \nu(z_{\parallel} - \zeta_{\parallel})$, this leads to the scaling of time scales in the two directions

$$\tau_{\parallel} \sim \frac{\xi^2}{\frac{\mathrm{d}F(v)}{\mathrm{d}v}} \sim \xi^{2+(\beta-1)/\nu} = \xi^{z_{\parallel}},$$
(30)

$$\tau_{\perp} \sim \frac{\xi^2}{\frac{F(v)}{v}} \sim \xi^{2+\beta/\nu} = \xi^{z_{\perp}}.$$
 (31)

As a consequence

$$z_{\perp} = z_{\parallel} + \frac{1}{\mu} \equiv z_{\parallel} + 2 - \zeta_{\parallel}.$$
 (32)

While EK follow NF [31] in introducing a mean-field theory, and then expanding around it, this was not necessary in the field-theoretic work of Refs. [36, 37, 82]. Here we follow the latter approach. What is yet missing is a treatment of the transversal directions. We write this in differential form as

$$\eta \mathrm{d}u_{\perp}(x,t) = \left[c \nabla^2 u_{\perp}(x,t) - m^2 u_{\perp}(x,t) \right] \mathrm{d}t + \mathrm{d}f_{\perp}(x,u_{\parallel}). \tag{33}$$

The last term can be rewritten in different ways:

$$df_{\perp}(x, u_{\parallel}) = \partial_{u_{\parallel}} f_{\perp}(x, u_{\parallel}) du_{\parallel}$$

= $\partial_{u_{\parallel}} f_{\perp}(x, u_{\parallel}) \partial_{t} u_{\parallel}(x, t) dt.$ (34)

We now suppose that $\partial_{u_{\parallel}} f_{\perp}(x, u_{\parallel})$ is a white noise in u_{\parallel} , s.t.

$$\mathrm{d}f_{\perp}(x, u_{\parallel}) = \sqrt{\sigma}\kappa(x, u_{\parallel})\mathrm{d}u_{\parallel},\tag{35}$$

$$\langle \kappa(x, u)\kappa(x', u')\rangle = \delta^d(x - x')\delta(u - u').$$
 (36)

Alternatively, we can use a white noise in time,

$$df_{\perp}(x, u_{\parallel}) = \sqrt{\sigma \dot{u}_{\parallel}(x, t)} \zeta(x, t) dt, \qquad (37)$$

$$\langle \zeta(x,t)\zeta(x',t')\rangle = \delta^d(x-x')\delta(t-t'). \tag{38}$$

Let us derive some consequences of these equations. First of all, consider the motion of the center of mass of $u_{\perp}(x,t)$. We claim it satisfies the stochastic differential equation

$$\partial_t u_{\perp}(t) := \partial_t \frac{1}{L^d} \int_x u_{\perp}(x, t)$$

$$= -m^2 u_{\perp}(t) + L^{-d/2} \sqrt{\sigma \dot{u}_{\parallel}(t)} \zeta(t), \quad (39)$$

$$\langle \zeta(t)\zeta(t')\rangle = \delta(t - t'). \quad (40)$$

To prove the equivalence to Eqs. (37)-(38), one first checks the second moment of the driving term,

$$\left\langle \frac{1}{L^d} \int_x \sqrt{\sigma \dot{u}_{\parallel}(x,t)} \zeta(x,t) \frac{1}{L^d} \int_y \sqrt{\sigma \dot{u}_{\parallel}(y,t')} \zeta(y,t') \right\rangle$$

$$= \frac{1}{L^{2d}} \int_x \sigma \dot{u}(x,t) \delta(t-t') \equiv \frac{\sigma}{L^d} \dot{u}(t) \delta(t-t'). \quad (41)$$

Second, one uses that the process is Gaussian, implying that this is the only cumulant one has to check. In the same way, one derives that $u_{\perp}(t)$ is a Gaussian process with variance (in the limit of $m \to 0$),

$$\frac{1}{2}\left\langle \left[u_{\perp}(t) - u_{\perp}(t')\right]^{2}\right\rangle = \frac{\sigma}{L^{d}}|u_{\parallel}(t) - u_{\parallel}(t')|. \tag{42}$$

At m>0, the center of mass performs an Ornstein-Uhlenbeck process as a function of w. As a result we obtain, see appendix ${\bf B}$

$$\Delta_{\perp\perp}(w) := m^4 L^d \overline{u_\perp(w'+w)u_\perp(w')}^c$$
$$= \sigma m^2 e^{-m^2 w}. \tag{43}$$

Eq. (42) immediately implies relations (79) and $s_{\perp} \sim \sqrt{s_{\parallel}}$ observed in Fig. 8. More precisely

$$\left\langle S_{\perp}^{2}\right
angle \left|_{S_{\parallel}}=2\sigma S_{\parallel}.
ight.$$
 (44)

Let us finally address the question of the roughness exponent ζ_{\perp} . Differential equations (33) and (35) yield

$$\frac{1}{2} \left\langle \left[u_{\perp}(x, u_{\parallel}) - u_{\perp}(x', u_{\parallel}) \right]^{2} \right\rangle = \sigma |x - x'|^{2\zeta_{\perp}}, \quad (45)$$

$$\zeta_{\perp} = \frac{2 - d}{2}. \quad (46)$$

The parallel coordinate u_{\parallel} acts as a local time. The tricky point is whether this local time can be used as a global time. Our simulations show that this is indeed the case in d=1 [83]. Even if this cannot be rigorously asserted, we can show

that the 2-point function is indeed equivalent to the thermal one: The thermal equilibrium is characterized by a probability distribution of a monomer with coordinate u, given positions u_i , i = 1, ..., n of the n neighbors,

$$P(u|u_1, ..., u_n) = \exp\left(-\frac{\sum_{i=1}^n (u - u_i)^2 + m^2 u^2}{2T_{\text{eff}}}\right). \tag{47}$$

It is achieved by a Langevin equation for a selected monomer with coordinate u, here written in a discretized form and a time step $\delta t=1$

$$u(t+1) - u(t) = \sum_{i=1}^{n} [u_i - u(t)] - m^2 u(t) + \sqrt{2T_{\text{eff}}} \eta_t , (48)$$
$$\langle \eta_t \eta_{t'} \rangle = \delta_{t,t'}. \tag{49}$$

(This is a Kronecker- δ .) If the string (or manifold) is in thermal equilibrium, then running Eq. (48) for the selected monomer ensures that it remains in equilibrium. If it is not in equilibrium, then running Eq. (48) for the selected monomer ensures that it will get into equilibrium with its neighbors according to the measure (47). Running the same equation for each monomer in turn, and repeating the procedure for all monomers, one ensures that thermal equilibrium is reached for the whole string.

Let us finally comment on EK and their result that $\zeta_{\perp}=\zeta_{\parallel}-d/2$. There are several assumptions in their calculation which need to be questioned: The first and strongest is that $\Delta(u_{\parallel},u_{\perp})$ only depends on u_{\parallel} . The simplest interpretation is that $\Delta(u_{\parallel},u_{\perp})$ is constant in the perpendicular direction, or at least extremely long-ranged correlated, violating basic assumptions of the system, and the particle simulation in Ref. [81]. The next-to-simplest assumption is that $\Delta(u_{\parallel},u_{\perp})=\Delta(u_{\parallel})\delta(u_{\perp})$, i.e. extremely short-ranged correlated. In this case, however, the transversal dependence needs to appear in the FRG equations, which it does not. In this context let us remind that when one starts with short-ranged correlated disorder for N=1 in a simulation, one can clearly sees that $\Delta(w)$ acquires a finite range as m is decreased. This is correctly described by the FRG.

Next, we are doubtful about the appearance of z_{\parallel} in EK's Eq. (6.16): We believe that since $\Delta(u_{\parallel},u_{\perp})$ is a static quantity, its renormalization cannot contain information about the dynamical exponents. Rather, the rescaling term in Eq. (6.16) should reduce to $(\epsilon-2\zeta_{\perp})C_{\perp}(u)$, similar to what happens in Eq. (6.15), where the rescaling term simplifies to $(\epsilon-2\zeta_{\perp})C_{\perp}(u)$. With this in mind, we can rewrite Eq. (6.16) as

$$\zeta_{\perp} = \frac{4 - d}{2} + z_{\parallel} - z_{\perp}
= \begin{cases} \frac{4 - d}{2} & \text{if } z_{\perp} \to z_{\parallel} \\ \zeta_{\parallel} - \frac{d}{2} & \text{if } z_{\perp} \to z_{\parallel} + \frac{1}{\nu} \end{cases}$$
(50)

The first is the Larkin (dimensional-reduction) result, expected if the disorder is constant in space. The second is the one given by EK, which contradicts our result (46), and our numerical simulations, see Eq. (59), while Eqs. (46) and (59) agree.

B. Analytical results in the fast-flow regime

The standard argument for the amplitude of the 2-point function in the fast-flow regime is constructed as follows: First, one considers the 2-point function at equal times set to 0.

$$\langle u(q,0)u(-q,0)\rangle = \int_{t>0} \int_{t'>0} \Delta(v(t-t')) e^{-(q^2+m^2)(t+t')}.$$
(51)

For RF disorder, since $\Delta(w)$ monotonically and faster than exponentially decays to 0 for increasing w, one can approximate for large v

$$\Delta(vt) \simeq 2\mathcal{A}\delta(vt) \equiv \frac{2\mathcal{A}}{v}\delta(t),$$
 (52)

$$\mathcal{A} := \int_0^\infty \Delta(w) \, \mathrm{d}w. \tag{53}$$

With this, Eq. (51) reduces to

$$\langle u(q,0)u(-q,0)\rangle \simeq \frac{2\mathcal{A}}{v} \int_{t>0} e^{-2(q^2+m^2)t}$$

$$= \frac{T_{\text{eff}}^{\text{RF}}}{q^2+m^2}, \qquad T_{\text{eff}}^{\text{RF}} = \frac{\mathcal{A}}{v}. \quad (54)$$

For RB disorder, the situation is different as $\Delta(w) = -R''(w)$, where now R(w) is fast decaying. As a result

$$\Delta(v(t-t')) = -R''(v(t-t'))$$

$$= \frac{1}{v^2} \partial_t \partial_{t'} R(v(t-t'))$$

$$\simeq -\frac{2\mathcal{B}}{v^3} \delta''(t), \qquad (55)$$

$$\mathcal{B} = \int_{-\infty}^{\infty} R(w) dw. \qquad (56)$$

A numerical simulation shows that thermal noise correlated as $\delta''(t)$ leads to a non-vanishing variance for each site, uncorrelated between neighboring sites. It does not contribute to the structure factor. Thus the leading contribution should come from Eqs. (52)-(53), where the RG-flow from RB to RF is cut such that below, in section IV F, we observe $\mathcal{A} \simeq 1/v^2$. A proper theoretical explanation remains outstanding.

For a single over-damped particle in a one-dimensional force field with a finite correlation length, we solve the problem analytically in section C, yielding in agreement with the above scalings $T_{\rm eff}^{\rm RB} \sim 1/v^3$ and $T_{\rm eff}^{\rm RF} \sim 1/v$ at large velocities

IV. NUMERICAL RESULTS

A. Steady state in the quasistatic regime

We start analyzing the geometry of the elastic string in the quasistatic steady-state regime, using the cellular automaton. In Fig. 3 we show the structure factors for the parallel (a) and perpendicular (b) directions, as a function of the parameter m

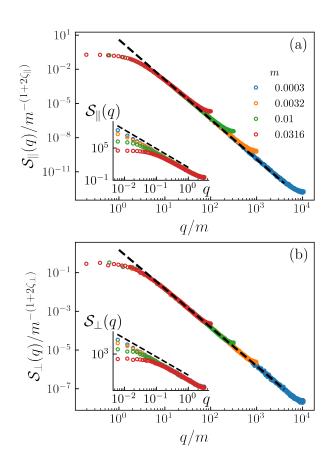


FIG. 3. Scaled structure factors for quasi-static motion as a function of confinement strength m for the parallel (a) and transverse (b) directions. Both directions display a self-affine structure up to the confinement length $L_m \sim m^{-1}$. Dashed lines in (a) and (b) correspond to power-laws $\mathcal{S}_{\parallel}(q) \sim q^{-(1+2\zeta_{\parallel})}$ and $\mathcal{S}_{\perp}(q) \sim q^{-(1+2\zeta_{\perp})}$, yielding the depinning roughness $\zeta_{\parallel} \approx 1.25$ and $\zeta_{\perp} \approx 0.5$ respectively. Insets show raw data for each case.

in Eq. (6). Since $v\to 0^+$, the depinning correlation length ξ is not set by the distance to f_c , but by the confining potential, $\xi_m\approx \frac{1}{m}$. Indeed, in both cases we observe that the string becomes flat beyond ξ , while below (but above the lattice constant here set to 1) a self-affine random-manifold regime is observed. The inset of the two figures validates the scalings

$$S_{\alpha}(q) \sim q^{-(1+2\zeta_{\alpha})} G_{\alpha}(q\xi_m),$$
 (57)

with $G_{\alpha}(y)=$ const for $y\ll 1$ ($\xi_m\gg 1/q$) and $G_{\alpha}(y)=y^{1+2\zeta_{\alpha}}$ for $y\gg 1$ ($\xi_m\ll 1/q$). These results also imply that $W_{\alpha}^2\sim \xi_m^{2\zeta_{\alpha}}\approx L^{2\zeta_{\alpha}}$ if mL is kept constant when increasing L. The roughness exponents in the two directions (see dashed lines) are different

$$\zeta_{\parallel} = 1.25 \pm 0.01,$$
(58)

$$\zeta_{\perp} = 0.50 \pm 0.01.$$
 (59)

These results are compatible with the earlier result of Ref. [72]. We find a value of $\zeta_{\parallel} \approx 5/4$, indistinguishable from the roughness exponent of the driven one-dimensional

quenched Edwards-Wilkinson interface obtained from numerical simulations [43, 70], and consistent with 2-loop functional renormalization group calculations [36, 37]. In the perpendicular direction the exponent is the same as for a moving line in presence of thermal noise. Both results are in agreement with our theoretical predictions in Eqs. (23) and (46).

The statistical tilt symmetry applied to the parallel direction implies that

$$\nu = \frac{1}{2 - \zeta_{\parallel}} = 1.33 \pm 0.02. \tag{60}$$

These results are consistent with the planar approximation of Ertas and Kardar [62], and with their 1-loop analysis. As we discussed in section III A, they are inconsistent with the higher-order results. In particular, they contradict EK's $\zeta_{\perp} = 5\zeta_{\parallel}/2 - 2$ [62].

As the model of [62] and its numerical implementation are equivalent to ours, the numerical discrepancy can be explained by noting that the x and L dependence of the correlation function

$$B_{\alpha}(x,L) := \overline{\langle [u_{\alpha}(x,t) - u_{\alpha}(0,t)]^2 \rangle} \bigg|_{L} \tag{61}$$

used in Ref [62] cannot detect roughness exponents larger than one for a fixed sample size L. As first observed in Ref. [69] for N=1 (\mathcal{A} is a number)

$$B(x,L) \simeq \mathcal{A}L^{2\zeta-2}x^2 \text{ if } \zeta > 1,$$
 (62)

$$B(x, L) \simeq \mathcal{A}x^{2\zeta} \text{ if } \zeta < 1.$$
 (63)

Taking into account that $\zeta_{\parallel}>1$ and $\zeta_{\perp}<1$, the first line applies to ζ_{\parallel} , and the second to ζ_{\perp} . Therefore, if we use the scaling $B(x)\sim x^{2\zeta}$ to determine ζ , its value is only correct when $\zeta<1$, but saturates at 1 whenever $\zeta>1$ (In practice, it is even difficult to see the exponent 1, and one tends to measure something slightly smaller [28]). We believe that this is what Ertas and Kardar [62] saw in their simulation.

Besides validating the planar approximation, the results of Eq. (58) imply that whenever the harmonic elasticity is an approximation for a more complicated elasticity, the model becomes physically unrealistic for large enough sizes L because local slopes $\overline{\langle (\mathrm{d}u_{\parallel}/\mathrm{d}x)^2\rangle}=B(1,L)\simeq L^{2\zeta_{\parallel}-2}$ diverge with L [69]. This motivates one to either include anharmonic corrections to the elasticity, or other effects such as overhangs and pinch-off loops to the model. This not withstanding, the predictions of Eq. (58) may describe the geometry at intermediate scales, below a putative crossover to a different regime. This scenario is present in recent experiments on creep [84] and depinning [61] displaying super-rough magnetic domain walls in ultrathin ferromagnetic films.

B. Relaxation from a flat initial condition in the quasistatic regime

In the quasistatic protocol we start with a flat initial condition $\mathbf{u}(z,t) = \{u_0,0\}$ such that $m^2(w-u_0) = f_c$. Since the flat string is uncorrelated from the disorder we also have

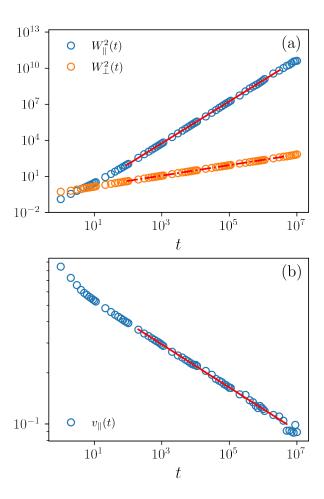


FIG. 4. Non-steady relaxation of the roughness (a) and velocity (b) from a flat initial configuration at constant force $f=f_c\approx 0.765,$ for a string with $L=2^{18}$ monomers, averaged over 8 disorder realizations. Red lines show power-laws fits in the universal relaxation regime. In (a) we fit $W_\alpha^2(t)\sim t^{2\zeta_\alpha/z_\alpha}$ with $\alpha=\parallel,\perp$ to obtain $z_\parallel=1.43\pm 0.01,\,z_\perp=2.27\pm 0.05.$ (b) From the relaxation of the mean velocity $v\sim t^{-\beta/\nu z_\parallel}=t^{-(2-\zeta_\parallel)\beta/z_\parallel}$ we obtain $\beta=0.24\pm 0.01.$

 $v(t=0)=f_c$. As observed for interfaces relaxing at depinning [43], before reaching the steady state the string is in a universal transient regime, which yields information about the critical exponents of the steady-state depinning transition. In particular u_{\perp} and u_{\parallel} each evolves with a different dynamical length, $\ell_{\parallel}(t)\sim t^{1/z_{\parallel}}$, and $\ell_{\perp}(t)\sim t^{1/z_{\perp}}$, controlling the relaxational dynamics.

Since the interface is initially flat, $W_{\alpha}^2(t=0)=0$. After a non-universal microscopic transient the global width reaches a universal transient regime described by $W_{\alpha}^2(t) \sim \ell_{\alpha}(t)^{2\zeta_{\alpha}}$, and hence

$$W_{\alpha}^{2}(t) \sim t^{2\zeta_{\alpha}/z_{\alpha}}.$$
 (64)

This holds as long as $\ell_{\alpha}(t) < \xi_m$. On the other hand, at long times it saturates as $W_{\alpha}^2(t) \sim \xi_m^{2\zeta_{\alpha}} \sim L^{2\zeta_{\alpha}}$ (the last relation holds provided mL is kept fixed). The power-law regime of Eq. (64) is confirmed in Fig. 4(a) were we show the evolution

of $W_{\parallel}^2(t)$ and $W_{\perp}^2(t)$. Using the known values of ζ_{\parallel} Eq. (58) and ζ_{\perp} Eq. (59) and by fitting in the appropriate (intermediate) range indicated in red, we get

$$z_{\parallel} = 1.43 \pm 0.01,$$
 (65)

$$z_{\perp} = 2.27 \pm 0.05. \tag{66}$$

This validates the relation (32)

$$z_{\perp} = z_{\parallel} + 1/\nu,\tag{67}$$

predicted in Ref. [62], as $z_{\parallel}+1/\nu-z_{\perp}=-0.09\pm0.08$. For reference, the analytical values proposed in [70] combined with the scaling relation (67) are

$$z_{\parallel} = \frac{10}{7} = 1.42857,\tag{68}$$

$$z_{\perp} = \frac{61}{28} = 2.17857. \tag{69}$$

In the same universal regime where Eq. (64) holds, the parallel center-of-mass velocity reaches a universal transient regime, where it vanishes as $v(t) \sim \ell_{\parallel}(t)^{-\beta/\nu}$, and hence

$$v(t) \sim t^{-\beta/\nu z_{\parallel}}. (70)$$

In Fig. 4(b) we show the fit to this regime and obtain, knowing ν from Eq. (60) and z_{\parallel} from Eq. (65),

$$\beta = 0.24 \pm 0.01. \tag{71}$$

This is indistinguishable from the result for the onedimensional interface [43]. It is compatible with the exact relation [70]

$$\beta = \nu(z_{\parallel} - \zeta_{\parallel}) = \frac{5}{21} = 0.238095...$$
 (72)

To conclude, these results validate the exponent relations due to the planar approximation [62] in the modified form of section III A.

Finally, a detailed geometrical view of the relaxation can be obtained from the structure factors. Using the exponents obtained in Fig. 5 we show that when $\ell_{\alpha}(t) < \xi_m$ the evolution of the structure factors accurately follows the scaling

$$S_{\alpha}(q,t) \sim q^{-(1+2\zeta_{\alpha})} F_{\alpha}(q\ell_{\alpha}(t)),$$
 (73)

with F(x)= const for $x\ll 1$, and $F(x)=x^{(1+2\zeta_{\alpha})}$ for $x\gg 1$. Therefore, the string progressively becomes self affine with exponents ζ_{α} up to the corresponding scales $\ell_{\alpha}(t)$. For larger distances the memory of the flat initial condition is preserved.

C. Depinning avalanches

We now describe the avalanche statistics in the quasistatic regime. We first compute the center-of-mass jumps, defined in Eq. (16). In the insets of Fig. 7 we see that jumps in both directions fairly follow a power-law decay with a cut-off

$$P_{\alpha}(S) \sim S^{-\tau_{\alpha}} G_{\alpha}(S/S_{m}^{\alpha}). \tag{74}$$

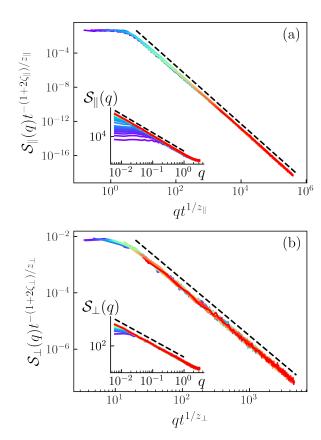


FIG. 5. Relaxation of the structure factors for L=1024 monomers and $m^2=10^{-6}$, showing $\mathcal{S}_{\alpha}(q)\sim q^{-(1+2\zeta_{\alpha})}G_{\alpha}(qt^{1/z_{\alpha}})$ scaling, in both directions $\alpha=\parallel,\perp$. Dashed lines indicate power laws and insets show corresponding raw data. (a) Rescaled $\mathcal{S}_{\parallel}(q)$ from $t=10^3$ (violet) to $t=5\times 10^7$ (red). The dashed line corresponds to $z_{\parallel}=1.43$ and $\zeta_{\parallel}=1.25$. (b) Rescaled $S_{\perp}(q)$ from $t=10^6$ (violet) to $t=5\times 10^7$ (red). The dashed line corresponds to $z_{\perp}=2.18$ and $\zeta_{\perp}=0.5$.

Here

$$S_m^{\alpha} := \frac{\langle S_{\alpha}^2 \rangle}{2\langle S_{\alpha} \rangle} \tag{75}$$

grows with decreasing m, and $G_{\alpha}(x)$ are cut-off functions, such that $G_{\alpha}(x) \sim \text{const}$ for $x \ll 1$ and $G_{\alpha}(x) \to 0$ roughly exponentially for x > 1. For the quantitative numerical analysis it is convenient [85] to define

$$s := \frac{S}{S_m},\tag{76}$$

$$p_{\alpha}(s) := P_{\alpha}(S) \frac{S_m^2}{\langle S \rangle}. \tag{77}$$

From the main panel and the inset of Fig. 6 we see that the cut-offs respectively scale as

$$S_m^{\parallel} \sim m^{-(d+\zeta_{\parallel})}, \qquad d = 1, \tag{78}$$

$$S_m^{\perp} \sim \sqrt{S_m^{\parallel}}. (79)$$

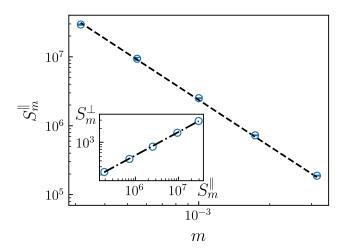


FIG. 6. Dependence of the avalanche size cutoff S_m^{α} with the confinement factor m. The dashed line corresponds to $S_m^{\parallel} \propto m^{-(D+\zeta_{\parallel})}$ with D=1. Inset: the dot-dashed line shows that cut-offs in each direction are strongly correlated as $S_m^{\perp} \propto \sqrt{S_m^{\parallel}}$.

In Fig. 7 we show the master curves in the two directions, as obtained by rescaling those in the insets for different values of m. The collapse for different m is better in the parallel direction than in the perpendicular one, probably due to the smaller range of sizes for the latter. Nevertheless, we can fit the avalanche exponents for s < 1 in both cases, leading to

$$\tau_{\parallel} = 1.09 \pm 0.03,$$
 (80)

$$\tau_{\perp} = 1.17 \pm 0.06.$$
(81)

The value of τ_{\parallel} is consistent with the planar approximation, as numerical simulations of avalanches for 1-dimensional interfaces present an indistinguishable value for τ [85]. The scaling relation $\tau = 2 - 2/(d+\zeta)$ of Narayan and Fisher [31] (for N=1) with the exponents of [70] (see Eq. (58)) yield

$$\tau_{\parallel} = 2 - \frac{2}{d + \zeta_{\parallel}} \xrightarrow{d \to 1} \frac{10}{9} = 1.11111...$$
(82)

For the scalar model (N=1) this scaling relation was conjectured by Narayan and Fisher [31] assuming a finite density of avalanches at the depinning threshold. It was rederived in Ref. [67] from FRG. This result significantly differs from the mean-field result $\tau_{\rm MF}=3/2$. Eq. (82) was tested numerically [85] and analytically via 1-loop FRG calculations [10].

To understand the value of τ_{\perp} , we remind Eq. (44), $\left\langle S_{\perp}^{2}\right\rangle |_{S_{\parallel}}=2\sigma S_{\parallel}$. From this we immediately obtain the scaling relation (79) for S_{m}^{\perp} . We have analyzed the joint pdf $p(s_{\parallel},s_{\perp})$ for a long sequence of avalanches, see Fig. 8. The strong correlation along the line $s_{\perp}\sim\sqrt{s_{\parallel}}$ confirms Eq. (44). This allows us to write $P_{\parallel}(S_{\parallel})\mathrm{d}S_{\parallel}=P_{\perp}(S_{\perp})\mathrm{d}S_{\perp}$ with $S_{\perp}\sim S_{\parallel}^{1/2}$. Assuming that for small arguments $P_{\parallel}(S_{\parallel})\sim$

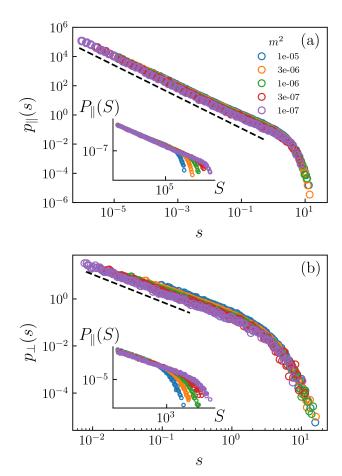


FIG. 7. Rescaled distribution of center-of-mass jumps for different m^2 and L=4096 monomers in the parallel (a) and perpendicular (b) directions, according to Eq. (77). Dashed lines indicate power-law fits $p_{\alpha}(s) \propto s^{-\tau}$ well below the cut-offs. Insets show raw data. The dashed-line fits yield $\tau_{\parallel}=1.09\pm0.03$ (a) and $\tau_{\perp}=1.17\pm0.06$ (b).

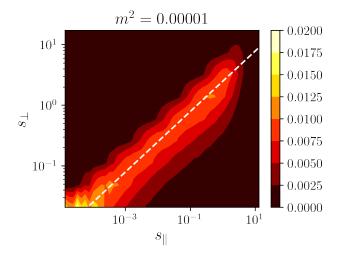


FIG. 8. Normalized count of events with center-of-mass jump sizes (s_\parallel,s_\perp) , highlighting their strong correlation. The white dashed-line corresponds to $s_\perp \propto \sqrt{s_\parallel}$.

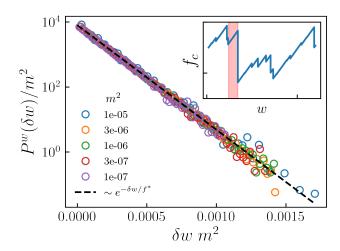


FIG. 9. Scaled "waiting time" distribution between consecutive avalanches in the parallel direction. The dashed-line indicates an exponential fit.

$$S_{\parallel}^{- au_{\parallel}}$$
 and $P_{\perp}(S_{\perp})\sim S^{- au_{\perp}}$, we obtain that

$$\tau_{\perp} = 2\tau_{\parallel} - 1 \xrightarrow{d \to 1} \frac{11}{9} = 1.22222...$$
(83)

The numerically obtained value reported in Eq. (81) is $\tau_{\perp} = 1.17 \pm 0.06$, in fair agreement with the one predicted by the scaling relation (83).

D. Waiting-time distribution

Consecutive avalanches are characterized by a "waiting-time" distribution P^w defined in Eq. (17). In Fig. 9 we show that this distribution follows an exponential decay, already observed in Ref. [86] (N=1, equilibrium),

$$P^{w}(\delta w) \approx \frac{m^2}{\delta f^*} e^{-\delta w m^2 / f^*}, \tag{84}$$

with $f^*\approx 0.000135$ a microscopic force. Therefore our avalanches are characterized by a mean waiting distance f^*/m^2 . If we choose $L=\xi_m\approx 1/m$, the mean waiting distance diverges with system size as $\langle \delta w \rangle \sim L^2$, implying a dominance of large-stress accumulation periods needed to trigger large avalanches of size $S_{\parallel}\sim L^{1+\zeta_{\parallel}}$. Interestingly, a pure exponential distribution is also found for the "Gumbel" universality class of a driven particle in a short-range correlated random-force landscape [81]. This contrasts with the Weibull and Frechet universality classes. Nevertheless, the critical force distribution for 1-dimensional interfaces in a box of size $L\times L^{\zeta}$ is slightly different from Gumbel [87, 88]. Since in our case we have (in the parallel direction) an aspect ratio $\xi_m \times \xi_m^{\zeta}$ (with $\xi_m \approx 1/m$) it would be interesting to derive P^w for such a distribution.

E. The renormalized force-force correlator

Finally, we discuss the force-force correlator of Eq. (18), which is a central quantity in the renormalization group calculations. In Fig. 10(a) we show that the correlator $\Delta_{\parallel} \equiv \Delta_{\parallel,\parallel}$ for different m can be collapsed using $\Delta_{\parallel}(w) \approx m^{\epsilon-2\zeta_{\parallel}}\tilde{\Delta}_{\parallel}(wm^{\zeta_{\parallel}})$ [77], with $\epsilon=4-d=3$ for our case and $\zeta_{\parallel}=1.25$ from Eq. (58).

By fitting $\tilde{\Delta}_{\parallel}(x)=ce^{-ax-bx^2-dx^3}$ in the range $w\in[0,1]$ we obtain: $a=2.38\pm0.07, b=0.8\pm0.2, c=0.188\pm0.001$ and $d=0.09\pm0.21$. As a consequence $\frac{\tilde{\Delta}_{\parallel}(0)\tilde{\Delta}_{\parallel}''(0)}{\tilde{\Delta}_{\parallel}'(0^+)^2}=0.71\pm0.09$. Increasing the fit range to $w\in[0,1.6]$ this value increases to 0.76 ± 0.06 . We thus estimate the scale-free universal ratio to be

$$\frac{\Delta_{\parallel}(0)\Delta_{\parallel}''(0)}{\Delta_{\parallel}'(0^{+})^{2}} \equiv \frac{\tilde{\Delta}_{\parallel}(0)\tilde{\Delta}_{\parallel}''(0)}{\tilde{\Delta}_{\parallel}'(0^{+})^{2}} = 0.74 \pm 0.07.$$
 (85)

This is larger than the 1-loop value of 2/3 predicted in Ref. [62]. It is fairly close to the value of 0.73(3) measured experimentally in 2-dimensional magnetic domain walls [60]. It can also be compared to the value predicted by FRG for short-range elasticity in the N=1 case [28, 60], where one gets 2/3 in d=4 (exact), 0.71 in d=3 (2-loop) 0.75 in d=2 (2-loop), 0.79 in d=1 (2-loop) and 0.822 (toy model in d=0). Within error bars the value of Eq. (85) agrees with the one predicted by the theory for N=1, and thus confirms the planar approximation.

In Fig. 10(b) we show that the correlator $\Delta_{\perp} \equiv \Delta_{\perp,\perp}$ can, for different m, fairly well be collapsed, within the statistical error bars, using a master curve $\Delta_{\perp}(w) = m^2 \tilde{\Delta}_{\perp}(wm^2)$, as anticipated in Eq. (43). By fitting the predicted exponential decay as $\tilde{\Delta}_{\perp}(w) = \sigma \exp(-dw)$ for all curves combined, we obtain $\sigma = 0.306 \pm 0.027$, $d = 1.50 \pm 0.25$. The value of σ is fairly close to the single-monomer standard deviation of perpendicular jumps, while the decay constant d comes out larger. However, we see that reducing m, the scaling function $\tilde{\Delta}(w)$ converges more and more to $\tilde{\Delta}_{\perp}(w) = \sigma \mathrm{e}^{-w}$ predicted in Eq. (43). The latter curve is shown in black dashed on Fig. 10(b). It seems convergence is slow, and the prediction (43) is reached only asymptotically. We may therefore suspect that the amplitude ratio (85) has also not yet converged. We defer an in-depth analysis to future work.

F. Crossover to the fast-flow regime

At finite velocities just above the depinning threshold, the steady-state correlation length is expected to diverges as $\xi_f \sim (f-f_c)^{-\nu} \sim v^{-\nu/\beta}$. As for interfaces [89], ξ_f is a characteristic geometrical crossover length. In Fig. 11(a) we show that for intermediate and large length scales, and different small steady-state velocities

$$S_{\parallel}(q) \sim q^{-(d+2\zeta_{\parallel})} \tilde{S}_{\parallel}(qv^{-\nu/\beta}).$$
 (86)

Here $\tilde{\mathcal{S}}_{\parallel}(x)=$ const for $x\gg 1$, while for $x\ll 1$ one has $\tilde{\mathcal{S}}_{\parallel}(x)\sim x^{2(\zeta_{\parallel}-\zeta_{\rm EW})}$, where $\zeta_{\rm EW}=(2-d)/2$ and $\zeta_{\parallel}=$

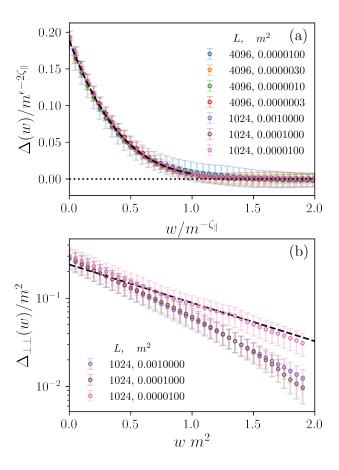


FIG. 10. (a) Scaled force-force correlator for components in the parallel direction (with $\epsilon=4-d=3$). The dashed line corresponds to a fit of $f(x)=ce^{-ax-bx^2-dx^3}$, yielding $a=2.38\pm0.07$, $b=0.8\pm0.2$, $c=0.188\pm0.001$ and $d=0.09\pm0.21$. (b) Scaled force-force correlator for components in the perpendicular direction. The dashed-line corresponds to a fit to $g(x)=a'\exp(-x)$, $a'=0.27\pm0.02$.

1.25 as given by Eq. (58). From the renormalization group viewpoint this result is in agreement with the crossover of the depinning fixed point towards an Edwards-Wilkinson regime, as predicted for the FL [62] and for the interface [33]. In other words, besides renormalizing the friction such that $v \sim (f-f_c)^\beta$, pinning forces on the coarse-grained FL above ξ_f are similar to thermal noise. What was derived for N=1 remains valid in the planar approximation.

In Fig. 11(b) we show the structure factor in the perpendicular direction for different velocities near the depinning threshold. Remarkably, there are no signatures of the correlation length $\xi_f \sim v^{-\nu/\beta}$. That is, for non-microscopic length scales we find that

$$S_{\perp}(q) \sim q^{-(1+2\zeta_{\perp})} \sim q^{-2},$$
 (87)

and we are not able to detect any geometrical crossover at $\xi_f(v)$, at variance with the clear crossover observed in $S_{\parallel}(q)$. The reason is that the assumptions entering Eq. (37) remain unchanged for large driving velocities v. We thus only observe a crossover imposed by the confining potential at $q\xi_m\approx 1$.

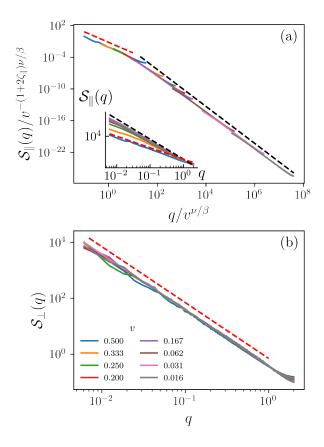


FIG. 11. Steady-state structure factors at finite imposed velocities for L=1024 and $m^2=0.00001$ for the longitudinal (a) and transversal directions (b). (a) Rescaled structure in the parallel direction. Red dashed lines indicate the fast-flow roughness exponent $\zeta_{\rm ff}=1/2$, and the black dashed lines the depinning roughness exponent $\zeta_{\parallel}=1.25$. The crossover between the two regimes occurs at the depinning correlation length $\xi_f \sim v^{-\nu/\beta}$. Inset shows raw non-scaled data. (b) The transverse structure factor appears to be independent of the imposed velocity and described by a roughness exponent $\zeta_{\perp}=0.5$ (red dashed line).

At large velocities, in the *fast-flow regime*, the depinning correlation length becomes small and it is expected that the pinning forces became a rapidly fluctuating uncorrelated noise acting on an otherwise flat moving elastic FL. We hence expect $S_{\alpha}(q) \sim (cq^2 + m^2)^{-1}$ at intermediate scales, $q\xi_m = q/m > 1$, and a crossover towards $S_{\alpha}(q) \sim \text{const}$ for $q\xi_m = q/m < 1$. In the insets of Fig. 12 we verify this by showing the structure factor as a function of v (not necessarily small), for the longitudinal and perpendicular directions. At variance to small v, $S_{\perp}(q)(cq^2 + m^2)$ is v-dependent. Its behaviour at small q motivates the study of effective temperatures. These are introduced from generalized fluctuation-dissipation theorems by using that the static linear response function in the α direction due to an external time-independent but q-dependent field in the α' direction is *exactly* given by

$$\chi_{\alpha,\alpha'} = \delta_{\alpha,\alpha'}\chi(q) \equiv \frac{\delta_{\alpha,\alpha'}}{cq^2 + m^2},$$
(88)

due to the statistical tilt symmetry [31]. In equilibrium (v =

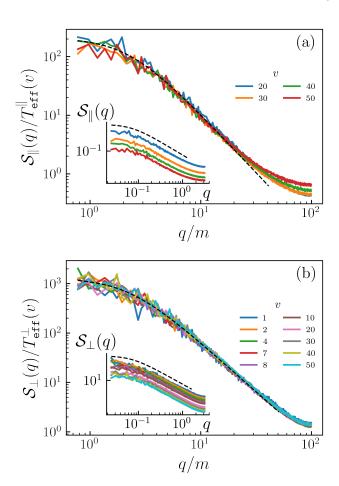


FIG. 12. Rescaled Structure factors at different large velocities for $m^2=0.001$ and L=1024 in the longitudinal (a) and transverse (b) directions. In both cases raw-data (insets) can be collapsed by using effective temperatures $T_{\rm eff}^{\parallel}(v)$ (a) and $T_{\rm eff}^{\perp}(v)$ (b). Dashed-lines correspond to the purely "thermal" $\sim (cq^2+m^2)^{-1}$ dependence.

0) at a finite temperature T, the fluctuation-dissipation theorem implies that $\mathcal{S}_{\alpha}(q) = T\chi(q)$. For the driven, out-of-equilibrium FL at zero temperature we can thus define anisotropic effective temperatures $T_{\mathrm{eff}}^{\alpha}$ ($\alpha = \parallel, \perp$)

$$T_{\text{eff}}^{\alpha}(v) := \frac{\mathcal{S}_{\alpha}(q)}{\chi(q)} = (cq^2 + m^2)\mathcal{S}_{\alpha}(q). \tag{89}$$

At large scales compared to the correlation length $\xi_f(v)$, and since $\mathcal{S}_{\alpha}(q) \sim (cq^2+m^2)^{-1}$, a single anisotropic effective temperature is sufficient for the whole regime. This is expected to hold for $2\pi\xi_f^{-1}\gg q$. It remains valid for the largest length scales, and when q< m, there is a crossover to a flat regime $S_{\alpha}(q)\simeq T_{\rm eff}(v)m^{-2}$, due to the confining potential. Therefore $\mathcal{S}_{\alpha}(q)/T_{\rm eff}^{\alpha}$ is independent of v for large length scales (small q) as shown in Fig. 12, and $T_{\rm eff}^{\alpha}(v)$ describes large-scale fluctuations in general.

Since large length scales are associated with a slow dynamics, these definitions may yield a *bona fide* temperature in a thermodynamic sense [90]. Using these definitions in the main panels of Fig. 12 we show that $S_{\alpha}(q)$ for different ve-

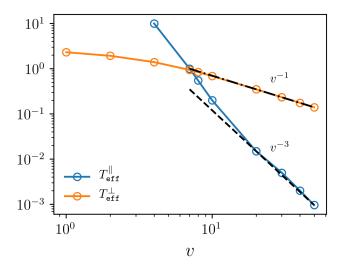


FIG. 13. Effective temperatures as a function of the velocity for $m^2=0.001$ and L=1024. Dashed lines indicate asymptotic dependencies.

locities v can be distinguished by $T_{\tt eff}^{\alpha}(v)$ for small enough q. For large q, only the parallel direction shows a deviation from the master curve, indicating that the parallel direction retains genuine non-equilibrium features at short length scales.

The velocity dependence of the two effective temperatures is shown in Fig. 13. They are direction dependent, and monotonously decrease with increasing v. They intersect at a characteristic velocity of $v\approx 7$, above which $T_{\rm eff}^{\perp}>T_{\rm eff}^{\parallel}$. At small v the transversal temperature $T_{\rm eff}^{\perp}$ saturates at $T_{\rm eff}^{\perp}=\sigma\approx 1/3$, as discussed above, explaining why on Fig. 11(b) no appreciable velocity dependence is observed. The observed asymptotic forms are $T_{\rm eff}^{\parallel}\sim v^{-3}$ and $T_{\rm eff}^{\perp}\sim v^{-1}$. The two-component Edwards-Wilkinson type of scaling with effective temperatures $T_{\rm eff}^{\alpha}$ of the structure factor at large velocities implies that the global width scales as

$$W_{\alpha}^2 \sim \frac{T_{\text{eff}}^{\alpha}}{m^{2\zeta_{\text{EW}}}} = \frac{T_{\text{eff}}^{\alpha}}{m},$$
 (90)

as verified in Fig. 14 for each direction, as a function of velocity.

The crossing of the effective temperatures is associated with the existence of an isotropic point for the global width, above which the FL tends to be elongated in the transverse direction, in contrast with the situation near depinning where they are elongated in the longitudinal direction. This was predicted in Ref. [16] from general arguments. To see this better, it is useful to compute the joint distribution function (13) of local displacements, which gives us a top view of the FL fluctuations in the co-moving frame. In the inset of Fig. 14 we do not only see the change of aspect ratio and the reduction of the global width with increasing v, but we also observe that the parallel-displacement distribution is asymmetric, with a more elongated tail at smaller velocities, in contrast to the symmetric distribution in the perpendicular direction. To characterize it we show in Fig. 15 the reduced distributions of Eq. (12) for a

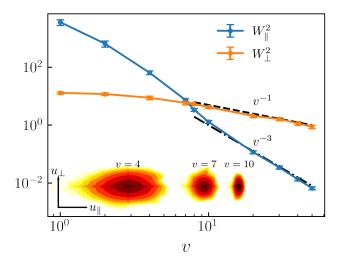


FIG. 14. Mean quadratic widths as a function of the velocity for $m^2=0.001$ and L=1024. Dashed-lines indicate asymptotic dependencies.

large range of velocities. Only for large v do they converge towards a Gaussian, as can be seen in the inset of Fig. 15(a): At low velocities (lighter colors) the distribution has asymmetric tails. As shown in the main panel the variance is controlled by the velocity-dependent effective temperatures of Fig. 13. These displacements translate into an appreciable skewness. In Fig. 16 we show the skewness, as a function of the velocity, defined as

$$\mu_{3,\alpha} = \frac{\overline{\left\langle \left[u(x,t) - u_{\alpha}(t) \right]^3 \right\rangle}}{W_{\alpha}^2(t)^{\frac{3}{2}}}.$$
 (91)

As expected, the perpendicular direction has an undetectable skewness while the longitudinal one presents a negative skewness at small v. It roughly vanishes as $\sim 1/v$ for large velocities. In Fig.17 we also show the kurtosis

$$k_{\alpha} = \frac{\overline{\left\langle \left[u(x,t) - u_{\alpha}(t) \right]^4 \right\rangle}}{W_{\alpha}^4(t)}.$$
 (92)

Within error bars we find $k_{\perp} \approx 3$ and $k_{\perp} \approx 3$ at large velocities, consistent with an approximately gaussian shape. Only in the longitudinal direction at low velocities we observe a departure from gaussian, $k_{\parallel} > 3$, though with a large error bar.

The above observations are a strong indication for the existence of a *large-deviation* function, encountered for depinning already in Ref. [91]. Provided the limit exists, the large-deviation function F(x) is defined as

$$F(x) := -\lim_{v \to \infty} \frac{\ln P(xv)}{v}.$$
 (93)

Since our data do not allow to evaluate F(x) precisely enough, we leave its determination for future work.

On the other hand, the inset of Fig. 15(b) shows that the perpendicular fluctuations are well approximated by a Gaussian

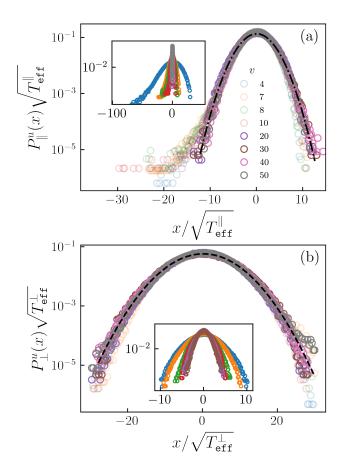


FIG. 15. Local displacement fluctuations around the center of mass in the parallel (a) and perpendicular directions (b). Lighter symbols are used for smaller velocities. Parameters of the simulations are $m^2=0.001$ and L=1024. Dashed-lines are Gaussian fits.

with a velocity-controlled variance,

$$P_{\perp}^{u}(x) \sim e^{-\frac{(x-u_{\perp})^2}{2W_{\perp}^2(v)}}$$
 (94)

Since $W_{\perp}^2(v) \approx T_{\rm eff}^\perp(v)/m$, see Eq. (90), the velocity dependence is exclusively controlled by the transverse effective temperature, as shown by the rescaled curves in the main panel of Fig. 15(b). These results show that the perpendicular direction can be described by an Edwards-Wilkinson equation with an effective temperature at all velocities. The longitudinal direction shows genuine non-equilibrium effects well beyond the depinning transition, which disappear roughly as 1/v for large v.

It is worth noting that these rare asymmetric parallel fluctuations may be more pronounced for strong pinning. For instance, when pinned by nanoparticles, the FL appears as a sequence of convex arcs in the direction of motion connecting localized pinned pieces [63], explicitly breaking the $\delta u_{\parallel}(z,t) \rightarrow -\delta u_{\parallel}(z,t)$ symmetry. This symmetry is however always broken at depinning [28, 92]. This kind of structure may explain both tails of the displacement distribution. Nevertheless, at very large velocities both directions display anisotropic Gaussian fluctuations. As discussed in the next

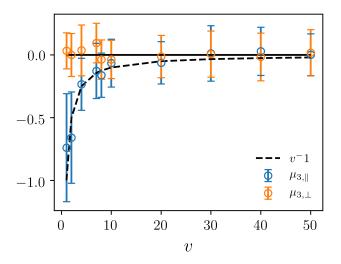


FIG. 16. Skewness of the two components of the local displacement distribution as a function of the velocity for $m^2=0.001$ and L=1024.

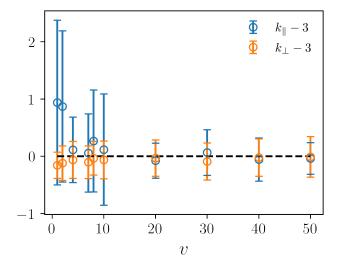


FIG. 17. Kurtosis of the local displacement fluctuations in both directions. The dashed-line correspond to a pure gaussian.

section, the anisotropy of these fluctuations is rather sensible to whether the microscopic disorder is RB or RF.

G. Random-field disorder

So far we discussed RB disorder acting on a vortex line, corresponding to short-range correlated pinning potentials. This type of disorder seems to be the only one relevant in experiments, and in particular for point disorder in bulk superconductors. While we do not know how to realize isotropic RF disorder corresponding to uncorrelated pinning forces, we nevertheless consider it here for comparison. We remind that the two types of disorder are differentiated by their correlators

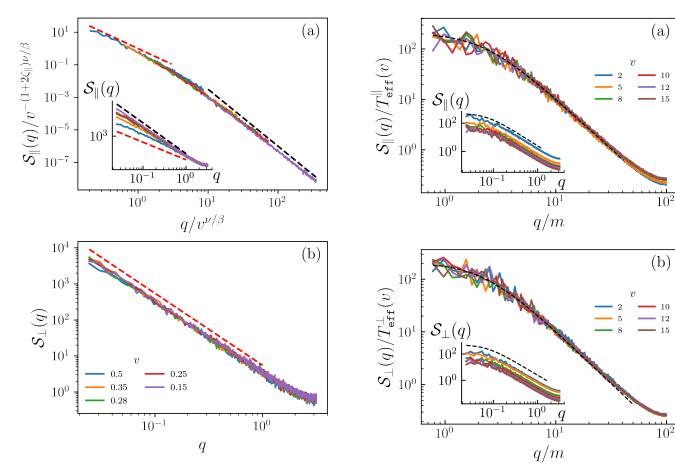


FIG. 18. Steady-state structure factor at low velocities in the parallel (a) and perpendicular (b) directions for RF disorder. (a) The inset shows raw data and the main panel the scaled structure factor in the parallel direction using the exponents obtained in for RB disorder. (b) Raw data in the perpendicular direction appears to be independent of v. See Fig. 11 for a direct comparison with the RB case.

FIG. 19. Rescaled structure factors at different (large) velocities for $m^2=0.001$ and L=1024 in the longitudinal (a) and transverse (b) directions, for RF disorder. In both cases raw data (insets) can be collapsed by using the effective temperatures $T_{\rm eff}^{\parallel}(v)$ (a) and $T_{\rm eff}^{\perp}(v)$ (b). Dashed lines correspond to a purely "thermal" $\sim (cq^2+m^2)^{-1}$ dependence. See Fig.12 for a comparison to the RB case.

(see section II A).

We first discuss the low-velocity regime near the depinning transition. In Fig. 18(a) we show, using the same exponents of Table I obtained for the RB case, that the steady-state structure factor scales as $\mathcal{S}_{\parallel}(q) \sim q^{-(1+2\zeta_{\parallel})}G(q\xi)$, $\xi \sim (f-f_c)^{-\nu}$, with $G(x) \sim x^{1+2\zeta_{\parallel}}$ for $x\gg 1$, and $G(x)\sim const$ for $x\ll 1$, provided $\xi<1/m$ and q>m. Since the same scaling was shown in Fig. 11(a) for the RB case, this result is again consistent with the planar approximation, and with the finding that RB and RF share the same depinning universality class [36, 37]. In Fig. 18(b) we show that $\mathcal{S}_{\perp}(q)\sim q^{-(1+2\zeta_{\perp})}$, with no clear signature of ξ , as observed before in Fig. 11(b) for the RB case.

The crossover to the fast-flow regime reveals some important differences between RB and RF. In Fig. 19 we show that effective temperatures in both directions are well-defined and rescale the structure factor for an extended range of velocities. This result can be compared directly to Fig. 12 for the RB case.

In Figs. 20 and 21 for the anisotropic effective temperatures

and global widths we see that at intermediate velocities the RF case is qualitatively similar to the RB case, in the sense that $T_{\rm eff}^{\parallel} > T_{\rm eff}^{\perp}$ and $W_{\parallel}^2 > W_{\perp}^2 \sim 1/v.$ For small velocities, the local displacement distribution is asymmetric in the longitudinal direction, as was shown in Fig. 15 for the RB case. In contrast, for larger velocities the curves for the different directions no longer cross at a characteristic velocity (see Figs. 20 and 21), but directly merge into an isotropic decay at large velocities, with $T_{\rm eff}^{\parallel} \approx T_{\rm eff}^{\perp} \sim 1/v$ and $W_{\parallel}^2 \approx W_{\perp}^2 \sim 1/v.$ Isotropic RF disorder thus produces isotropic fluctuations at large velocities, in contrast with the anisotropic fluctuations in the RB case.

V. DISCUSSION AND CONCLUSIONS

We studied depinning and flow of a flux lines with harmonic elasticity in an isotropic random medium with short-range correlated disorder. We report novel phenomena, such

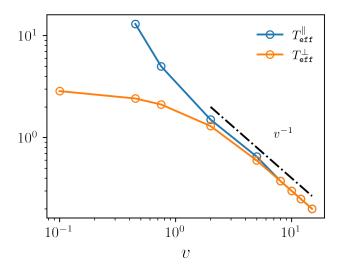


FIG. 20. Effective temperatures as a function of the mean velocity for RF disorder. Dashed-dotted lines indicate the asymptotic behavior.

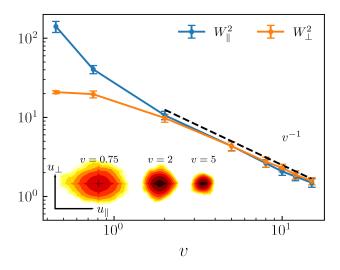


FIG. 21. Mean squared widths as a function of mean velocity for RF disorder. Dashed-dotted lines indicate the asymptotic behavior. The inset displays the joint local-displacement distribution for three velocities.

as the asymmetry of local parallel displacements at low velocities, the inversion of the aspect ratio of widths in the RB case, and important differences between RB and RF in the fast-flow regime.

For quasistatic driving we calculated several universal quantities. In Table I we summarize the values of all critical exponents that we measured, and the relations between them according to our numerical tests. Some critical exponents differ appreciably from previous reports. Our value $\beta=0.24(1)$ differs from $\beta\approx0.31$ or $\beta=0$ given Ref. [62], but is indistinguishable from the one for interfaces in two-dimensional random media [2]. The value $\zeta_{\parallel}\approx1.25(1)$ contrasts with $\zeta_{\parallel}=1$ from Ref. [62], agrees with the one reported in Ref. [63] for

strong disorder, and is indistinguishable from the one for one dimensional interfaces [2, 39, 41]. This result is physically relevant as $\zeta_{\parallel}>1$ implies the breakdown of linear elasticity at large length scales. Some proposed scaling relations do not pass our numerical tests, particularly $\zeta_{\perp} = \zeta_{\parallel} - 1/2$ [62] and $\zeta_{\perp} = 5\zeta_{\parallel}/2 - 2$ [26]. Other relations predicted in Ref. [62] are verified, as shown in Table I. We added the tested relation for τ_{\parallel} , which is identical to the one for interfaces in two dimensions [31, 85]. A new relation links τ_{\perp} to τ_{\parallel} . In spite of differences in the above scaling relations, the main message is that the Ertas-Kardar planar approximation is working well, provided we use the appropriate results for the (D, N) = (1, 1)case [43], and correct the roughness exponent for the transversal direction to $\zeta_{\perp} = (2-d)/2$. We explicitly verified that for (D, N) = (1, 2), microscopic RB and RF disorder lead to a single RF universality class at depinning, a result we expect from the planar approximation.

For intermediate driving velocities we show that the transverse local fluctuations are Gaussian, and that the structure of the elastic string is described by a single exponent $\zeta_{\perp} = 1/2$, together with a well-defined effective temperature that tends to saturate at small velocities and vanishes as $T_{\rm eff}^{\perp} \sim 1/v$ at large velocities. This supports the identification of a transverse "shaking temperature" in Ref. [16] as a limit of the transverse effective temperature we define, and which is valid for all finite velocities. Local longitudinal fluctuations are skewed at low velocities and become Gaussian at large ones, with an effective longitudinal temperature vanishing as $T_{\rm eff}^{\parallel} \sim 1/v^3$ for RB disorder. At large velocities the correlation length becomes small and the interface is essentially flat in the driving direction (see Fig. 14). This result is inconsistent with the prediction of a zero "longitudinal shaking temperature" in Ref. [16]. The difference may be attributed to the fact that the latter calculation neglects terms of order $\mathcal{O}(1/v^2)$ and for RB disorder only contains the leading term proportional to $\int_{\mathcal{U}} \Delta(u) = 0$. On the other hand, the $1/v^3$ behavior is inconsistent with the prediction of an "Edwards-Wilkinson temperature" proportional to 1/v [33], if we assume that the planar approximation holds in this regime. This discrepancy is due to the use of a v-independent RF disorder in the highvelocity regime, ignoring that the microscopic disorder is RB, and that the driving velocity reduces the effects of the RG flow bringing it to RF. In contrast, we confirmed numerically the analytical expectation that microscopic RF disor-

```
1.25 \pm 0.01
               Fig. 3(a)
0.5 \pm 0.01
               Fig. 3(b)
1.43 \pm 0.01
               Fig. 4(a)
2.27 \pm 0.05
               Fig. 4(a)
1.33 \pm 0.02
               Fig. 11(a)
               Fig. 4(b)
0.24 \pm 0.01
1.09 \pm 0.03
               Fig. 7(a)
1.17 \pm 0.06
               Fig. 7(b)
```

TABLE I. Critical depinning exponents obtained in this work. We also indicate the figure where the exponent was fitted or tested and the scaling relations that hold within our numerical uncertainty. The same exponents and relations hold both for RB and RF disorder.

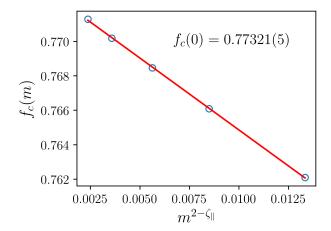


FIG. 22. Critical force as a function of mass behaves as $f_c(m) \sim m^2 u_m = m^{2-\zeta_{\parallel}}$. Extrapolated to m=0 it gives the zero-mass critical force $f_c(0)=0.77321(5)$.

der produces an isotropic effective temperature vanishing as $T_{\tt eff}^{\parallel} \approx T_{\tt eff}^{\perp} \sim 1/v.$

The same dependencies in the effective temperatures are observed in the diffusion of a single monomer driven in 2d RB disorder [74]. This suggests that the longitudinal effective temperature in the large-velocity regime is controlled by what happens for a single monomer. We confirm that in the comoving frame the string can be described as a two-component Edwards-Wilkinson line with uncorrelated noise controlled by v as predicted in [16, 33], with an anisotropy that depends as discussed on the microscopic disorder. These results show that the nature of the microscopic disorder can be detected by observing the anisotropic fluctuations in the fast-flow regime.

Our results should be relevant for flux lines or other elastic lines in random media, such as polymers driven in random quenched media, or cracks. Since cracks have long-range elasticity, we expect the transversal roughness to be logarithmic. This agrees with [93], and was experimentally observed in [94].

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Appendix A: Force-controlled driving versus velocity-controlled driving

Using the velocity-controled driving we performed a set of simulations for different masses and velocities. In Fig. 11

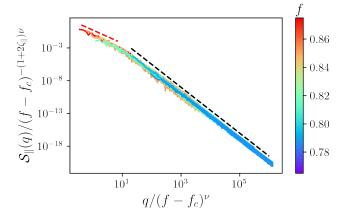


FIG. 23. Scaled steady-state structure factor for the constant-force ensemble according to the deppinning length $l_{\parallel} \sim v(f)^{-\frac{\nu}{\beta}} = [f-f_c(0)]^{-\nu}$ with $\zeta_{\parallel}=1.25$, $\nu=1.33$ and zero-mass critical force $f_c(0)=0.77321(5)$. Red dashed lines indicate the fast-flow roughness exponent $\zeta_{\rm ff}=0.5$, and the black dashed-lines the depinning roughness exponent $\zeta_{\parallel}=1.25$.

we show the depinning transition with exponents $\zeta_{\parallel}=1.25$, $\nu=1.33$ and $\beta=0.33$. The critical force fluctuates around $f_c(m)=m^2\langle vt-u_{\parallel}\rangle$, and this scales with the mass as $f_c(m)\approx m^2u_m=m^{2-\zeta_{\parallel}}$ as shown in Fig. 22. Fitting this relation, we extract the zero-mass critical force $f_c(0)=0.77321(5)$.

With the same parameters we perform a set of simulations in the force-controled driving ensamble. In Fig. 23 we show the structure factor for the parallel direction scaled according to the depping length $l_{\parallel} \sim v(f)^{-\frac{\nu}{\beta}} = (f-f_c)^{-\nu}$ with the same scaling exponents ζ_{\parallel} , ν and critical force $f_c = 0.7656$. The distance to the zero-mass critical force is $|f_c(0) - f_c| \approx 0.0076$.

Appendix B: Correlations of an Ornstein-Uhlenbeck process

We wrote the equation of motion

$$\begin{split} \partial_{u_{\parallel}} u_{\perp}(x,u_{\parallel}) &= -m^2 u_{\perp}(x,u_{\parallel}) + \nabla^2 u_{\perp}(x,u_{\parallel}) \\ &+ \sqrt{2\sigma} \eta(x,t), \end{split} \tag{B1}$$

$$\left\langle \eta(x, u_{\parallel}) \eta(x', u_{\parallel}') \right\rangle = \delta^d(x - x') \delta(u_{\parallel} - u_{\parallel}').$$
 (B2)

Integrating over x, dividing by L^d and replacing $u_{\parallel} \to w$ yields

$$\partial_w u_{\perp}(w) = -m^2 u_{\perp}(w) + \sqrt{\frac{2\sigma}{L^d}} \eta(w),$$
 (B3)

$$\langle \eta(w)\eta(w')\rangle = \delta(w-w').$$
 (B4)

This is an Ornstein-Uhlenbeck process, solved by

$$u_{\perp}(w) = \int_{-\infty}^{w} \mathrm{d}w_1 \,\mathrm{e}^{-m^2(w-w_1)} \xi(w_1).$$
 (B5)

It leads to force correlations

$$\Delta_{\perp}(w - w') = m^{4}L^{d} \langle u_{\perp}(w)u_{\perp}(w') \rangle
= m^{4}L^{d} \int_{-\infty}^{w} dw_{1} \int_{-\infty}^{w'} dw_{2} e^{-m^{2}(w+w'-w_{1}-w_{2})} \langle \eta(w_{1})\eta(w_{2}) \rangle
= 2\sigma m^{4} \int_{-\infty}^{\min(w,w')} d\tilde{w} e^{-m^{2}(w+w'-2\tilde{w})}
= \sigma m^{2} e^{-m^{2}|w-w'|}.$$
(B6)

Appendix C: Single-monomer diffusion in the commoving frame

Let us consider an overdamped particle driven by a force f in a one-dimensional space with quenched random forces:

$$\eta \dot{x} = F(x) + f,\tag{C1}$$

where F(x) is a short-range correlated quenched random force field such that

$$\overline{F(x)} = 0, (C2)$$

$$\overline{F(x)F(x')} = f_0^2 g(|x - x'|/d_0).$$
 (C3)

Here d_0 is a characteristic length, f_0 a characteristic force amplitude, and g(u) a rapidly decaying function of unit range and unit amplitude. Without loss of generality we can adimensionalize the equation of motion by measuring distances in units of d_0 , forces in units of f_0 and time in units of $\tau_0 = \eta d_0/f_0$, such that

$$\dot{x} = F(x) + f. \tag{C4}$$

We consider two toy models, one for RF and one for RB disorder, in which the force fields are piecewise constant.

1. RF disorder

To construct a RF disorder such that $\int_{\mathcal{U}} g(y) > 0$ we take

$$F(x) = R_{[x]} \tag{C5}$$

where [...] denotes the integer part. The R_n are uniformly distributed random numbers in the interval [-1,1] such that

$$\overline{R_n R_m} = \frac{\delta_{n,m}}{3}.$$
 (C6)

From Eq. (C4) the time spent in the interval n is

$$\Delta t_n = \frac{1}{R+f},\tag{C7}$$

where $R \equiv R_n$, and hence

$$\langle \Delta t \rangle = \frac{1}{2} \int_{-1}^{1} \frac{1}{R+f} dR = \frac{1}{2} \ln \left(\frac{f+1}{f-1} \right), \quad (C8)$$

$$\langle \Delta t^2 \rangle = \frac{1}{2} \int_{-1}^{1} \frac{1}{(R+f)^2} dR = \frac{1}{f^2 - 1}.$$
 (C9)

The mean velocity is

$$v = \frac{1}{\langle \Delta t \rangle} = \frac{2}{\ln\left(\frac{f+1}{f-1}\right)},\tag{C10}$$

displaying a depinning transition at f=1, while for $f\gg 1$, $v\simeq f$ as expected. The (differential) mobility is

$$\mu := \frac{\mathrm{d}v}{\mathrm{d}f} = \frac{4}{(f^2 - 1)\ln\left(\frac{f+1}{f-1}\right)^2},$$
(C11)

such that $\mu \to \infty$ when $f \to 1$ and $\mu \to 1$ when $f \to \infty$. The diffusion constant in the commoving frame is $D \equiv \langle [x-vt]^2 \rangle / t = \langle [1-v\Delta t]^2 \rangle$ and can be expressed in terms of $\langle \Delta t \rangle$ and $\langle \Delta t^2 \rangle$ as

$$D = \frac{\langle \Delta t^2 \rangle - \langle \Delta t \rangle^2}{\langle \Delta t \rangle^3}.$$
 (C12)

Using the generalized Einstein relation we get the effective temperature as $T_{\tt eff} = D/\mu$.

We are interested in the $f\gg 1$ fast-flow behaviour of D and $T_{\rm eff}$. Expanding in powers of 1/f we get

$$D \simeq \frac{1}{3f} + \frac{7}{45f^3} + \mathcal{O}(f^{-5}),$$
 (C13)

$$T_{\text{eff}} \simeq \frac{1}{3f} + \frac{2}{45f^3} + \mathcal{O}(f^{-5}).$$
 (C14)

Recovering the physical dimensions, we get at the lowest order, that $D \sim (d_0^2/\tau_0)(f_0/f) = (f_0d_0/\eta)(f_0/f) \sim d_0v_0^2/v$, with $v_0 = f_0/\eta$.

2. RB disorder

To model RB disorder with $\int_y g(y) = 0$, we define the random forces in Eq. (C4) as

$$F(x) = R_{[x]} sign(x - [x] - 1/2) + f.$$
 (C15)

As above R_n are iid random variables, uniformly distributed in [-1,1]. By repeating the procedure of the previous section we obtain $\langle \Delta t \rangle$ as for the RF case, and thus identical v and μ as a function of f. However, $\langle \Delta t^2 \rangle$ is different,

$$\langle \Delta t^2 \rangle = \frac{1}{2} \int_{-1}^{1} \left[\frac{1}{2(f+R)} + \frac{1}{2(f-R)} \right]^2 dR$$
$$= \frac{1}{4f} \left[\frac{2f}{f^2 - 1} + \ln\left(\frac{f+1}{f-1}\right) \right]. \tag{C16}$$

This leads to different asymptotic behaviors,

$$D \simeq T_{\text{eff}} \simeq \frac{4}{45f^3} + \mathcal{O}(f^{-5}).$$
 (C17)

Recovering physical dimensions we get $D \sim d_0 v_0 (f_0/f)^3 \sim d_0 v_0^4/v^3$, with $v_0 = f_0/\eta$.

Appendix D: Notations

TABLE II. Glossary

Symbols	
d	Internal dimension
\tilde{D}	Space dimension
$\stackrel{D}{N}$	
1 V	Number of displacement
	components
α	Direction respect to the driving
	force $\alpha = \perp, \parallel$
x	Internal coordinate
t	Time
v	Mean velocity in the direction of
U	the drive
ζ_{lpha}	Depinning roughness exponents
$\zeta_{ t ff}$	Fast flow roughness exponent
β	Velocity exponent
z_{lpha}	Dynamical exponents
ν	Depinning correlation length
	exponent
<i>T</i>	-
$ au_lpha$	Avalanche exponents
u _	Local displacement vector
\mathbf{F}_{p}	Local pinning force vector
\mathbf{f}	Driving force vector
$T_{ t eff}^{lpha}$	Effective temperatures
u_{α}	Local displacement components
V	Random quenched potential
u	Center of mass displacement in the
	drive direction
f_{c}	Critical depinning force
m^2	Curvature of the driving parabolic
	potential
w	Center of the driving parabolic
w	potential
δw	Discretization of w
χ	Static linear response function as-
	sociated with the structure factor
$\mu_{3,\alpha}$	Skewness parameter of local dis-
	placement fluctuations
k_{α}	Kurtosis parameter of local dis-
	placement fluctuations
W_{α}	Global widths
Δ	Microscopic pinning force-force
	correlator
R	Microscopic pinning potential-
	potential correlator
B_{α}	Displacement correlation function
p_{α}	Avalanche size reduced PDFs
S_{lpha}	Structure factors
${\cal S}^{lpha}_m$	
	Avalanche size cutoffs
ξ_m	Confining potential characteristic
	length-scale
P_{α}	PDF of center of mass jumps in the
	α direction
δu_{α}	local displacement relative to cen-
o wa	ter of mass
_	
c*	Effective noise intensity
f^*	Characteristic force in the waiting
	time distribution
P^w	Waiting time PDF
P^u	Local displacement PDF
ξ	Depinning correlation length
RB	Random-Bond type of disorder
RF	Random-Field type of disorder
FRG	Functional Renormalization Group

- [1] G. Durin and S. Zapperi, The Barkhausen effect, in *The Science of Hysteresis*, edited by G. Bertotti and I. Mayergoyz (Amsterdam, 2006) p. 51, arXiv:cond-mat/0404512 [cond-mat].
- [2] J. Ferré, P. Metaxas, A. Mougin, J.-P. Jamet, J. Gorchon, and V. Jeudy, Universal magnetic domain wall dynamics in the presence of weak disorder, Comptes Rendus Physique 14, 651 (2013), disordered systems / Systèmes désordonnés.
- [3] G. Durin, F. Bohn, M. Correa, R. Sommer, P. L. Doussal, and K. Wiese, Quantitative scaling of magnetic avalanches, Phys. Rev. Lett. 117, 087201 (2016), arXiv:1601.01331.
- [4] W. Kleemann, Universal domain wall dynamics in disordered ferroic materials, Annu. Rev. Mater. Res. 37, 415 (2007).
- [5] P. Paruch and J. Guyonnet, Nanoscale studies of ferroelectric domain walls as pinned elastic interfaces, Comptes Rendus Physique 14, 667 (2013).
- [6] D. Bonamy, S. Santucci, and L. Ponson, Crackling dynamics in material failure as the signature of a self-organized dynamic phase transition, Phys. Rev. Lett. 101, 045501 (2008).
- [7] L. Ponson, Depinning transition in the failure of inhomogeneous brittle materials, Phys. Rev. Lett. **103**, 055501 (2009).
- [8] C. Le Priol, J. Chopin, P. Le Doussal, L. Ponson, and A. Rosso, Universal scaling of the velocity field in crack front propagation, Phys. Rev. Lett. 124, 065501 (2020).
- [9] S. Moulinet, A. Rosso, W. Krauth, and E. Rolley, Width distribution of contact lines on a disordered substrate, Phys. Rev. E 69, 035103 (2004), cond-mat/0310173.
- [10] P. L. Doussal, Sinai model in presence of dilute absorbers, J. Stat. Mech., P07032 (2009), arXiv:0906.0267.
- [11] R. Planet, S. Santucci, and J. Ortín, Avalanches and non-gaussian fluctuations of the global velocity of imbibition fronts, Phys. Rev. Lett. 102, 094502 (2009).
- [12] S. Atis, A. K. Dubey, D. Salin, L. Talon, P. Le Doussal, and K. J. Wiese, Experimental evidence for three universality classes for reaction fronts in disordered flows, Phys. Rev. Lett. 114, 234502 (2015).
- [13] E. Bayart, I. Svetlizky, and J. Fineberg, Fracture mechanics determine the lengths of interface ruptures that mediate frictional motion, Nature Physics 12, 166 EP (2015).
- [14] A. Nicolas, E. E. Ferrero, K. Martens, and J.-L. Barrat, Deformation and flow of amorphous solids: a review of mesoscale elastoplastic models, (2017), arXiv:1708.09194.
- [15] J. P. Sethna, M. K. Bierbaum, K. A. Dahmen, C. P. Goodrich, J. R. Greer, L. X. Hayden, J. P. Kent-Dobias, E. D. Lee, D. B. Liarte, X. Ni, K. N. Quinn, A. Raju, D. Z. Rocklin, A. Shekhawat, and S. Zapperi, Deformation of crystals: Connections with statistical physics, Annu. Rev. Mater. Res. 47, 217 (2017).
- [16] T. Nattermann and S. Scheidl, Vortex-glass phases in type-II superconductors, Adv. Phys. 49, 607 (2000), cond-mat/0003052.
- [17] T. Giamarchi and S. Bhattacharya, Vortex phases, in 2001 Cargese school on "Trends in high magnetic field science" (Springer-Verlag, 2002).
- [18] P. Le Doussal, Novel phases of vortices in superconductors, Int. J. Mod. Phys. B 24, 3855 (2010).
- [19] W.-K. Kwok, U. Welp, A. Glatz, A. E. Koshelev, K. J. Kihlstrom, and G. W. Crabtree, Vortices in high-performance high-temperature superconductors, Rep. Prog. Phys. 79, 116501 (2016).
- [20] A. U. Thomann, V. B. Geshkenbein, and G. Blatter, Vortex dynamics in type-ii superconductors under strong pinning conditions, Phys. Rev. B 96, 144516 (2017).

- [21] I. A. Sadovskyy, A. E. Koshelev, W.-K. Kwok, U. Welp, and A. Glatz, Targeted evolution of pinning landscapes for large superconducting critical currents, PNAS 116, 10291 (2019).
- [22] S. Eley, A. Glatz, and R. Willa, Challenges and transformative opportunities in superconductor vortex physics, J. Appl. Phys. 130, 050901 (2021).
- [23] T. Schulz, R. Ritz, A. Bauer, M. Halder, M. Wagner, C. Franz, C. Pfleiderer, K. Everschor, M. Garst, and A. Rosch, Emergent electrodynamics of skyrmions in a chiral magnet, Nature Physics 8, 301 EP (2012).
- [24] E. A. Jagla and A. B. Kolton, A mechanism for spatial and temporal earthquake clustering, J. Geophys. Res. Solid Earth 115, 10.1029/2009JB006974 (2010).
- [25] E. A. Jagla, F. P. Landes, and A. Rosso, Viscoelastic effects in avalanche dynamics: A key to earthquake statistics, Phys. Rev. Lett. 112, 174301 (2014).
- [26] M. Kardar, Nonequilibrium dynamics of interfaces and lines, Phys. Rep. 301, 85 (1998), cond-mat/9704172.
- [27] D. S. Fisher, Collective transport in random media: from superconductors to earthquakes, Phys. Rep. 301, 113 (1998), condmat/9711179.
- [28] K. Wiese, Theory and experiments for disordered elastic manifolds, depinning, avalanches, and sandpiles, ROPP accepted (2021), arXiv:2102.01215.
- [29] See [16] for a general description.
- [30] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, Dynamics of interface depinning in a disordered medium, J. Phys. II (France) 2, 1483 (1992).
- [31] O. Narayan and D. Fisher, Threshold critical dynamics of driven interfaces in random media, Phys. Rev. B 48, 7030 (1993).
- [32] H. Leschhorn, T. Nattermann, S. Stepanow, and L.-H. Tang, Driven interface depinning in a disordered medium, Annalen der Physik 509, 1 (1997), arXiv:cond-mat/9603114.
- [33] P. Chauve, T. Giamarchi, and P. L. Doussal, Creep and depinning in disordered media, Phys. Rev. B 62, 6241 (2000), cond-mat/0002299.
- [34] A. Kolton, S. Bustingorry, E. Ferrero, and A. Rosso, Uniqueness of the thermodynamic limit for driven disordered elastic interfaces, J. Stat. Mech. 2013, P12004 (2013), arXiv:1308.4329.
- [35] X. Cao, S. Bouzat, A. B. Kolton, and A. Rosso, Localization of soft modes at the depinning transition, Phys. Rev. E 97, 022118 (2018).
- [36] P. Chauve, P. L. Doussal, and K. Wiese, Renormalization of pinned elastic systems: How does it work beyond one loop?, Phys. Rev. Lett. 86, 1785 (2001), cond-mat/0006056.
- [37] P. L. Doussal, K. Wiese, and P. Chauve, 2-loop functional renormalization group analysis of the depinning transition, Phys. Rev. B 66, 174201 (2002), cond-mat/0205108.
- [38] A. Fedorenko and S. Stepanow, Universal energy distribution for interfaces in a random-field environment, Phys. Rev. E 68, 056115 (2003).
- [39] H. Leschhorn, Interface depinning in a disordered medium numerical results, Physica A: Statistical Mechanics and its Applications 195, 324 (1993).
- [40] L. Roters, A. Hucht, S. Lübeck, U. Nowak, and K. Usadel, Depinning transition and thermal fluctuations in the random-field Ising model, Phys. Rev. E 60, 5202 (1999).
- [41] A. Rosso, A. K. Hartmann, and W. Krauth, Depinning of elastic manifolds, Phys. Rev. E 67, 021602 (2003).
- [42] A. Rosso, P. Le Doussal, and K. Wiese, Numerical calculation of the functional renormalization group fixed-point functions at

- the depinning transition, Phys. Rev. B **75**, 220201 (2007), cond-mat/0610821.
- [43] E. Ferrero, S. Bustingorry, and A. Kolton, Non-steady relaxation and critical exponents at the depinning transition, Phys. Rev. E 87, 032122 (2013), arXiv:1211.7275.
- [44] S. Ramanathan and D. S. Fisher, Onset of propagation of planar cracks in heterogeneous media, Phys. Rev. B **58**, 6026 (1998).
- [45] S. Zapperi, P. Cizeau, G. Durin, and H. Stanley, Dynamics of a ferromagnetic domain wall: Avalanches, depinning transition, and the Barkhausen effect, Phys. Rev. B 58, 6353 (1998).
- [46] A. Rosso and W. Krauth, Roughness at the depinning threshold for a long-range elastic string, Phys. Rev. E 65, 025101 (2002).
- [47] O. Duemmer and W. Krauth, Depinning exponents of the driven long-range elastic string, Journal of Statistical Mechanics: Theory and Experiment 2007, P01019 (2007).
- [48] L. Laurson, X. Illa, S. Santucci, K. Tore Tallakstad, K. J. Måløy, and M. J. Alava, Evolution of the average avalanche shape with the universality class, Nature Communications 4, 3927 (2013).
- [49] H.-H. Boltz and J. Kierfeld, Depinning of stiff directed lines in random media, Phys. Rev. E 90, 012101 (2014).
- [50] L.-H. Tang, M. Kardar, and D. Dhar, Driven depinning in anisotropic media, Phys. Rev. Lett. 74, 920 (1995).
- [51] A. Fedorenko, P. Le Doussal, and K. Wiese, Statics and dynamics of elastic manifolds in media with long-range correlated disorder, Phys. Rev. E 74, 061109 (2006), cond-mat/0609234.
- [52] S. Bustingorry, A. B. Kolton, and T. Giamarchi, Random-manifold to random-periodic depinning of an elastic interface, Phys. Rev. B 82, 094202 (2010).
- [53] L. A. N. Amaral, A. L. Barabsi, and H. E. Stanley, Universality classes for interface growth with quenched disorder, Phys. Rev. Lett. 73, 62 (1994).
- [54] A. Rosso and W. Krauth, Origin of the roughness exponent in elastic strings at the depinning threshold, Phys. Rev. Lett. 87, 187002 (2001), cond-mat/0104198.
- [55] T. Goodman and S. Teitel, Roughness of a tilted anharmonic string at depinning, Phys. Rev. E **69**, 062105 (2004).
- [56] P. Le Doussal and K. J. Wiese, Functional renormalization group for anisotropic depinning and relation to branching processes, Phys. Rev. E 67, 016121 (2003).
- [57] Y. J. Chen, S. Zapperi, and J. P. Sethna, Crossover behavior in interface depinning, Phys. Rev. E 92, 022146 (2015).
- [58] L. E. Aragón, A. B. Kolton, P. L. Doussal, K. J. Wiese, and E. A. Jagla, Avalanches in tip-driven interfaces in random media, EPL (Europhysics Letters) 113, 10002 (2016).
- [59] A. Glatz, T. Nattermann, and V. Pokrovsky, Domain wall depinning in random media by ac fields, Phys. Rev. Lett. 90, 047201 (2003).
- [60] C. ter Burg, F. Bohn, F. Durin, R. Sommer, and K. Wiese, Force correlations in disordered magnets, (2021), arXiv:2109.01197 [cond-mat.dis-nn].
- [61] L. Albornoz, E. Ferrero, A. Kolton, V. Jeudy, S. Bustingorry, and J. Curiale, Universal critical exponents of the magnetic domain wall depinning transition, (2021), arXiv:2101.06555.
- [62] D. Ertas and M. Kardar, Anisotropic scaling in threshold critical dynamics of driven directed lines, Phys. Rev. B 53, 3520 (1996).
- [63] A. E. Koshelev and A. B. Kolton, Theory and simulations on strong pinning of vortex lines by nanoparticles, Phys. Rev. B 84, 104528 (2011).
- [64] L. Civale, Pushing the limits for the highest critical currents in superconductors, PNAS 116, 10201 (2019).
- [65] A. Middleton, Asymptotic uniqueness of the sliding state for charge-density waves, Phys. Rev. Lett. 68, 670 (1992).

- [66] A. Dobrinevski, P. Le Doussal, and K. Wiese, Non-stationary dynamics of the Alessandro-Beatrice-Bertotti-Montorsi model, Phys. Rev. E 85, 031105 (2012), arXiv:1112.6307.
- [67] A. Dobrinevski, P. Le Doussal, and K. Wiese, Avalanche shape and exponents beyond mean-field theory, EPL 108, 66002 (2014), arXiv:1407.7353.
- [68] A. Rosso and W. Krauth, Monte Carlo dynamics of driven strings in disordered media, Phys. Rev. B 65, 012202 (2001), cond-mat/0102017.
- [69] H. Leschhorn and L.-H. Tang, Comment on "Elastic string in a random potential", Phys. Rev. Lett. 70, 2973 (1993).
- [70] P. Grassberger, D. Dhar, and P. K. Mohanty, Oslo model, hyperuniformity, and the quenched Edwards-Wilkinson model, Phys. Rev. E 94, 042314 (2016).
- [71] A. Shapira and K. Wiese, unpublished.
- [72] A. E. Koshelev and V. M. Vinokur, Dynamic melting of the vortex lattice, Phys. Rev. Lett. 73, 3580 (1994).
- [73] A. B. Kolton, R. Exartier, L. F. Cugliandolo, D. Domínguez, and N. Grønbech-Jensen, Effective temperature in driven vortex lattices with random pinning, Phys. Rev. Lett. 89, 227001 (2002).
- [74] A. B. Kolton, Pinning induced fluctuations on driven vortices, Physica C 437-438, 153 (2006), proceedings of the Fourth International Conference on Vortex Matter in Nanostructured Superconductors VORTEX IV.
- [75] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Vortices in high-temperature superconductors, Rev. Mod. Phys. 66, 1125 (1994).
- [76] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (Dover Publications, 2004).
- [77] P. Le Doussal and K. Wiese, How to measure Functional RG fixed-point functions for dynamics and at depinning, EPL 77, 66001 (2007), cond-mat/0610525.
- [78] P. L. Doussal, K. Wiese, S. Moulinet, and E. Rolley, Height fluctuations of a contact line: A direct measurement of the renormalized disorder correlator, EPL 87, 56001 (2009), arXiv:0904.4156.
- [79] K. Wiese, M. Bercy, L. Melkonyan, and T. Bizebard, Universal force correlations in an RNA-DNA unzipping experiment, Phys. Rev. Research 2, 043385 (2020), arXiv:1909.01319.
- [80] C. ter Burg and K. Wiese, Mean-field theories for depinning and their experimental signatures, Phys. Rev. E 103, 052114 (2021), arXiv:2010.16372.
- [81] P. L. Doussal and K. Wiese, Driven particle in a random landscape: disorder correlator, avalanche distribution and extreme value statistics of records, Phys. Rev. E 79, 051105 (2009), arXiv:0808.3217.
- [82] P. L. Doussal, K. Wiese, and P. Chauve, Functional renormalization group and the field theory of disordered elastic systems, Phys. Rev. E 69, 026112 (2004), cond-mat/0304614.
- [83] We learned from L. Ponson that the perpendicular roughness for fracture in d=1 is $\zeta_{\perp}\simeq 0$, consistent with the "thermal" exponent for LR elasticity $\zeta_{\perp}=(1-d)/2$.
- [84] M. Grassi, A. B. Kolton, V. Jeudy, A. Mougin, S. Bustingorry, and J. Curiale, Intermittent collective dynamics of domain walls in the creep regime, Phys. Rev. B 98, 224201 (2018).
- [85] A. Rosso, P. Le Doussal, and K. J. Wiese, Avalanche-size distribution at the depinning transition: A numerical test of the theory, Phys. Rev. B 80, 144204 (2009).
- [86] P. Le Doussal, A. Middleton, and K. Wiese, Statistics of static avalanches in a random pinning landscape, Phys. Rev. E 79, 050101 (R) (2009), arXiv:0803.1142.
- [87] C. Bolech and A. Rosso, Universal statistics of the critical depinning force of elastic systems in random media, Phys. Rev.

- Lett. 93, 125701 (2004), cond-mat/0403023.
- [88] A. Fedorenko, P. Le Doussal, and K. Wiese, Universal distribution of threshold forces at the depinning transition, Phys. Rev. E **74**, 041110 (2006), cond-mat/0607229.
- [89] A. B. Kolton, A. Rosso, T. Giamarchi, and W. Krauth, Creep dynamics of elastic manifolds via exact transition pathways, Phys. Rev. B 79, 184207 (2009).
- [90] L. F. Cugliandolo, The effective temperature, Journal of Physics A: Mathematical and Theoretical 44, 483001 (2011).
- [91] P. Le Doussal, A. Petković, and K. Wiese, Distribution of velocities and acceleration for a particle in Brownian corre-

- lated disorder: Inertial case, Phys. Rev. E **85**, 061116 (2012), arXiv:1203.5620.
- [92] J. Sparfel and K. Wiese, Skewness at depinning, and conformal invariance, unpublished (2021).
- [93] S. Ramanathan, D. Ertas, and D. Fisher, Quasistatic crack propagation in heterogeneous media, Phys. Rev. Lett. 79, 873 (1997).
- [94] D. Dalmas, A. Lelarge, and D. Vandembroucq, Crack propagation through phase-separated glasses: Effect of the characteristic size of disorder, Phys. Rev. Lett. 101, 255501 (2008).