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**Pickands' constant at first order in an expansion around Brownian motion. (English. English summary)**

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Gaussian processes have played important and powerful roles in the modeling of many natural phenomena, and in diverse applications such as those in telecommunication, finance and insurance. The extreme values of a Gaussian process, e.g., the tail of the distribution for large values of the maximum, are essential in statistical estimation and inference. These values are expressed in terms of the Pickands constant $\mathcal{H}_\alpha$. As such, the Pickands constant has attracted much research interest over decades.

To obtain a precise definition of the Pickands constant and appreciate its universality, let's consider a stationary Gaussian process $X_t$ with mean $\mathbb{E}(X_t)=0$, and normalized squared variance $\mathbb{E}(X_t^2)=1$. Suppose that the covariance function $r(t):=\mathbb{E}(X_sX_{s+t})$ is independent of $s$, that $r(t)<1$ for all $t>0$ and that $r(t)\simeq 1-|t|^\alpha$ as $t\to 0$. Then the Pickands constant is defined as

$$\mathcal{H}_\alpha=\lim_{T\to\infty}\frac{1}{T}\mathbb{E}\left(\exp\left(\max_{0<t<T}X_t\right)\right).$$

In 1969, Pickands proved that

$$\mathbb{P}\left(\max_{t\in[0,T]}X_t>u\right)\simeq\Psi(u)Tu^{2/\alpha}\mathcal{H}_\alpha,\quad \text{as } u\to\infty,$$

where

$$\Psi(u):=\frac{1}{\sqrt{2\pi}}\int_u^\infty\exp\left(-\frac{x^2}{2}\right)dx.$$

While there exist several representations of the Pickands constant $\mathcal{H}_\alpha$, which depends only on $\alpha$, its exact value is unknown, except for two special Gaussian processes, namely $\mathcal{H}_1=1$ for a standard Brownian motion, and $\mathcal{H}_2=1/\sqrt{\pi}$ for an affine process. Despite significant effort to determine the values of $\mathcal{H}_\alpha$, no analytical result for $\alpha\neq 1,2$ has been obtained, but many numerical approximations have been achieved by many methods.

In this paper, the authors derive the new exact result $\mathcal{H}_\alpha=1-(\alpha-1)\gamma_E+\mathcal{O}(\alpha-1)^2$, where $\gamma_E$ is the Euler's constant. Their approach is beautiful and effective. In short, they extend a recent perturbative approach to a fractional Brownian motion (fBm) to include a drift term, and use the path integral, evaluated perturbatively around the Browning motion, to obtain the result claimed. Moreover, their method also yields other interesting results, such as the full distribution of a maximum, and not only its limiting behaviors for large arguments, as well as the probability distribution of the maximum of an fBm with an unconstrained endpoint.

The paper is carefully written, neatly organized and nicely presented for readers to follow easily and logically. It is self-contained, with relevant previously known results summarized in earlier sections, and detailed derivations included in the appendices. The paper is highly recommended for all researchers who work on or have interests in Gaussian processes or similar stochastic processes. *Sujin Suwanna*