MR2381600 (2009c:82030) 82B44 82B28 82B41

Wiese, K. J. [Wiese, Kay Jörg] (F-ENS-LTP);
Le Doussal, P. [Le Doussal, Pierre] (F-ENS-LTP)

Functional renormalization for disordered systems. Basic recipes and gourmet dishes. (English summary)

Markov Process. Related Fields **13** (2007), *no. 4,* 777–818.

The paper provides an introductory review of the so-called functional renormalization group for disordered systems. The systems are described by a Hamiltonian $\mathcal{H} = \mathcal{H}_{\rm el} + \mathcal{H}_{\rm DO}$ containing, besides the elastic energy term

$$\mathcal{H}_{\rm el}[u] = \frac{1}{2}\int dx\,(\nabla u(x))^2\,,$$

with $x \in \mathbf{R}^d$ and the displacement field $u \in \mathbf{R}^N$, a disorder term

$$\mathcal{H}_{\rm DO}[u] = \int dx\, V(x, u(x)),$$

identified by the disorder distribution function $R(u)$ such that the correlation is given by

$$\overline{V(x,u)\,V(x',u')} = \delta(x-x')\,R(u-u').$$

The renormalization group leads to a flow equation for the function R, which is computed up to 1-loop or 2-loop order approximation. After a finite renormalization, non-analyticity phenomena (formation of a cusp) occur. The authors also discuss how to measure non-analyticity in simulations and experiments. *Guido Gentile*

References

1. K.J. Wiese (2002) Disordered systems and the functional renormalization group: A pedagogical introduction. *Acta Physica Slovaca* **52,** 341–353. cond-mat/0205116.
2. K.J. Wiese (2003) The functional renormalization group treatment of disordered systems: a review. *Ann. Inst. H. Poincaré* **4,** 473–496. cond-mat/0302322. MR2037575
3. P. Le Doussal (2004) Functional renormalisation for disordered systems. www.lancs.ac.uk/users/esqn/windsor04/program.htm.
4. P. Le Doussal (2006) Functional RG for disordered elastic systems. online.kitp.ucsb.edu/online/sle06/ledoussal1/.
5. K.J. Wiese (2006) Functional renormalization for disordered systems: basic recipes and Gourmet dishes. online.kitp.ucsb.edu/online/sle06/wiese/.
6. S. Lemerle, J. Ferré, C. Chappert, V. Mathet, T. Giamarchi and P. Le Doussal (1998) Domain wall creep in an Ising ultrathin magnetic film. *Phys. Rev. Lett.* **80,** 849–853. MR1606412
7. S. Moulinet, C. Guthmann and E. Rolley (2002) Roughness and dynamics of a contact line of a viscous fluid on a disordered substrate. *Eur. Phys. J. A* **8,** 437–43.
8. K.B. Efetov and A.I. Larkin (1977) *Sov. Phys. JETP* **45,** 1236.
9. A.I. Larkin (1970) *Sov. Phys. JETP* **31,** 784–786.
10. G. Parisi and N. Sourlas (1979) Random magnetic fields, supersymmetry, and negative dimensions. *Phys. Rev. Lett.* **43,** 744–5.

11. F.J. Wegner and A. Houghton (1973) Renormalization group equation for critical phenomena. *Phys. Rev. A* **8,** 401–412.
12. D.S. Fisher (1986) Interface fluctuations in disordered systems: $5-\varepsilon$ expansion. *Phys. Rev. Lett.* **56,** 1964–1997.
13. P. Le Doussal (2006) Finite temperature Functional RG, droplets and decaying Burgers turbulence. *Europhys. Lett.* **76,** 457–463. cond-mat/0605490. MR2299914
14. P. Le Doussal, K.J. Wiese and P. Chauve (2004) Functional renormalization group and the field theory of disordered elastic systems. *Phys. Rev. E* **69,** 026112 (42 pages). cont-mat/0304614. MR2037575
15. A.A. Middleton, P. Le Doussal and K.J. Wiese (2007) Measuring functional renormalization group fixed-point functions for pinned manifolds. *Phys. Rev. Lett.* **98,** 155701 (4 pages). cond-mat/0606160.
16. L. Balents, J.P. Bouchaud and M. Mézard (1996) The large scale energy landscape of randomly pinned objects. *J. Phys.* I (*France*) **6,** 1007–1020.
17. P. Le Doussal and K.J. Wiese (2007) How to measure Functional RG fixed-point functions for dynamics and at depinning. *EPL* **77,** 66001 (5 pages). cond-mat/0610525.
18. A. Rosso, P. Le Doussal and K.J. Wiese (2007) Numerical calculation of the functional renormalization group fixed-point functions at the depinning transition. *Phys. Rev. B* **75,** 220201 (4 pages). cond-mat/0610821.
19. P. Chauve, T. Giamarchi and P. Le Doussal (2000) Creep and depinning in disordered media. *Phys. Rev. B* **62,** 6241–6267. cond-mat/0002299.
20. L. Balents and P. Le Doussal (2005) Thermal fluctuations in pinned elastic systems: field theory of rare events and droplets. *Ann. Phys.* **315,** 213–303. cond-mat/0408048. MR2115659
21. L. Balents and P. Le Doussal (2004) Broad relaxation spectrum and the field theory of glassy dynamics for pinned elastic systems. *Phys. Rev. E* **69,** 061107 (33 pages). cond-mat/0312338.
22. P. Le Doussal (2006) Chaos and residual correlations in pinned disordered systems. *Phys. Rev. Lett.* **96,** 235702.
23. L. Balents and D.S. Fisher (1993) Large-N expansion of $4-\varepsilon$-dimensional oriented manifolds in random media. *Phys. Rev. B* **48,** 5949–5963.
24. P. Chauve, P. Le Doussal and K.J. Wiese (2001) Renormalization of pinned elastic systems: How does it work beyond one loop? *Phys. Rev. Lett.* **86,** 1785–1788. cond-mat/0006056.
25. S. Scheidl (2000–2004) Private communications about 2-loop calculations for the random manifold problem.
26. Y. Dincer (1999) Zur Universalität der Struktur elastischer Mannigfaltigkeiten in Unordnung. Master's Thesis, Universität Köln.
27. P. Chauve and P. Le Doussal (2001) Exact multilocal renormalization group and applications to disordered problems. *Phys. Rev. E* **64,** 051102/1–27. cond-mat/0006057. MR1871829
28. A.A. Middleton (1995) Numerical results for the ground-state interface in a random medium. *Phys. Rev. E* **52,** R3337–40.
29. P. Le Doussal and K.J. Wiese (2005) 2-loop functional renormalization for elastic manifolds pinned by disorder in N dimensions. *Phys. Rev. E* **72,** 035101 (R) (4 pages). cond-mat/0501315.
30. M. Kardar, G. Parisi and Y.-C. Zhang (1986) Dynamic scaling of growing interfaces. *Phys. Rev. Lett.* **56,** 889–892.
31. E. Marinari, A. Pagnani and G. Parisi (2000) Critical exponents of the KPZ equation via multi-surface coding numerical simulations. *J. Phys. A* **33,** 8181–8192.

MR1804037
32. P. Le Doussal and K.J. Wiese (2002) Functional renormalization group at large $1/N$ for random manifolds. *Phys. Rev. Lett.* **89,** 125702. cond-mat/0109204v1.
33. M. Mézard and G. Parisi (1991) Replica field theory for random manifolds. *J. Phys.* I (*France*) **1,** 809–837.
34. P. Le Doussal and K.J. Wiese (2004) Derivation of the functional renormalization group β-function at order $1/N$ for manifolds pinned by disorder. *Nucl. Phys. B* **701,** 409–480. cond-mat/0406297. MR2102693
35. T. Nattermann, S. Stepanow, L.H. Tang and H. Leschhorn (1992) Dynamics of interface depinning in a disordered medium. *J. Phys.* II (*France*) **2,** 1483–1488.
36. O. Narayan and D.S. Fisher (1993) Threshold critical dynamics of driven interfaces in random media. *Phys. Rev. B* **48.** 7030–7042.
37. P. Le Doussal, K.J. Wiese and P. Chauve (2002) 2-loop functional renormalization group analysis of the depinning transition. *Phys. Rev. B* **66,** 174201 (34 pages). cond-mat/0205108.
38. K.J. Wiese (2005) Supersymmetry breaking in disordered systems and relation to functional renormalization and replica-symmetry breaking. *J. Phys. A* **17,** S1889—S1898. cond-mat/0411656.
39. J. Zinn-Justin (1989) *Quantum Field Theory and Critical Phenomena.* Oxford University Press, Oxford. MR1079938
40. J. Kurchan (1992) Supersymmetry in spin glass dynamics. *J. Phys.* I (*France*) **2,** 1333.
41. D.S. Fisher (1985) Random fields, random anisotropies, nonlinear sigma models and dimensional reduction. *Phys. Rev. B* **31,** 7233–7251.
42. D.E. Feldman (2000) Quasi-long-range order in the random anisotropy Heisenberg model: Functional renormalization group in $4-\varepsilon$ dimensions. *Phys. Rev. B* **61,** 382–390.
43. D.E. Feldman (2002) Critical exponents of the random-field $O(N)$-model. *Phys. Rev. Lett.* **88,** 177202 (4 pages). cond-mat/0010012.
44. P. Le Doussal and K.J. Wiese (2006) Random field spin models beyond one loop: a mechanism for decreasing the lower critical dimension. *Phys. Rev. Lett.* **96,** 197202. cond-mat/0510344.
45. G. Tarjus and M. Tissier (2006) A unified picture of ferromagnetism, quasilong range order and criticality in random field models. *Phys. Rev. Lett.* **96,** 087202. cond-mat/0511096.
46. G. Tarjus and M. Tissier (2006) Two-loop functional renormalization group of the random field and random anisotropy $O(N)$ models. *Phys. Rev. B* **74** (2006) 214419 (18 pages). cond-mat/0606698.
47. T. Giamarchi and P. Le Doussal (1995) Elastic theory of flux lattices in the presence of weak disorder. *Phys. Rev. B* **52,** 1242–1270.
48. F. Kühnel, P. Le Doussal and K.J. Wiese (2006) Unpublished.
49. P. Le Doussal (2006) Talk at IHEP. Conference *"Renormalization nonperturbative: de la physique de la matière condensée à la cosmologie",* Feb. 2006.
50. G. Tarjus and M. Tissier (2004) Nonperturbative functional renormalization group for random-field models: The way out of dimensional reduction. *Phys. Rev. Lett.* **93,** 267008.
51. A. Rosso, W. Krauth, P. Le Doussal, J. Vannimenus and K.J. Wiese (2003) Universal interface width distributions at the depinning threshold. *Phys. Rev. E* **68,** 036128 (4 pages). cond-mat/0301464.
52. P. Le Doussal and K.J. Wiese Higher correlations, universal distributions and finite size scaling in the field theory of depinning. *Phys. Rev. E* **68,** 046118 (15

pages). cond-mat/0301465.
53. A. Fedorenko and S. Stepanow (2003) Universal energy distribution for interfaces in a random-field environment. *Phys. Rev. E* **68,** 056115 (6 pages).
54. A. Fedorenko, P. Le Doussal and K.J. Wiese (2006) Universal distribution of threshold forces at the depinning transition. *Phys. Rev. E* **74,** 041110 (12 pages). MR2283971
55. C.J. Bolech and A. Rosso (2004) Universal statistics of the critical depinning force of elastic systems in random media. *Phys. Rev. Lett.* **93,** 125701 (4 pages). cond-mat/0403023.
56. A. Rosso and W. Krauth (2001) Origin of the roughness exponent in elastic strings at the depinning threshold. *Phys. Rev. Lett.* **87,** 187002.
57. L.A.N. Amaral, A.L. Barabasi and H.E. Stanley (1994) Critical dynamics of contact line depinning. *Phys. Rev. Lett.* **73,** 62–65.
58. L.-H. Tang, M. Kardar and D. Dhar (1995) Driven depinning in anisotropic media. *Phys. Rev. Lett.* **74,** 920–923. MR1324959
59. P. Le Doussal and K.J. Wiese (2003) Functional renormalization group for anisotropic depinning and relation to branching processes. *Phys. Rev. E* **67,** 016121 (20 pages). cond-mat/0208204. MR1959152
60. L.-H. Tang and H. Leschhorn (1992) Pinning by directed percolation. *Phys. Rev. A* **45,** R8309–8312.
61. S.V. Buldyrev, A.-L. Barabasi, F. Caserta, S. Havlin, H.E. Stanley and T. Vicsek (1992) Anomalous interface roughening in porous media: experiment and model. *Phys. Rev. A* **45,** R8313–8316.
62. S.C. Glotzer, M.F. Gyure, F. Sciortino, A. Coniglio and H.E. Stanley (1994) Pinning in phase-separating systems. *Phys. Rev. E* **49,** 247–258.
63. A. Fedorenko and S. Stepanow (2003) Depinning transition at the upper critical dimension. *Phys. Rev. E* **67,** 057104 (4 pages). cond-mat/0209171.
64. K.J. Wiese and P. Le Doussal 3-loop FRG study of pinned manifolds. In preparation.
65. P. Le Doussal, K.J. Wiese, E. Raphael and R. Golestanian (2006) Can non-linear elasticity explain contact-line roughness at depinning? *Phys. Rev. Lett.* **96,** 015702. cond-mat/0411652.
66. D. Carpentier and P. Le Doussal (1998) Disordered xy models and Coulomb gases: renormalization via traveling waves. *Phys. Rev. Lett.* **81,** 2558–2561.
67. P. Le Doussal and G. Schehr (2007) Free fermions, functional RG and the Cardy Ostlund fixed line at low temperature. *Phys. Rev. B* **75,** 184401 (2007) (8 pages) cond-mat/0607657.
68. L. Balents (2001) Private communication.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.