Quantum fluids

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M1 ICFP 2017-2018

(I) Bose-Einstein condensation as the macroscopic population of a single-particle state

1. **Introduction : What is a quantum fluid ?**

   The condition $n\lambda_{dB}^3 \geq 1$; Particle indistinguishability and symmetry of the wave function under particles exchange; Fermions, bosons and composite particles (e.g. atoms)

2. Recalls of statistical mechanics
   (a) The statistical operator
   (b) Statistical ensembles
   (c) Bose and Fermi laws for the ideal gas
   (d) Classical limit of the quantum statistics

3. **Bose-Einstein condensation of the ideal gas**
   (a) Saturation of the excited states population
   (b) Condensed fraction and mean occupation numbers $\leftrightarrow$ Boltzmann
   (c) Some thermodynamical quantities
   (d) Condensation in a harmonic potential
   (e) One-body density matrix, spatial density and condensation condition

4. **Interactions in cold gases**
   (a) Reminder of scattering theory : scattering amplitude and Born approximation
   (b) Low-energy collisions and s-wave scattering length
   (c) Real potential and model potential : the regularized-delta potential

5. **The condensate wave function in the mean field approximation**

(a) Equation of motion for the one-body density matrix
(b) Hartree-Fock approximation in the absence of a condensate
(c) Hartree-Fock approximation in the presence of a BEC
(d) Time-dependent Gross-Pitaevskii equation

6. **Hydrodynamical description**
   (a) Hydrodynamic equations: derivation and interpretation
   (b) Sound waves
   (c) Hydrodinamic modes in a trap
7. **Quantum vortices**
   (a) What happens when you rotate a superfluid
   (b) Structure of a vortex
   (c) Energetic considerations
   (d) Hydrodynamic equations in a rotating frame
   (e) Dynamical instability for vortex formation

(II) **Finite temperature systems and superfluidity**

1. **Linearization of Gross-Pitaevskii equation**
   (a) Symmetries and spectral properties of the GPE linearized operator
   (b) Eigenmodes and spectrum in the homogeneous case
   (c) The Landau criterium of superfluidity
2. **Bogoliubov approach and quasi particles**
   (a) Time evolution of a perturbation
   (b) Quadratized hamiltonian
   (c) Description in terms of quasi particles
3. **Superfluidity**
   (a) Hess-Fairbank effect, Normal fraction and superfluid fraction
   (b) Exemple of calculation of the superfluid fraction
   (c) The Landau criterion revisited
   (d) Metastability of super currents

Bibliography

1. Cours Phytem
   Physique statistique et quantique
   Alice Sinatra
   http://www.phys.ens.fr/~sinatra/Qstat2.pdf

2. Cours au Collège de France
   Claude Cohen-Tannoudji
   1997-1998, 1998-1999, 1999-2000, 2000-2001, 2001-2002
   http://www.phys.ens.fr/cours/college-de-france/

3. Course at Les Houches Summer School
   Yvan Castin
   Bose-Einstein condensates in atomic gases: simple theoretical results
   arXiv:cond-mat/0105058

4. Book "Quantum Liquids"
   Anthony Leggett
   Oxford University Press

Example of Tutorials

1. Density operator
2. Bose-Einstein condensation of the ideal gas
3. Correlation functions for the Bose gas
4. The Gross-Pitaevskii equation
5. Dynamics of a BEC : vortices
6. The Josephson effect
7. Superfluidity
8. Two fluids model and sound propagation

Previous exams and homeworks
http://www.phys.ens.fr/~sinatra/AliceSinatrateaching.html