Laboratoire Kastler Brossel
Physique quantique et applications

Quantum enhanced sensing of a spatially extended field

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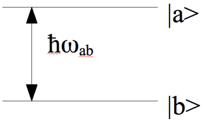
- "Quantum-enhanced multiparameter estimation and compressed sensing of a field"
  Y. Baamara, M. Gessner, A.S. SciPost Physics 14, 050 (2023).
- "Multiparameter estimation with an array of entangled atomic sensors"
  Y. Li, L. Joosten, Y. Baamara, P. Colciaghi, A.S., P. Treutlein, T. Zibold (2025).

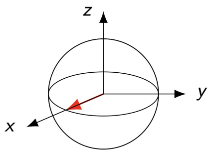
DQ-mat colloquium, Hannover, November 2025

Atomic sensors and quantum projection noise

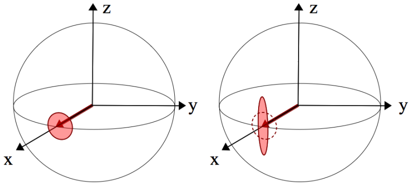
Two-level system



spin $1/2$



N atoms : collective spin $N/2$



projection noise : $s_y = \pm\frac{1}{2}$, $\Delta s_y = \frac{1}{2}$ $\quad \Delta S_y = \frac{\sqrt{N}}{2}$ $\quad \Delta S_y = \xi\frac{\sqrt{N}}{2}$

Angular uncertainty on the collective spin position : $\delta\Theta = \frac{\Delta S_y}{\langle S_x \rangle}$

- **Coherent spin state for independent atoms :**

$$\langle S_x \rangle = \frac{N}{2} \quad \text{and} \quad \Delta S_y = \frac{\sqrt{N}}{2} \quad \Longrightarrow \quad \boxed{(\delta\Theta)_{\mathrm{CSS}} = \frac{1}{\sqrt{N}}}$$

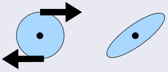
- **Spin-squeezed state $\xi < 1$ for correlated atoms :**

$$\langle S_x \rangle \simeq \frac{N}{2} \quad \text{and} \quad \Delta S_y = \xi\frac{\sqrt{N}}{2} \quad \Longrightarrow \quad \boxed{(\delta\Theta)_{\mathrm{SSS}} = \frac{\xi}{\sqrt{N}}}$$

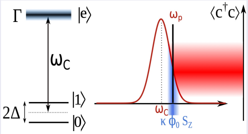
Different methods to realise spin squeezing

Non-linear evolution from atom-atom intercations

$$H_{NL} = \hbar \chi S_z^2$$

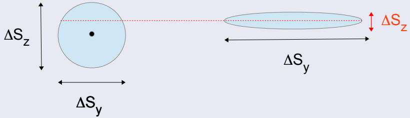


Cavity backaction : interaction with light in a cavity



$$H = \hbar(\omega_{at} + \kappa \phi_0 c^\dagger c) S_z \quad ; \phi_0 \propto \frac{g_0^2}{\Delta}$$

Quantum non demolition measurement



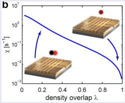
Realisation of a spin-squeezed state

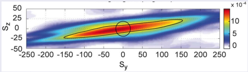
Non linear Hamiltonian $H_{NL} = \hbar\chi S_z^2$

- **Two components BEC :** $N \simeq 10^2 - 10^3$, $\xi^2 \simeq 0.15$

  Oberthaler, *Nonlinear atom interferometer surpasses classical precision limit* Nat. (2010).

  Treutlein, *Atom-chip-based generation of entanglement for quantum metrology* Nat. (2010).





- **Cavity backaction :** $N \simeq 10^2 - 10^4$, $\xi^2 \simeq 0.3$

  Vuletìc, *Implementation of cavity squeezing of a collective atomic spin* PRL (2010).

  Vuletìc, *Entanglement on an optical atomic-clock transition* Nat. (2020).

Spin squeezing by QND measurement

- **Large spin squeezing :** $N \simeq 10^5$, $\xi^2 \simeq 10^{-2}$

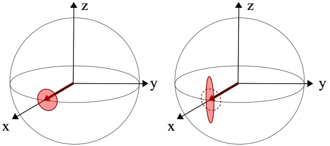
  Kasevich, *Measurement noise 100 times lower than the quantum-projection limit using entangled atoms* Nat. (2016).

- **Long-lived spin squeezeing**

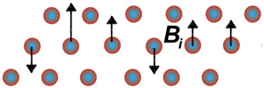
  Reichel, *Observing spin-squeezing under spin-exchange for a second*, PRX Quantum (2023).

Single/multi-parameter quantum metrology

Atom clock : all the atoms measure a single parameter ω_{ab}, angular precession frequency of the collective spin.



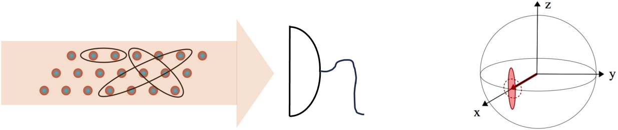
Field mapping. The local precession frequencies $\propto$ local values of a field B_i, are the parameter to be measured.



This talk : use a spin squeezed state divided in different spatial modes as a ressource for multiparameter quantum metrology.

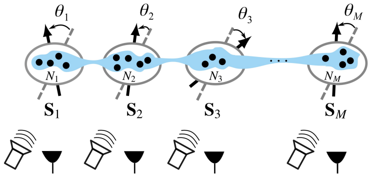
Talk plan : 2 multiparameter q-metrology problems

1. **Field mapping with single atoms**



- Ressources : μ realisations of a N-atom spin-squeezed state
- Capabilities : local spin manipulation and **collective measurements**.

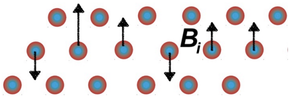
2. **Sensor network, each sensor is a collective spin**



- Ressources : μ realisations of a N-atom spin-squeezed state
- Capabilities : local spin manipulation and **local measurements**.

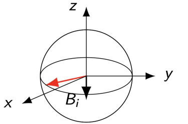
Single atoms as field sensors

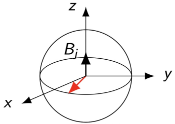
- **Estimation of N "parameters" : $\theta_i \propto B_i$ with N two-level atoms**



Initial state of the atoms (CSS)

$$|\psi_0\rangle = |Ox\rangle^{\otimes N}$$

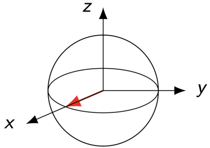




Encoding N parameters

$$\boxed{\hat{U}(\vec{\theta}) = e^{-\mathrm{i}\sum_i \theta_i \hat{s}_{z,i}}}$$

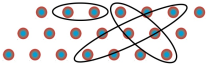
- **Quantum noise (repeated measurements on a spin $1/2$)**



- Mesure $s_x \rightarrow \langle s_x \rangle = 1/2$ et $\Delta s_x = 0$
- Mesure $s_y \rightarrow \langle s_y \rangle = 0$ et $\Delta s_y = \frac{1}{2}$
- For μ repetitions $\sum \langle s_x \rangle = \frac{\mu}{2}$ et $\Delta \left(\sum s_y\right) = \frac{\sqrt{\mu}}{2}$

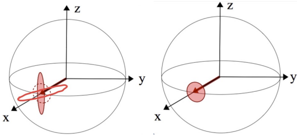
Correlated spins and collective measurements

Quantum gain on a **the spatial average** $\Theta = \sum_i \theta_i / N$ can be obtained by correlating the spins :



Encoding N parameters θ_i on a **spin-squeezed ensemble of N atoms**

$$\boxed{|\psi\rangle = e^{-\mathrm{i}\sum_\mathrm{i} \theta_\mathrm{i}\, \hat{\mathrm{s}}_{\mathrm{z},\mathrm{i}}} |\psi_0\rangle} \quad \text{with} \quad |\psi_0\rangle = e^{-\mathrm{i}\Omega S_x} e^{-\mathrm{i}t\mathrm{S}_\mathrm{z}^2} \left[|Ox\rangle^{\otimes N} \right]$$

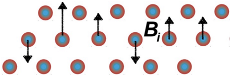


Squeezing of $S_y = \sum \hat{s}_{y,i}$ allows estimation of Θ with **quantum gain**

$$\langle\psi|S_y|\psi\rangle \approx \frac{N}{2}\Theta \quad \textbf{and} \quad \Delta\Theta = \frac{1}{\sqrt{\mu}}\frac{\xi}{\sqrt{N}} \quad \textbf{where} \quad \xi < 1$$

N.B. S_y is the maximally squeezed linear combination of $\hat{s}_{y,i}$. Any orthogonal combination is anti-squeezed.

Multiparameter estimation with quantum gain



$$\boxed{\hat{U}(\vec{\theta}) = e^{-\mathrm{i}\sum_i \theta_i \hat{s}_z^i}}$$

To determine $\theta_1, \ldots \theta_N$ we need N independent linear combinations

The previous scheme (measure of S_y) gives only access to the spatial average ⊖

Way out :

- N independent combinations $\sum_i \epsilon_i \theta_i$ with $\epsilon_\mathrm{i} = \pm 1$
- After squeezing, local spin flips to introduce minus signs in front the parameters

$$\boxed{\mathrm{e}^{\mathrm{i}\pi \hat{\mathrm{s}}_{\mathrm{x,k}}} e^{-\mathrm{i}\sum_i \theta_i \hat{s}_{z,i}} \mathrm{e}^{-\mathrm{i}\pi \hat{\mathrm{s}}_{\mathrm{x,k}}} = e^{-\mathrm{i}(\theta_1 \hat{s}_{z,1} + \theta_2 \hat{s}_{z,2} + \ldots - \theta_\mathrm{k} \hat{s}_{z,k} + \ldots + \theta_N \hat{s}_{z,N})}}$$

- Measure S_y which gives now access to $\sum \epsilon_i \theta_i$

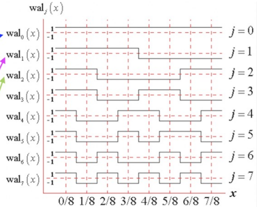
Parameter combinations and Walsh-Hadamard

Walsh-Hadamard transform (orthogonal, symmetric, involutive)

$$\tilde{\theta}_j = \sum_k [\mathcal{H}_m^{-1}]_{jk}\theta_k \quad \textbf{and} \quad \theta_k = \sum_j [\mathcal{H}_m]_{kj}\tilde{\theta}_j$$

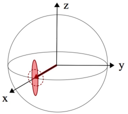
Number of parameters $M \equiv 2^m$ $\quad H_0 = 1, \quad H_m = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathcal{H}_{m-1} & \mathcal{H}_{m-1} \\ \mathcal{H}_{m-1} & -\mathcal{H}_{m-1} \end{pmatrix}$

$$H_3 = \frac{1}{2^{3/2}}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix}$$



- **Same uncertainty for each parameter estimator**

$$\boxed{\Delta\theta_k = \frac{\xi}{\sqrt{\mu_c}}} \quad \textbf{where} \quad \boxed{\xi^2 = \frac{N\Delta S_y^2}{|\langle S_x\rangle|^2}}$$



μ_c **= number of measurements of each Hadamard combination**

Measurement of a 2D scalar field



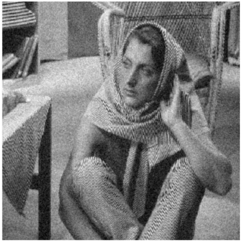
Original signal $\vec{\theta}$ $\Rightarrow$ $\hat{U}(\vec{\theta}) = e^{-\mathrm{i}\sum_i \theta_i \hat{s}_{i,z}}$ $N = 512 \times 512$

Reconstruction: measure N Hadamard coefficients ($\mu_c = 10$ repetitions)

- **Coherent spin state CSS**: $\Delta\theta_i^{\mathrm{CSS}} = \frac{1}{\sqrt{\mu_c}}$
- **Spin squeezed state SSS:** $H_{NL} = \chi S_z^2$

$$\Delta\theta_i^{\mathrm{SSS}} = \frac{\xi}{\sqrt{\mu_c}} \quad ; \quad t_{\mathrm{opt}} \sim \frac{1}{N^{2/3}} \quad ; \quad \xi(t_{\mathrm{opt}}) \sim \frac{1}{N^{1/3}} = \frac{1}{64}$$

Quantum gain : imperfect squeezing



Original signal $\Rightarrow$ $\hat{U}(\vec{\theta}) = e^{-\mathrm{i}\sum_i \theta_i \hat{s}_{i,z}}$ $N = 512 \times 512$

Imperfect spin squeezed SSSD, with collective dephasing $\gamma/\chi = 5$

$$\frac{\partial \rho}{\partial t} = -\mathrm{i}[\chi S_z^2, \rho] + \gamma \left[S_z \rho S_z - \frac{1}{2}\{S_z^2, \rho\} \right]$$

$$\Delta\theta_i^{\mathrm{SSSD}} = \frac{\xi}{\sqrt{\mu_c}} \quad ; \quad t_{\mathrm{opt}} \sim \frac{1}{N^{3/5}} \quad ; \quad \xi(t_{\mathrm{opt}}) \sim \frac{1}{N^{1/5}} \simeq \frac{1}{12}$$

Compressed sensing : only $\mathcal{L}_\mathcal{H} < N$ Hadamard coefficients are measured

$N = 512 \times 512$

CSS, $\mathcal{L}_\mathcal{H} = 64^2$

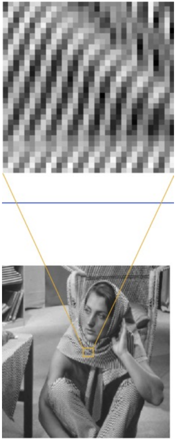
SSS, $\mathcal{L}_\mathcal{H} = 64^2$

SSSD

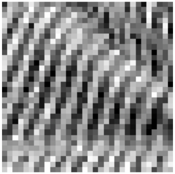
Y. Baamara, M. Gessner, A. Sinatra, "Quantum-enhanced multiparameter estimation and compressed sensing of a field", SciPost Physics, (2023).

Smaller size ($N = 1024$) with imperfect squeezing

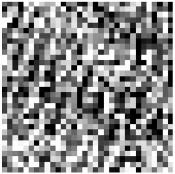
- **Original signal**
  $N = 32 \times 32$



- **Reconstruction with an imperfect squeezed state SSSD : $\gamma = \chi$, $\mu_c = 10$ repetitions**



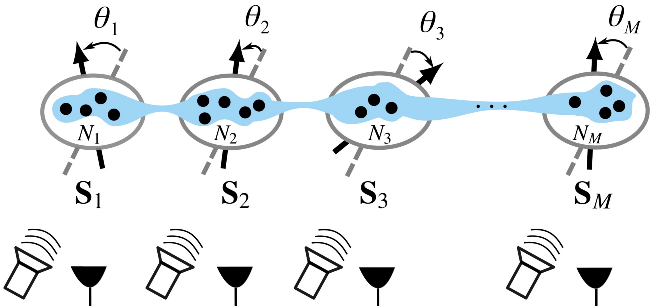
- Reconstruction with a coherent spin state CSS, $\mu_c = 10$ repetitions, for comparison



More atoms per site and local measurements

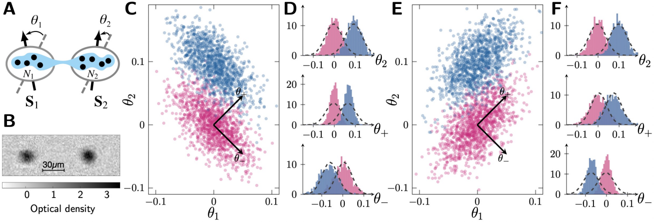
Redefinition of the ressources

- Spin-squeezed ensemble N atoms distributed in $M < N$ modes
- μ repetitions in total, where the θ_k are locally imprinted and the collective spins S_k are locally manipulated and measured



Theory-Experiment collaboration with Philipp Treutlein

Reshape quantum correlations by local manipulation



- Initial state $N = 1450$, ^{87}Rb, $\xi^2 = N\Delta S_z/|\langle S_x\rangle|^2 = -6.5$dB
- θ_1, θ_2 small rotation angles of S_1, S_2 around y-axis, estimated as:

$$\theta_1 = \frac{S_1^z}{\langle S_1^x\rangle} \quad ; \quad \theta_2 = \frac{S_2^z}{\langle S_2^x\rangle}$$

- Figure : $\theta_1 = 0$ and $\theta_2 = 0$ (pink) and $\theta_1 = 0$ and $\theta_2 \neq 0$ (blue).
- In the initial state $(\theta_1 + \theta_2) \propto S_z$ estimated with quantum gain (left)
- After the local π-pulse $e^{-i\pi \hat{S}_2^x}$, that inverts the sign of S_2^z, $(\theta_1 - \theta_2) \propto (S_1^z - S_2^z)$ is estimated with quantum gain (right).

Same spin-squeezed state ξ^2, N atoms, μ repetitions

Covariance matrix of parameter estimators, N/M atoms per mode

$$\Sigma_{kl} = \mathrm{Cov}(\theta_k, \theta_l) \quad ; \quad \Sigma_{kl} = \frac{\delta_{kl}}{\mu N/M}$$ **for a coherent spin state**

1. **Limited noise-reduction due to local squeezing**

$$\mathrm{Var}\,\theta_k \overset{\xi^2\to 0}{\sim} \frac{1}{\mu N/M}\left(\frac{M-1}{M}\right)$$

2. **Diagonalizing Σ:**

One **Squeezed combination** $(1,1,1\ldots)/\sqrt{M}$ $\quad \mathrm{Var}\,\theta^{\mathrm{sq}} = \dfrac{\xi^2}{\mu N/M}$

$(M-1)$ **Hadamard unsqueezed combinations** $\quad \mathrm{Var}\,\theta^{\mathrm{unsq}} \gtrsim \dfrac{1}{\mu N/M}$

Joint parameter estimation protocol

1. Share realizations (M groups of μ/M) to reshape the squeezing (π-pulses) before phase imprinting
2. For each realization use all the information from local measurements

Construct the unbiased estimator

$$\theta_k = \left(\frac{1+\alpha}{2}\right)\check{\theta}_k^{\mathrm{sq}} + \left(\frac{1-\alpha}{2}\right)\frac{1}{M-1}\sum_{i=1}^{M-1}\check{\theta}_k^{\mathrm{unsq,i}}$$

After minimization over α

$$\mathrm{Var}\,\theta_k = \frac{1}{\frac{1}{\mathrm{Var}\,\check{\theta}_{\mathrm{sq}}} + (M-1)\frac{1}{\mathrm{Var}\,\check{\theta}_{\mathrm{unsq}}}} = \bar{\sigma}^H$$

where $\bar{\sigma}^H$ is the harmonic average of eigenvalues of the initial Σ

We show that this result optimal (Cramér-Rao) with our resources

Experimental results for $M = 2$ entangled sensors

Joint measurement of two parameters θ_1 and θ_2 with quantum gain

Quantum gain (dB) =

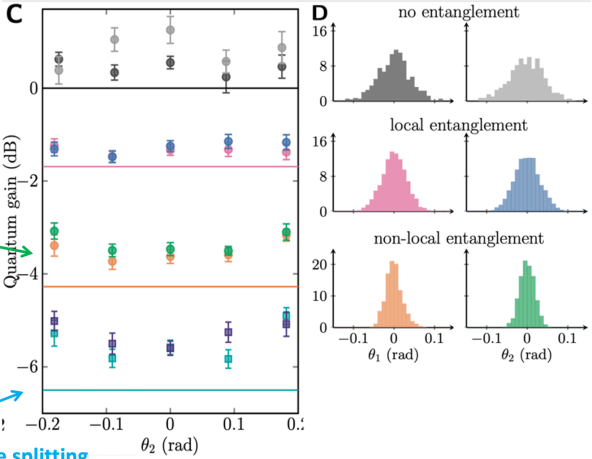
$$10 \log_{10} \left[\frac{Var\,\theta_i}{(Var\,\theta_i)_{css}} \right]$$

Using only local squeezing : Limited gain

Using both local and non-local correlations : Gain = $M\xi$ / $[1+(M-1)\,\xi^2]$

When a single combination of parameters is measured with a gain = ξ

Squeezing ξ of the BEC before splitting



Y. Li, L. Joosten, Y. Baamara, P. Colciaghi, A. Sinatra, P. Treutlein, T. Zibold, *und. ev.* (2025)

Quantum gain with local measurements

Quantum gain in the joint estimation of all parameters

$$\boxed{\frac{\mathrm{Var}\,\theta_k}{(\mathrm{Var}\,\theta_k)_{CSS}} \overset{N\gg 1}{\simeq} \frac{M}{\frac{1}{\xi^2}+(M-1)} = \frac{M\xi^2}{1+(M-1)\xi^2}}$$

- $M\xi^2$ is the noise reduction when using, for each combinations, only the μ/M realizations in which the combination is squeezed
- One can improve on this result using all the information, in which case $\mathrm{Var}\,\theta_k = \bar{\sigma}^H \simeq \bar{\lambda}^H$ harmonic average of $(\mu\mathcal{F})^{-1}$.

Quantum gain in the estimation of some parameter combinations

- ξ^2 is the noise reduction when estimating a single combination
- $\mathcal{L}_{\mathcal{H}}\xi^2$ is the noise reduction for $\mathcal{L}_{\mathcal{H}}$ combinations
- Compressed sensing, Spatial Fourier, Correlation functions ... $\mathcal{L}_{\mathcal{H}} \ll M \ll N$