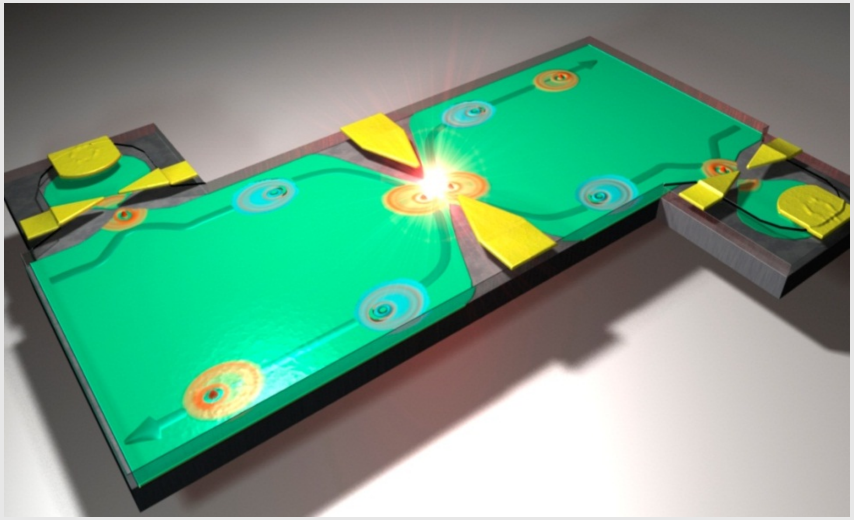
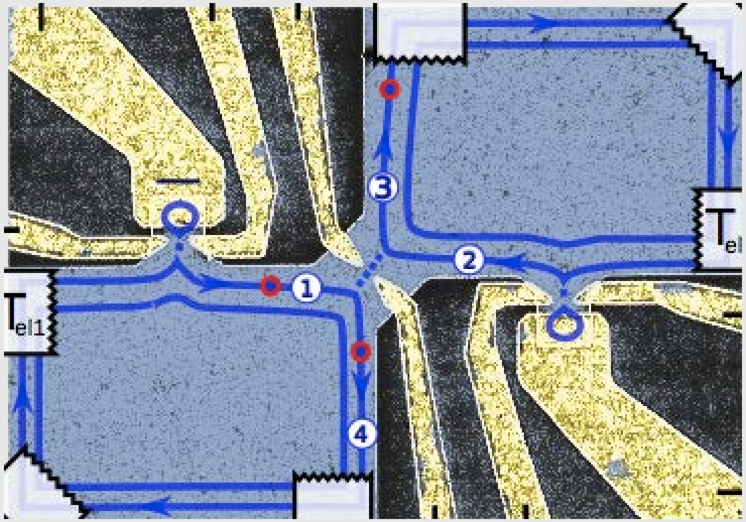
Electron quantum optics with edge states

Objective of the mini-course : an introduction





Create, manipulate and detect single electron wave pacquets

« à la manière de »

Quantum optics with single photon states in the 90's

(Materials + Tools) + (Principles + Implementation)

First lecture : 2d electron materials & devices

Part 1 : 2d electron systems

- High-mobility semiconducting heterostructures
- Quantum Hall Effect
- Electrons in graphene

Part 2 : 2d electron devices

- Electron reservoir and quantum transport
- Scattering by a quantum point contact
- The mesoscopic capacitor

Semiconductor Nanostructures, Thomas Ihn, Oxford University Press (2010)

Second lecture : quantum optics with edge states

Part 3 : Introduction to electrons optics

- Electron optics principles
- Single electron sources
- The mesoscopic capacitor source: average current and noise

Part 4 : Quantum optics experiments

- Partitioning single electrons
- Colliding indistinguishable electrons
- Decoherence and charge fractionalization

Electron quantum optics in ballistic chiral conductors,
Bocquillon et al., Annalen der Physik 526, 1 (2014)

laboratoire pierre aigrain
électronique et photonique quantiques

2D electron materials

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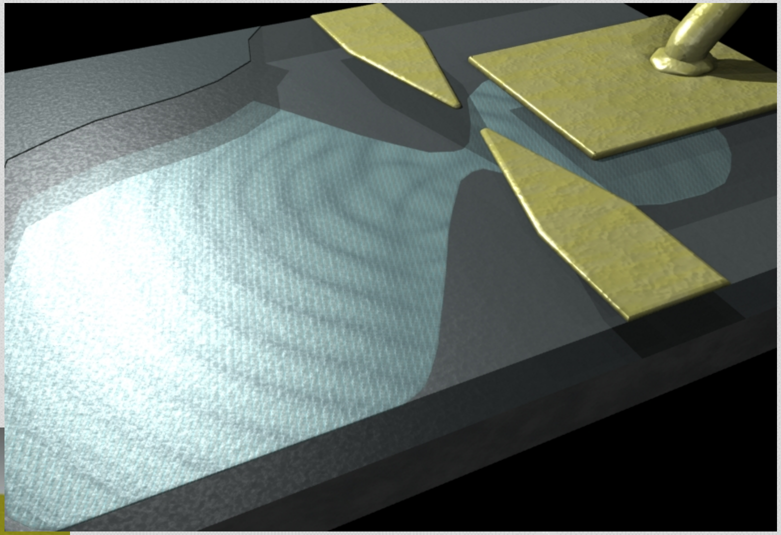
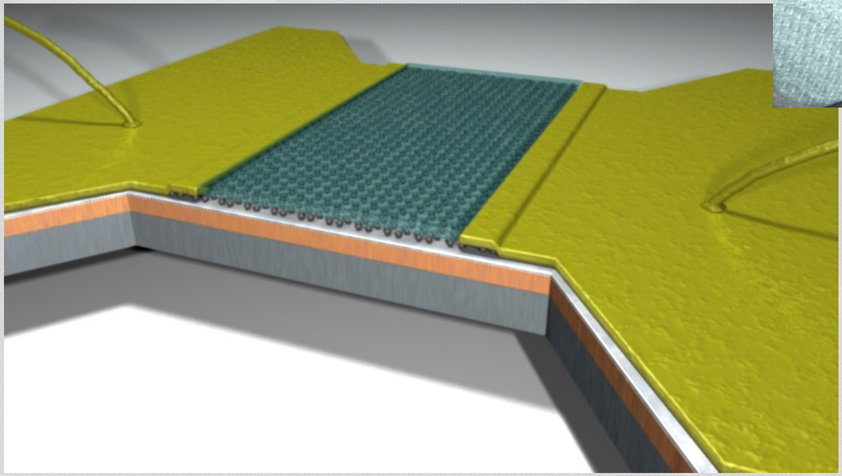




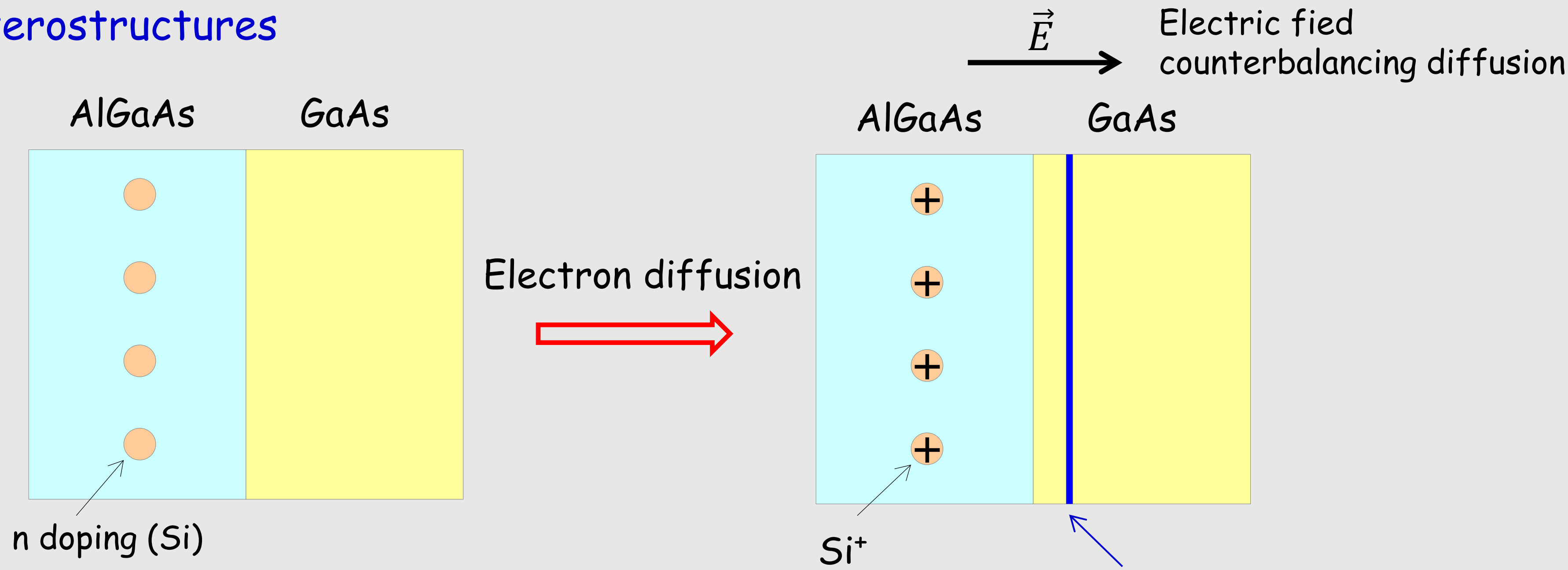
Gwendal Fève

Jean-Marc Berroir

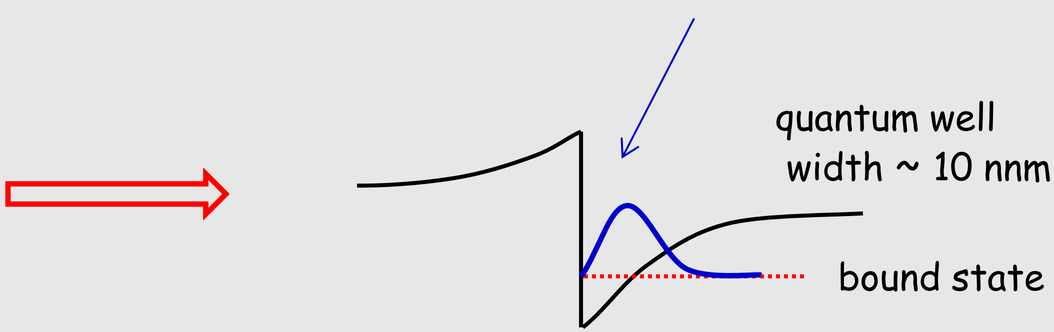
Principle of 2D electron system

modulation doped heterostructures

equilibrium

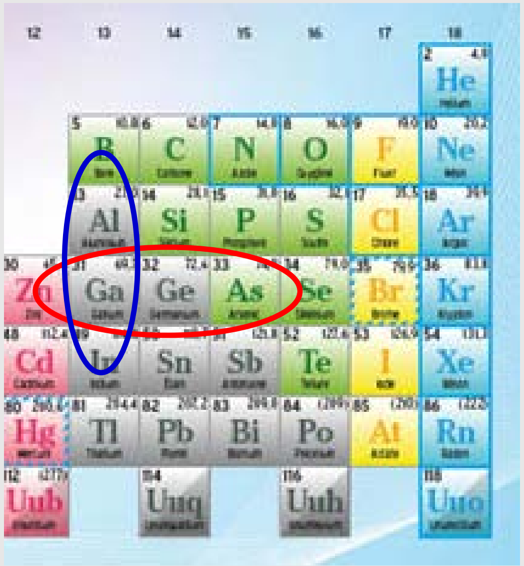


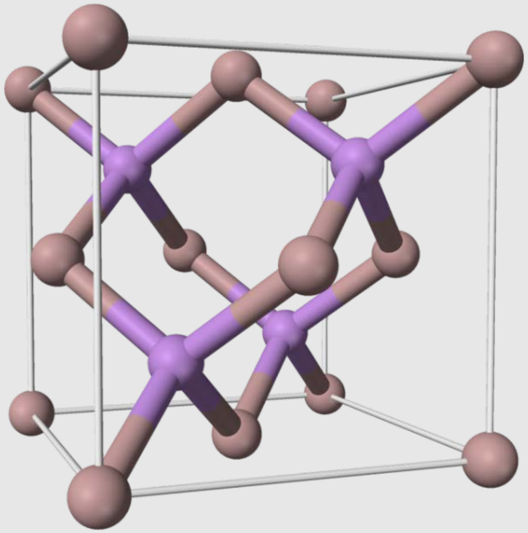
conduction band profile

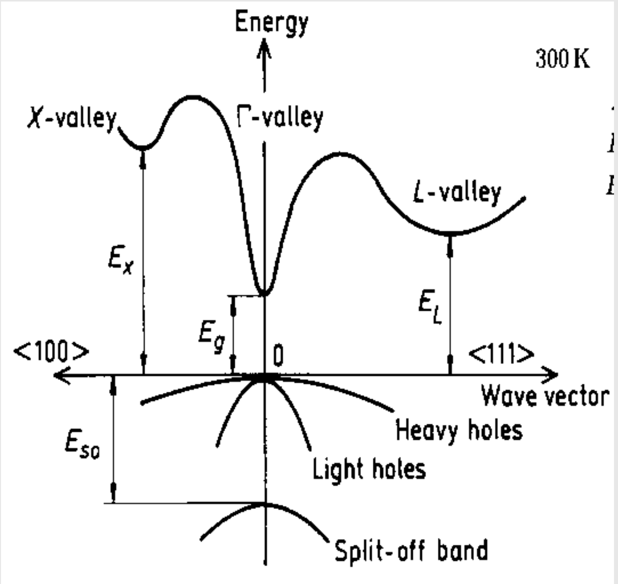


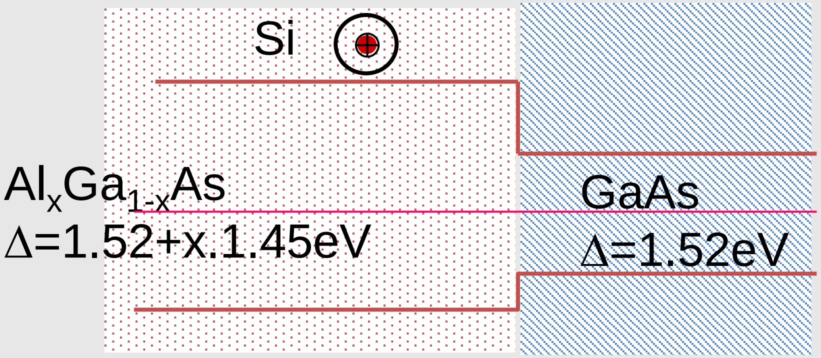
The AlGaAs/GaAS heterostructure

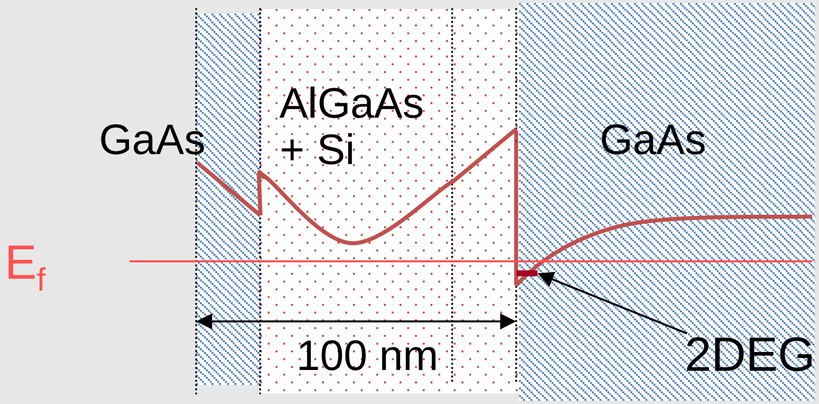
Schrödinger electrons : $\varepsilon = \frac{\hbar^2 k^2}{2m^*}$ Effective mass : $m^* = 0{,}067 \times m_0$



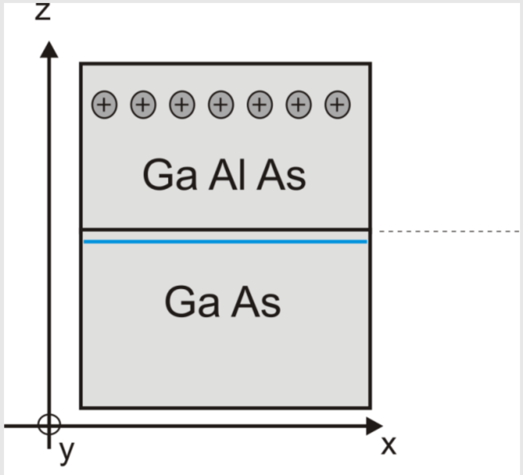


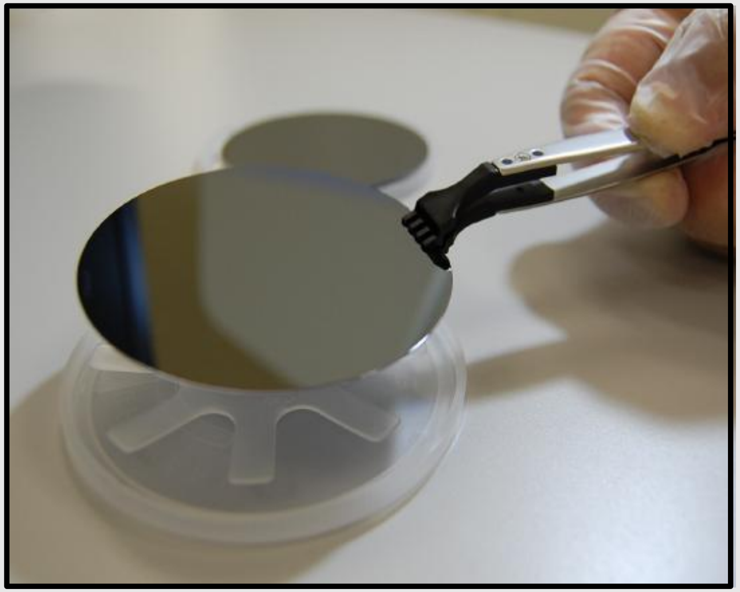


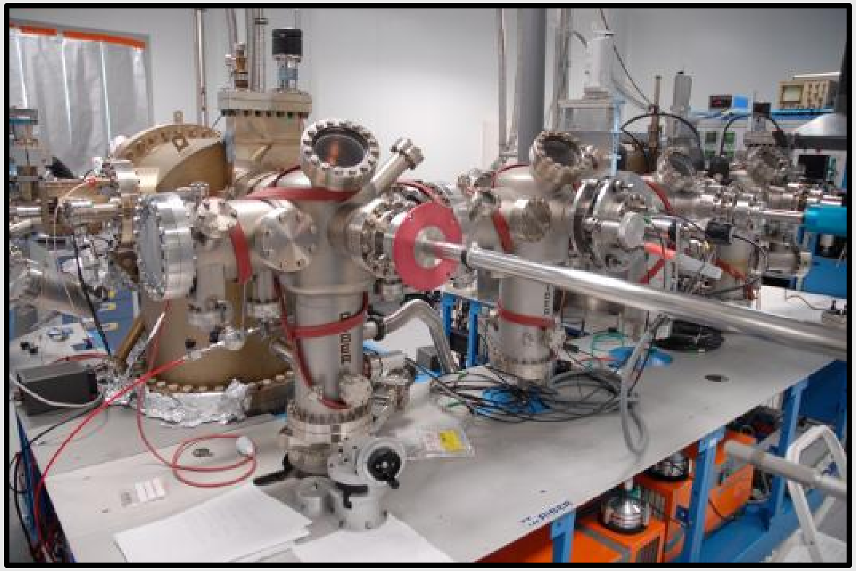




Molecular Beam Epitaxy







$\lambda_F \sim 50$ nm

$l_e \sim 10\text{-}20$ µm

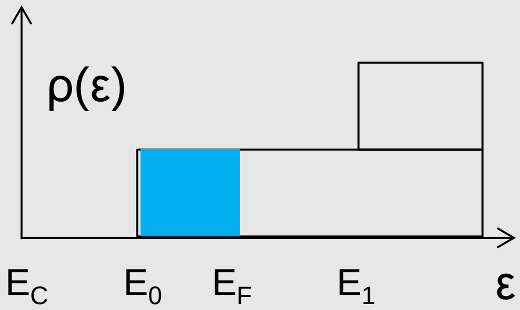
$l_\phi \sim 15$ µm

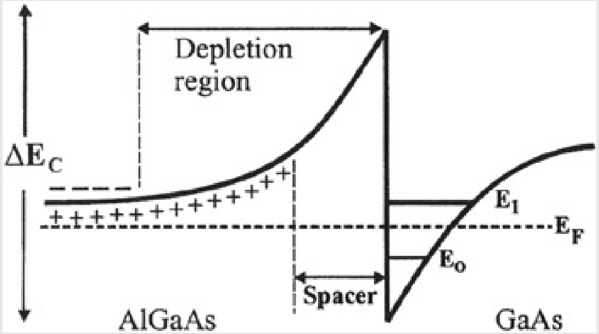
($T_\sim 100$ mK)

Laboratory of Photonics and Nanostructures, CNRS-LPN, Marcoussis, France

A (massive) 2d electron gas

- Density of states : $\rho(\varepsilon) = g_s g_v m^*/2\pi\hbar^2$ $\quad g_s = 2;\ g_v = 1$
  $(e^2\rho(\varepsilon_F) \approx 44\ fF\ \mu m^{-2} \equiv 2.6\ nm\ GaAs)$





- Electron density : $n \approx 10^{11} - 10^{12}\ cm^{-2}$ (determined by doping)

- Fermi-energy : $\varepsilon_F = \frac{2\pi\hbar^2 n}{g_s g_v m^*}$ $\quad (\approx 10\ meV\ at\ 3\ 10^{11} cm^{-2})$

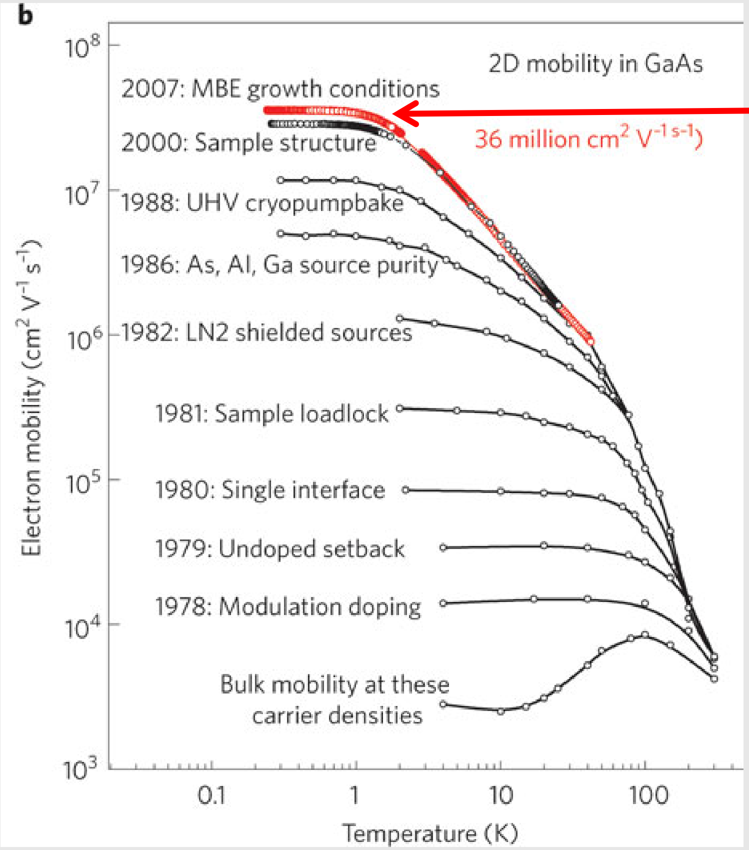
- Fermi wave lentgh: $\lambda_F = \sqrt{\frac{\pi g_s g_v}{n}}$ $\quad (\approx 25\ nm\ at\ 3\ 10^{11} cm^{-2})$ (>> metals !!)

- Fermi velocity : $v_F = \frac{\hbar k_F}{m^*}$ $\quad (\approx 7\ 10^4\ \ m\ s^{-1} at\ 3\ 10^{11} cm^{-2})$ (<< metals !!)

Conductivity / mobility of diffusive 2DEGs

- Good conductivity : $\sigma = R^{-1} L/W$ $(\sigma^{-1} \approx 1 - 10^4 \;\; Ohms)$
- High mobility : $\mu(n) \equiv v_D/E = \sigma/ne$ $(\mu \approx 10^3 - 10^7 \; cm^2V^{-1}s^{-1})$
- Diffusion const. : $\sigma = e^2\rho(\varepsilon_F) \times D(\varepsilon_F)$ $(D \approx 10 - 10^5 \; cm^2 s^{-1})$
- Diffusion length : $l_e = 2D/v_F$ $(l_e \approx 30 \; nm - 300\mu m$!!)

High-mobility GaAs



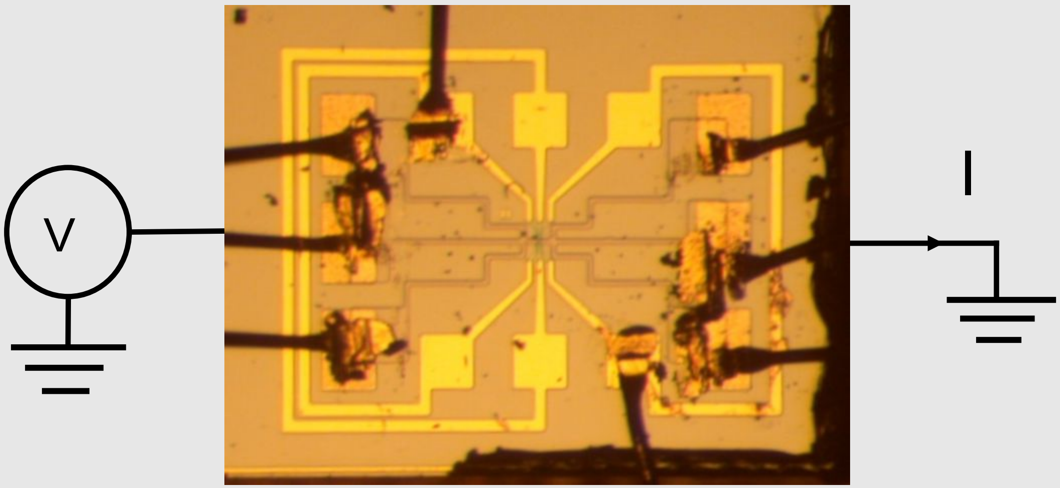
Ultra-clean limit (low-temp. physics)

Strong effect of temperature (phonons)

Room temperature (Nokia phones, …)S

N.D. Schlom, L.N. Pfeiffer, Nature Materials 9, 881–883 (2010)

Toward ballistic transport
Shubnikov-de-Haas

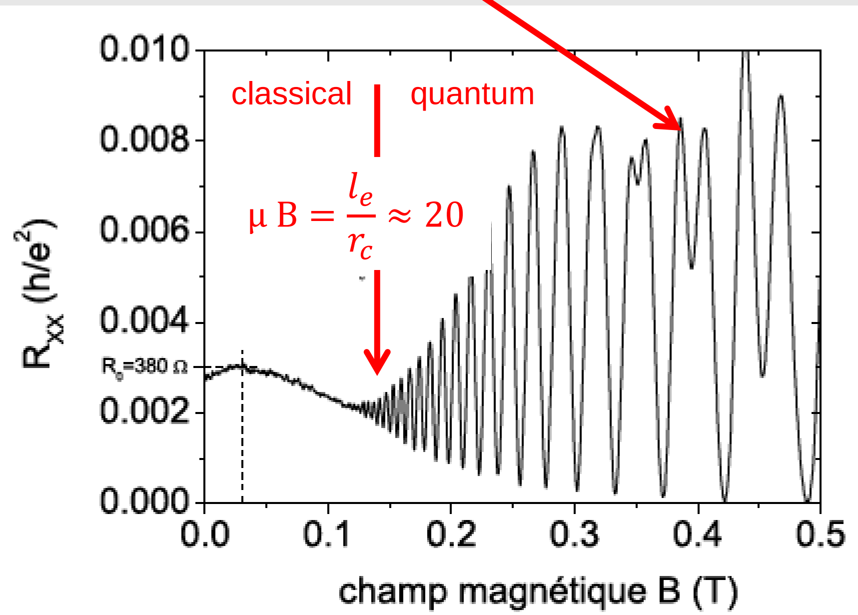


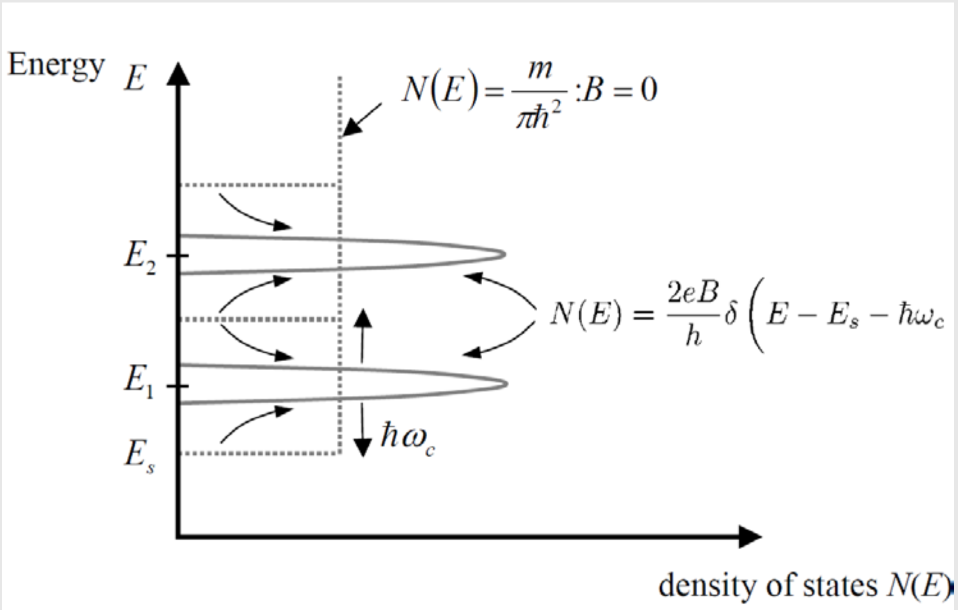
$$R_{XY} = (V_1 - V_3)/I = -B/ne$$

$$R_{XX} = (V_1 - V_2)/I = 1/ne\mu \times L/W$$

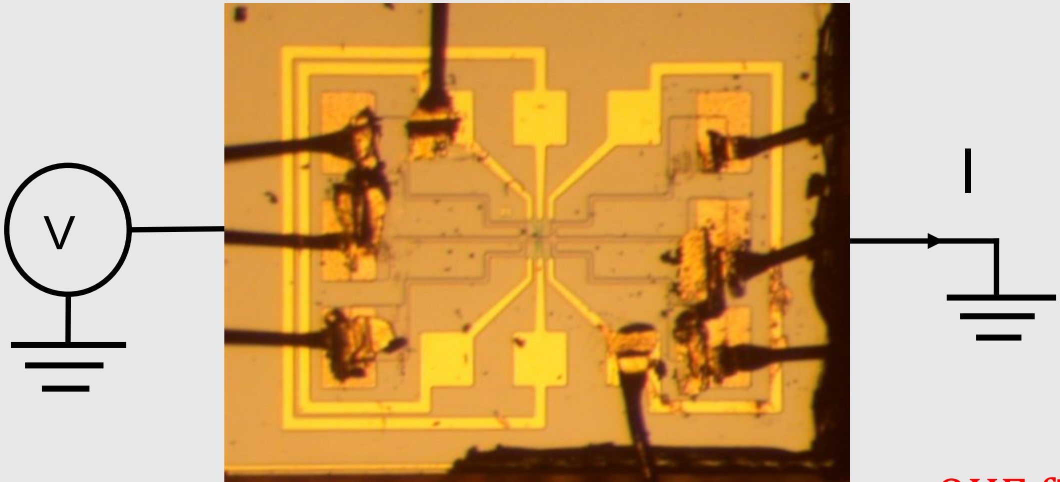
$$\mu \approx 1{,}3\ 10^6\ cm^2 V^{-1} s^{-1}$$
$$n \approx 1{,}35\ 10^{11}\ cm^{-2}$$

Zeeman splitting





Ballistic transport
Quantum Hall effect



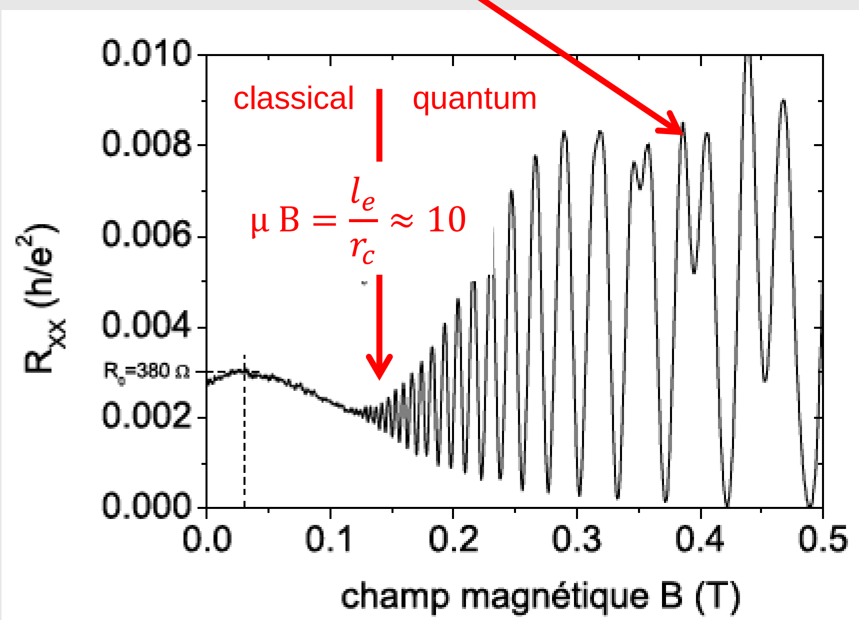
$$R_{XY} = (V_1 - V_3)/I = -B/ne$$

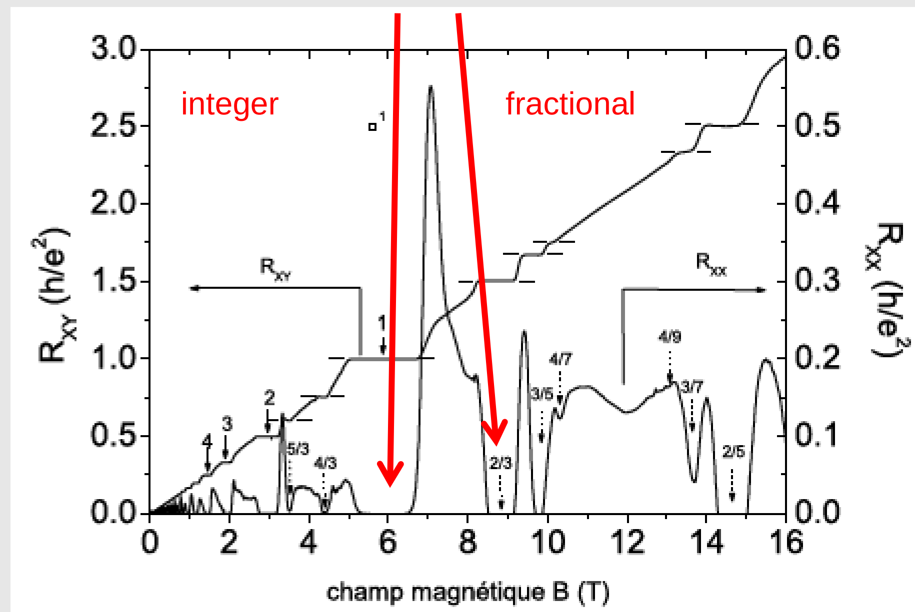
$$R_{XX} = (V_1 - V_2)/I = 1/ne\mu \times L/W$$

$$\mu \approx 1{,}3\ 10^6\ cm^2\,V^{-1}s^{-1}$$
$$n \approx 1{,}35\ 10^{11}\ cm^{-2}$$

Zeeman splitting

QHE filling factor: $\nu = n\,\varphi_0/B \quad (\varphi_0 = e/h)$

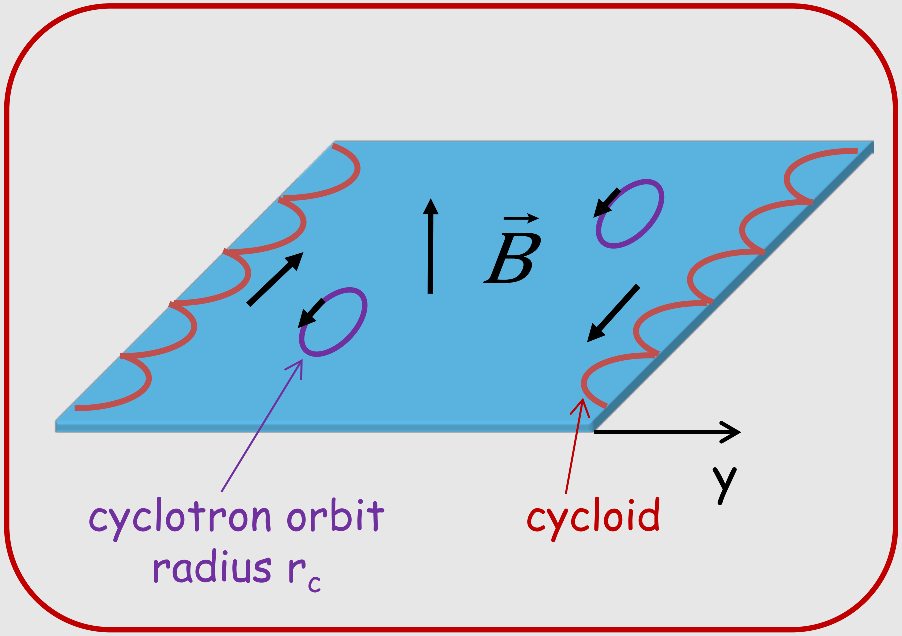




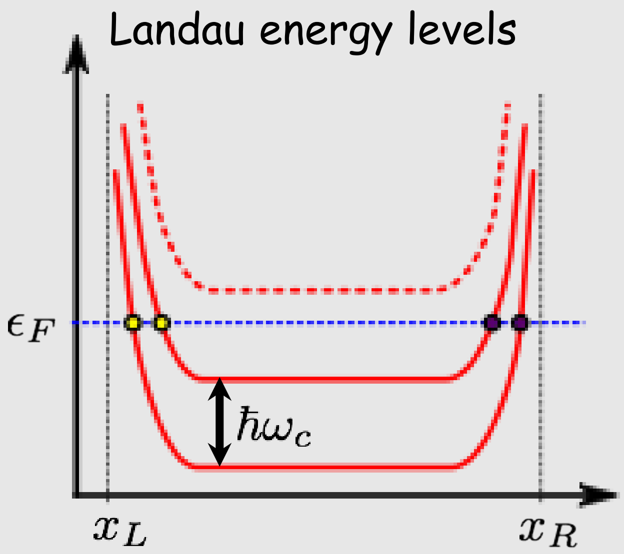
2DEG under perpendicular magnetic field

Cyclotron frequency : $\omega_c = eB/m = v_F/r_c$ Hamiltonian : $H = \frac{1}{2m}(p + eA)^2 + V(y)$ ...../.....

Classical motion



Quantum mechanical motion



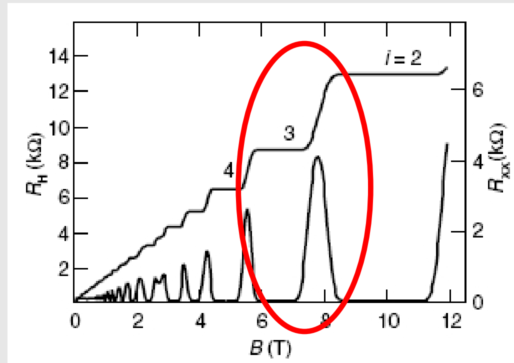
Flux quantum : $\varphi_0 = h/e$; Magnetic length : $l_B = \varphi_0/2\pi B$; Landau levels : $E = \left(n + \frac{1}{2}\right)\hbar\omega_c$

Drift velocity : $v_D = E/B$ (see T. Ihn) Edge channels follow equipotential lines

Edge states : ballistic transort

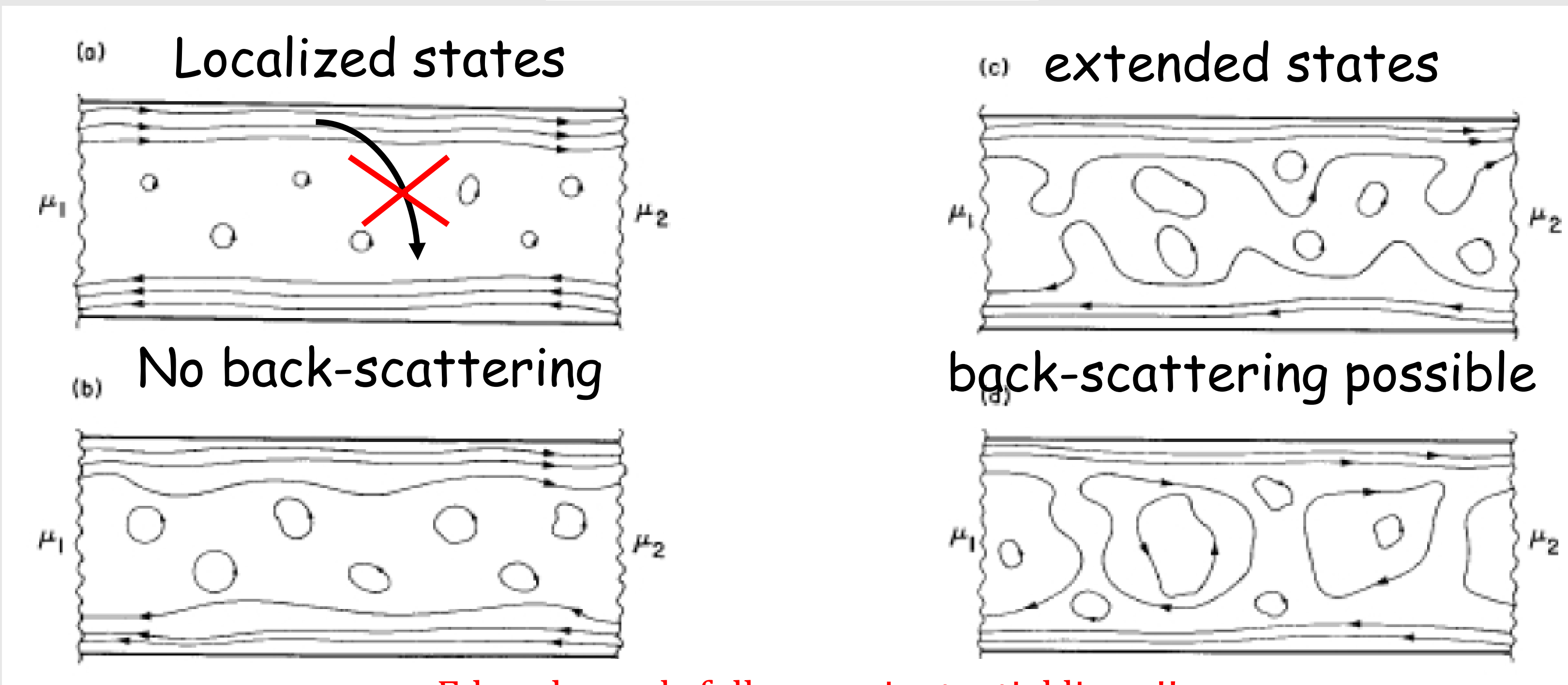
Ballistic

: $R_{XX} = 0$; $R_{XY} = h/3e^2$



Diffusive

$R_{XX} \neq 0$; $h/3e^2 < R_{XY} < h/2e^2$

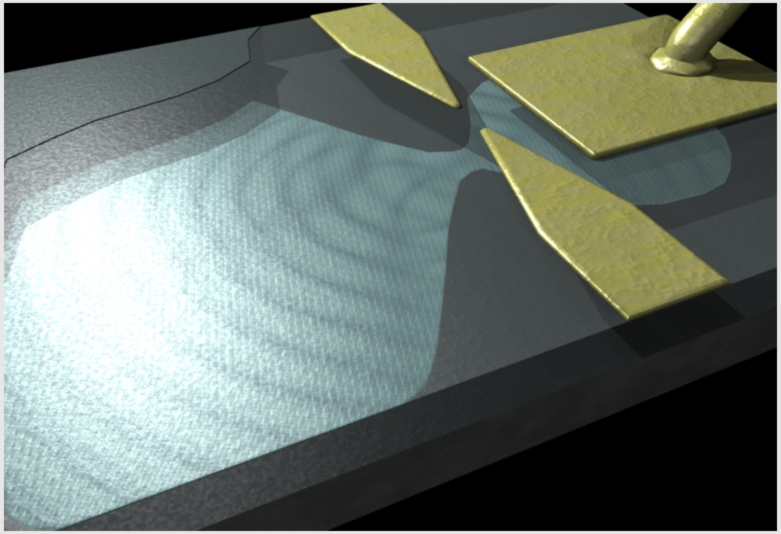


Edge channels follow equipotential lines !!

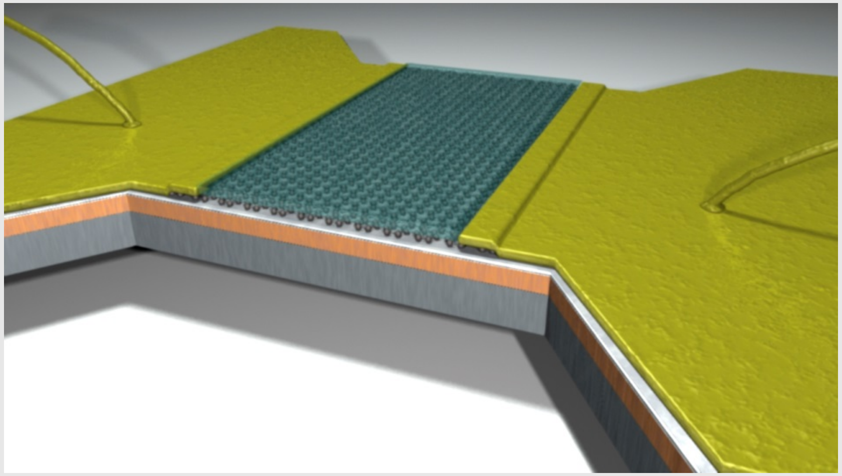
http://www.metas.ch/LesHouches/downloads/talks/22_Buttiker.pdf

Graphene

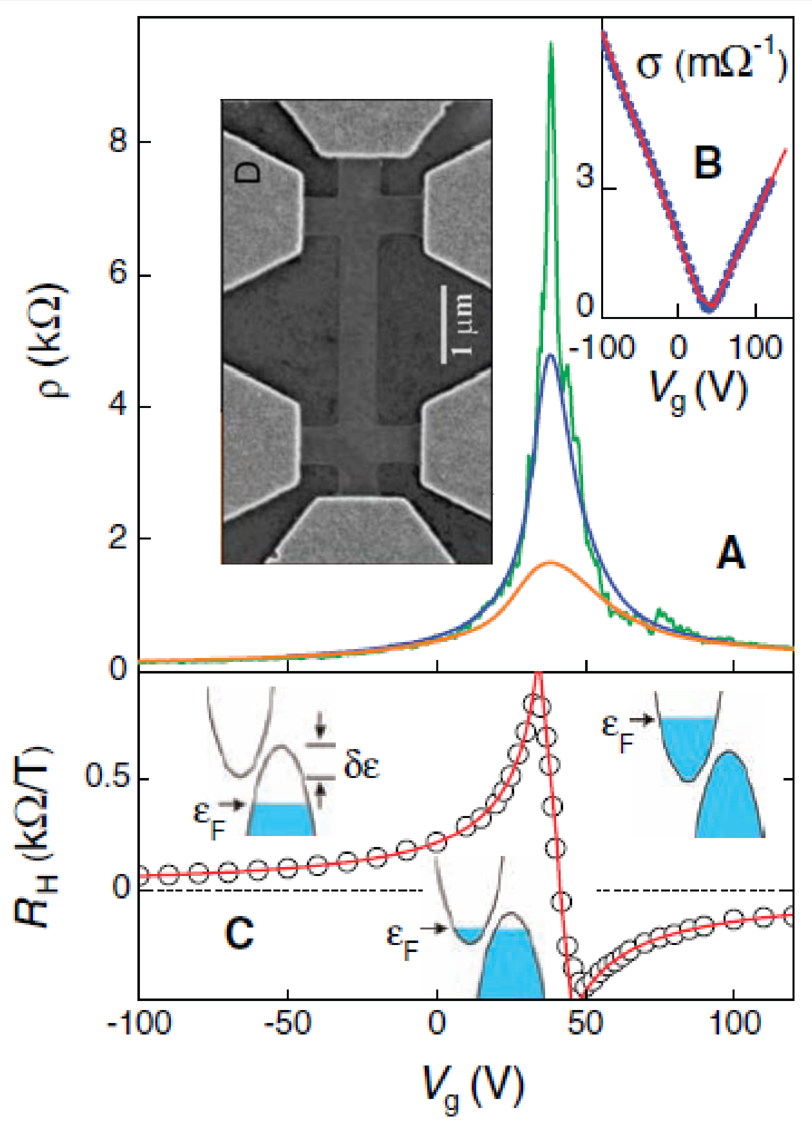
2degs in semiconducting heterostructures

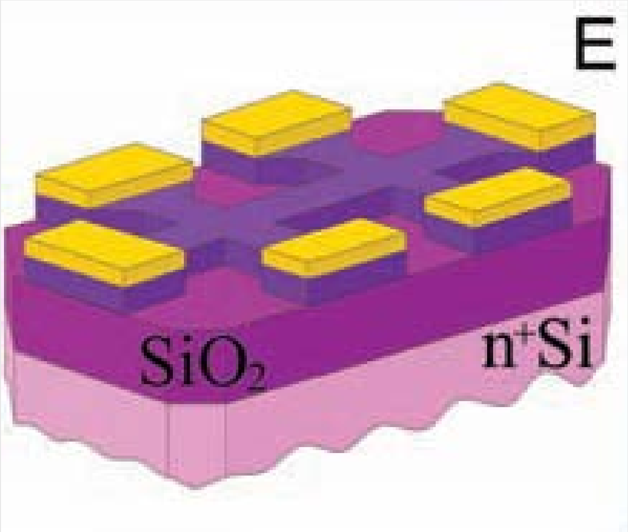


2degs in graphene

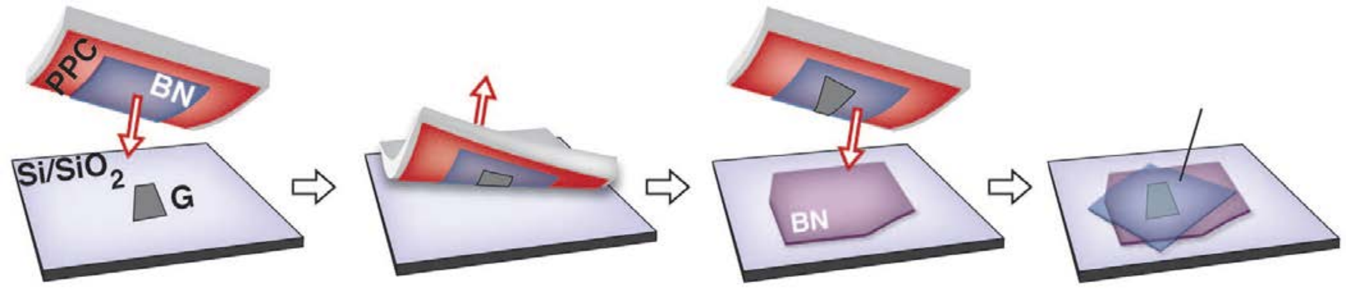


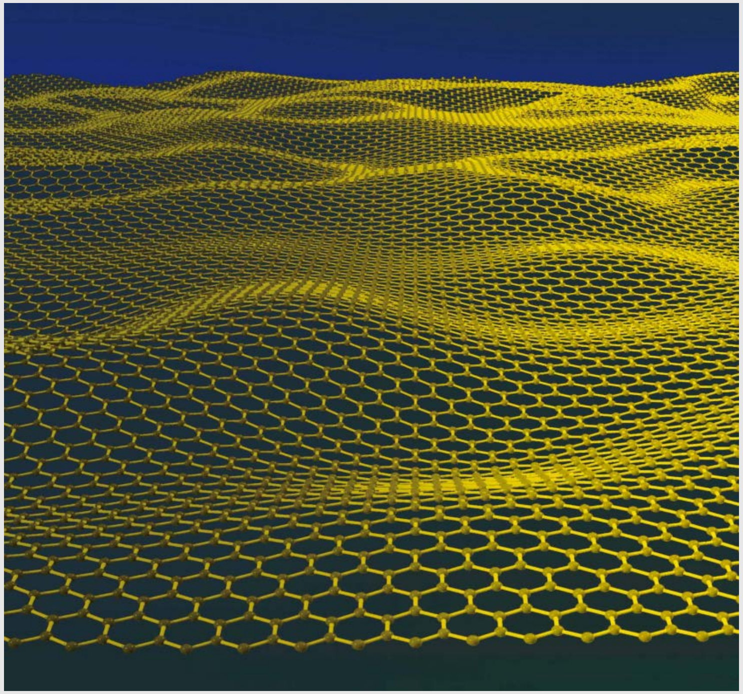
Electrons in 2D graphene
(K. Novoselov et al., Science 2004)

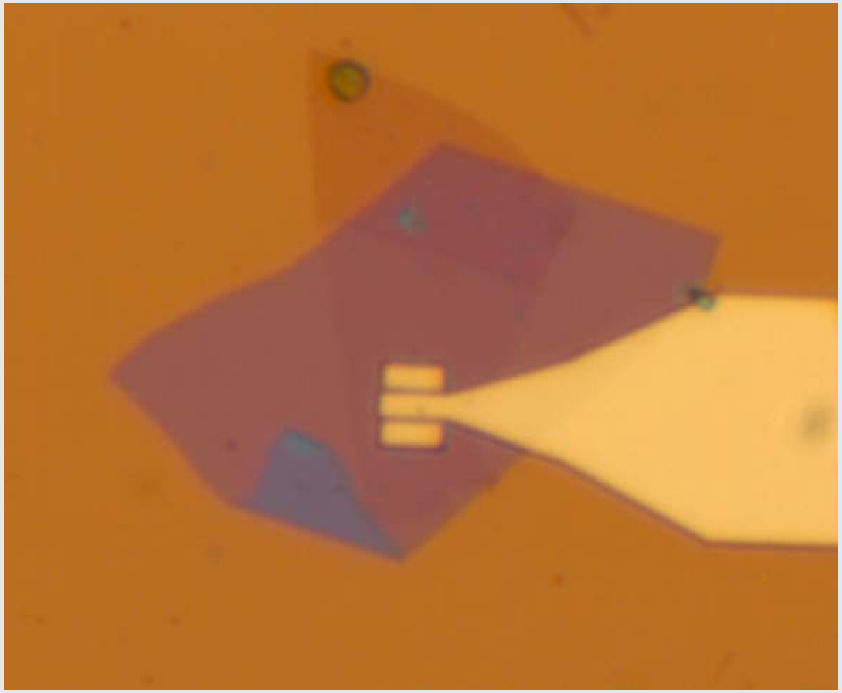




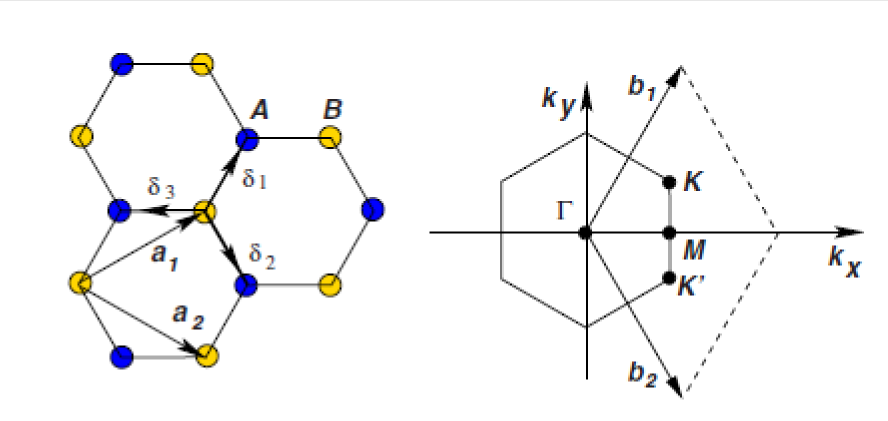
High-mobility graphene

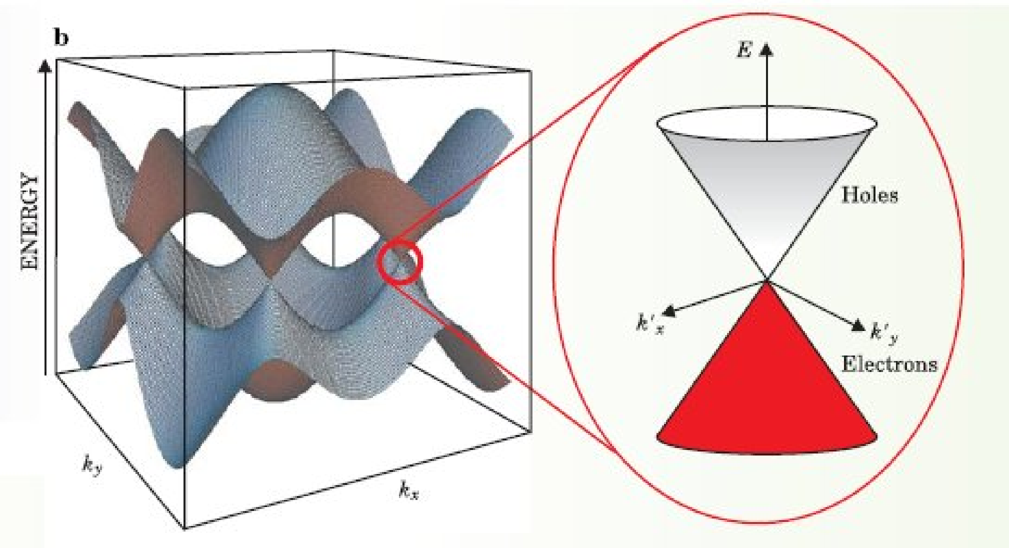




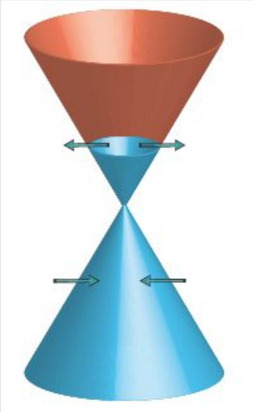


Band structure of Graphene
(tight-binding model, Wallace 1947)





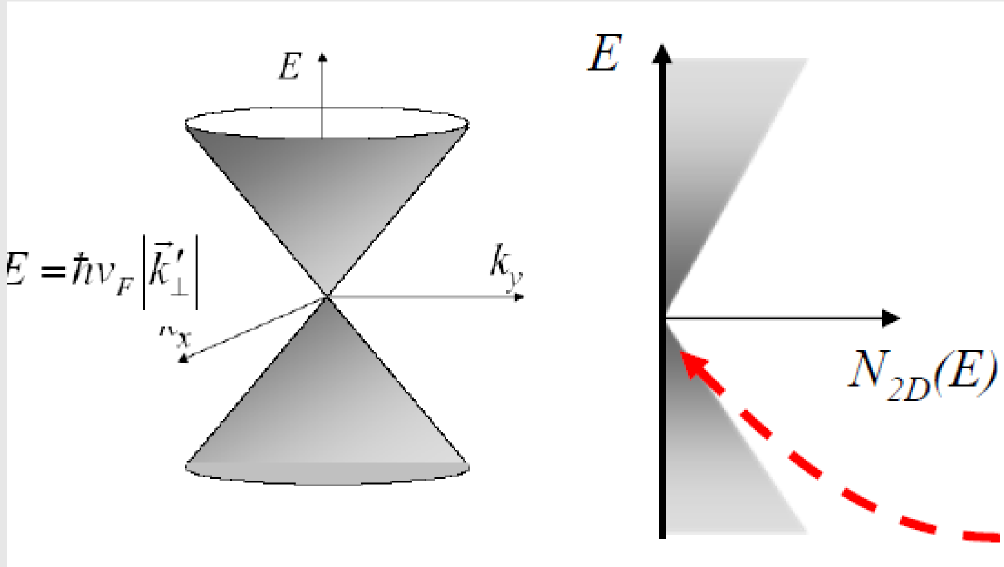
- Dirac electrons : $\varepsilon = \hbar v_F q$ (linear dispersion relation)
- Large Fermi velocity : $v_F = 10^6\, ms^{-1}$
- Effective mass : $m^* = 0$ (no band-gap)
- 2-component spinnor ! : $\psi = e^{iqr} \binom{1}{e^{i\varphi}}$ (chiral transport)



Schrödinger vs Dirac electrons

Dirac electrons :

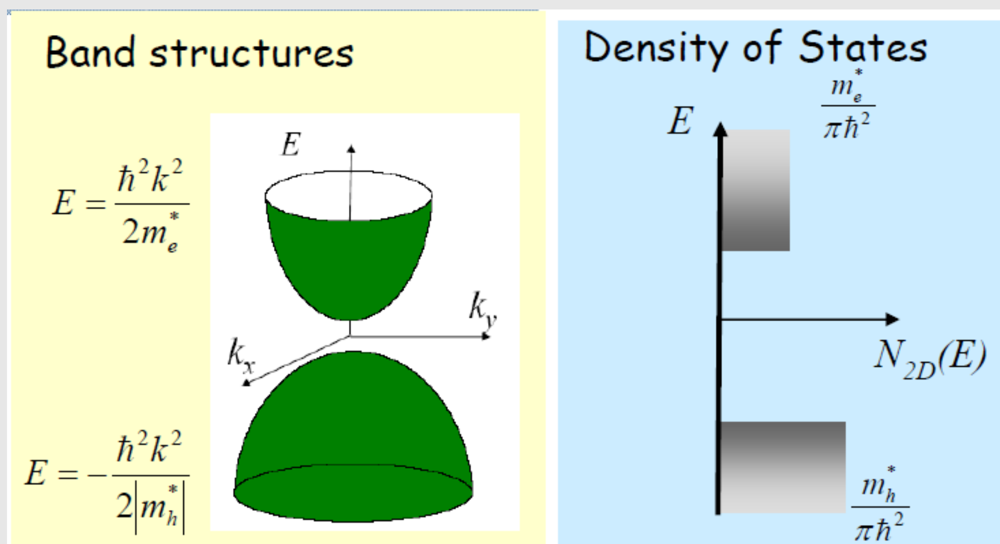
Linear dispersion
Mass-less fermions
Constant Fermi-velocity
Electron/hole symmetry
No band-gap !!
Wave function spinnor



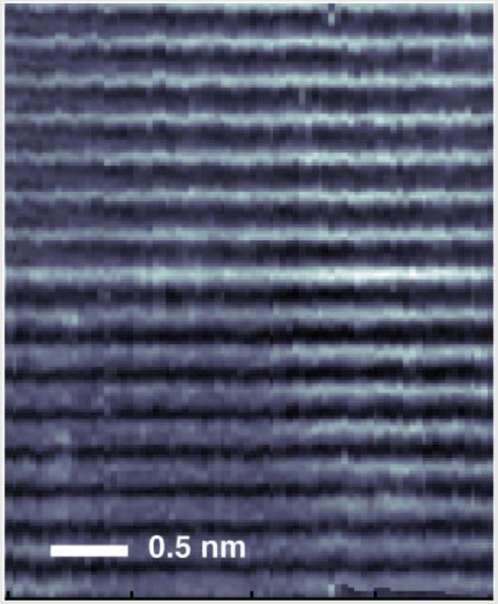
Dirac point

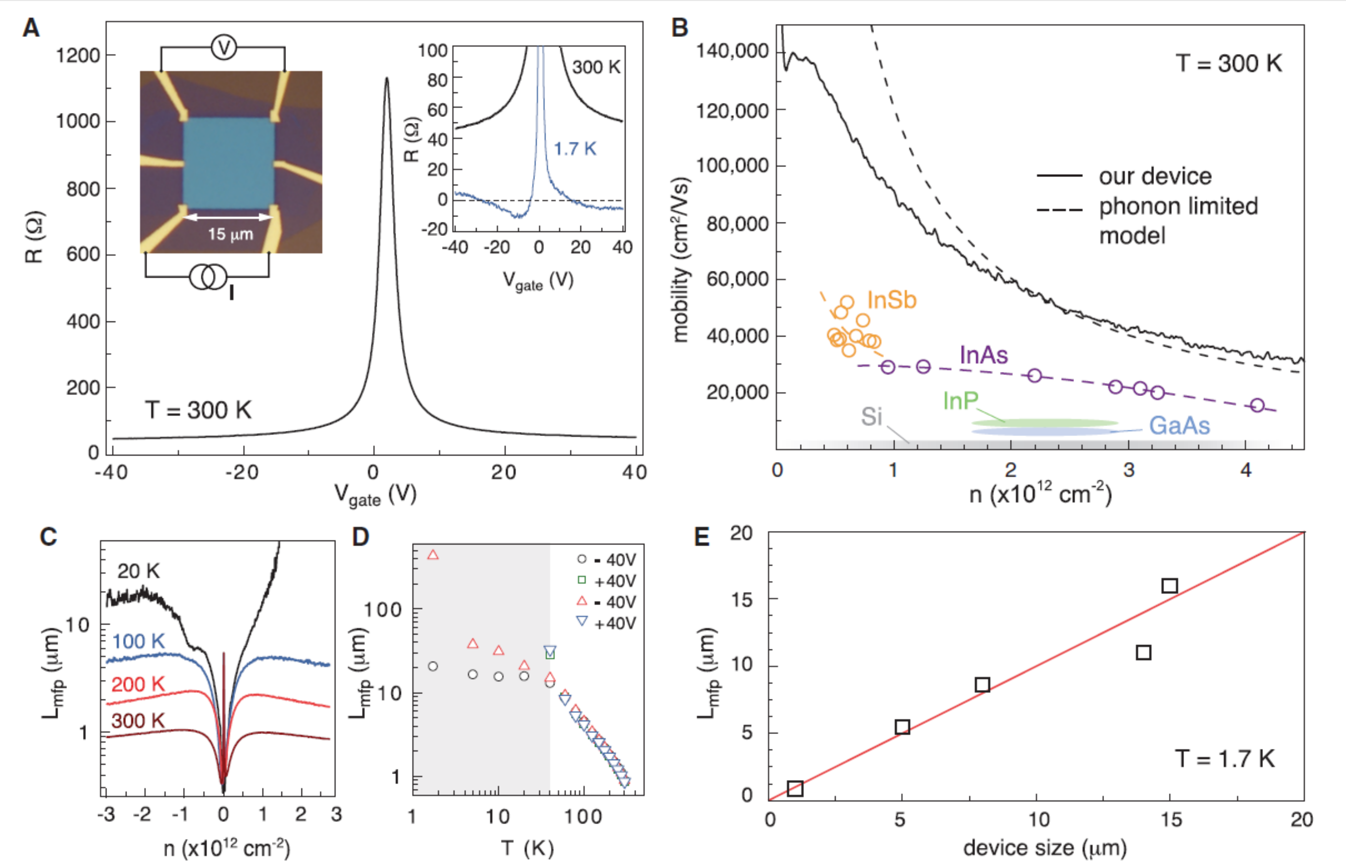
Schrödinger electrons :

parabolic dispersion
Massive fermions
Electrons ≠ holes
band-gap
Wave function scaler



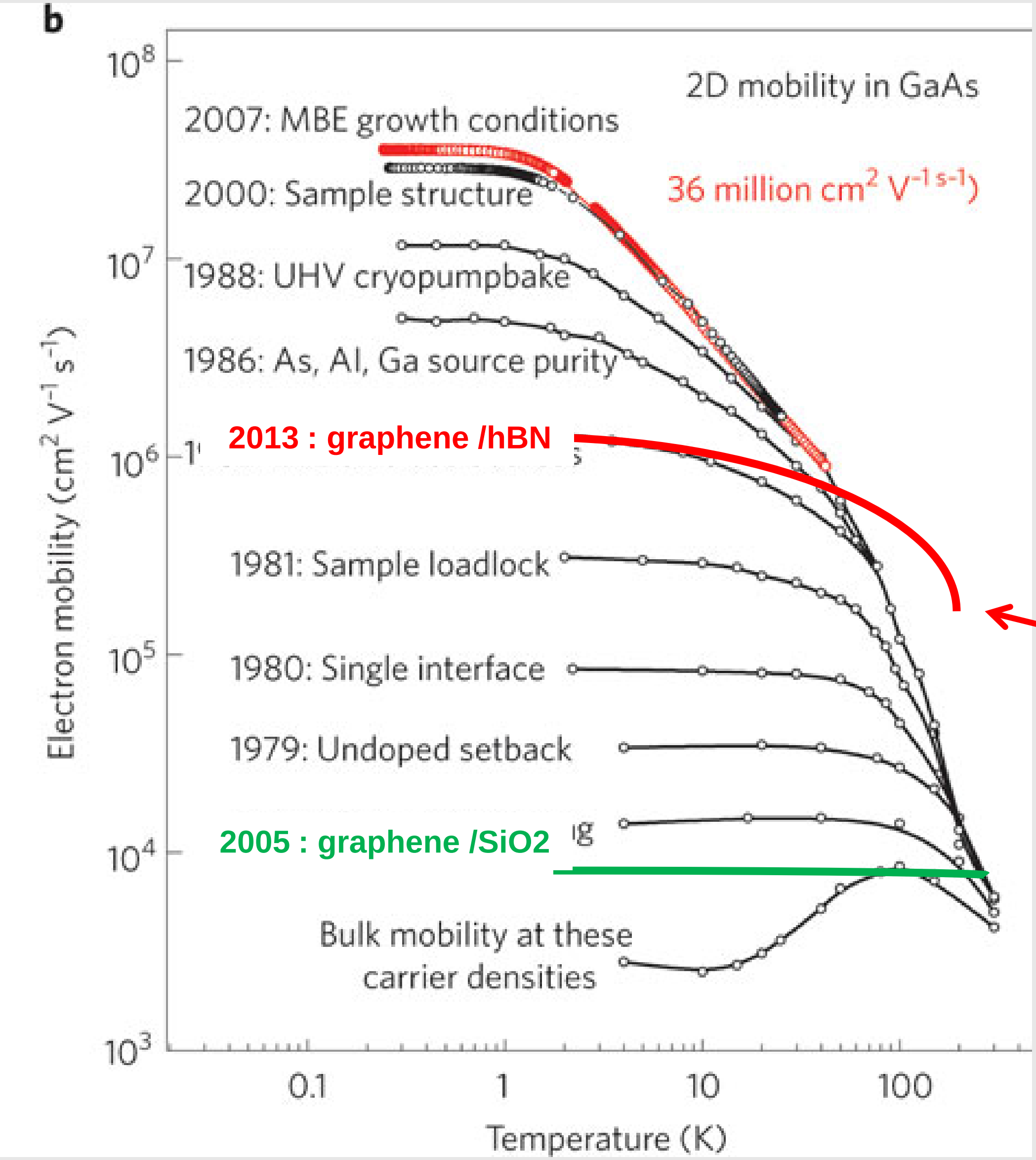
Ballistic graphene





L. Wang et al., Science 342, 614 (2013)

Comparison with AlGaAs/GaAs 2DEGs



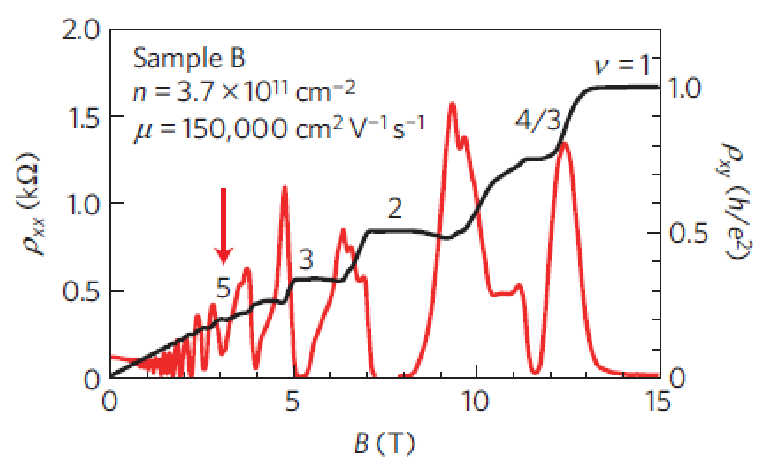
Very weak effect of phonons

Room temperature applications ?

More 2D electron materials

Schrödinger 2d electrons

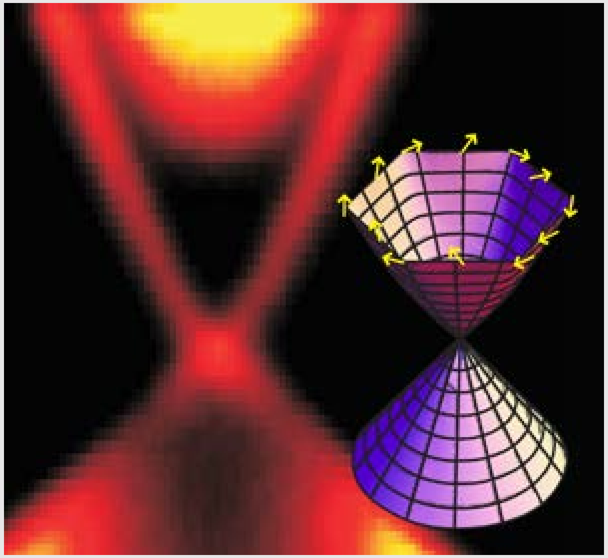
- 1980's : Si MOS-FETs
- 1990's : III-V GaAS, InAs, InP, GaN,
- 2010's : LaAlO3-SrTiO3, ZnO,
- 2005's : Bilayer graphene



A. Tsukazaki , Nat. Mat. 9, 889 (2010)

Dirac 2d electrons

- 2005's : graphene
- 2010's :Topological Insulators, Bi2Se3
- 20XX's : New materials to be discovered .....



B. D. Hsieh, Nature 460, 1101 (2009)

2d electron devices

Part 1 : 2d electron systems

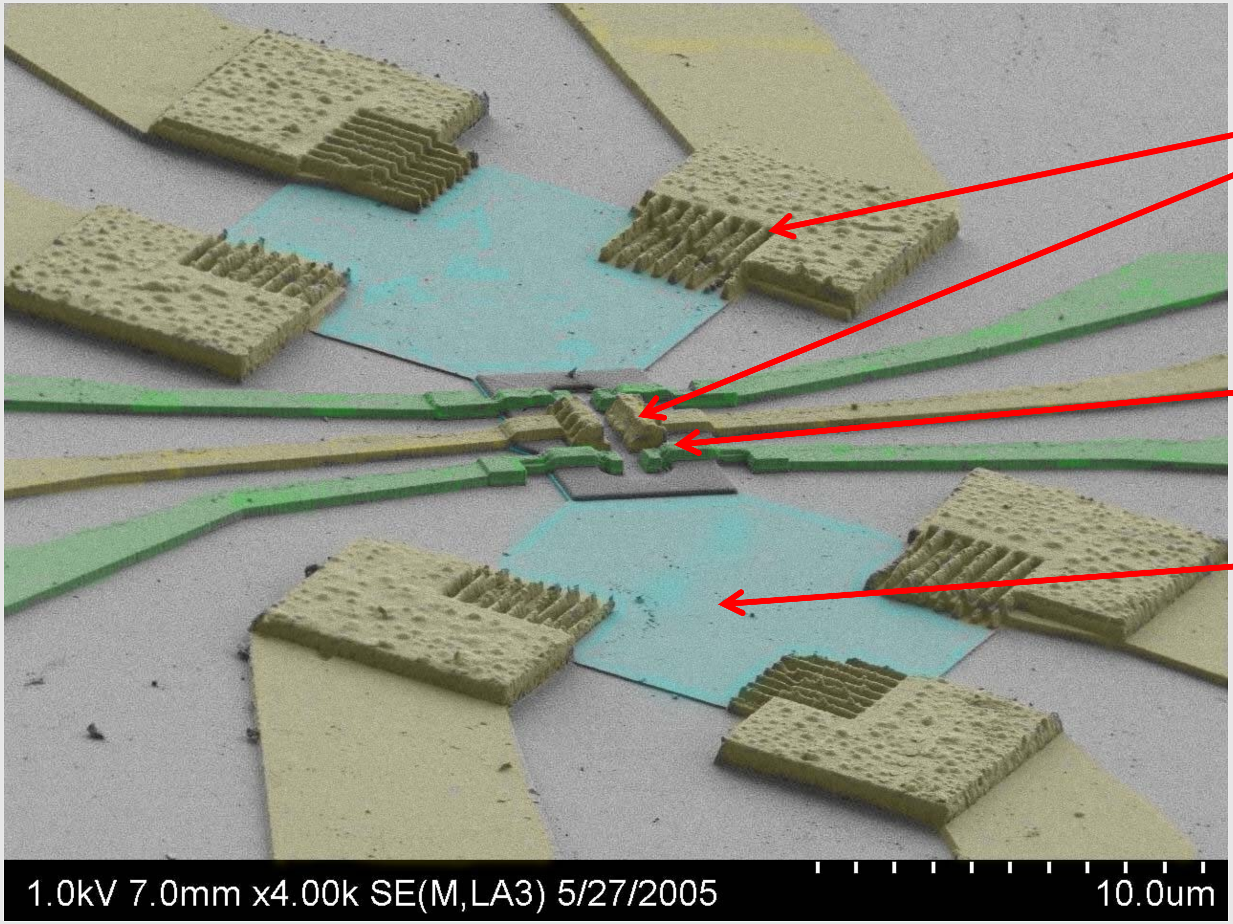
- High-mobility semiconducting heterostructures
- Quantum Hall Effect
- Electrons in graphene

Part 2 : 2d electron devices (GaAs only)

- Electron reservoir and quantum transport
- Scattering by a quantum point contact
- The mesoscopic capacitor

Semiconductor Nanostructures, Thomas Ihn, Oxford University Press (2010)

Example 1 : a Mach-Zehnder interferometer



Contacts
(mustard yellow)

Split gates
(green)

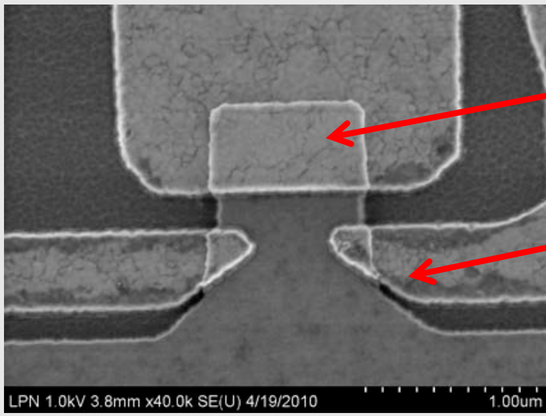
2D electron gas
(blue)

D. Mailly, Laboratory of Photonics and Nanostuctures, CNRS-Marcousssis, France

Example 2 : our Hanbury Brown and Twiss interferometer

Single Electron AC-Source

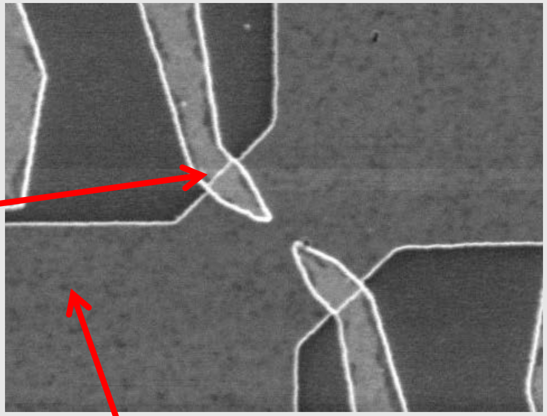
Electron Beam Splitter



SEM image

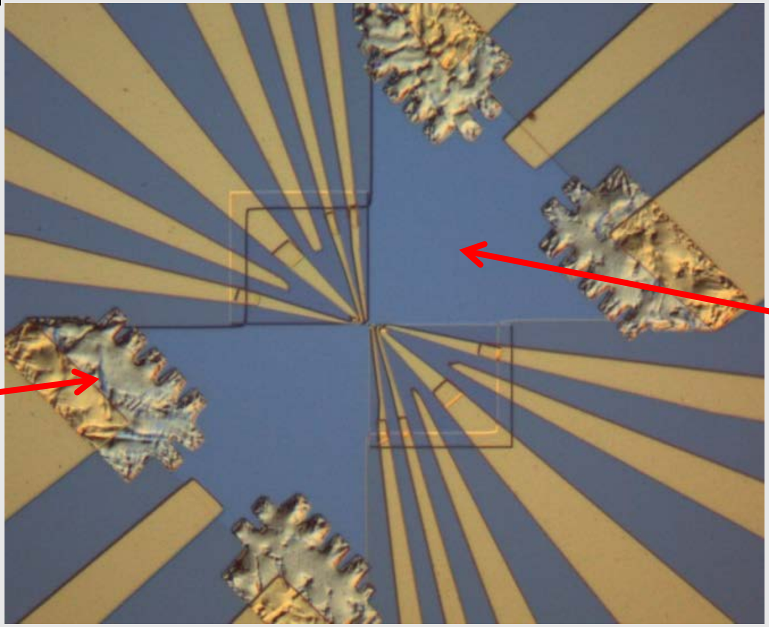
mesoscopic capacitor

split gates of the
Quantum Point Contact



SEM Image

Optical image

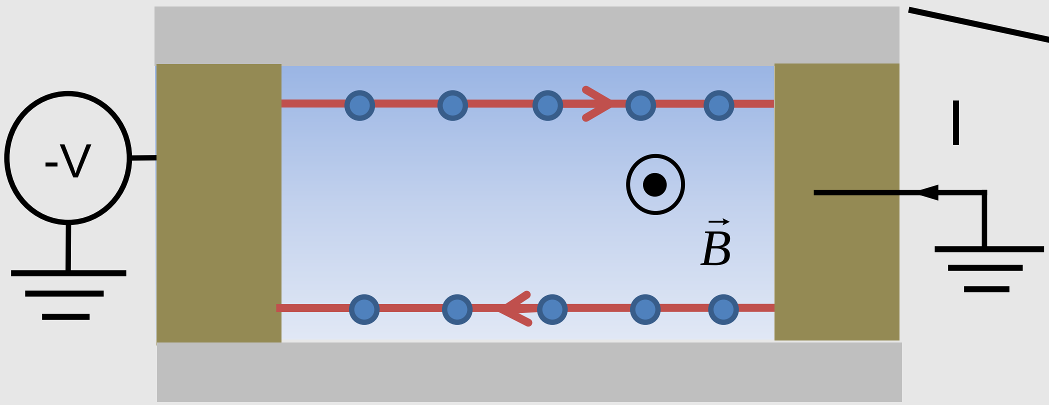


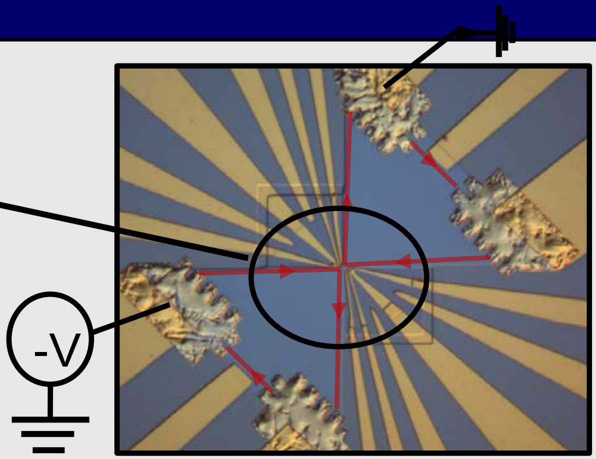
source contact
Direct Current
reservoir

burried 2DEG

Y. Jin, Laboratory of Photonics and Nanostuctures, CNRS-Marcousssis, France

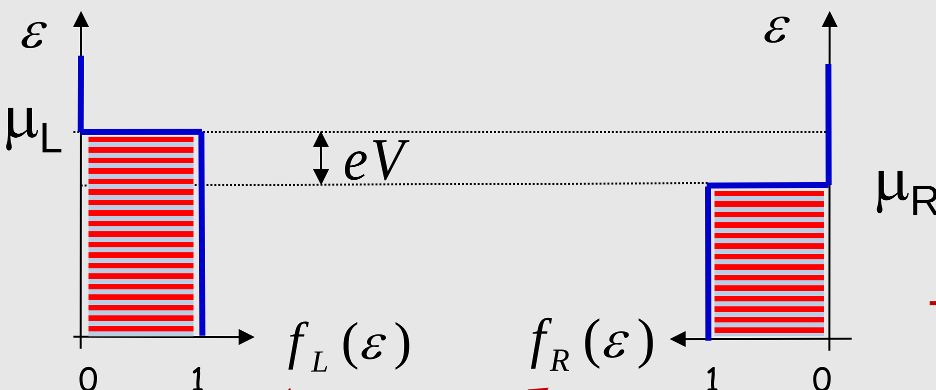
Conductance quantization (one edge channel)





$$I = \frac{e^2}{h} V$$

conductance quantum :
$(e^2/h = 38{,}740459\ \mu S)$



$$S(\omega) = 2\int \langle \Delta I(t)\Delta I(t+\tau) \rangle e^{i\omega\tau} d\tau$$

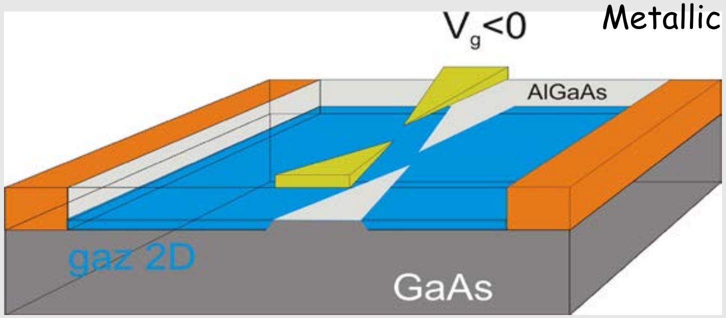
$$\langle \Delta I^2 \rangle = \int_0^{+\infty} S(\omega) d\omega$$

$S(\omega = 0) = 0$ no noise !

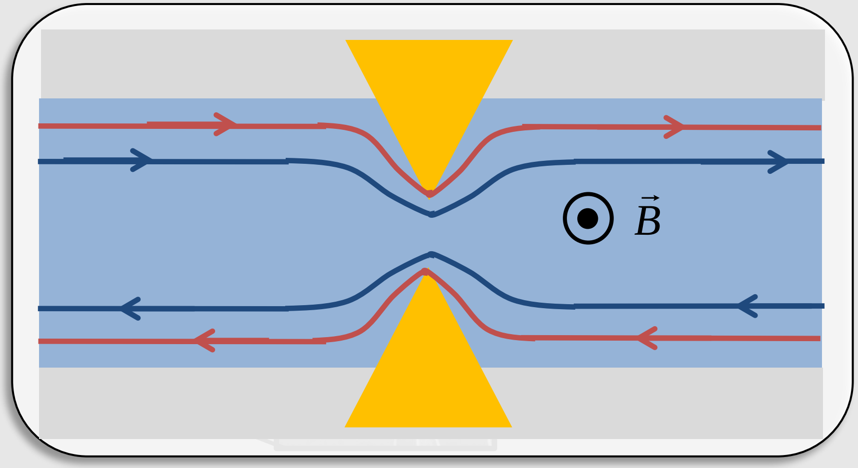
contact ⟺ black body for photons

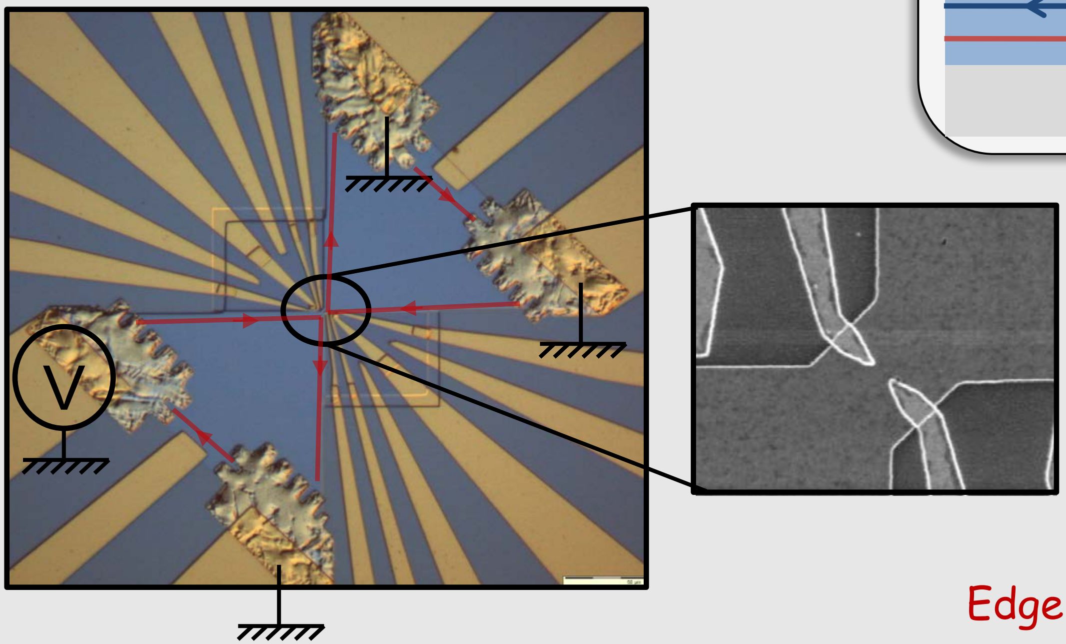
Fermi Dirac distribution

The Quantum Point Contact (QPC)



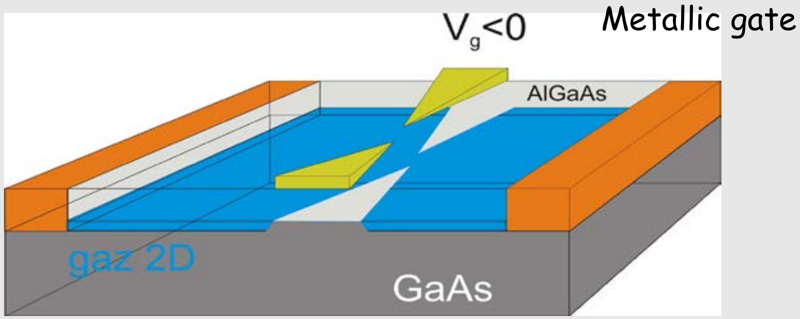
$$V_g = V_{g1}$$

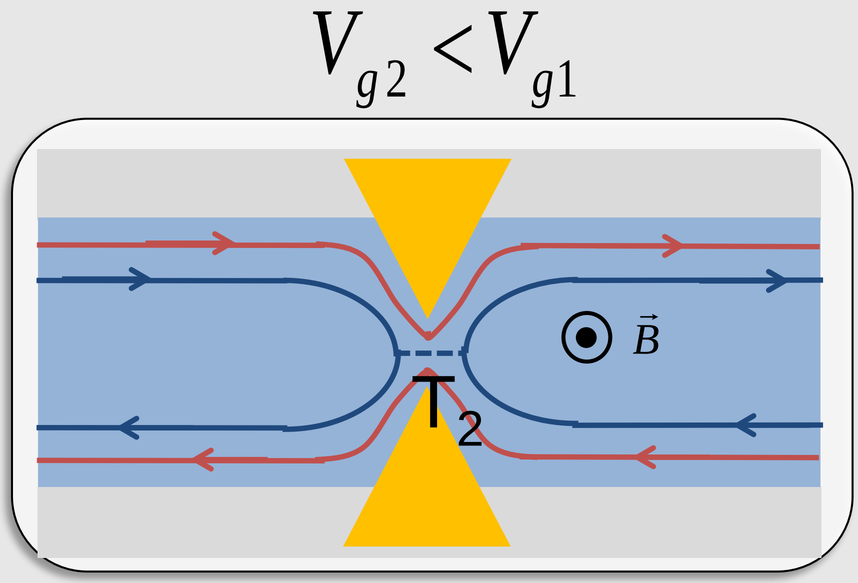


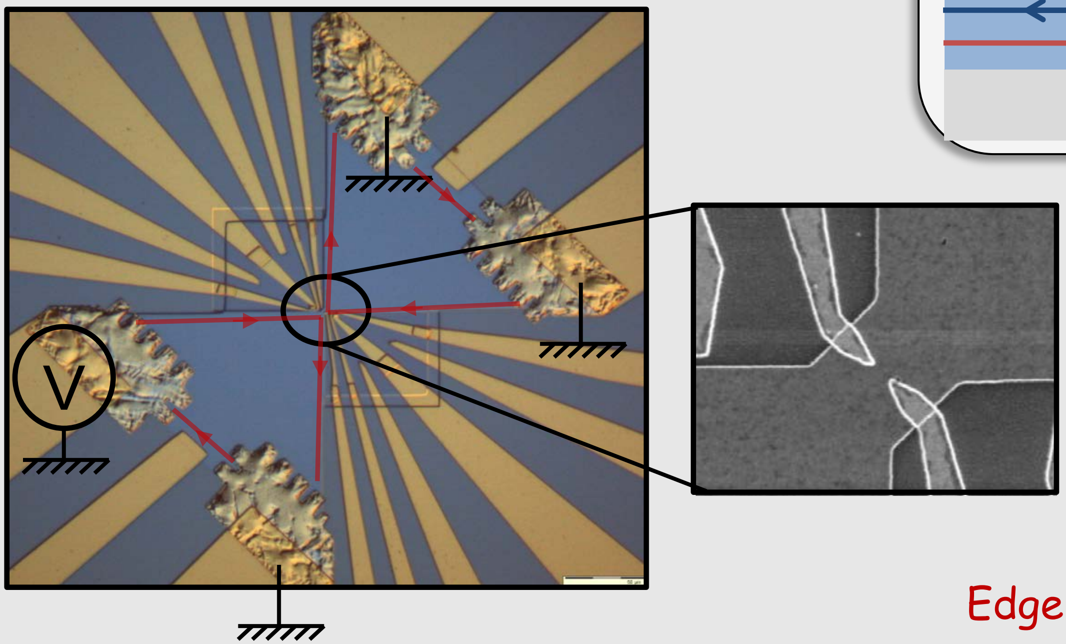


Edge channels follow equipotential lines

The Quantum Point Contact (QPC)

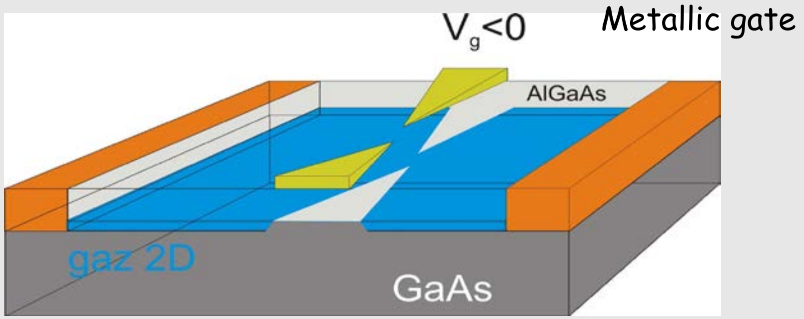




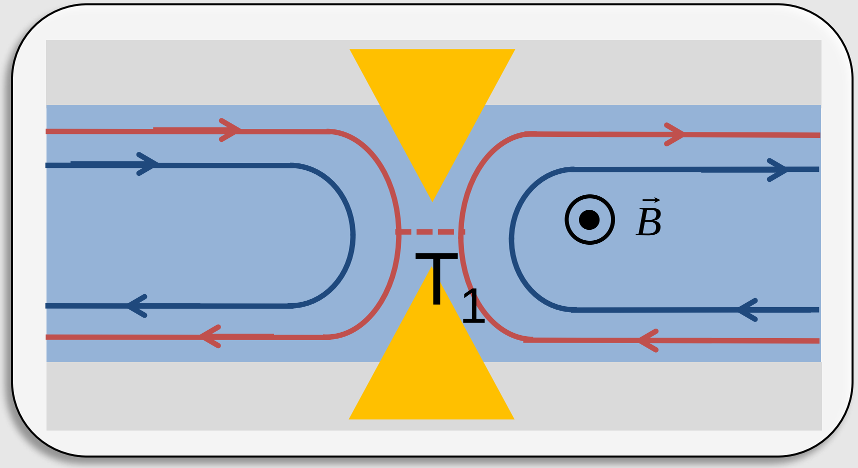


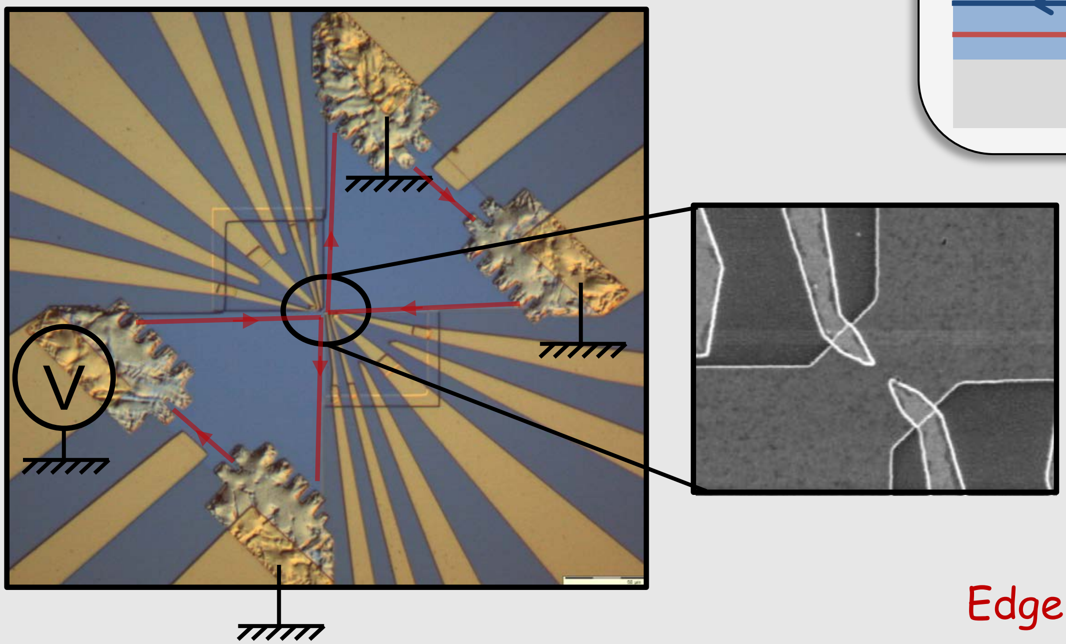
Edge channels follow equipotential lines

The Quantum Point Contact (QPC)



$$V_{g3} < V_{g2} < V_{g1}$$



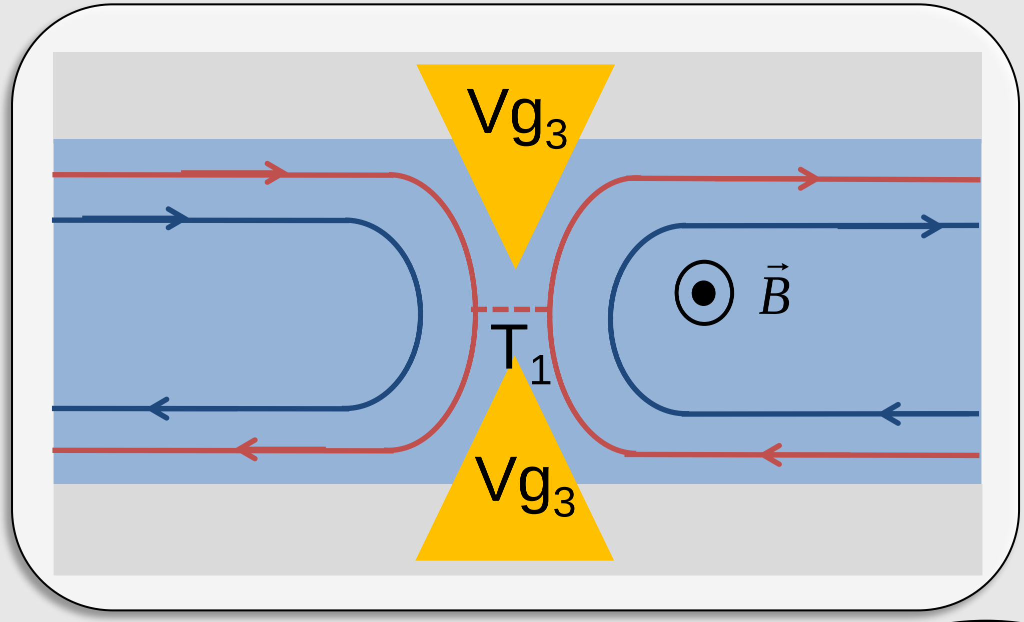


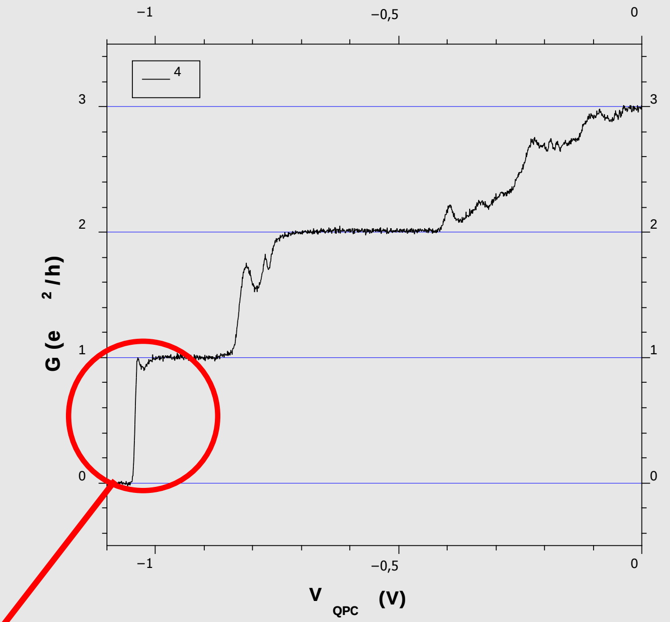
Edge channels follow equipotential lines

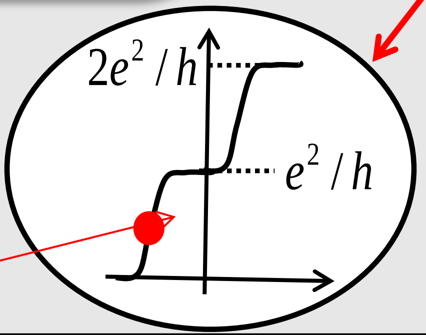
The transmission of a Quantum Point Contact (QPC)

Conductance quantization

$$G = \frac{e^2}{h} \sum_n T_n$$



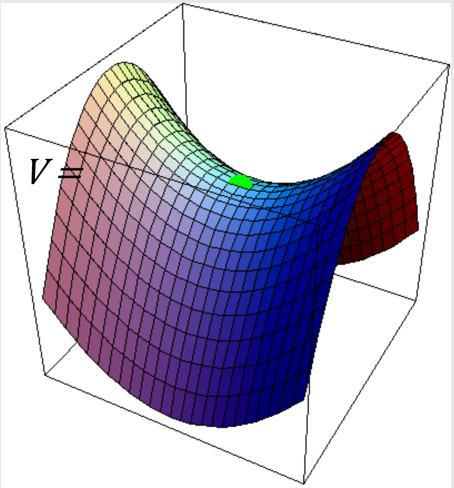




Tunable electronic beam splitter

T_1=1/2

Models of QPC transmission

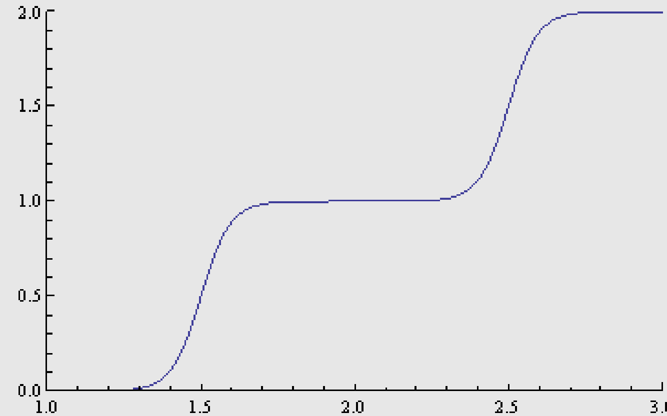


saddle point barrier :

$$V = V_0 - \frac{1}{2}\, m{\omega_x}^2\, x^2 + \frac{1}{2}\, m{\omega_y}^2\, y^2$$

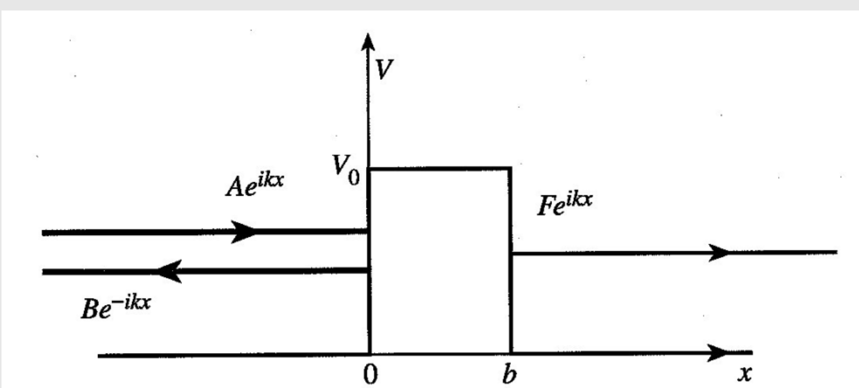
$$T_n = \left[1 + Exp[(\epsilon - \left(n + \tfrac{1}{2}\right)\hbar\omega_y)/\hbar\omega_x]\right]^{-1}$$

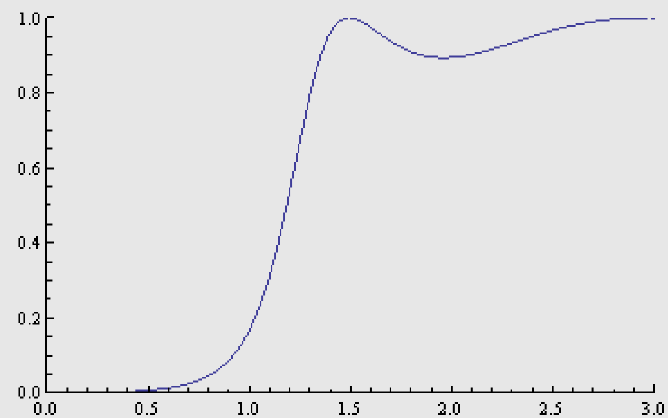
$$T_n = \left[1 + Exp - \pi\left(\omega_y/\omega_x\right)(y/l_B)^2\right]^{-1}$$



M. Büttiker , Phys. Rev. B41, 7906 1990

1D barrier : $T_n = \left[1 + \frac{{V_0}^2}{4E(V_0-\epsilon)} sinh^2\,(\alpha b)\right]^{-1}$





See e.g. http://www2.le.ac.uk/departments/physics/people/mervynroy/lectures/tunnelling-results.pdf

QPC shot noise

conductance is transmission

$$I = \frac{e}{h} \int T(E)[f_L - f_R]dE = \mathrm{T}\,\frac{e^2}{h}\,\mathrm{V}$$

transmission is stochastic => current is noisy

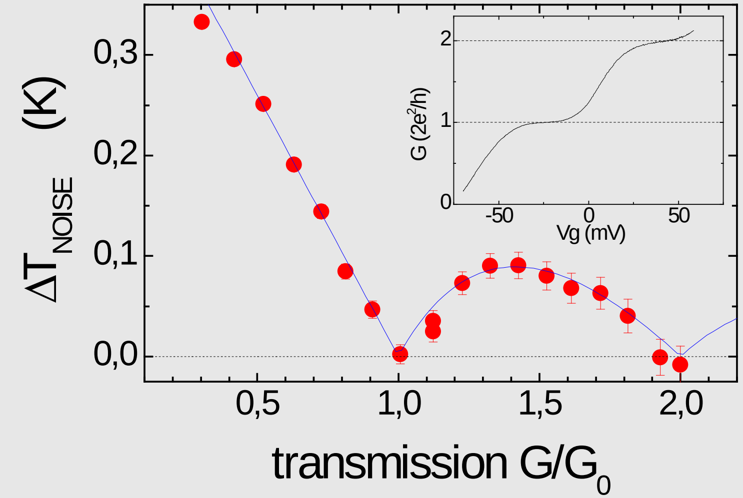
$$S_I = 2\frac{e^2}{h} \int\{T[f_L(1-f_L)] + R[f_R(1-f_R)] + RT(f_L - f_R)^2\}dE$$

equilibrium : temperature θ

$$S_I = 4\frac{e^2}{h}\,T \int k\theta f'(E)dE = 4Gk\theta$$

transport : partition noise

$$S_I = 2eT(1-T)\,\frac{e^2}{h}V = 2eI\,(1-T)$$

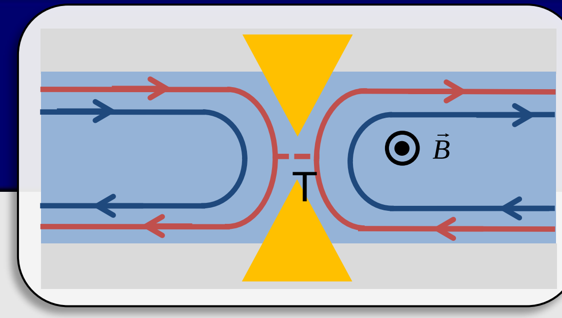


Several modes

$$G = \frac{e^2}{h}\sum_n T_n \quad ; \quad S_I = \frac{2e^2}{h}\sum_n T_n(1-T_n) \quad ; \quad F = \frac{S_I}{2eI} = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}$$

Y. Blanter, M. Buttiker, Phys. Rep. 336, 1 (2000); Glattli's group, e.g. PRL 90, 176803, (2003)

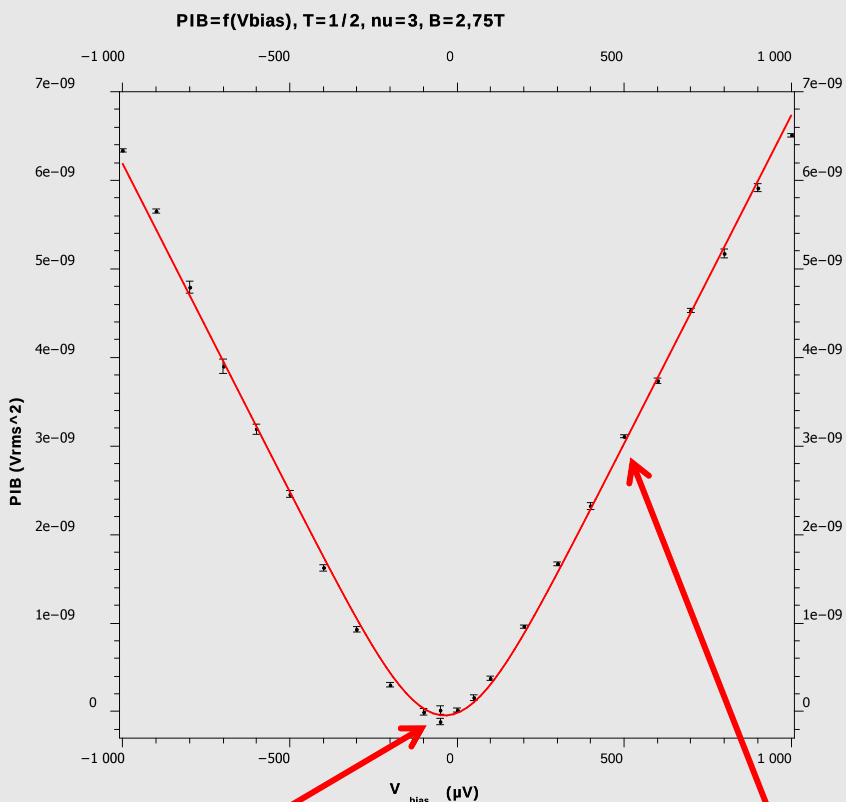
Exemples of QPC noise

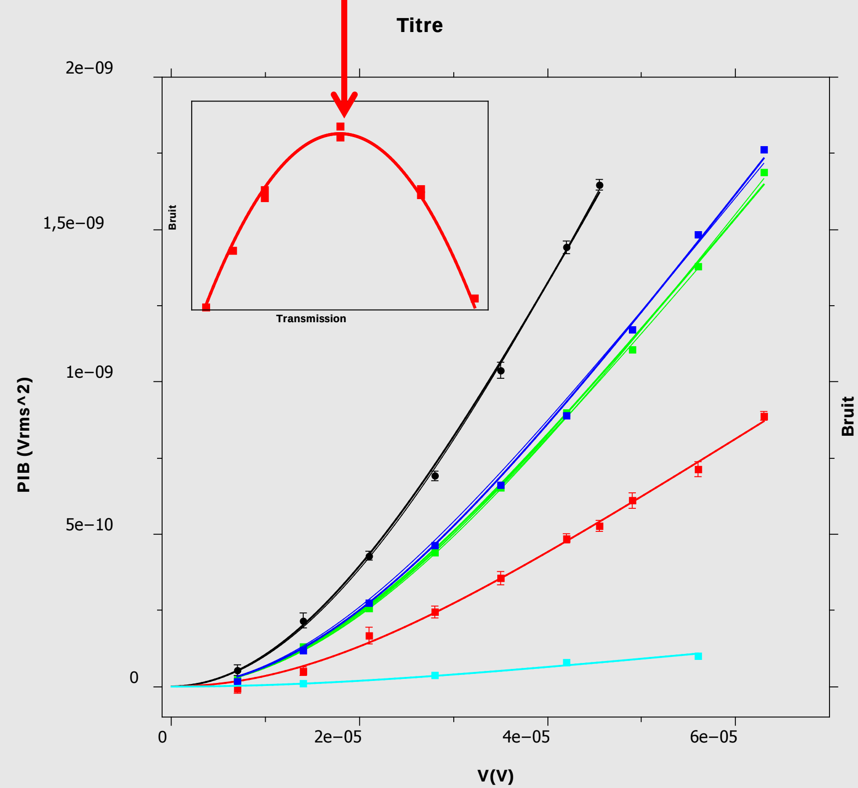


Thermal crossover

$$S_I = 2eI \tanh^{-1} \frac{eV}{2k\theta_e}$$

partition noise : $S_I \propto T(1-T)$

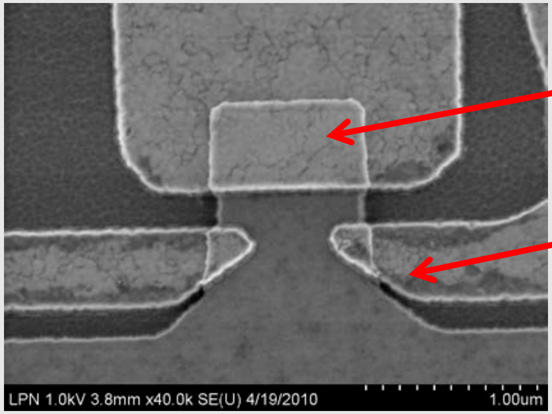




Thermal noise

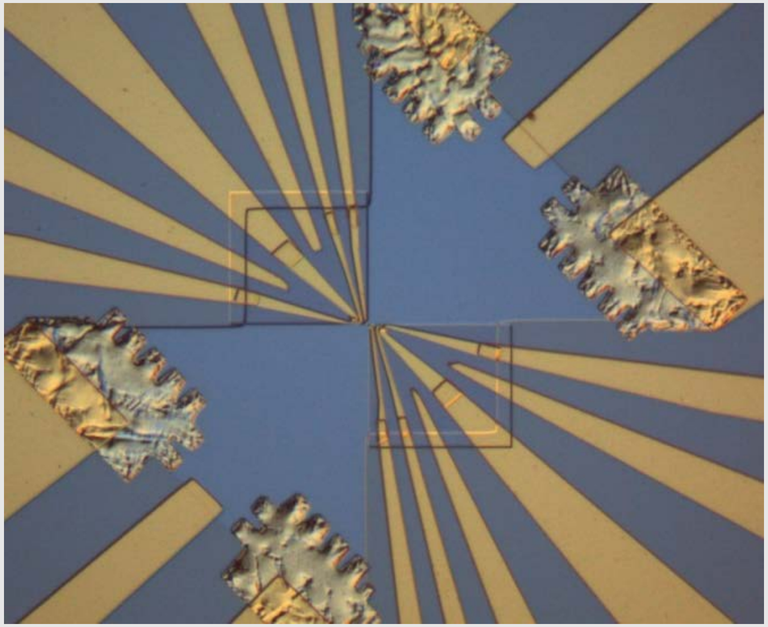
partition noise

Single Electron Source

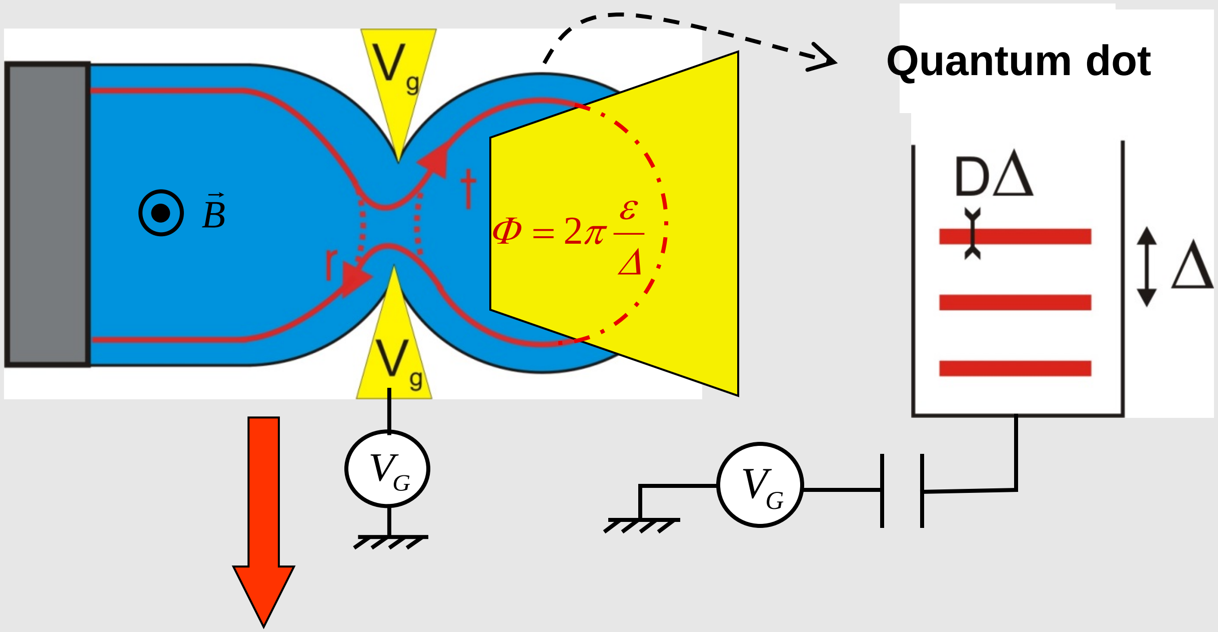


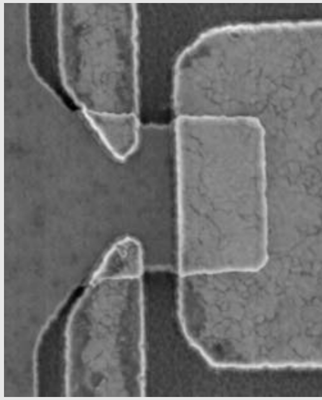
mesoscopic capacitor

split gates of the Quantum Point Contact



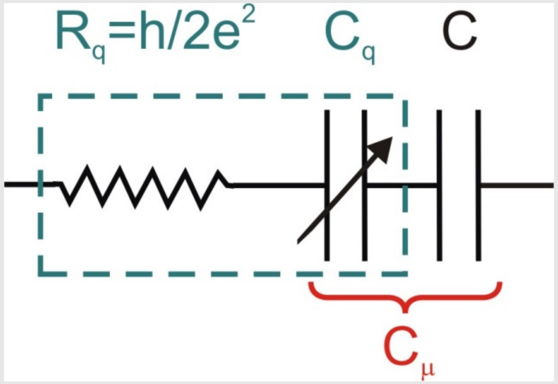
The Mesoscopic capacitor





Numbers

$\Delta \approx 2\,K; \quad D = 0 \rightarrow 1$



Quantum capacitance effect !

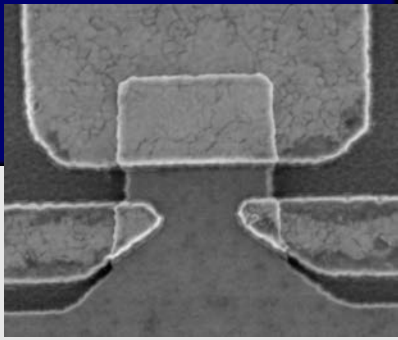
$$C_q = e^2 \rho(\varepsilon_F)$$

Quantization of the charge relaxation resistance

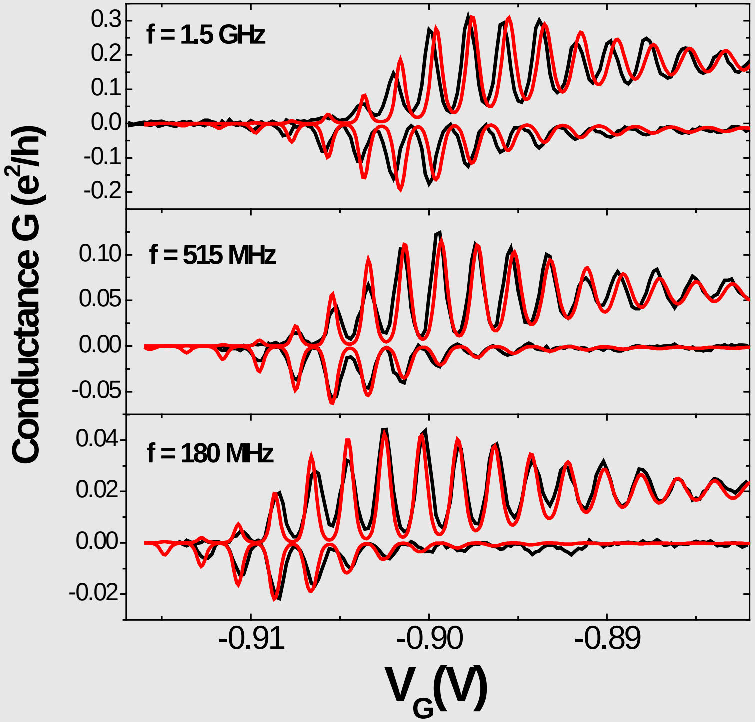
$$R_q = \frac{h}{2e^2}$$

Christen-Büttiker PRL 70, 4114 (1993)
Gabelli et al. Science 313, 499 (2006)

The charge relaxation resistance

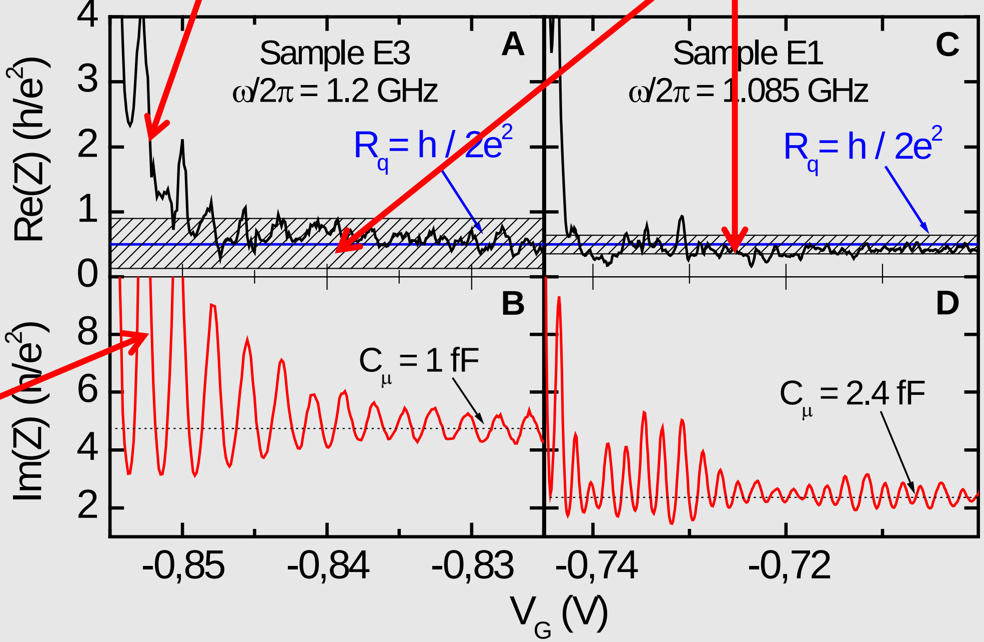


Raw admittance measurements



$R_q = \frac{h}{De^2}$ (incoherent)

$R_q = \frac{h}{2e^2}$ (coherent)



$\rho(\varepsilon_F)$ oscillations in the dot

Gabelli et al., Science 313, 499 (2006)
Gabelli et al., Rep. Prog. Phys. 75, 126504 (2012)

Next time :quantum optics with edge states

Part 3 : Introduction to electrons optics

- Electron optics principles
- Single electron sources
- The mesoscopic capacitor source : average current and noise

Part 4 : Quantum optics experiments

- Partitioning single electrons
- Colliding indistinguishable electrons
- Decoherence and charge fractionalization

Electron quantum optics in ballistic chiral conductors,
Bocquillon et al., Annalen der Physik 526, 1 (2014)