

# L'ancrage des vortex

## vu par la réponse linéaire

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### 1. Introduction

- systèmes élastiques et désordre
- corrélations spatiales (ligne de contact)
- réponse linéaire (onde de densité de charge, réseaux de vortex)

### 2. Ancrage de surface

- fréquence d'ancrage
- longueur de glissement
- ancrage collectif faible de surface

### 3. Vortex dans UPt<sub>3</sub>

- Les vortex en symétrie  $E_{2u}$
- La dissipation en limite propre
- L'observation de vortex continus

y(IPMC-Grenoble, 26 Avril 2001)

## Système élastique et désordre (GDR 2284)

	dynamique	statique	susceptibilité AC
MAGNETIC DOMAIN WALLS (MDW-1D) (Lemerle et al., Images Phys. 00')	<p>Graph of velocity <math>V</math> (in <math>\mu\text{m/s}</math>) versus magnetic field <math>H</math> (in kOe). The plot shows two distinct regions: "régime thermodynamiquement actif" (active thermodynamic regime) at low fields and "régime visqueux" (viscous regime) at higher fields. A critical field <math>H_c</math> is indicated where the velocity begins to increase.</p>	<p>Graph of width <math>W(L)</math> (in <math>\mu\text{m}</math>) versus length <math>L</math> (in <math>\mu\text{m}</math>). The data points show a peak around <math>L \approx 20 \mu\text{m}</math>.</p>	
MENNICHUS CONTACT LINE (CL-1/2D) (Rolley et al., Images Phys. 00')		<p>Graph of width <math>W(L)</math> (in <math>\mu\text{m}</math>) versus length <math>L</math> (in <math>\mu\text{m}</math>). Multiple curves are shown for various temperatures: 30 K, 90 K, 89 K, 72 K, 11 K, and 0.67 K. The curves shift to the right as temperature increases.</p>	
CHARGE DENSITY WAVES (CDW-1/2D) (Grüner, RMP 88')	<p>Graph of normalized current <math>I/I_0</math> versus electric field <math>E</math> (in V/cm). The plot shows a hysteresis loop with a plateau at <math>E = 10 \text{ V/cm}</math>. Parameters: <math>T = 130 \text{ K}</math>, <math>I_0 = 10^{-10} \text{ A}</math>.</p>		<p>Graph of dielectric constant <math>\epsilon'</math> versus <math>\log(\omega/2\pi)</math>. Multiple curves are shown for temperatures 64 K, 10 K, and 98 K. The curves exhibit a peak at low frequencies and then decrease.</p>
WIGNER CRYSTAL (WC-2D) (Williams et al., Images Phys. 92')	<p>Graph of DC current <math>i</math> (in nA) versus voltage. The plot shows a non-linear relationship with a sharp peak at zero voltage. Parameters: <math>n_s = 9.05 \times 10^{16} \text{ cm}^{-2}</math>, <math>B = 25.5 \text{ T}</math>.</p>		<p>Graph of real part of conductivity <math>\text{Re}(\sigma_{xx})</math> (in <math>\mu\text{S}</math>) versus frequency <math>f</math> (in GHz). Multiple curves are shown for different magnetic fields <math>B</math> (in T): 5 (0.325), 7.42 (0.297), 8.81 (0.250), 11.59 (0.190), and 14.37 (0.153).</p>
QUANTIZED VORTEX LINES (VL-2/3D) (Blatter et al., RMP 94')	<p>Graph of energy <math>U</math> versus current <math>I</math> (in A). The plot shows a series of curves with a minimum at <math>I = 0</math> and a maximum at <math>I = I_c</math>.</p>	<p>Micrograph showing a regular hexagonal lattice of quantized vortex lines in a superconductor.</p>	<p>Graph of ac penetration depth <math>\lambda_{ac}</math> (in <math>\mu\text{m}</math>) versus normalized frequency <math>\Omega/2\pi</math> (in Hz). The plot shows a characteristic dip at <math>\Omega/2\pi = \Delta/\hbar</math> (B_0 = 0.25 T, T = 4.2 K).</p>

Mais aussi . . . / . . . histérésis, relaxation, bruit, . . .

## Corrélations spatiales

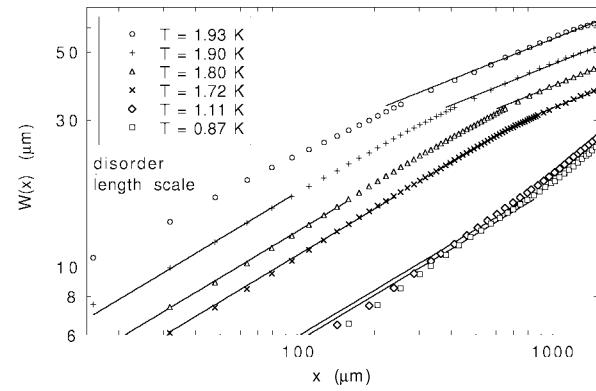
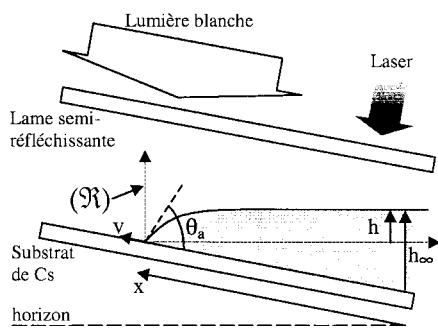
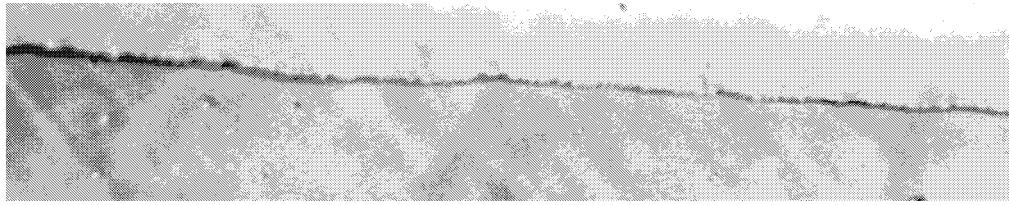
..... distorsions statiques  $U(r)$ , domaine de Larkin .....

$$F = C (U/L)^2 L^D - \varepsilon_p (U/\xi)^2 L^{D/2} \quad \text{élasticité locale}$$

$$W(L) = <(U(x+L) - U(x))^2> \sim L^\alpha \quad (L \gg L_C)$$

paroi de domaine (D=1):  $\alpha = 2/3$

..... ligne de Contact (D=1, non-local) .....



$$F = (\sigma \sin^2 \theta) U^2 - \varepsilon_p (U/\xi)^2 L^{1/2} \quad W \sim L^{1/2} \rightarrow L^{1/3} \quad (L_C \sim 10 \mu m)$$

$$F = C (U/L)^2 L^{D+1} - \varepsilon_p (U/\xi)^2 L^{D/2} \quad \text{élasticité non-locale}$$

**ancrage en surface (D=2, non-local):  $L_C \rightarrow \infty !!!$**

Susceptibilité linéaire :  $U(\mathbf{r}) + u(k,\omega)$

“approche classique”

.... permittivité (isolants : CDW, WC ) ....

$$qE = m\ddot{u} + m\dot{u}/\tau + Ku$$

$$\sigma = q\dot{u}/nE = i\omega\epsilon \quad ; \quad \epsilon = \epsilon_0/(1 + i\omega\tau - \omega^2/\omega_0^2)$$

$$\text{énergie } \epsilon_0 E^2/2 \quad (\epsilon_0 = \frac{nq^2}{K} \sim 10^6\text{-}10^7 \epsilon_0)$$

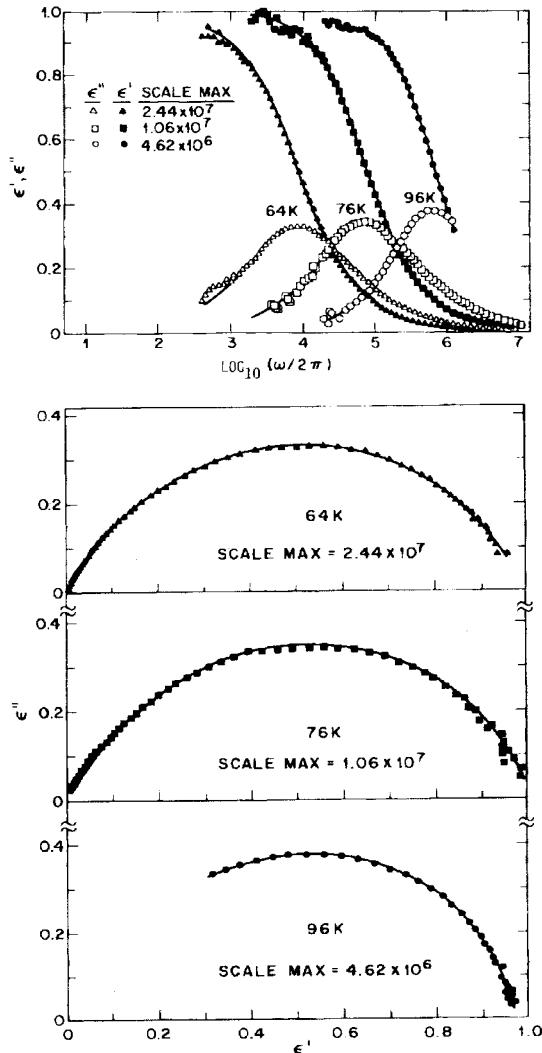
.. inductivité (Vortex supraconducteurs) ..

$$\mathbf{J} \times \boldsymbol{\varphi}_0 = \eta\dot{u} + Ku + \dots$$

$$\rho = n\varphi_0\dot{u}/J = i\omega\mu_0\lambda^2 \quad ; \quad \lambda^2 = \lambda_0^2/(1 + i\omega\tau)$$

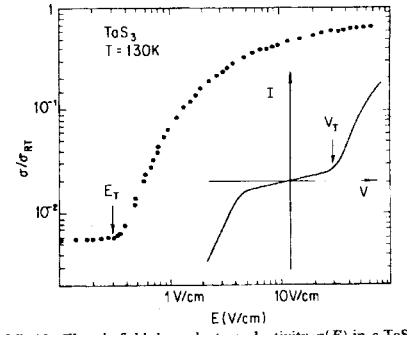
$$\text{énergie } \mu_0\lambda_0^2 J^2/2 \quad (\lambda_0^2 = \frac{n\varphi_0^2}{K} \sim 10^2\text{-}10^6 \lambda_L^2)$$

## relaxation vitreuse (CDW)

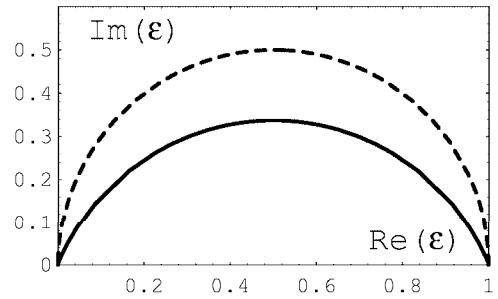


Cava et al. PRB 84:

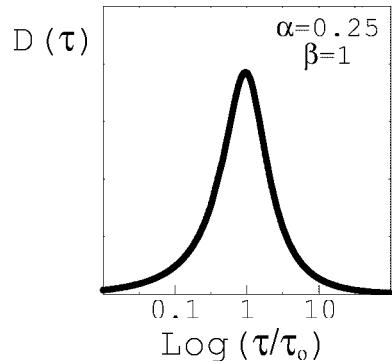
$$\epsilon = \frac{\epsilon_0}{(1 + (i\omega\tau)^{1-\alpha})^\beta}$$



Cole-Cole plot :



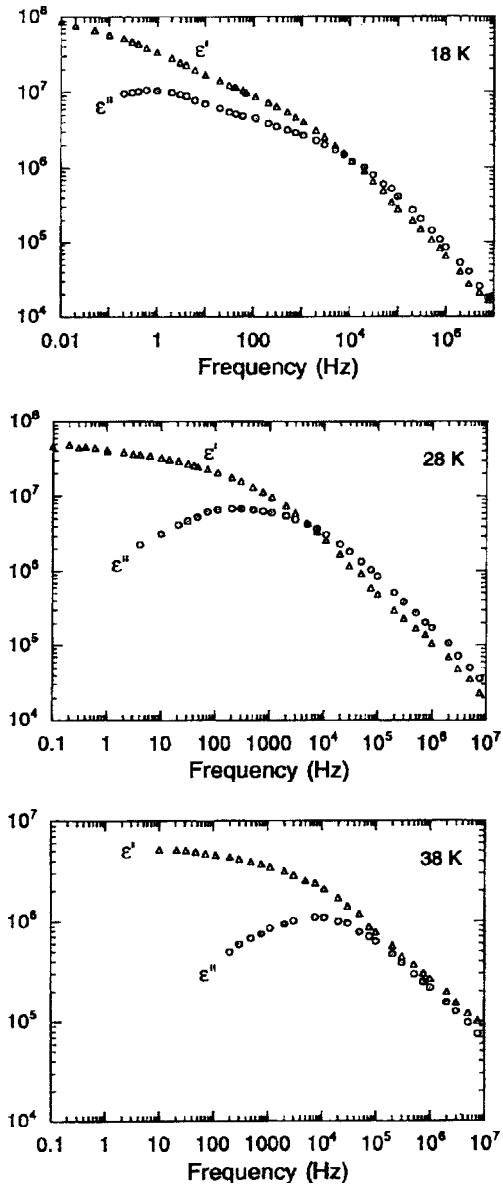
distribution Lognormale :



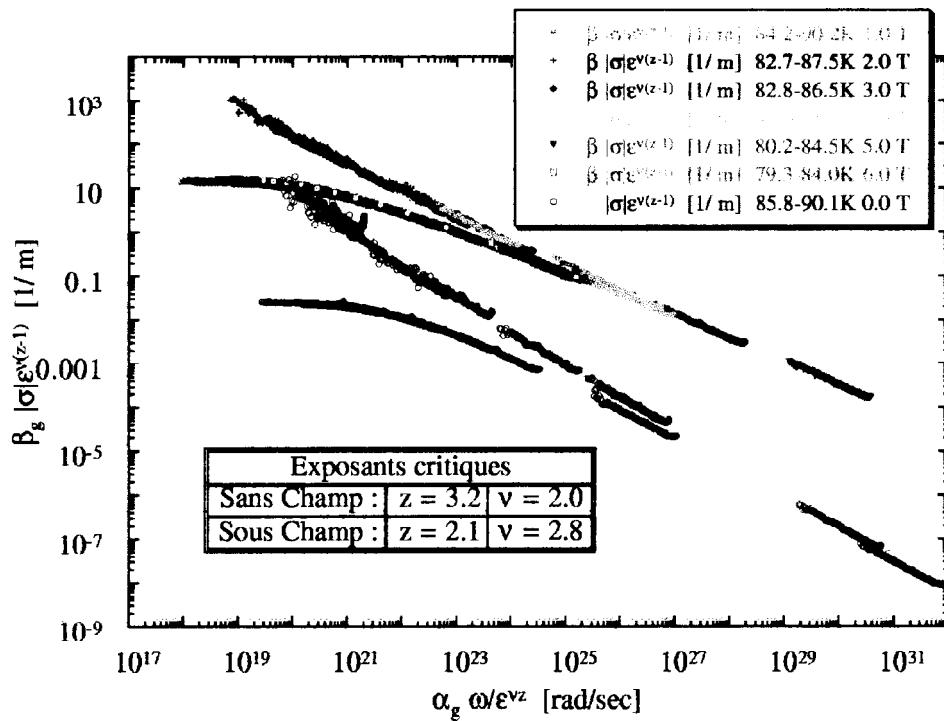
..... lois de puissance .....

$$\epsilon'' \sim (i\omega\tau)^{1-\alpha} \quad (\omega \rightarrow 0)$$

$$\epsilon'' \sim (i\omega\tau)^{\beta(1-\alpha)} \sim (i\omega\tau)^{-0.7} \quad (\omega \rightarrow \infty)$$



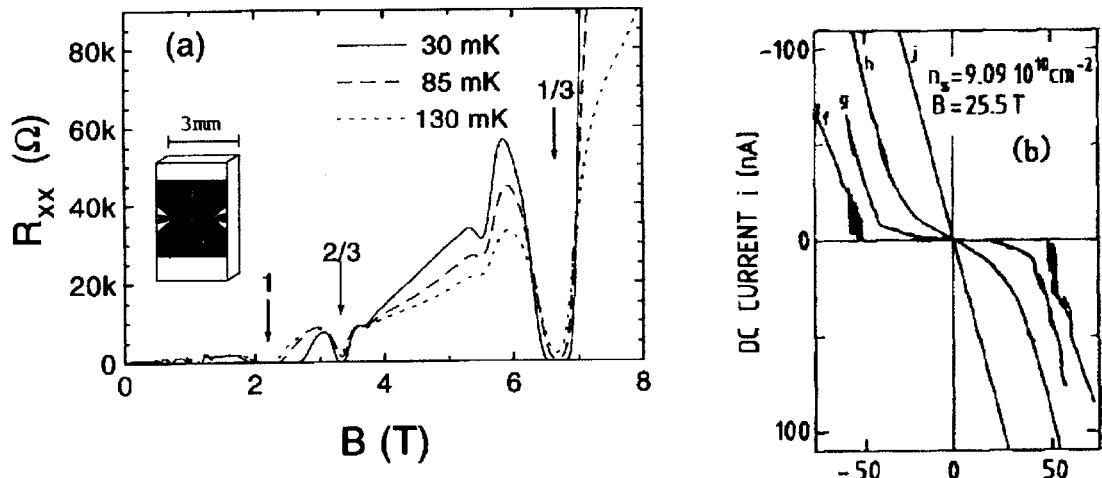
..... vortex dans YBCuO .....



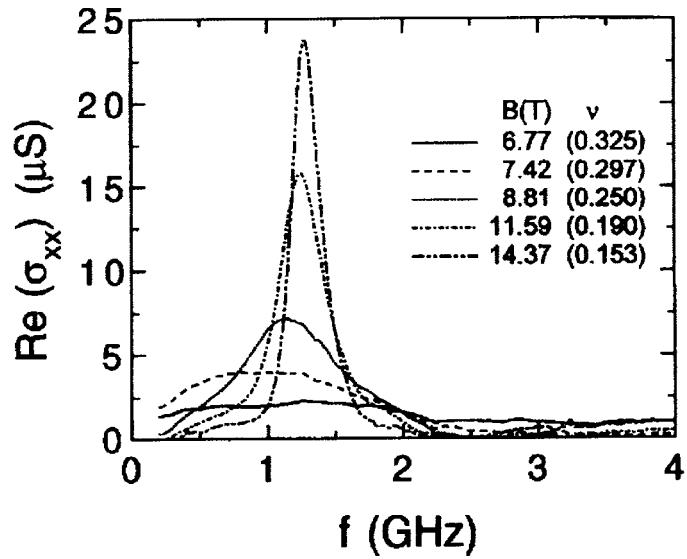
Reuss et al. thèse 2000

$$\sigma \sim (i\omega\tau)^{-(0.6-0.7)} \quad (\text{transition à haute fréquence})$$

## Cristal de Wigner (amortissement faible)



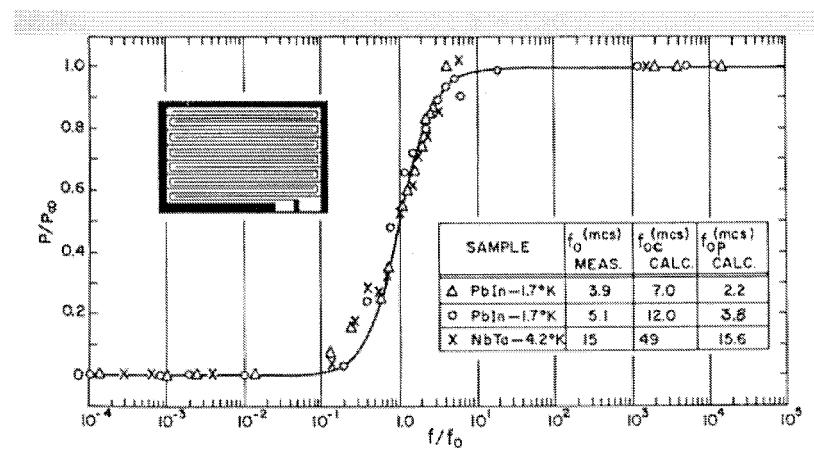
$$\text{résonance : } \omega_{peak} = \omega_0^2 / \omega_c$$



étude des modes élastiques  $u(k,\omega)$  avec  $k \neq 0$

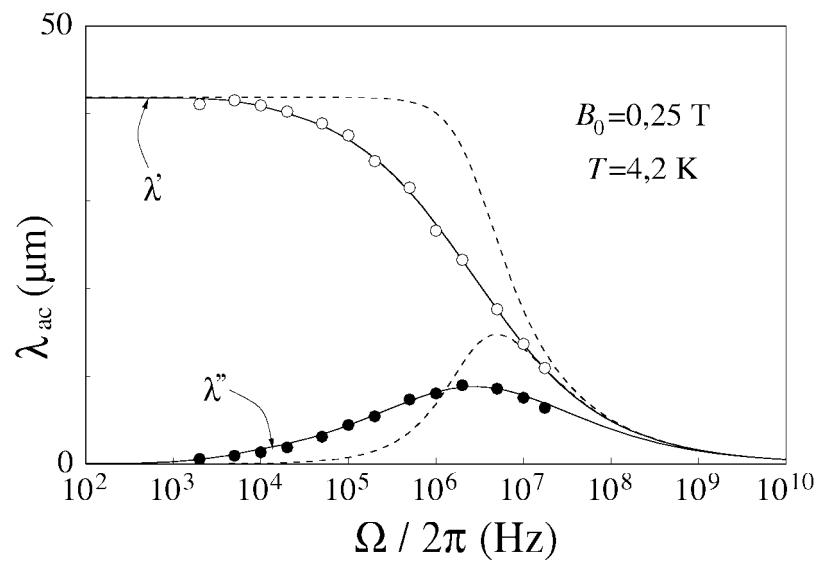
## Vortex supraconducteurs

### AC résistivité GITTELMAN-ROSENBLUM-66



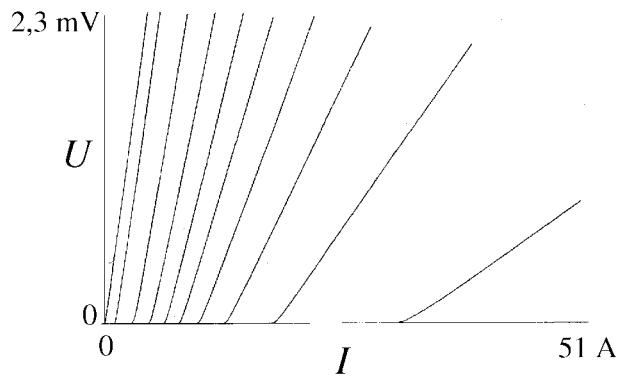
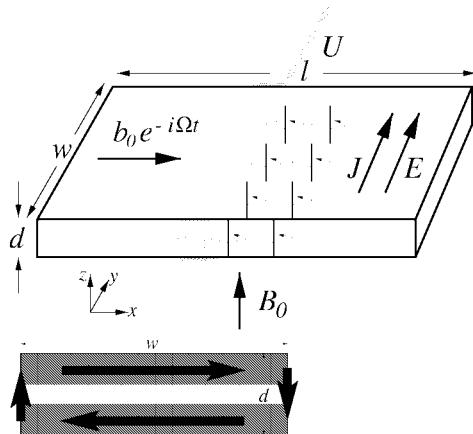
écrantage, effets non-locaux, ... ?

inductance de surface (3D) :  $Z = \frac{E}{h} = i\omega\mu_0\lambda(\omega)$

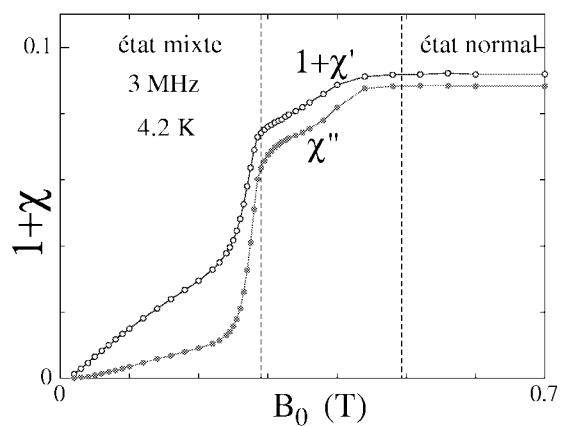
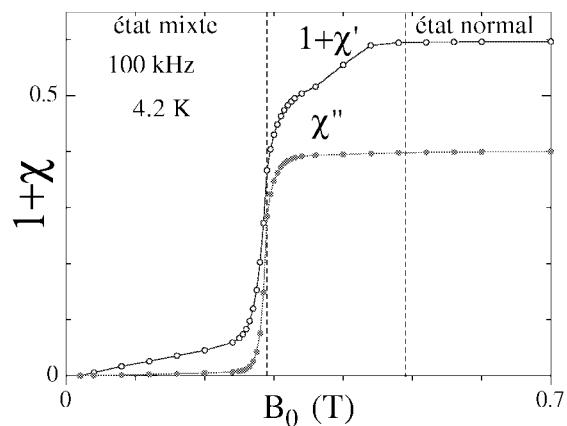


Modèles en présence	
THÉORIE LOCALE	$\frac{1}{\lambda_{ac}^2} = \frac{1}{\lambda_C^2} - \frac{2i}{\delta_f^2}$
GITTELMAN-ROSENBLUM	
ACTIVATION THERMIQUE	$\frac{1}{\lambda_{ac}^2} = \frac{1}{\lambda_C^2 + i\delta_f^2 e^{-U_0/kT}} - \frac{2i}{\delta_f^2}$
MARTINOLI; BRANDT; COFFEY-CLEM	
MODÈLE À 2 NIVEAUX	$\frac{\delta\lambda_{ac}^2}{\lambda_C^2} = \frac{kT(\log(\Omega_p/\Omega)+i)}{U_c \log^{\beta\zeta}(\Omega_p/\Omega)}$
KOSHELEV-VINOKUR ; FISCHER-NATTERMANN	
VISCOÉLASTIQUE	$\frac{1}{\lambda_{ac}} = \frac{1}{L_S(1+i\Omega_\eta/\Omega)} + \frac{1-i}{\delta_f}$
CHIEN-MARCHETTI	
HYDRODYNAMIQUE	$\frac{1}{\lambda_{ac}} = \frac{1}{L_S} + \frac{1-i}{\delta_f}$
Ci-DESSOUS	

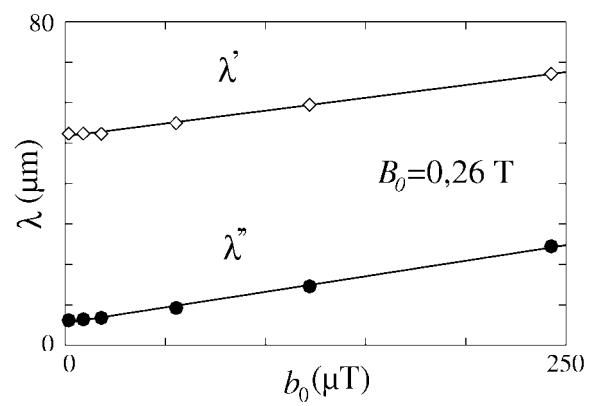
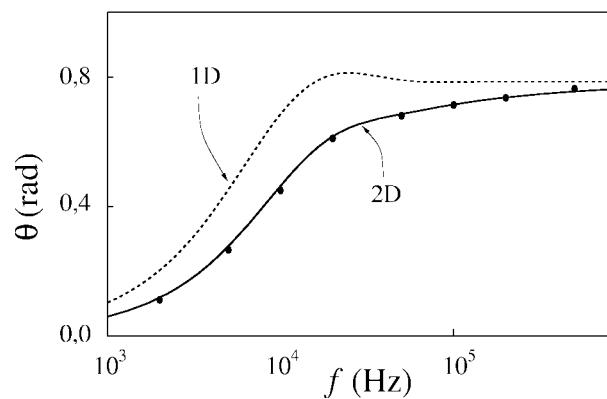
## Impédance de surface (Ann. Phys. Fr. 25, 00')



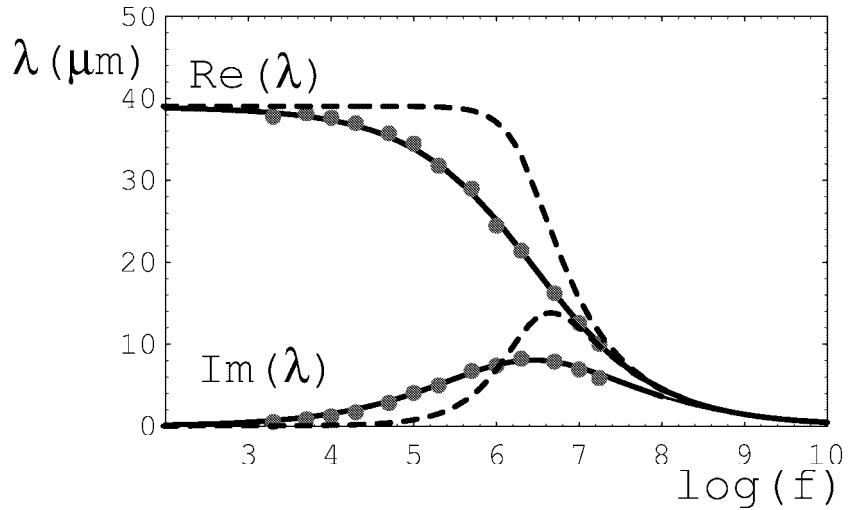
..... basse fréquence ..... haute fréquence



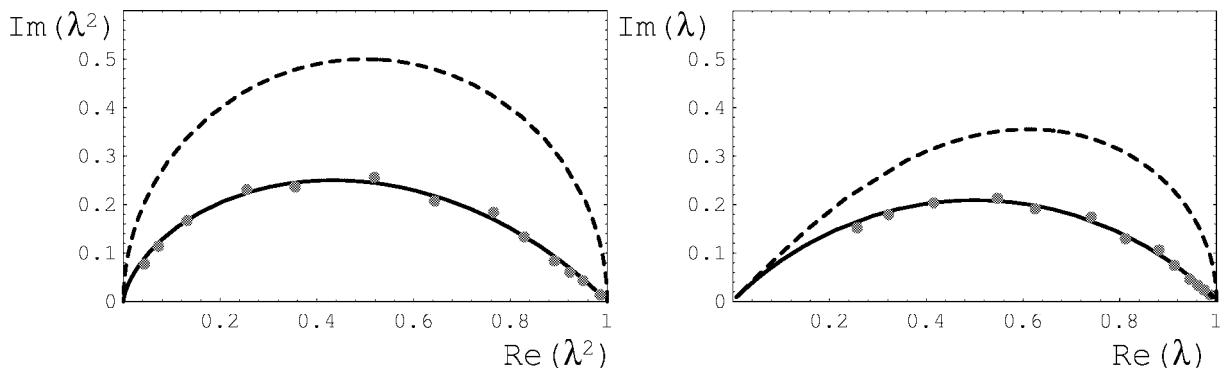
..... étalonnage phase ( $\pm 0.2$  deg.) ..... test de linéarité



## Spectre de fréquence (PbIn) PRL 97'



inductivité ( $\mu_0\lambda^2$ ) ..... inductance de surface ( $\mu_0\lambda$ )



$$\lambda^2 = \frac{\lambda_0^2}{(1+(i\omega\tau)^{0.5})^2} \dots \frac{1}{\lambda} = \frac{1}{\lambda_0} + \frac{\sqrt{-2i}}{\delta_f}$$

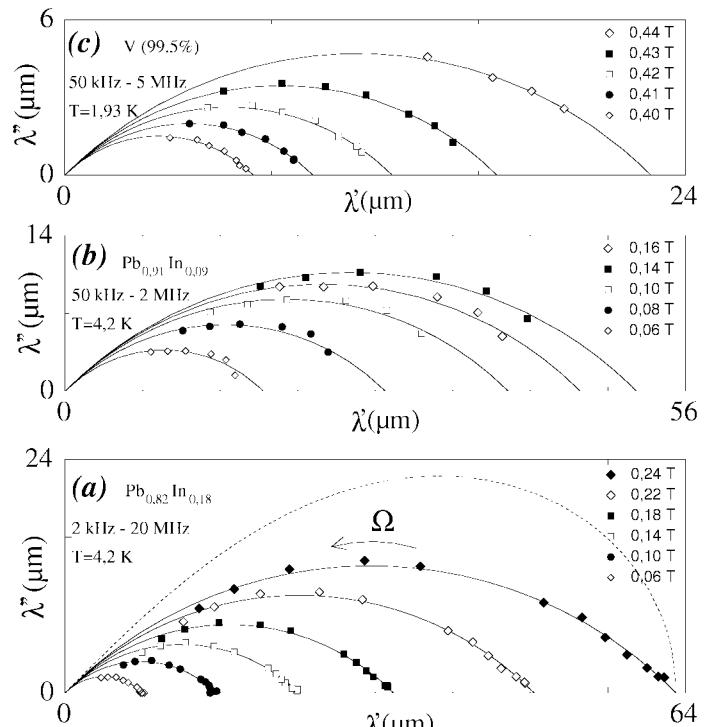
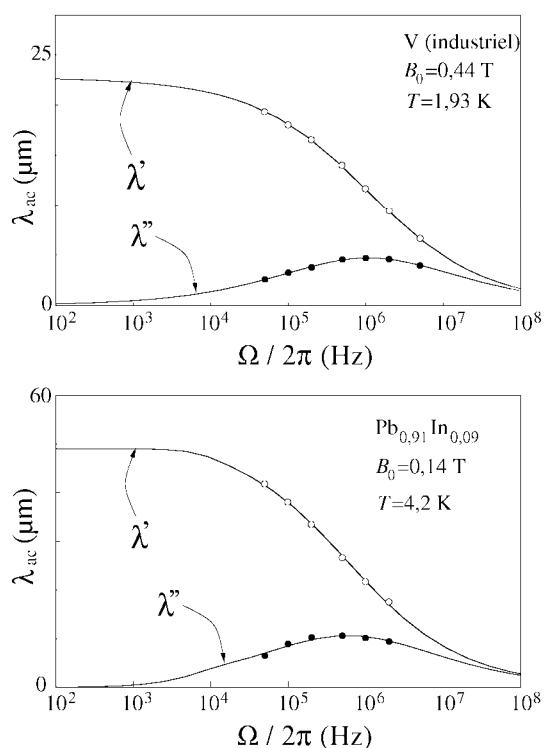
réponse vitreuse avec  $\alpha = 0.5$  et  $\beta = 2$

ou

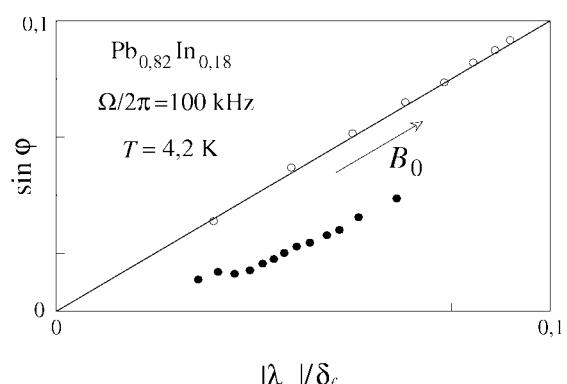
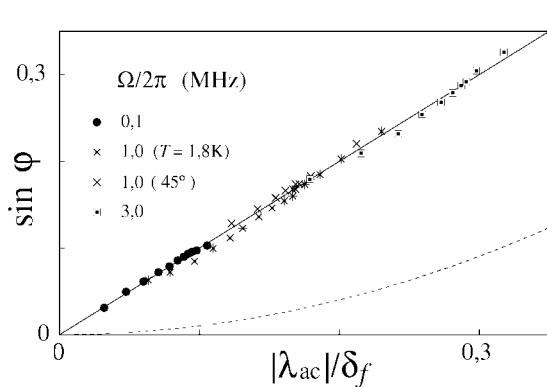
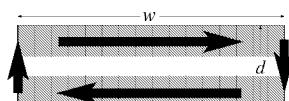
ancrage de surface?

## Ancrage de surface

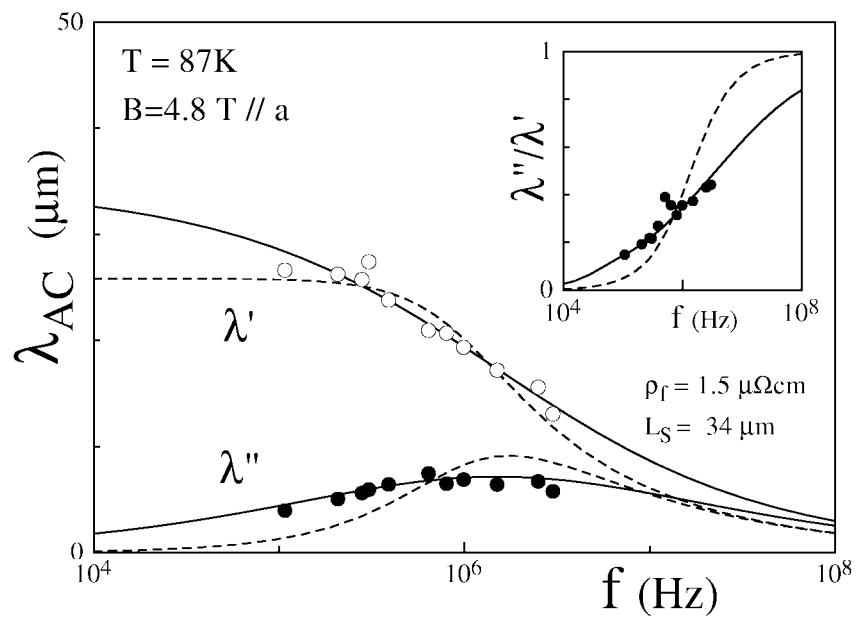
.....universalité (PbIn, Nb, V, YBaCuO, ...) .....



.....non-localité .....



## cristaux YBaCuO



..... électrodynamique à 2 modes .....

$$dF = -S \, dT + \mu_e \, dN - \frac{m}{e} \mathbf{J}_s \cdot d\mathbf{V}_s + \epsilon_0 \mathbf{E} \cdot d\mathbf{E} + \frac{1}{\mu_0} \mathbf{B} \cdot d\mathbf{B} + \boldsymbol{\varepsilon} \cdot d\boldsymbol{\omega}$$

$$\boldsymbol{\omega} = n_v \varphi_O \boldsymbol{\nu} = \mathbf{B} - \frac{m}{e} \boldsymbol{\nabla} \times \mathbf{V}_s$$

dissipation ..... équilibre

$$\mathbf{E} = \boldsymbol{\omega} \times \mathbf{v}_L = \beta \omega [J_s + \boldsymbol{\nabla} \times \boldsymbol{\varepsilon}]_{\perp} \quad J_s + \boldsymbol{\nabla} \times \boldsymbol{\varepsilon} = 0$$

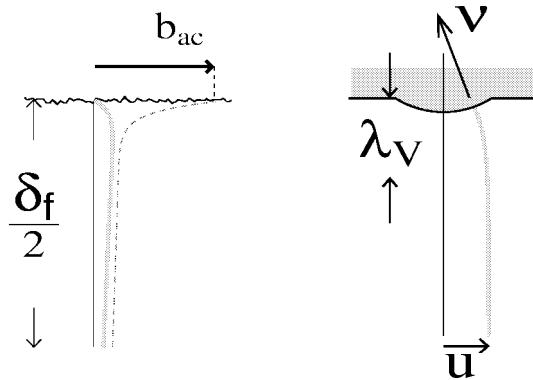
profondeur de peau classique ..... longueur élastique

$$\lambda_1 = (1+i)\delta_f/2 \quad \lambda_2 = \sqrt{\mu_0 \lambda_L^2 \varepsilon / \omega} \sim \sqrt{\varphi_0 / B}$$

..... réponse d'un demi-espace de vortex .....

$$B(z,t) = b_0 \zeta e^{i(k_1 z - \Omega t)} + b_0 (1 - \zeta) e^{i(k_2 z - \Omega t)} \text{ 2 modes}$$

$l = -u(0)/\nu(0)$  ..... longueur de glissement



$$\frac{1}{Z} = \frac{h_1 + h_2}{E_1 + E_2} \simeq \frac{h_1 + h_2}{E_1} \quad \frac{1}{\lambda_{ac}} = \frac{1}{L_S} + \frac{1-i}{\delta_f}$$

$$\frac{1}{L_S} = \frac{\mu_0 \varepsilon}{\omega} \frac{\nu(0)}{u(0)} \quad \text{terme croisé}$$

## Ancrage collectif faible de surface

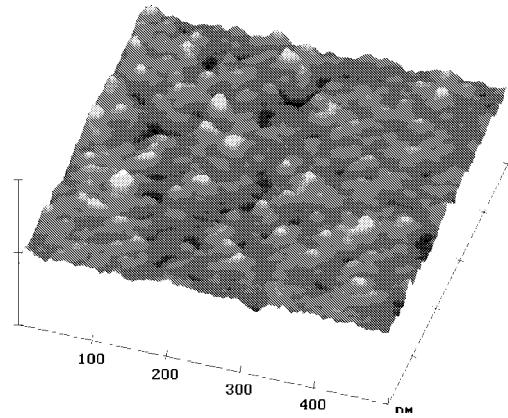
.....Nb à rugosité contrôlée  $\zeta(x,y)$  .....

polissage électrochimique  $\alpha_c \sim 0.1$  deg.

dépolissage Ar 500eV (IBE: 90 min)

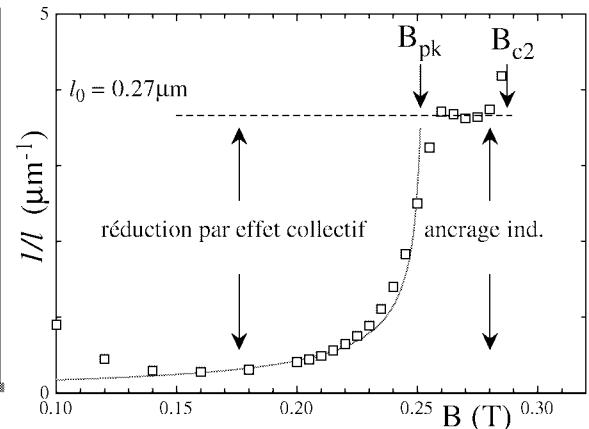
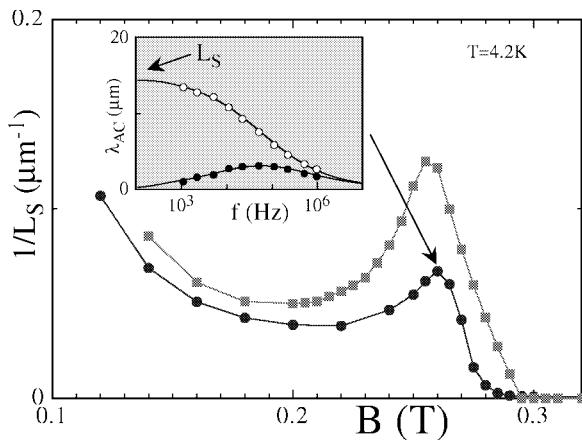
cliché AFM (Lacaze GPS-Paris7)

$$S_{\zeta\zeta} \sim 1/k$$



inductance de surface ..... longueur de glissement

longueur de phénoménologique  $1/l = 1/L_S \mu_0 \varepsilon / \omega$



transition d'amorphisation de surface?

## calcul en perturbation

les conditions aux limites désordonnées  $\zeta(r)$ , induisent des distorsions longitudinales et transverses  $U(k,z)$  de grande longueur d'onde ( $k \ll Q$ )

$$\frac{\partial U_m(R_i)}{\partial z} + \frac{\partial \zeta}{\partial x_m}(R_i) = 0 \quad \text{distorsions statiques}$$

$$\frac{\partial u_m(R_i)}{\partial z} + \frac{\partial^2 \zeta}{\partial x_m \partial x_n}(R_i) u_n(R_i) = 0 \quad \text{petits mouvements}$$

écart quadratique moyen

$$\langle U^2 \rangle = \sum_{Q \neq 0} Q^2 S_{\zeta\zeta}(Q) \int (l_\infty^T(k))^2 \pi k dk$$

longueur de glissement

$$l^{-1} = \frac{\nu(0)}{u(0)} = \sum_{Q \neq 0} Q^4 S_{\zeta\zeta}(Q) \int l_\infty^T(k) \pi k dk$$

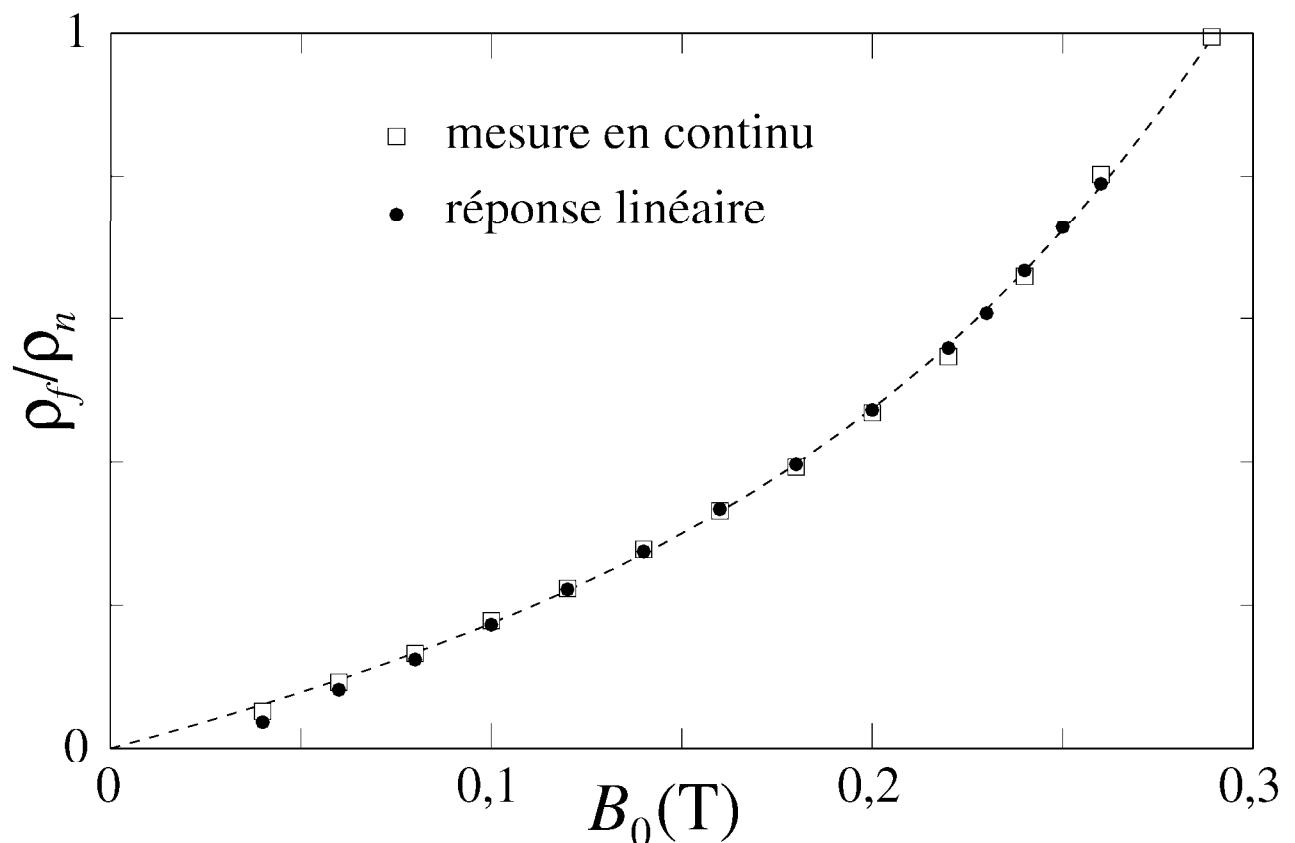
2 modes longitudinaux ..... 2 modes transverses

$$k^{-1}, \quad \lambda_V \quad (B \text{ continu}) \quad \sqrt{\frac{C_{11}}{C_{66}}} k^{-1}, \quad \lambda_V \quad (J_\perp(0) = 0)$$

pénétration apparente

$$l_\infty^T(k) = \frac{u_T(k,0)}{\nu_T(k,0)} \approx \frac{\mu_0 \varepsilon}{\sqrt{C_{66}}} \frac{\sqrt{1+k_x^2 \lambda^2}}{k_x} \gg l_\infty^L(k) \simeq k^{-1}$$

## limite haute fréquence



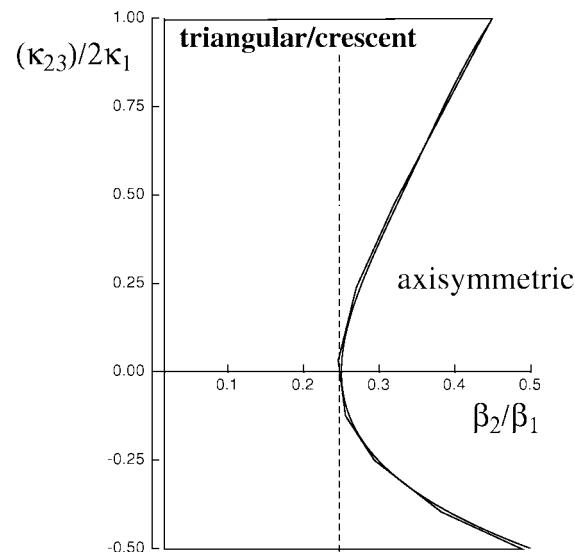
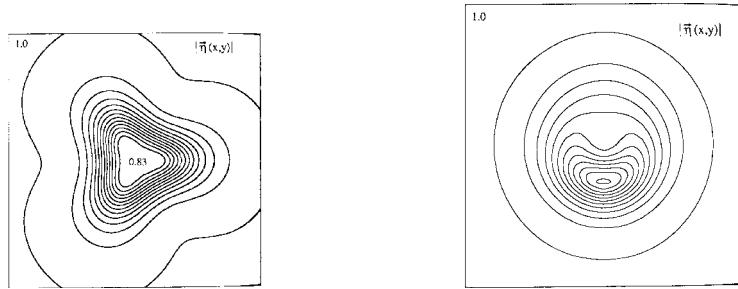
## Vortex en symétrie $E_{2u}$

Tokuyasu et al. PRB90', Sauls Adv. Phys. 94', Luk'Yanchuck-Zhitomirsky Supercon. Rev.95'

$$\Delta(\mathbf{p}_F) = \eta_1 (p_x^2 - p_y^2)p_z + \eta_2 2p_x p_y p_z \quad \boldsymbol{\eta}(\mathbf{r}) = (\eta_1, \eta_2)$$

$$F = \alpha(T)(\boldsymbol{\eta} \cdot \boldsymbol{\eta}^*) + \beta_1(\boldsymbol{\eta} \cdot \boldsymbol{\eta}^*)^2 + \beta_2|\boldsymbol{\eta} \cdot \boldsymbol{\eta}|^2 + \kappa_1(D_i\eta_j)(D_i\eta_j)^* + \kappa_2(D_i\eta_i)(D_j\eta_j)^* + \dots$$

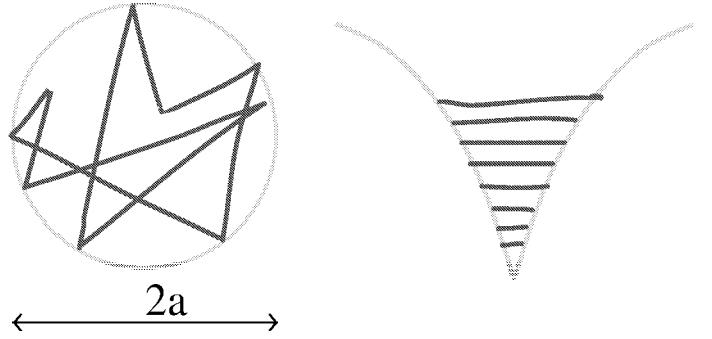
Une troisième longueur caractéristique	$\tilde{\xi} = \xi(\beta_1/\beta_2)^{1/2}$
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## Dissipation de cœur

Kopnin et al. PRB95', JLTP98'

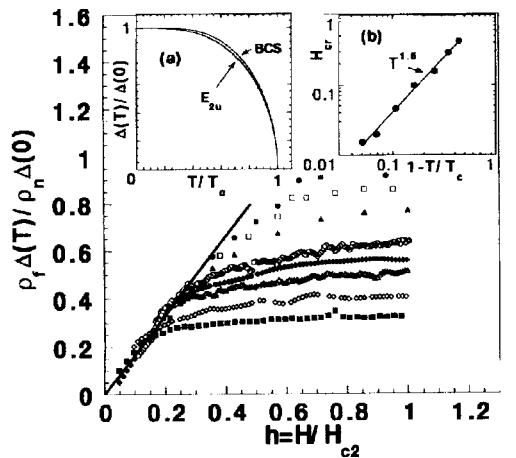
réseau dilué ( $H \ll H_{c2}$ )  
 cœur axisymétrique  $a \sim \xi$   
 régime quasi-ballistique  
 minigap  $\omega_0 = \frac{\xi^2}{E_F} \times \frac{\Delta^2}{a^2}$



$$\frac{B}{\varphi_0} \sigma_f \simeq \sigma_n \frac{\Delta_{max}(T)}{k_B T_c} \frac{\langle \tilde{\Delta}^2(p_F) \rangle}{2\pi a^2} \propto \omega_0$$

Kambe et al. PRL99'

forte résistivité  
 scaling  
 Gap anisotrope  $\rightarrow \langle \tilde{\Delta}^2(p_F) \rangle \simeq 0.3$

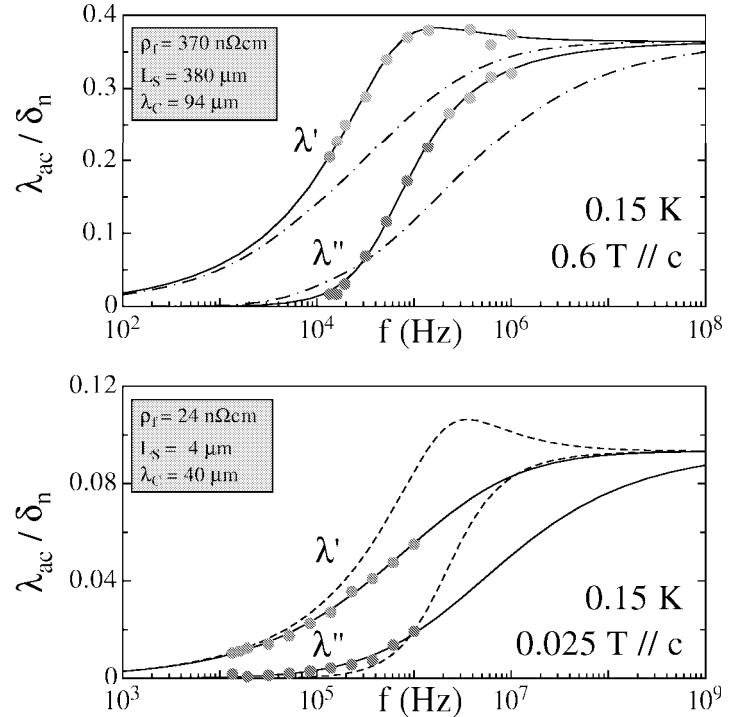


## Réponse linéaire

Spectre de fréquence

$$\frac{1}{\lambda_{AC}} = \frac{1}{L_S} + \sqrt{\frac{1}{\lambda_C^2} - \frac{2i}{\delta_f^2}}$$

$$\frac{\rho_f}{\rho_n} = \left( \frac{\delta_f}{\delta_n} \right)^2$$

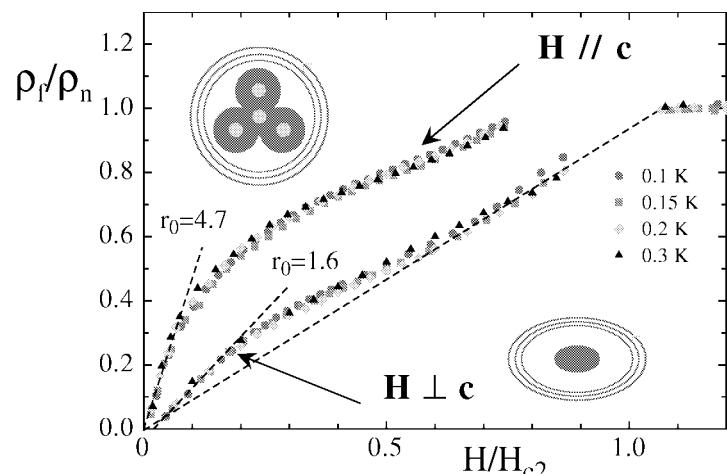


$r_{0\perp}$  idem Kambe

$r_{0\parallel}$  cinq fois trop grand

rayon de cœur  $a_\parallel^2 \simeq 5\xi_a^2$

trèflc avec  $\beta_2/\beta_1 \simeq 0.2$



## Collaborations

- N. LÜTKE-ENTRUP, P. MATHIEU, Y. SIMON (groupe VORTEX)
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- E.B. SONIN (Institut Racah, Jérusalem)
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