Critical-Current Fluctuations and Flux-Flow Noise in Type-II Superconductors

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(Received 4 September 1992)

By relying on a recent theory of transport in the mixed state and a related model of pinning in soft materials, the instantaneous critical current is introduced as a well-defined part I_1 of the total constant current. This nondissipative and time-dependent I_1 is assumed to be randomly distributed near the sample surface, and is described as a 2D turbulent homogeneous flow. Auto- and cross-correlation functions of both voltage and magnetic field noises are predicted and are in full agreement with experiments.

PACS numbers: 74.40.+k, 74.60.Ge, 74.60.Jg

Flux-flow noise has been extensively studied, particularly during the 1960's and 1970's [1-6]. The spectrum and amplitude of the noise voltage $\delta V(t)$, typically 10^{-8}-10^{-11} V/(Hz)$^{1/2}$ in the 0-10 kHz bandwidth, were regarded as significant parameters revealing the more or less irregular motion of vortex lines (VL) in the presence of pinning. The reader is referred to the excellent critical review by Clem [7]. In view of the experimental difficulties and the increasing intricacy of interpretations, this scope has been practically discarded since about 1980.

It should be noted that *soft samples* were used, classically shaped as foils or strips perpendicular to the applied magnetic field. Low to moderate critical currents I_c are required to avoid spurious sources of noise due to excessive dissipation, such as flicker noise [7]. Therefore, at working temperatures (4.2 K [1,2,5] or close to the λ point [6]), the authors had to operate at the onset of the *I-V* curve; in this region, where flux flow is essentially inhomogeneous along the sample [8], measurements yield unreliable results.

Recently [9], we have reconsidered the problem of noise, both theoretically and experimentally. We have performed a series of experiments in superfluid helium in order to widely explore the flux-flow regime. Voltage noise, magnetic field noise around the sample, and thermal Joule noise [10] were investigated, as well as their cross correlations. By relying on a phenomenological theory of vortex motion, recently published by two of us, Mathieu and Simon (MS) [11], we were able to work out a consistent theory of noise, the principle of which is brought forward in this Letter.

Roughly speaking, two mechanisms have been advanced to account for the voltage noise quantitatively. In their pioneering work, van Ooijen and van Gurp [1] assumed that flux bundles nucleated randomly at one edge and moved rigidly across the sample with constant velocity v_L. This process gives rise to a time sequence of statistically independent pulses, the cutoff frequency of the noise spectrum being related to the time of flight of bundles. This analogy with the electronic "shot noise" was attractive, but more and more adjustable parameters were introduced to fit the data; besides the flux bundle size φ_b, others include the fraction of pinned vortices [2], or a distribution of subpulse times [6]. Somewhat artificially, φ_b is found to be a rapidly decreasing function of current, falling off below the flux quantum φ_0 [2,6]. Moreover, the predicted dependence on the configuration of voltage leads is not observed. The strong vortex density fluctuations δn involved in this picture are questionable since they ignore strong interactions that tend to restore the uniformity of the vortex array. In any case, they would imply a large and easily detectable magnetic field noise in the low-frequency range [9], which clearly is not observed either [9].

To interpret the observed smallness of the noise in pure Nb and Va foils, Heiden *et al.* suggested an alternative approach which emphasizes the role of local 2D velocity fluctuations δv_L in an otherwise quasiperfect moving lattice ($\delta n \approx 0$). An effective number m of VL participating in the fluctuation surrounding a pinning site, and a site density n_p are introduced. Fluctuations δv_L at different pinning sites are assumed to be statistically independent. The amplitude and spectrum of $\delta v_L(t)$, as well as n_p and m, are undetermined parameters governing the noise amplitude and spectrum. It turns out that the only quantitative predictions of this model that concern the geometrical dependence of voltage-voltage correlations for two movable contact pairs are not confirmed by experiment [7,9].

Thus, the intricate relationship of noise-to-vortex motion remains rather obscure [12,13]. Quoting Clem [7] in his conclusion: "a suitable theory of noise is likely to be challenging. Ideally, such a theory should be intimately related to an appropriate theory for critical currents." It is in this connection that we wish to make a point. In flux-flow or noise models, whether concerned with individual vortices [7] or with an elastic continuum [12], the detailed distribution of currents through the vortex lattice is disregarded as well as the related local equilibrium of vortices around pinning centers when $I \le I_c$. Usually I_c is not identified as some well-defined part of the transport current, but rather as the right coefficient obtained by equating either VI_c with an estimate of the dissipation, or J_cB with some mean pinning force density [12]. We believe that an analysis of the current distribution, such as the one presented below, is the key to the puzzle of noise and critical currents.

Here we need to recall the main outlines of the MS phenomenological theory [11,14]. The mixed state is regarded as a continuum on the scale of the vortex spacing a, and the vortex array is described locally by the vector $\boldsymbol{\omega}=n\varphi_0\mathbf{v}$, where $\mathbf{v}$ is a unit vector along VL. In contrast with Clem's kinematics [7], which starts by investigating the motion of individual vortices, such a macroscopic analysis is not restricted to 2D motions of straight VL. This is a crucial point in our interpretation of critical currents. The local mean density of the free energy F is expressed as a function of a reduced set of macroscopic variables, in particular the electromagnetic field $\mathbf{E}$, $\mathbf{B}$, the supercurrent $\mathbf{J}_s$ (or the superfluid velocity $\mathbf{V}_s$), and $\boldsymbol{\omega}$. The equations for thermodynamic equilibrium are obtained by minimizing the magnetic free enthalpy of the sample, and a complete set of transport equations is derived [11]. MS use the same rigorous standard method as that used by Bekarevitch and Khalatnikov in their theory of vortex motion in rotating He II [15]. The originality of the MS theory essentially lies in obtaining and relying on the following three basic equations, namely, the London macroscopic equation (1), the equation (2) for local vortex equilibrium, and the boundary equation (3):

$$\boldsymbol{\omega}=\mathbf{B}-m/e\,\mathrm{curl}\mathbf{V}_s\,, \quad (1)$$

$$\mathbf{J}=\mathbf{J}_s=-\mathrm{curl}\boldsymbol{\varepsilon}\,, \quad (2)$$

$$\boldsymbol{\varepsilon}\times\mathbf{N}=0\,. \quad (3)$$

Here m and e are the electronic mass and charge, $\mathbf{N}$ is the outward normal unit vector, and the *vortex potential* $\boldsymbol{\varepsilon}$ is defined as $\partial F/\partial\omega$. The equation of state $\varepsilon(\omega,T)$ is directly related to the reversible magnetization curve [16]. Equation (1) distinguishes $\boldsymbol{\omega}$ from $\mathbf{B}$, and justifies taking them as independent local variables; its counterpart in rotating He II is $\boldsymbol{\omega}=n\kappa\mathbf{v}=\mathrm{curl}\mathbf{V}_s$, where κ is the quantum of circulation. Equation (2), or $\mathbf{J}_s+\mathrm{curl}\boldsymbol{\varepsilon}=0$, states in a macroscopic way that the supercurrent at the vortex cores is zero, including the contribution induced by the vortex itself if it is curved. As in rather general cases $\boldsymbol{\varepsilon}\simeq\varepsilon\mathbf{v}$ is along VL, Eq. (3) means that the VL must end perpendicular to the sample surface. As shown in Fig. 1, a systematic bending of VL in xz planes can exist at equilibrium provided that a supercurrent flows in the y direction. From Eqs. (1) and (2) and from Maxwell equations, it is easily seen that such a distortion of the vortex array takes place, and associated $\mathbf{J}_s$ flow, only over a small depth $d\sim\lambda(1+\omega/\mu_0\varepsilon)^{-1/2}$ from the surface, while $\mathbf{J}_s\simeq0$ and $\boldsymbol{\omega}\simeq\mathbf{B}$ in the bulk [11]. For instance, if a slab is inclined to the applied field [Fig. 1(a)], the boundary condition (3) entails two perturbed layers, where equilibrium supercurrents are nothing but Meissner-like diamagnetic currents. We have shown [17] that a similar effect exists in a rotating cavity in He II, in agreement with experiment (see Fig. 1 in Ref. [16]).

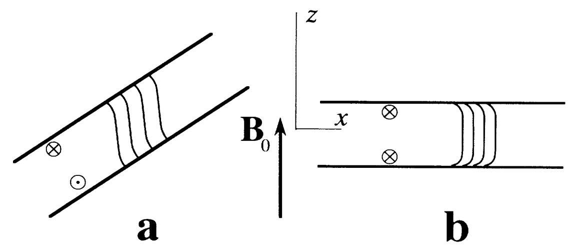

FIG. 1. Nondissipative currents associated with a distortion of the vortex lattice. VL are shown. Magnetic field lines (not shown) are curved in opposite directions. (a) Diamagnetic currents in a slab inclined to the applied magnetic field. There is no pinning. The sample, including the surface, is perfect. (b) Subcritical transport currents in a rough slab normal to the field. The surface shown in this case is the mean smoothed surface. In the perturbed layer, vortex lines are not field lines, in accordance with the London macroscopic equation (1).

A continuum description excludes inhomogeneities on the scale of a. However, surface irregularities on a scale comparable to or smaller than a are unavoidable in soft samples as well. Therefore MS suggest that Eq. (3) should be replaced by an inequality in the form $|\boldsymbol{\varepsilon}\times N|\leq\varepsilon\sin\theta_c$, where $\mathbf{N}$ is now normal to the mean smoothed surface. So a large number of metastable or nondissipative configurations can occur, as frequently observed in disordered systems or in systems subject to irregular boundary conditions. Let us mention for example the effect of surface roughness on the wettability, resulting in contact-angle hysteresis [18]. As shown in Fig. 1(b), this new boundary condition allows a net transport supercurrent to flow near the surface [11,16]. Thus connecting I_c with $\varepsilon(\omega,T)$ and $\sin\theta_c\lesssim1$, we are able to account for the order of magnitude of critical currents in soft samples, as well as for their field and temperature dependence [16]. We stress that, in both Figs. 1(a) and 1(b), VL and magnetic field lines separate and curve in opposite directions across the two perturbed layers, consistent with Eq. (1).

When the current density $\mathbf{J}$ departs from $-\mathrm{curl}\boldsymbol{\varepsilon}$, the VL move. So it is convenient to separate $\mathbf{J}$ into two parts as $\mathbf{J}_1+\mathbf{J}_2$, where $\mathbf{J}_1=-\mathrm{curl}\boldsymbol{\varepsilon}$ is the nondissipative part of the current [11]. $\mathbf{J}_1$ is defined, at any point and time, as the supercurrent (including diamagnetic currents) that would come into equilibrium with the vortex array in its instantaneous configuration. In quasistationary conditions, covering ac transport currents in the acoustic range of frequencies, and the whole investigated spectrum of flux-flow noise (0–10 kHz), the MS transport equations take a simplified form. They yield $\mathbf{v}_L=-\rho_f/\omega(\mathbf{v}\times\mathbf{J}_2)$ and $\mathbf{E}=-\mathbf{v}_L\times\boldsymbol{\omega}=\rho_f\mathbf{J}_{2\perp}$, where $\rho_f(\omega)$ is the flux-flow resistivity [11]. Whereas the currents $\mathbf{J}_1$ may be regarded as idealized *surface currents* $\mathbf{i}_1(i_1A/m)$ on the scale of the sample, the currents $\mathbf{J}_2$ are bulk distributed.

A picture of the dc flux flow, including a possible mechanism of noise, follows at once from the above interpretation of pinning in soft samples. The observed shape of the V-I curve at large currents, $V=R_f(I-I_c)$, suggests that the surface retains its ability to carry a constant nondissipative current, at least on the average. Thus, by assuming that the vortex array moves uniformly while maintaining its critical-state configuration, such as sketched in Fig. 1(b), MS could explain most of the dc properties, in particular the dc Joule effect [11,14]. Of course, the rigid and uniform motion of VL, especially near the surface, is unrealistic; it only represents a time-average picture of the vortex flow. By handling it as if it were strictly steady, MS just follow a familiar procedure in the hydrodynamics of turbulence where, in many practical situations, the mean flow developments of real interest. Concerning noise, however, we cannot ignore the irregularities of the vortex flow. The motion of VL slipping through the surface defects must be irregular, giving rise to *large local fluctuations* of the VL bending and associated $\mathbf{J}_1$'s. Nevertheless the relative fluctuations of the *instantaneous critical current* $\langle I_1\rangle$, *defined* here as the flux I_1 of $\mathbf{J}_1$ through a cross section averaged over the length of the sample, should be considerably reduced by a well-known statistical effect, to typically 10^{-3}–10^{-4}. At constant $I=I_1+I_2$, correlative fluctuations of the bulk dissipative current, $\delta I_2=-\delta I_1$, entail a small voltage noise $\delta V=R_f\langle\delta I_2\rangle=-R_f\langle\delta I_1\rangle$. This two-step mechanism explains the smallness of the voltage noise in pure Nb, despite comparable noise sources $\delta\mathbf{J}_1$ such as revealed by the field noise [9] (see below). More quantitatively, we shall describe surface currents $\mathbf{i}_1$ as a 2D *homogeneous turbulent flow*.

As the current density distribution $\mathbf{J}(x,y,z)$ fluctuates, the small magnetic field b due to the transport current also is noisy. The low-frequency noise δb (typically $100\phi_0$ rms in a 1 mm^2 pick-up coil) has been measured recently by one of us [9] and by Yeh and Kao [19]. It should be noted that $\delta b(t)$ is too small for any induction term $-\partial\phi/\partial t$ to be significant in δV (except perhaps in very pure metals, as $R_f\to 0$). This is corroborated by the fact that no influence of the measuring circuit geometry is observed. To account for the minuteness of the field noise, Yeh and Kao present a new and strongly amended version of the shot noise model. But, as a result, the voltage noise is no longer explained, while a successful theory ought to account for both noises.

Let us consider a simple standard situation, so as to get rid of unimportant end effects: The sample is taken as a "soft" piece $\Delta y=L$ of an otherwise "hard" thin strip of width $\Delta x=W$. Both ends of the strip serve as equipotential current leads. Thus the voltage V across this sample (and δV) is unambiguously defined, and can be written as the integral of E_y (δE_y) over the sample volume, divided by the cross-sectional area. Small contributions of the field noise (or $\delta\omega$) in δE_y, though detectable [9], may be neglected as a first excellent approximation, so that $\delta E_y\approx\rho_f\delta J_{2y}$ and $\delta V\approx R_f\langle\delta I_2\rangle$ as stated above. On the other hand, the field noise may be investigated with a variety of coils. For definiteness, let $\Phi(t)$ be the noisy magnetic flux through a rectangular loop $L'\times W$ lying in the xy plane of the strip, and overlapping the sample ($L'\gtrsim L$).

Dividing the $\mathbf{i}_1$ field into mean and fluctuating parts, we write

$$\mathbf{i}_1=i_c\mathbf{y}+\mathbf{u}(\mathbf{r},t)\,. \tag{4}$$

Assuming uniform surface conditions, $i_c=$const, and $I_c\approx 2Wi_c$. The (only useful) y-y component of the current correlation tensor, for two points of the surface separated by the space vector $\mathbf{R}(X,Y)$, reads

$$\overline{u_y(\mathbf{r},t)u_y(\mathbf{r}-\mathbf{R},t-\tau)}=u^{*2}C(\mathbf{R},\tau)\,. \tag{5}$$

Here u^* is the *intensity of the turbulence.* The space-time correlation coefficient $C(\mathbf{R},\tau)$ is assumed to decrease rapidly as R increases (for any τ); its space integral over XY can be written as $c^2f(\tau)$, where $f(0)=1$. This assigns a *length scale* c to the turbulence. We presume that $a\ll c\ll W$. Then a straightforward calculation gives the auto- and cross-correlation functions

$$C_{VV}(\tau)=2R_f^2(W/L)u^{*2}c^2f(\tau)\,, \tag{6}$$

$$C_{\phi\phi}(\tau)=(\mu_0^2/6)LWu^{*2}c^2f(\tau)\,, \tag{7}$$

$$C_{V\phi}(\tau)=0\,. \tag{8}$$

At a given operating point (T,B,I), we always observed that voltage and field noise spectra exhibit the same f dependence, interpreted here as that of noise sources $\mathbf{i}_1$ [i.e., the Fourier transform of $f(\tau)$]. So their amplitudes may be characterized by the rms values δV^* and $\delta\Phi^*$. By measuring $\delta V^{*2}=C_{VV}(0)$, and comparing with Eq. (6), where $f(0)=1$, we get the "elementary fluctuating current" u^*c. As I is increased *along the linear part* of the V-I curve, we find that $u^*c\simeq$const (typically 1 mA), while the spectra retain their shape. This conforms with the idea that any further increase of the bulk current I_2 does not affect the well-established turbulent flow of the surface current I_1. We again emphasize that this steady noise is settling only when the entire vortex array is moving. The correlation of pinning and noise levels has long been noticed [7,20]. For instance, while the critical current of a $Pb_{82}In_{18}$ strip decreased rapidly as the field was increased, $u^*c/I_c\simeq$const in an intermediate range of fields ($0.2<B/B_{c2}<0.8$). Taking $u^*\sim i_c$, as expected in a fully developed turbulence, we obtain $c\sim 1\ \mu$m, consistent with our assumptions. On the other hand, δV^* and $\delta\Phi^*$ have been measured for several samples cut from the same PbIn foil. Except for end effects in W/L, we find that $\delta V^{*2}\propto L/W$, and $\delta\Phi^{*2}\propto LW$, in accordance with Eq. (6) [where $R_f\propto L/W$] and Eq. (7), if u^*c is assumed to be size independent (homogeneous turbulence).

The main achievement of the above description is to account for the interrelation between voltage and field noises. From Eqs. (6) and (7), the amplitude ratio $\delta V^*/\delta\Phi^*$ is entirely predicted in terms of known parameters, in full agreement with experiment. Furthermore, measurements of cross-power spectra confirm the absence of correlation as required by Eq. (8). Experimental details are planned to be published elsewhere. These results are at variance with predictions of shot-noise models, where $\delta V(t) \sim (v_L/W)\delta\Phi(t)$ (strict equality would be obtained in the approximate rectangular-pulse model first proposed in Ref. [1]), so that δV and $\delta\Phi$ should be totally correlated. Moreover, the expected ratio $\delta\Phi^*/\delta V^*$ should be much larger than that observed by a factor of 10^3–10^4 (depending on the sample size).

Furthermore, the flux-flow noise in PbIn and Nb bars of large cross section (4×4 mm^2) has been investigated by arranging several coils around the sample [9]. The analysis of auto- and cross-power spectra in this geometry proves the existence of surface-volume fluctuations of the current distribution. It is worth noting that Nb samples used were single crystals of the same quality as those in which a quasiperfect crystal of vortex lines was observed by neutron diffraction [21]; the vortex crystal was preserved in flux flow [22]. Nevertheless critical currents and field noise levels are quite comparable to those observed in standard PbIn polycrystals. The smallness of δV in pure Nb is not connected with the perfection of the vortex lattice, as suggested by Heiden *et al.* [5], but is merely due to the relatively low resistivities.

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