Anomalous transverse voltages in the superconducting surface sheath

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The critical state and transport properties of the superconducting sheath are investigated, in slabs parallel to the applied magnetic field, when it makes an arbitrary angle θ with the direction of the applied current. The observation of critical currents governed by surface defects, and linear current-voltage characteristics, corroborate the conclusion advanced by several authors in the past, that the surface sheath of a real rough surface is populated by quantized vortices or "flux spots," which should exhibit the same pinning and flux-flow properties as usual vortices. Nevertheless, transverse voltages measured above H_{c2} are at variance with the well-known general relation between macroscopic fields, $\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}$, contrary to those observed in the mixed state. Making allowance for the observed anisotropy of the surface critical current in parallel field, a model is proposed that accounts successfully for the unexpected distribution of electric currents and fields above H_{c2}. We emphasize that experimental results cannot be reconciled with the existence of surface vortices, and our analysis would lead to serious difficulties in interpreting Joule dissipation, unless we rely on some unorthodox conclusions of a continuum theory of vortex motion developed recently by two of us. Measurements reported in this paper support the correctness of this theory.

I. INTRODUCTION

This paper is concerned with the critical state and transport properties of a slab in a parallel magnetic field, particularly above H_{c2}. Figure 1 shows several current-voltage characteristics, such as commonly observed on increasing the magnetic field across H_{c2}. The persistence of a critical current indeed confirms the capacity of the superconducting surface sheath for carrying a reasonably large current density, i_c A/m, without dissipation.

At first sight, the unchanging shape of the I-V curve suggests that some aspects or rules of the flux flow in the mixed state should extend beyond H_{c2}. In particular, and as has long been noticed,[1,2] i_c is governed by (surface) defects. Moreover, Hart and Swartz[2] (HS) inferred from their experiments, that the superconducting layer of a rough (real) surface should be permeated by an array of quantized *flux spots.* As pointed out by several authors,[2–4] many of the properties of the vortex lines in the mixed state, in connection with pinning and flux flow, may well pertain to these flux spots or surface vortices. While developing our own argument, we recall in Sec. II how this opinion prevailed at the end of the 1960s. Recently two of us have proposed a continuum theory of vortex motion in the mixed state,[5,6] which we shall refer to below as the MS theory. As discussed in Sec. II, we believe that most of the ideas underlying the MS theory may be generalized to describe the critical state and vortex motion in the surface sheath, even though we are dealing with a quasi-two-dimensional (2D) situation.

In the mixed state, under steady-state conditions, a simple and well-known equation relates the electric field $\mathbf{E}$ and the vortex line velocity $\mathbf{v}_L$, viz. $\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}$. This equation, relating macroscopic fields, has been derived by Josephson[7] in its stricter form, $\mathbf{E}'=-\mathbf{v}_L\times\mathbf{B}$, where $\mathbf{E}'=\mathbf{E}+\nabla\mu/e$ is the gradient of the electrochemical potential; the circulation of $\mathbf{E}'$ is just what is measured by a voltmeter. Josephson's method of proof is one of great generality. In the author's own terms, it "is applicable to (any) systems which are inhomogeneous with respect to composition or flux line density." $\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}$ implies the existence of strong transverse voltages in the mixed state, when $\mathbf{B}$ makes an arbitrary angle θ with the applied current (Fig. 2), in full agreement with experiment.[8] We

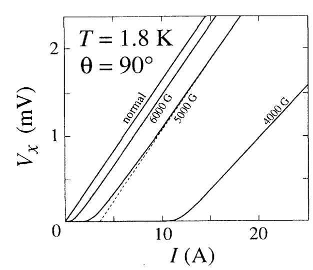

FIG. 1. Longitudinal voltage-current characteristics, $V_x(I)$, of a Pb-In 17,5 at. % slab, for different values of the parallel applied field B_0, and $\theta=\pi/2$ (see Fig. 2). The slab dimensions along the xyz directions shown in Fig. 2 are, respectively, $L=30$ mm, $W=8$ mm, and $t=1$ mm. The distance between voltage probes is $\Delta x\simeq 13$ mm. At $T=1.8$ K, $H_{c2}=4700$ Oe. The normal resistivity is $\rho_n=10.1\,\mu\Omega\,$cm. For $B_0>B_{c2}$ the limiting slope of the V-I curve at large currents is the normal resistance $R_n=\rho_n\Delta x/Wt$. At 5000 G, trailing of the curved part of the V-I curve reveals a relatively large dispersion or critical current along the sample, as explained in the text. By extrapolating at $V=0$ the linear part of the characteristic at 5000 G (dashed line) we obtain the mean critical current $I_c=3.5$ A. When $\mathbf{B}_0$ is aligned with the applied current ($\theta=0$), I_c is reduced to 2.8 A at 5000 G. On dividing these two values of I_c by $2W$, we get representative mean values of the extreme current densities a and b of the critical curve $i_c(\varphi)$ of Fig. 5.

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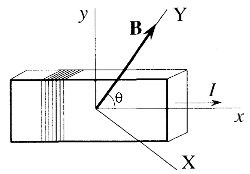

FIG. 2. The geometry of the sample. The magnetic field **B** is aligned with the xy faces of the slab, and can be inclined at an arbitrary angle θ to the direction x of the applied current. A 100–200 turns coil, wound directly on the slab as shown, presents mutual inductance with current loops along yz planes. It is designed to confirm, through a low-frequency modulation experiment, the occurrence of circulating transverse currents above H_{c2}.

then expect that the transverse field suddenly falls to zero at H_{c2}, unless vortex motion takes place in the surface sheath. If so, and in so far as $\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}$ is a fundamental relationship pertaining to any kind of vortex motion, a similar effect ought to have been observed above H_{c2}, in spite of the fact that nearly the whole of the sample is in the normal state.

Transverse-voltage measurements are reported in Sec. III. We did observe transverse fields above H_{c2}, but their behavior is totally at variance with the equation $\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}$. Nevertheless, as explained in Sec. III, this is not inconsistent with the presence of vortices in the surface sheath, provided that the MS equation relating $\mathbf{E}'$ and $\mathbf{v}_L$ [Eq. (7) below] is used in the place of $\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}$. In most situations, in particular those regarding bulk flux flow, both equations are very much the same.[6] Thus, the surface superconducting sheath appears, in this respect, as an unusual case well suited to support the correctness of the MS approach.

By relying on the observed anisotropy of surface critical currents, a model is proposed (Sec. IV) which well accounts for transverse voltages above H_{c2}. As a proof, a mutual inductance experiment, has been designed to probe the predicted current distribution. From these experiments it emerges that steady distributions of transverse electric fields and currents may occur, where a bulk normal Joule effect is balanced by a *negative* surface Joule effect. This rather surprising result again is well explained by the MS theory of flux-flow dissipation as discussed in Sec. IV.

II. THE CRITICAL STATE OF THE SURFACE SHEATH: A PRELIMINARY DISCUSSION

Earlier attempts to interpret i_c as a fundamental thermodynamic property of the surface sheath have been highly unsatisfactory. Abrikosov[9] and Park[10] calculated the maximum current density i_m, which can be passed through the surface sheath, in accordance with the Ginzburg-Landau equations. But $i_c \ll i_m$ by 1 or 2 orders of magnitude.[2] While i_m certainly makes sense as a theoretical upper bound, it was clear that something else limited surface currents far below.

Fink and Barnes,[11] then Park,[12] defined the critical state of the surface sheath as that state for which the magnetic free-enthalpy difference G_s-G_n between the superconducting state and the normal state is zero. Critical current densities obtained in this way are much lower than those given by Abrikosov[9] and Park,[10] and turn out to be the order of magnitude observed. However, besides the fact that minimum free-energy arguments are quite questionable when dealing with transport phenomena, severe objections have been made to this line of reasoning.[1,2] Though deriving unlike expressions for i_n, both models[11,12] make it to be dependent on the foil thickness and/or width; such a size dependence of i_c has never been observed.[2] Another common prediction is worth mentioning, as it is inconsistent with our own results. In an experiment reported in Sec. IV, we have been led to destroy surface superconductivity on one face of the slab by nickel plating. Now, according to both models,[11,12] the measured critical current in this case should be reduced by a factor of $\sqrt{2}$. As far as both faces of the slab have been prepared alike, we instead observe that I_c is smaller by a factor of 2 (Fig. 1), as clearly expected if i_c is caused by surface defects. But the most direct criticism is still to argue that no transition to the normal state is observed,[2] as readily seen by mere inspection of the I-V curve at large currents, where $V/I \neq R_n$, the normal resistance of the sample.

All four of the above-mentioned theories[9–12] suppose that the local magnetic field be everywhere parallel to a perfect planar surface on the scale of the coherence length $\xi(T)$. HS rightly noticed that such ideal conditions are unattainable, in practice, so that the field has a nonzero normal component over most of the surface. Therefore, in discussing their results, HS proposed that the magnetic field, locally inclined to the surface, crosses the superconducting sheath as an array of quantized flux spots. Not long after, Kulik,[3] following Abrikosov's approach, showed that the superconducting layer, in an inclined field, actually has a vortex structure similar to the Abrikosov lattice. The mapping of the magnetic field sketched in Fig. 2 of Kulik's paper illustrates and confirms the flux spot picture. Confining himself to rectangular unit cells, Kulik obtained a square vortex lattice at equilibrium, whose period d is given by

$$d^2 B \sin\alpha=\phi_0 , \tag{1}$$

where ϕ_0 is the flux quantum. $B\sin\alpha$ is the normal component of the magnetic field. For small inclination angles α, $d=(\phi_0/B\alpha)^{1/2}$. For instance, taking $B=5000$G, and $\alpha=1°$, $d\lesssim 5000$ Å. However careful the alignment of the whole slab with the field, one cannot avoid large-scale roughness of the surface, and thereby penetration of magnetic flux through the sheath. So it is to be expected that a large number of Kulik's vortices populate the surface. The generation of surface vortices, which may be pinned, enter a critical state, and move, provides an attractive explanation for all observations.

Let us return, for further comments, to the set of I-V characteristics as shown in Fig. 1. The curved part at the foot of the characteristic, though of minor importance, is a constant feature, which gives indirect evidence of the prominent part of defects, above and below H_{c2} alike. As the distribution of defects is hardly homogeneous in practice, each segment x of the sample has a different critical current, $I_c(x)$, in a finite range, say from I_c' to I_c''. Thus the curved region of the overall characteristic, between I_c' and I_c'', results from the sum of individual linear characteristics. This fact is easily checked by using a series of close voltage probes.[13] It is easily seen that the critical current I_c, usually obtained by extrapolating the linear part of the I-V curve, represents the mean value of $I_c(x)$ between the voltage probes. In discussing experimental results, it will be convenient to assume that the distribution of surface defects is *homogeneous* and *isotropic*. Under such ideal conditions, the I-V curve should display the standard broken shape: $V=0$ for $I<I_c$, and $V=R(I-I_c)$ for $I>I_c$. Below H_{c2}, $R=R_f<R_n$ is the field-dependent flux-flow resistance. Near H_{c2}, the conductivity difference, $\sigma_f-\sigma_n$, is an effect associated with the relaxation time of the order parameter.[6,14] Vortex motion, if it occurs in the surface sheath, must entail similar time-relaxation effects. Because of the smallness of the volume involved, however, their contribution to the total dissipation is not significant, so that R cannot be distinguished, within experimental accuracy, from the normal resistance R_n (see Fig. 1).

The constant intercept of the linear part of the characteristic suggests that the surface sheath, above H_{c2}, retains its ability to carry a constant nondissipative current I_1. That is its maximum value I_c, as much as allowed by surface defects, while the voltage is proportional to the dissipative excess current $I_2=I-I_1=V/R_n$.

The MS theory just leads to the same naive interpretation of the customary flux-flow regime in the mixed state.[5,6] Yet there is an objection to be made: How does one explain the large part $VI_1=VI_c$ of the Joule power VI, if I_1 itself does not contribute to dissipation? There is no inconsistency, however, as discussed at some length in Ref. 6. Whether below or above H_{c2}, we are clearly faced with the same problems. We shall return to this point at the end of Sec. IV.

To begin with, we shall avoid arguing about the detailed nature of the pinning and dissipative mechanisms. First of all we wish to consider the previous and less complicated question of how electric fields and currents are distributed in the "vortex flow" regime above H_{c2}, especially when the magnetic field is inclined at arbitrary angles θ. In whichever model of transport, a current $I>I_c$ may be separated into a surface part $I_1=I_c$ (current density $\mathbf{i}_1$) and a bulk part I_2 (normal current density $\mathbf{J}_2=\sigma_n\mathbf{E}$). We may accept as an experimental fact the existence of associated Joule effects: $\mathbf{E}\cdot\mathbf{i}_1(W/m^2)$ and $\mathbf{E}\cdot\mathbf{J}_2(W/m^3)$, but the very nature of dissipative mechanisms, however, will come into question inevitably, when we discuss our experimental results.

III. TRANSVERSE VOLTAGES: EXPERIMENTAL RESULTS

Longitudinal and transverse voltages, V_x and V_y, have been measured in a series of 10–20 lead-indium slabs aligned with the applied magnetic field, for various values of the angle θ, as shown in Fig. 2. Typically, the dimensions L, W, and t of the slab, along the x, y, and z directions, respectively (Fig. 2), are $L=3$ cm, $W\lesssim 1$ cm, and $t=0.5-1$ mm. Cast ingots of the solid solution Pb-In 17.5 at. % were annealed for 2 weeks under purified argon ($\sim 10^{-4}$ mm Hg) within about 10 °C of the solidus point. Slabs were spark cut, either directly from the ingot, or from 0.5–1 mm sheets obtained by rolling, or by pressing a piece of ingot between glass microscope slides in a hydraulic press. Pressed samples exhibited a mirror-like finish, a good parallelism of the faces, and yet relatively large critical currents above H_{c2} ($i_c\gtrsim 1$A/cm). Samples were used as rolled or as pressed, or after various surface treatments (mechanical or chemical polishing, electroplating).

For the sake of the discussion, we shall refer to simple *standard conditions,* in which samples are assumed to exhibit a *homogeneous and isotropic* distribution of (surface) defects, as stated in Sec. II, as also *bulk homogeneity.* Such conditions entail a translational symmetry along xy planes (Fig. 2). In the vortex state macroscopic space charges and Bernoulli effects are negligibly small, so that $\mu=$const in a homogeneous sample, and, in any case, $\mathbf{E}'=\mathbf{E}$. Moreover, translational symmetry and *steady* conditions ($\nabla\times\mathbf{E}=0$, $\nabla\cdot\mathbf{E}=0$) require $\mathbf{E}$ to be *uniform* everywhere inside the sample.

In the mixed state, strong transverse voltages in an inclined field ($\theta\neq\pi/2$) have long been observed,[8] and experimental results fully agree with theoretical predictions. Let us recall them briefly, for comparison with the unexpected and contrasted behavior of transverse fields above H_{c2}. Except for the intricate situation where $\mathbf{B}$ is aligned with the applied current ($\theta\to 0$), bulk flux flow in the mixed state is well understood. Vortex lines (density n, direction $\boldsymbol{\nu}$), defined as the lines on which the order parameter ψ vanishes, lie along the direction of the magnetic field $\mathbf{B}$ so that

$$\mathbf{B}=n\phi_0\boldsymbol{\nu}\ , \tag{2}$$

and all theories of vortex motion state that, under stationary (and standard) conditions,[6]

$$\mathbf{E}'=\mathbf{E}=-\mathbf{v}_L\times\mathbf{B}\ , \tag{3}$$

$$\mathbf{E}=\rho_f(B)\,\mathbf{J}_{2\perp}\ , \tag{4}$$

where ρ_f is the flux-flow resistivity, and $\mathbf{J}_{2\perp}$ is the component of the bulk current normal to vortex lines. In Eqs. (2) and (3), $\mathbf{B}$ stands for the field *inside* the sample. Due to magnetization, $\mathbf{B}$ differs from the applied external field $\mathbf{B}_0=\mu_0\mathbf{H}_0$. Note, however, that, in the parallel flat slab geometry, $\mathbf{B}$ and $\mathbf{B}_0$ remain parallel, and $B=\text{const}(\theta)$. Equation (3) alone prescribes a strong constraint on the direction of the electric field, since $\mathbf{E}$ must be normal to $\mathbf{B}$:

$$E_y=-E_x/\tan\theta\ . \tag{5}$$

If $\mathbf{J}_2$ is assumed to flow in the x direction, we have

$$E_x=E\,\sin\theta=\rho_f J_2\sin^2\theta\ . \tag{6}$$

As $\rho_f = \text{const}\ (\theta)$ in the geometry used, measurements of the apparent flux-flow resistivity, E_x/J_2 or dV_x/dI, for $10° \lesssim \theta < 90°$, entirely confirmed the predicted $\sin^2\theta$ dependence. As $\theta \to 0$, too large critical currents prevented us from observing a linear flux-flow regime.

It is noteworthy that the flux-flow constraint (5) must hold so long as some vortex structure exists. If, for instance, $\theta = \pi/4$, $E_y = -E_x$, so that $\mathbf{E}$ makes a constant $\pi/4$ angle with the applied current *up to* H_{c2}, despite the fact that the sample is approaching the normal state continuously ($\psi \to 0$ and $\mathbf{B} \approx \mathbf{B}_0$). Equation (3) breaks down at H_{c2}, and $\mathbf{E}$ is expected to line up suddenly with the direction of the applied current as it should be in the normal state, giving rise to jumps in longitudinal and transverse voltages. That is just what we observed. As shown in Fig. 3, when H_0 is increased at constant I, $|V_y|$ first increases (in the same ratio as V_x), and falls off abruptly at H_{c2} (over about 20–30 G). However, significant transverse voltages persist above H_{c2}, according to a mechanism to be determined. Whatever it may be, it has to be ascribed to surface superconductivity: nickel plating both faces of the slab indeed makes V_y vanish above H_{c2} within experimental accuracy. It is to be noted that, because H_{c2} is very sensitive to the indium concentration, unannealed samples may exhibit a blurred transition over a few hundred G (see, for example, the data of Ref. 8). In spite of a narrow liquidus-solidus range, careful annealing is required to make the alloy composition uniform. In practice, we found that a sharp decrease of $|V_y|$ at H_{c2} was the best way of testing bulk homogeneity, ensuring that the whole bulk sample has entered the normal state.

Let us now examine the outstanding features of transverse voltages above H_{c2}, as compared with those observed below H_{c2}. First of all, V_y *changes sign* ($V_y > 0$ in the case of Fig. 2, where $I = I_x > 0$). Moreover, while the longitudinal characteristic $V_x(I)$ has the common shape, V_x increasing linearly at large currents, the transverse characteristic $V_y(I)$ shows *a flat plateau* (Fig. 4). This implies that, at given θ, the direction of the electric field is now current and field dependent: E_y/E_x may take any positive value, though rapidly decreasing with increasing I or B ($I \to \infty$ or $B \to B_{c3}$, $E_y/E_x \to 0$).

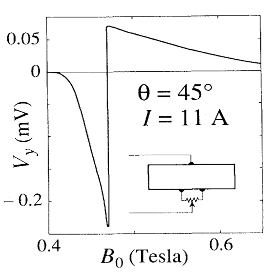

FIG. 3. The transverse voltage V_y as function of the magnetic field at $T = 1.8$ K, and constant current $I = 11$ A. Orientations of the applied magnetic field and current are those shown in Fig. 2. Voltage contacts are carefully aligned perpendicular to the direction of the applied current, or else a potentiometer set up is used as shown in the inset. Anyway, it is advisable to verify that V_y, contrary to V_x, is an odd function of θ. V_x and V_y are both even functions of B, and odd functions of I. Negative transverse voltages below H_{c2} are fully explained by usual flux-flow equations. The unexpected observation of positive transverse voltages above H_{c2} are the subject of this paper. The abruptness of the voltage jump at H_{c2}, over about 20–30 G, attests a good homogeneity in indium concentration after annealing.

Seeing that a vanishing vortex structure near H_{c2} still forces the orientation of $\mathbf{E}$, we are not surprised that the superconducting sheath, thin though it may be, can affect the electric field direction. Moreover, referring to steady standard conditions, where $\mathbf{E}$ is uniform throughout the sample, we are aware that any constraint on $\mathbf{E}$ in the sheath must extend to the bulk. But the only reliable equation that presumably applies to any systems involving flux line motion (see Sec. I), $\mathbf{E} = -\mathbf{v}_L \times \mathbf{B}$, happens to be inconsistent with our experimental results above H_{c2}. In particular, E_y has the wrong sign. Whatever way we turn to interpret voltages along the surface sheath, we first have to remove this difficulty, unless we renounce surface vortices.

As far as its main ideas are readily extended to the surface vortex state, the phenomenological MS theory of the mixed state provides a satisfactory answer. The originality of the MS theory essentially lies in realizing from the outset[5] that (i) at equilibrium, vortices must terminate perpendicular to the surface sample; (ii) Eq. (3) is not of general validity, and holds only for curlfree (in particular, in the absence of) supercurrents. In a thermodynamic treatment of the mixed state, regarded as a continuum, the vector $\boldsymbol{\omega} = n\phi_0\boldsymbol{\nu}$, which describes the local density and direction of vortex lines, and the macroscopic magnetic field $\mathbf{B}$ must be considered as local independent variables: for a given value of $\boldsymbol{\omega}$ at some point M, $\mathbf{B}(M)$ still may be varied by any change in the distribution of currents elsewhere. Most of equilibrium and transport problems may be reexamined from this point of view. Consider, for example, the equilibrium of a perfect cylinder or sphere in an external field $\mathbf{B}_0$ (see Fig. 2 in Ref. 6): it is found that vortex lines curve in over a small depth near the surface to end normal to the boundary, while field lines in this perturbed layer are bent *in the opposite direction.* A condition for the local equilibrium of vortices[5,6] requires that supercurrents to be associated with the local distortion of the vortex array, while $\mathbf{J} \simeq 0$ and $\boldsymbol{\omega} \simeq \mathbf{B}$ in the bulk. These are nothing but the Meissner-like diamagnetic currents. Now, consider the

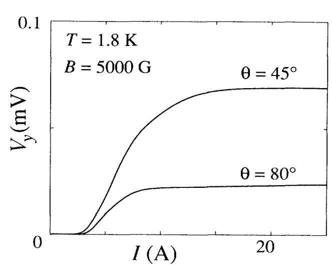

FIG. 4. Transverse voltage-current characteristics, $V_y(I)$, above H_{c2}, showing the voltage saturation at large currents. No transverse voltage is observed for $\theta = 0°$ and $\theta = 90°$. The effect has a maximum for $\theta \lesssim 45°$.

case of a rough slab in perpendicular field: there are no diamagnetic currents. But in the presence of surface roughness on a scale comparable to or smaller than the vortex spacing, there are many ways for the vortices to end normal to the actual surface, allowing for a large number of metastable or nondissipative solutions.[6] Associated supercurrents in this case may well appear as nondissipative transport currents $\mathbf{J}_1$. Again, across a thin surface layer, vortex lines (i.e., the singular lines $\psi=0$) strongly deviate from magnetic field lines, and $\boldsymbol{\omega}\neq\mathbf{B}$ (see Fig. 3 of Ref. 6).

Using a standard rigorous method, MS derived a complete set of transport equations. In particular, under stationary conditions, the equation [Eq. (39) of Ref. 6]

$$(\mathbf{E}'\simeq)\mathbf{E}=-\mathbf{v}_L\times\boldsymbol{\omega} \tag{7}$$

is obtained as a straightforward consequence of conservation laws.[5,6] Equation (7) should be substituted into Eq. (3) for more general applications.

It should be noted that the normal ending of vortices at the sample surface is consistent with (required by) the Ginzburg-Landau boundary condition, $\partial|\psi|/\partial n=0$, where $\partial/\partial n$ is the normal derivative at the surface. Kulik's vortices do not make an exception to this rule, though this is not pointed out by the author. By simple inspection of Kulik's solution for the order parameter in the surface sheath, it is seen that the lines $\psi=0$ are indeed normal to the surface, whatever the angle α between surface and field may be. Since surface vortices are very short and melt away rapidly in the normal bulk, they cannot change direction through the surface sheath. According to Eq. (1) the vortex density $n=a^{-2}$, and thus ω, are α dependent, so that ω should be a highly variable function of position, in particular ω_z changes sign, along a rough xy face of the slab. The vector $\boldsymbol{\omega}$, however, keeps close to the z direction. Therefore, it is clear that any direction of the electric field in the xy plane becomes compatible with the new transport equation (7).

IV. ANISOTROPY OF SURFACE CRITICAL CURRENTS: A MODEL OF TRANSVERSE VOLTAGES ABOVE H_{c2}

Due to Ohm's law, $\mathbf{J}_2=\sigma_n\mathbf{E}$, the existence of a transverse field $E_y>0$ at large currents implies that of a transverse normal current $J_{2y}=\sigma_n E_y>0$. This is a marked difference with bulk currents $\mathbf{J}_2$ in the mixed state, which are flowing in the x direction. Since no net current can flow in the y direction ($I_y=0$), surface currents must flow on both faces in the negative y direction so that

$$2i_{1y}+tJ_{2y}=0\ , \tag{8}$$

while the applied current $I>I_c$ is

$$I=I_x=W(2i_{1x}+tJ_{2x})\ , \tag{9}$$

where $I_c=2Wi_{1x}>0$ and $J_{2x}>0$. Therefore, for large currents, the surface current density $\mathbf{i}_1$ makes an angle with the x direction, except for $\theta=0$ and $\pi/2$. If we succeed in explaining the clockwise rotation of $\mathbf{i}_1$, we should state, conversely, that a normal current must result from the backflow of the transverse surface currents, giving rise to transverse voltages:

$$V_y=-2Wi_{1y}/t\sigma_n\ . \tag{10}$$

For $\theta=0$ and $\pi/2$, no transverse voltages are observed, as expected from symmetry considerations. This fact warrants that no preferred direction exists on the surface except for that of the magnetic field itself. For $I>I_c$,

$$i_1=i_{1x}=I_c/2W\ . \tag{11}$$

As discussed in Sec. II, we only have access, in actual samples, to average values of critical currents between voltage probes, and all current densities i_1 and J_2 in the above standard equations should be replaced by appropriate mean values. Nevertheless, as we systematically observed that I_c ($\theta=\pi/2$) were 20–30 % larger than I_c ($\theta=0$), we may reasonably conclude that surface critical currents are locally *anisotropic:* the maximum current density $i_1=i_c$, that the surface is capable of carrying without dissipation, depends on the angle φ between $\mathbf{i}_1$ and $\mathbf{B}$. Thus, an ideal standard sample would be characterized, for given values of T and B, by a critical curve $i_c(\varphi)$ such as sketched in Fig. 5; extreme values $i_c(\pi/2)=a>b=i_c(0)$ correspond to $I_c(\theta=\pi/2)=2Wa$ and $I_c(\theta=0)=2Wb$, respectively.

Once we have accepted the anisotropy of surface critical currents as an experimental evidence, it is a simple matter to establish the connection between this anisotropy and the observed behavior of the transverse voltage-current characteristics above H_{c2}. For this purpose we still refer to simple standard conditions, where the critical state of the surface sheath, at any point of the surface, is uniformly described by the same curve $i_c(\varphi)$. Then we make the reasonable assumption that transport currents are so distributed as to minimize the total power input $V_xI_x=V_xI$: at I constant, this means V_x minimum, in particular, $V_x=0$ as far as allowed by critical properties of the surface. Low nondissipative surface currents can flow in the x direction up to $i_{1x}=i_c(\theta)=OM$ (Fig. 5). The transport current $I=2Wi_{1x}$ can be further in-

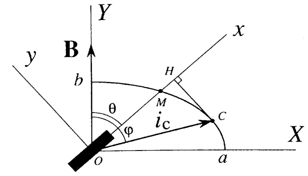

FIG. 5. Graphical construction of the critical state of the surface sheath in inclined fields. As shown by experiment, the critical surface current density $i_c(A/m)$ is anisotropic, and depends on the angle φ between the direction of the magnetic field and that of the surface current density $\mathbf{i}_1$. Assuming homogeneous surface conditions (referred to as standard conditions in the text), any point of the xy faces should be characterized by the same theoretical curve $i_c(\varphi)$. $\mathbf{i}_i=\mathbf{OC}$ represents the current density in the critical state, defined as that state achieving the maximum transport current i_{1x} for zero longitudinal voltage.

creased, while maintaining $V_x=0$, provided that the surface current $\mathbf{i}_1$ rotates clockwise along the critical curve. When $\mathbf{i}_1=\mathbf{OC}$ (Fig. 5), the critical current is attained: $I=I_c(\theta)=2W\,OH$. Any excess current $I-I_c$ will appear as bulk normal currents $\mathbf{J}_{2x}>0$. As $\mathbf{i}_1$ is rotating, a transverse voltage V_y arises, which is associated to the normal backflow in accordance with Eq. (10). V_y comes to saturation for $i_{1y}=HC$ (Fig. 5). The curve traced out by the vector $\mathbf{OH}$, from A to B, is the so-called pedal curve of the curve $i_c(\varphi)$. If the critical curve is approximated by an ellipse with semiaxes a and b, it is easily shown that HC has a maximum $a-b$ near $\theta=\pi/4\,[\tan\theta=(b/a)^{1/2}]$. Hence, Eq. (10) yields the maximum transverse voltage to be expected:

$$V_{y\,\max}=(2W\,HC_{\max})/t\sigma_n \simeq 2W(a-b)/t\sigma_n=\Delta I_c/t\sigma_n \ , \qquad (12)$$

where $\Delta I_c=I_c(\pi/2)-I_c(0)$. $V_{y\,\max}$ can be estimated by using the experimental mean values of critical currents; taking, for instance, data from Figs. 1 and 4 at 5000G, we find $\Delta I_c=0{,}7$ A and, from Eq. (12), $V_{y\,\max}=71\ \mu$V, which is close to the measured saturation value for $\theta=\pi/4$ (Fig. 4). In view of the dispersion of critical currents in actual samples, perhaps such a quantitative agreement is fortuitous, but the predicted order of magnitude remains significant.

This model well accounts for the main features of transverse voltages above H_{c2}: sign, amplitudes, and saturation at large currents. This could have been our conclusion, but we cannot evade a last difficulty, in connection with the transverse current distribution implied by our interpretation. Before coming to this point, let us report the result of an experiment designed to confirm the presence of the transverse superconducting-normal counterflow.

A pickup coil consisting of n turns of wire was wound close to the slab surface as shown in Fig. 2. Surface currents i_{1y} on both faces, and return normal currents J_{2y}, form double current loops in the yz planes. By symmetry, the magnetic flux through the coil as a result of these transverse currents is zero. By nickeling one face of the slab, the superconducting sheath is destroyed on this face, so that transverse currents i_{1y} and J_{2y} now form a single loop: I_c and V_y are reduced by a factor of 2, but a magnetic flux $(\sim n\mu_0 i_{1y}Wt)$ links the coil. If then the sample is driven by a modulated current $I+I^*e^{i\omega t}$ along the rising part of the $V_y(I)$ characteristic, V_y and i_{1y} are modulated accordingly, and we expect an induced emf in the coil, which we estimate roughly as

$$(\text{pickup signal})e\sim\omega n\mu_0 i^*_{1y}Wt\sim\omega n\mu_0 V^*_y t^2\sigma_n \ . \qquad (13)$$

The frequency ω is low enough to ensure a quasistatic modulation of the characteristics, as checked experimentally. With $I^*\simeq0.4$ A, $\omega/2\pi=130$ Hz, $n=200$ turns, and data of Figs. 1 and 4, Eq. (13) predicts $e\sim1-10\ \mu$V, a signal readily measured by a lock-in amplifier. We did observe such a signal under the required conditions. That is, at $I^*=$const, the signal vanishes, as expected, in the normal state, *in the mixed state*, and, immediately above H_{c2}, for either subcritical or large currents (as V_y is saturated). It vanishes also at any current and field, when either no face or both faces are nickel plated.

Let us return to the graphic construction of the transverse current in a standard sample (Fig. 5). As $\mathbf{i}_1$ rotates from $\mathbf{OM}$ to $\mathbf{OC}$, V_y increases, while $V_x=0$. Then, as I is further increased, $V_y=$const, while V_x starts rising. Indeed, we are prevented from observing such an ideal behavior, for the critical-current dispersion makes both $V_x(I)$ and $V_y(I)$ characteristics spread out over a large current interval (Figs. 1 and 4). Nevertheless, we are entitled to consider the standard case as being physically possible. Thus, within a short current interval $(OM<I/2W<OH)$, $\mathbf{E}$ may be a purely transverse field $(E_x=0)$. Under these conditions, the bulk normal Joule effect $\mathbf{E}\cdot\mathbf{J}_2$ turns out to be exactly balanced by a *negative surface Joule effect,* $\mathbf{E}\cdot\mathbf{i}_1=E_y i_y<0$, associated with vortex flow in the surface sheath. Note that, according to Eq. (7), electric field and vortex motion are inseparable. This is a most unusual situation. In dc experiments we are not accustomed to obtain circulating steady currents in an imperfect conductor without needing the emf of some generator to keep them going. Note, however, that no physical law is violated. As pointed out at the end of Sec. II, the difficulty lies elsewhere, in connection with the usual interpretation of terms such as $\mathbf{E}\cdot\mathbf{i}_1$ in the Joule effect.

Consider an operating point (I,V) in the flux-flow regime along any longitudinal characteristic, either below or above H_{c2}. VI represents the electrical power input to the sample. In steady conditions, energy is transferred at the same rate to the heat reservoir, as required by the law of energy conservation. It is also generally stated that VI is the rate at which energy is *dissipated* within the portion of circuit considered. For instance, the part $V(I-I_c)=V^2/R_f$ of the Joule effect, in the mixed state, is clearly associated with the viscous vortex flow: the dissipation rate by unit length of vortex can be written as $\eta\mathbf{v}_L^2$, where $\eta=B\phi_0/\rho_f$ is a viscosity coefficient.[15] The remaining part of the Joule effect, $VI_1=VI_c$, which is the integral sum of terms such as $\mathbf{E}\cdot\mathbf{i}_1$, is not so easily accounted for (including above H_{c2}), but it is usually thought of as resulting from the same damping mechanism as $\eta\mathbf{v}_L^2$: elastic instabilities at the pinning sites should give large local fluctuations of $\mathbf{v}_L$, so that $\overline{\mathbf{v}_L^2}>(\overline{\mathbf{v}}_L)^2$, where $\overline{\mathbf{v}}_L=E/B$ stands for the mean velocity of the vortex array. Thus, $\eta(\overline{\mathbf{v}_L})^2$ should correspond to $V(I-I_c)$, while $\eta\delta\mathbf{v}_L^2$ should be responsible for the extra dissipation VI_c. Though very ingenious, this model is not entirely satisfactory,[16] but that is not the point.

As argued in Sec. II, there is no reason to believe that the mechanism underlying the Joule effect VI_c is different above and below H_{c2}. Now, if $\mathbf{E}\cdot\mathbf{i}_1$ has to be associated with some dissipative mechanism, it must be *positive,* a condition which is inconsistent with the intermediate regime described above. The model for critical currents and surface Joule effect in soft materials,[5,6] that proceeds from the MS theory of transport, avoids this further paradox.

In conclusion, we briefly recall the relevant arguments of the MS theory with regard to pinning and dissipation in soft samples. The current density **J** is separated into two parts, as $\mathbf{J}_1+\mathbf{J}_2$, where $\mathbf{J}_1=-\mathrm{curl}\varepsilon\boldsymbol{\nu}$ is the *nondissipative part* of the supercurrent. Here $\varepsilon(\omega)$ is a "vortex potential" closely related to the reversible magnetization curve.[6] $\mathbf{J}_1$ is defined at any point and time as the supercurrent (including diamagnetic currents) that would come into equilibrium with the vortex array in its instantaneous configuration. $\mathbf{J}_1$ does not enter into the dissipative function, but it may contribute to transport currents. As stated in Sec. III, in the presence of surface irregularities, there are many vortex configurations allowing nondissipative transport currents $\mathbf{J}_1$ to flow close to the surface. On the scale of the sample these currents can be regarded as surface currents $\mathbf{i}_1$. The instantaneous critical current *is defined* as the flux I_1 of $\mathbf{J}_1$ through a cross-section averaged over the measured length of the sample. Small fluctuations of I_1 ($\sim 10^{-3}-10^{-4}$), resulting from stronger local fluctuations, are responsible for the flux-flow noise.[17] Critical-current data,[18] as also a number of old or more recent experiments,[6,17] have confirmed this point of view. Now, an important convective term $\varepsilon(\omega)\omega\mathbf{v}_L$ appears in the general expression of the *energy flux*.[6] In usual flux-flow regimes, because of the mean steady bending of the vortex lines near the surface, the surface normal component of $\varepsilon\omega\mathbf{v}_L$ is pointing outwards, and contributes to the heat ejected to the surrounding medium (see Fig. 4 of Ref. 6). In any circumstances, the net outward flux of energy due to the term $\varepsilon\omega\mathbf{v}_L$ is just VI_1, but clearly, nothing in this description requires it to be necessarily positive.

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