Inhibition of the dynamo effect by phase fluctuations

2006 Europhys. Lett. 76 602

(http://iopscience.iop.org/0295-5075/76/4/602)

EUROPHYSICS LETTERS 15 November 2006

Europhys. Lett., **76** (4), pp. 602–608 (2006)
DOI: 10.1209/epl/i2006-10313-4

Inhibition of the dynamo effect by phase fluctuations

F. Pétrélis and S. Fauve

Laboratoire de Physique Statistique de l'Ecole Normale Supérieure, CNRS UMR 8550
24 rue Lhomond, 75005 Paris, France

received 7 June 2006; accepted in final form 20 September 2006
published online 18 October 2006

PACS. 47.65.-d – Magnetohydrodynamics and electrohydrodynamics.
PACS. 47.20.-k – Flow stabilities.

Abstract. – We study the effect of velocity fluctuations on the generation of magnetic field by a cellular flow of an electrically conducting fluid. When the magnetic field can grow at large scale compared to the scale of the flow (limit of scale separation), the onset of dynamo action is analytically predicted. Whereas an amplitude modulation of the flow may either increase or decrease the dynamo threshold, we show that fluctuations of the phase of the cellular flow always inhibit the dynamo process.

Introduction. – The generation of magnetic field by the flow of an electrically conducting fluid, *i.e.* the dynamo process, was first studied to understand magnetic fields of stars or of the Earth and other planets [1]. More recent aspects also concern the applicability of dynamo theory to explain galactic and extra-galactic fields [2]. The main difficulty in studying these problems results from the high level of turbulence of the involved flows $\boldsymbol{v}(\boldsymbol{r}, t)$. Several strongly different scales make realistic direct numerical simulations impossible and we have not yet rigorous analytical tools to handle a bifurcation problem from a fully turbulent flow regime. Two recent experimental observations of fluid dynamos have displayed a very interesting feature: the observed dynamo threshold was found in good agreement with the one computed as if the mean flow $\overline{\boldsymbol{v}}(\boldsymbol{r})$ were acting alone, *i.e.* neglecting turbulent fluctuations $\tilde{\boldsymbol{v}}(\boldsymbol{r}, t) = \boldsymbol{v}(\boldsymbol{r}, t) - \overline{\boldsymbol{v}}(\boldsymbol{r})$ [3, 4]. We have suggested that this results from the strong confinement of these flows by solid boundaries. Then, the flow consists of small-scale turbulent fluctuations superposed on a stationary large-scale mean flow. We have shown that in such cases, there is no shift in threshold of the dynamo generated by the mean flow at first order in the fluctuation level [5]. There is no shift at second order either if the fluctuations have no helicity. Recent numerical simulations [6] have displayed agreement with our prediction.

When a turbulent flow is not externally confined and thus can develop in a fully three-dimensional way, it is well known that its r.m.s. velocity fluctuations are of integral scale, *i.e.* comparable to the largest velocity scale [7]. The above argument is then of little help to predict a dynamo threshold for the full velocity field $\boldsymbol{v}(\boldsymbol{r}, t)$ after having computed the one of $\overline{\boldsymbol{v}}(\boldsymbol{r})$. Indeed, no method presently exists to relate the threshold of $\boldsymbol{v}(\boldsymbol{r}, t)$ to the one of $\overline{\boldsymbol{v}}(\boldsymbol{r})$ when the fluctuations are not small compared to the mean flow. In non-confined flows, some large fluctuations are related to the erratic motion of large eddies. Instead of using Reynolds decomposition, $\boldsymbol{v}(\boldsymbol{r},t) = \overline{\boldsymbol{v}}(\boldsymbol{r}) + \tilde{\boldsymbol{v}}(\boldsymbol{r},t)$, it is then tempting to model this type of disturbances writing $\boldsymbol{v}(\boldsymbol{r},t) = \overline{\boldsymbol{v}}[\boldsymbol{r} + \boldsymbol{s}(\boldsymbol{r},t)] + \tilde{\boldsymbol{u}}(\boldsymbol{r},t)$, thus keeping into the mean field the motion of the large eddies. In the language of cellular flows, $\boldsymbol{s}(\boldsymbol{r},t)$ represents phase perturbations.

We show here using simple examples in the context of mean-field magnetohydrodynamics, that large-scale fluctuations due to random displacement of eddies within a cellular flow (phase fluctuations), always increase the dynamo threshold, whereas fluctuations of the amplitude of the velocity field can shift the threshold in both directions.

Amplitude modulation. – We consider the flow of an electrically conducting fluid with velocity field $\boldsymbol{v}(\boldsymbol{r},t)$,

$$\boldsymbol{v} = \boldsymbol{v}_0(y,z) + \boldsymbol{v}_f(y,z,t) = \begin{pmatrix} V\left(\cos(ky) - \cos(kz)\right)\left(1 + \delta_v \cos(\omega_v t + \phi_v)\right) \\ U \sin(kz)\left(1 + \delta_u \cos(\omega_u t + \phi_u)\right) \\ U \sin(ky)\left(1 + \delta_u \cos(\omega_u t + \phi_u)\right) \end{pmatrix}. \tag{1}$$

The constant part of this flow, $\boldsymbol{v}_0$ ($\delta_u = \delta_v = 0$), is the G. O. Roberts' flow [8]. It consists of a square periodic array of counter-rotating eddies in the y-z plane, with axial flow in each of them such that all vortices have helicity of the same sign (we take $U > 0$, $V > 0$). This flow is close to the time-averaged flow of the Karlsruhe experiment [3]. Such a flow is a quite efficient dynamo because a large-scale magnetic field can be generated by an alpha-effect [9]. Indeed, scale separation enables the magnetic field to grow even if $R_m = \|\boldsymbol{v}\|/(\eta k)$, the magnetic Reynolds numbers at the eddy scale, is small. When R_m is large, recent numerical simulations have displayed large variations of the alpha effect which can no longer be simply related to the helicity of the velocity field [10]. In the following we restrict ourselves to the limit of scale separation. Then, R_m is small at dynamo onset so that analytical progresses can be performed.

We first study the effect of the amplitude modulation $\boldsymbol{v}_f$ on the dynamo threshold calculated with $\boldsymbol{v}_0$. Note that the modulation of the flow is made with different amplitudes δ_u, δ_v, frequencies ω_u, ω_v and phases ϕ_u, ϕ_v for the axial (along the x-axis) and the azimuthal velocity (with y and z components).

Following Roberts, we define the average

$$\langle f \rangle = \frac{1}{L^3 T} \int f(x,y,z,t)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}t\,, \tag{2}$$

where the integration is performed on a volume L^3 and on a time T such that L is larger than any length scale of the flow and T is longer than any time scale of the flow. Assuming scale separation between mean and fluctuating fields, we write the magnetic field as

$$\boldsymbol{B} = \langle \boldsymbol{B} \rangle + \boldsymbol{b}\,. \tag{3}$$

The induction equation gives for the mean and fluctuating magnetic fields

$$\partial_t \langle \boldsymbol{B} \rangle = \nabla \times \langle \boldsymbol{v} \times \boldsymbol{b} \rangle + \eta \nabla^2 \langle \boldsymbol{B} \rangle\,, \tag{4}$$

$$\partial_t \boldsymbol{b} = \nabla \times (\boldsymbol{v} \times \langle \boldsymbol{B} \rangle) + \nabla \times (\boldsymbol{v} \times \boldsymbol{b} - \langle \boldsymbol{v} \times \boldsymbol{b} \rangle) + \eta \nabla^2 \boldsymbol{b}\,. \tag{5}$$

where $\eta = (\mu_0 \sigma)^{-1}$ is the magnetic diffusivity. Since the mean magnetic field evolves at a larger scale L than the velocity field ($kL \gg 1$), the latter equation can be written as

$$\partial_t \boldsymbol{b} - \eta \nabla^2 \boldsymbol{b} = \langle \boldsymbol{B} \rangle \cdot \nabla \boldsymbol{v}\,, \tag{6}$$

provided that $\boldsymbol{b} \ll \langle \boldsymbol{B} \rangle$.

The small-scale magnetic field is thus linear in the velocity field so that we write $\boldsymbol{b} = \boldsymbol{b}_0 + \boldsymbol{b}_f$ where $\boldsymbol{b}_0$ is created by $\boldsymbol{v}_0$ and $\boldsymbol{b}_f$ by $\boldsymbol{v}_f$. The forced solution of eq. (6) is

$$\boldsymbol{b}_0 = \frac{1}{\eta k} \begin{pmatrix} V\left(\langle B_z \rangle \sin(kz) - \langle B_y \rangle \sin(ky)\right) \\ U \langle B_z \rangle \cos(kz) \\ U \langle B_y \rangle \cos(ky) \end{pmatrix} . \tag{7}$$

The components of $\boldsymbol{b}_f$ can be written

$$b_{f\,i} = A_i b_{0\,i} \,, \tag{8}$$

with A_i defined by

$$\begin{aligned} A_x &= \delta_v \frac{\eta k^2}{\omega_v^2 + (\eta k^2)^2} \left(\omega_v \sin(\omega_v t + \phi_v) + \eta k^2 \cos(\omega_v t + \phi_v)\right) , \\ A_y &= A_z = \delta_u \frac{\eta k^2}{\omega_u^2 + (\eta k^2)^2} \left(\omega_u \sin(\omega_u t + \phi_u) + \eta k^2 \cos(\omega_u t + \phi_u)\right) . \end{aligned}$$

We now have to calculate the average $\langle \boldsymbol{v} \times \boldsymbol{b} \rangle$. It can be written as

$$\langle \boldsymbol{v} \times \boldsymbol{b} \rangle = \langle \boldsymbol{v}_0 \times \boldsymbol{b}_0 \rangle + \langle \boldsymbol{v}_f \times \boldsymbol{b}_0 \rangle + \langle \boldsymbol{v}_0 \times \boldsymbol{b}_f \rangle + \langle \boldsymbol{v}_f \times \boldsymbol{b}_f \rangle . \tag{9}$$

The first term is related to the alpha-effect of the basic flow. Indeed we have

$$\langle \boldsymbol{v}_0 \times \boldsymbol{b}_0 \rangle = \alpha_0 \begin{pmatrix} 0 \\ \langle B_y \rangle \\ \langle B_z \rangle \end{pmatrix} , \tag{10}$$

where $\alpha_0 = -UV/(\eta k)$ relates the average of the electromotive force to the averaged magnetic field. The second and third terms are zero because they are long-time averages of the products of a constant with a fluctuating term. The fourth term is

$$\langle \boldsymbol{v}_f \times \boldsymbol{b}_f \rangle = \alpha_f \begin{pmatrix} 0 \\ \langle B_y \rangle \\ \langle B_z \rangle \end{pmatrix} , \tag{11}$$

where

$$\alpha_f = -\frac{\eta k^3 U V \delta_u \delta_v}{2(\omega_u^2 + (\eta k^2)^2)} \delta(\omega_u, \omega_v) \cos(\phi_u - \phi_v) \tag{12}$$

and $\delta(\omega_u, \omega_v)$ is zero if $\omega_u \neq \omega_v$ and is one if $\omega_u = \omega_v$. We can relate α_f to α_0 by

$$\alpha_f = \alpha_0 \frac{(\eta k^2)^2 \delta_u \delta_v}{2(\omega_u^2 + (\eta k^2)^2)} \delta(\omega_u, \omega_v) \cos(\phi_u - \phi_v) . \tag{13}$$

We now look for unstable modes of the form $\langle \boldsymbol{B} \rangle = \mathcal{B} e^{iKx}$, where K is supposed to be small compared to k such that the averaged magnetic field evolves on a larger scale than the velocity field. Writing each component of eq. (4), we find that $\mathcal{B}_y + i\mathcal{B}_z$ becomes neutral if

$$\left| \frac{\alpha_0 + \alpha_f}{\eta K} \right| = 1 . \tag{14}$$

If the modulation of the velocity field is assumed to be small, the onset is at lowest order

$$\frac{UV}{\eta^2 kK} = 1 - \delta_u \delta_v \frac{(\eta k^2)^2}{2(\omega_u^2 + (\eta k^2)^2)} \cos(\phi_u - \phi_v) \delta(\omega_u, \omega_v) \,. \tag{15}$$

At this order in the expansion, there is a shift in the onset only if the modulations have the same pulsation. The shift can be positive or negative and its sign is determined by that of $\delta_u \delta_v \cos(\phi_u - \phi_v)$. For in-phase modulations, the onset is lowered, whereas it is increased if the pulsations are out of phase. This effect can be understood by evaluating the helicity of the fluctuating field. If it has the same sign as the basic flow, the alpha effects cooperate and the onset is lowered. In contrast, if the helicities have opposite signs, the onset is increased. The amplitude of the shift decreases with the frequency of the modulation. It varies like $(\omega^2/(\eta k^2)^2 + 1)^{-1}$. Note that this result is different from the one obtained for a modulated Ponomarenko dynamo [11].

Phase fluctuations. – Up to now, we dealt with a modulation of the cellular flow uniform in space and coherent in time. In contrast, it is expected that turbulent fluctuations act randomly in time and space to modify the cellular flow. In the following we investigate the effect on the dynamo onset of random phase perturbations of the cellular flow. To wit, we successively study two Roberts flows modified by phase fluctuations. The first case is a time-dependent phase fluctuation and we write the velocity field as

$$\boldsymbol{v} = \begin{pmatrix} V\left(\cos(ky+\phi) - \cos(kz+\psi)\right) \\ U \sin(kz+\psi) \\ U \sin(ky+\phi) \end{pmatrix}, \tag{16}$$

where ψ and ϕ are two random functions that depend on time only. This amounts to switching randomly in time the origin of the flow.

The averaged and fluctuating magnetic fields are solution of equations (4), (5). Assuming that $\boldsymbol{b} \ll \boldsymbol{B}$, the fluctuating magnetic field is solution of eq. (6). After calculating $\langle \boldsymbol{B} \rangle \cdot \nabla \boldsymbol{v}$, the latter equation is transformed into an ordinary differential equation by Fourier transforming on the y and z variables. We obtain the solution for $\boldsymbol{b}$ in the form

$$\boldsymbol{b} = \begin{pmatrix} Vk\left(\langle B_z \rangle \mathcal{L}(\sin(kz+\psi)) - \langle B_y \rangle \mathcal{L}(\sin(ky+\phi))\right) \\ Uk \langle B_z \rangle \mathcal{L}(\cos(kz+\psi)) \\ Uk \langle B_y \rangle \mathcal{L}(\cos(ky+\phi)) \end{pmatrix}, \tag{17}$$

where $\mathcal{L}$ is a linear operator defined by

$$\mathcal{L}(f) = e^{-\eta k^2 t} \int_0^t e^{\eta k^2 t'} f(t') \mathrm{d}t' \,. \tag{18}$$

We then compute the alpha effect $\langle \boldsymbol{v} \times \boldsymbol{b} \rangle$. Here again, the average is performed on time and space assuming that there is a time and length scale separation between the flow and the averaged magnetic field scales. With this assumption, we get an effective alpha effect:

$$\langle \boldsymbol{v} \times \boldsymbol{b} \rangle = -UVk \begin{pmatrix} 0 \\ \langle B_y \rangle \left\langle \int_0^t e^{-\eta k^2 (t-t')} \cos(\phi(t) - \phi(t')) \mathrm{d}t' \right\rangle \\ \langle B_z \rangle \left\langle \int_0^t e^{-\eta k^2 (t-t')} \cos(\psi(t) - \psi(t')) \mathrm{d}t' \right\rangle \end{pmatrix}. \tag{19}$$

Assuming the gradients to be small, *i.e.* $\partial_t\psi/(\eta k^2) \ll 1$ and $\partial_t\phi/(\eta k^2) \ll 1$, the integrals can be expressed in term of time derivatives of the phases,

$$\langle \boldsymbol{v} \times \boldsymbol{b} \rangle = -\frac{UV}{\eta k} \begin{pmatrix} 0 \\ \langle B_y \rangle \,(1 - \frac{1}{\eta^2 k^4} \langle (\partial_t \phi)^2 \rangle) \\ \langle B_z \rangle \,(1 - \frac{1}{\eta^2 k^4} \langle (\partial_t \psi)^2 \rangle) \end{pmatrix} . \tag{20}$$

We first remark that this effective alpha effect is smaller than the alpha effect of the unmodulated flow. Therefore, the dynamo onset is postponed to

$$\frac{UV}{\eta^2 kK} = 1 + \frac{1}{\eta^2 k^4} \langle (\partial_t \phi)^2 \rangle + \frac{1}{\eta^2 k^4} \langle (\partial_t \psi)^2 \rangle . \tag{21}$$

Equation (21) is obtained when the phase fluctuations are random in time but act coherently in space. We now turn to a space-dependent phase that drives a random detuning between the cells of the flow. Indeed we expect that one of the effects of turbulence on a periodic flow will be to reduce the power spectrum density of the velocity field at wavenumber k. Random fluctuations acting on the phase are a possible though rough model of this effect. To investigate this situation, we consider a Roberts flow for which the origin of the cellular flow depends randomly on the axial coordinate and write

$$\boldsymbol{v} = \begin{pmatrix} V\,(\cos\,(ky + \phi) - \cos\,(kz + \psi)) \\ U\,\sin\,(kz + \psi) - \frac{V}{k} \partial_x \phi \cos\,(ky + \phi) \\ U\,\sin\,(ky + \phi) + \frac{V}{k} \partial_x \psi \cos\,(kz + \psi) \end{pmatrix} , \tag{22}$$

where ϕ and ψ are functions of x only. Derivatives of the phases appear explicitly in the expression of the velocity in order to insure incompressibility of the flow. Assuming the gradients to be small, *i.e.* $\partial_x\psi/k \ll 1$ and $\partial_x\phi/k \ll 1$, the calculation of the fluctuating magnetic field is performed perturbatively. We write $\boldsymbol{b} = \boldsymbol{b}_0 + \boldsymbol{b}_1 + \cdots$ and transform eq. (6) into

$$\begin{aligned} -\eta \nabla_h \boldsymbol{b}_0 &= -\eta(\partial_{y,y} + \partial_{z,z})\boldsymbol{b}_0 = \langle \boldsymbol{B} \rangle \cdot \nabla \boldsymbol{v} , \\ -\nabla_h \boldsymbol{b}_1 &= \partial_{x,x} \boldsymbol{b}_0 , \end{aligned} \tag{23}$$

where $\nabla_h f = \partial_{y,y} f + \partial_{z,z} f$ is the horizontal Laplacian operator. Because of the harmonic dependence of $\langle \boldsymbol{B} \rangle \cdot \nabla \boldsymbol{v}$, we have

$$\begin{aligned} \boldsymbol{b}_0 &= \frac{\langle \boldsymbol{B} \rangle \cdot \nabla \boldsymbol{v}}{\eta k^2} , \\ \boldsymbol{b}_1 &= \frac{\partial_{x,x} \boldsymbol{b}_0}{k^2} . \end{aligned} \tag{24}$$

We then compute the alpha effect

$$\langle \boldsymbol{v} \times \boldsymbol{b} \rangle = -\frac{UV}{\eta k} \begin{pmatrix} \langle B_x \rangle \frac{\langle (\partial_x \phi)^2 \rangle + \langle (\partial_x \psi)^2 \rangle}{k^2} \\ \langle B_y \rangle \,(1 - \frac{\langle (\partial_x \phi)^2 \rangle}{k^2}) \\ \langle B_z \rangle \,(1 - \frac{\langle (\partial_x \psi)^2 \rangle}{k^2}) \end{pmatrix} . \tag{25}$$

In that limit, the effect of phase fluctuations is to reduce the part of the alpha effect that drives the instability and the onset of dynamo action is postponed to

$$\frac{UV}{\eta^2 kK} = 1 + \frac{\langle (\partial_x \psi)^2 \rangle}{k^2} + \frac{\langle (\partial_x \phi)^2 \rangle}{k^2} . \tag{26}$$

Note that the averaged helicity of the flow is

$$\langle \boldsymbol{v} \cdot \nabla \times \boldsymbol{v} \rangle = UVk \left(2 + \frac{\langle (\partial_x \phi)^2 \rangle}{k^2} + \frac{\langle (\partial_x \psi)^2 \rangle}{k^2} \right), \tag{27}$$

such that it is increased by phase fluctuations, but this does not result in an increase of the part of the alpha-effect that drives the instability.

The results (21) and (26) are valid for both random and deterministic functions ϕ and ψ, provided that their scale of variation is much larger than the one of the flow and much smaller than the one of the whole system.

Note that the time (respectively x) average of (16) (respectively (22)) gives a mean velocity field $\langle \boldsymbol{v} \rangle$ that depends on $\langle \cos\psi \rangle$, $\langle \sin\psi \rangle$, $\langle \cos\phi \rangle$ and $\langle \sin\phi \rangle$. Thus, these terms are involved in the dynamo threshold of $\langle \boldsymbol{v} \rangle$ that differs from the predictions (21) and (26) which involve phase gradients, η and k.

Conclusion. – We have studied the effect of velocity fluctuations on the dynamo onset for a cellular flow. We have first considered a Roberts flow subject to an amplitude modulation and then studied the effect of phase fluctuations. In the limit of length scale separation, the onset can be calculated explicitly even with modulation. For a modulation of the flow amplitude, the onset shift is proportional to the product of the axial velocity modulation with the azimuthal one. The sign of the displacement depends on the phase difference between the modulations: if they act in phase, the onset is lowered; it is increased if they act out of phase. In the presence of phase fluctuations, the results are different. In the cases we have studied, the part of the alpha effect that drives the instability is lowered by phase fluctuations. This results in an increase of the dynamo onset at second order in fluctuations amplitude.

However, we emphasize that even if phase fluctuations inhibit the dynamo modes generated by the mean flow in the absence of fluctuations and thus shift their onset to larger velocity amplitudes, we cannot in general rule out the generation of other dynamo modes by the fluctuations themselves. For instance, beyond the regime of small R_m at the scale of the flow assumed in the present paper, it is known that fluctuations of the form $\psi(t) = \sin\omega t$, $\phi(t) = \cos\omega t$ in eq. (16) can generate fast dynamo modes [12].

Finally, a related problem concerns the dependence of the dynamo threshold R_{mc} on the kinematic Reynolds number Re of the flow. This is actively studied using numerical simulations [6, 13, 14]. If the flow is forced such that a large-scale velocity field is prescribed, one observes that R_{mc} first increases and then tends to saturate to a constant value when Re is increased (for instance, by decreasing the viscosity ν, the other parameters being fixed). A possible explanation is as follows: the initial increase is due to fluctuating eddies as described here. When Re is large, these large-scale fluctuations do not increase anymore; a further increase in Re amounts to add fluctuations close to the Kolmogorov scale. These fluctuations do not affect the dynamo threshold because they are of small scale and vanishing energy compared to the one of the mean flow.

REFERENCES

[1] ROBERTS P. H. and GLATZMAIER G. A., *Rev. Mod. Phys.*, **72** (2000) 1081.
[2] WIDROW L. M., *Rev. Mod. Phys.*, **74** (2002) 775.
[3] STIEGLIETZ R. and MÜLLER U., *Phys. Fluids*, **13** (2001) 561.
[4] GAILITIS A., LIELAUSIS O., PLATACIS E., DEMENT'EV S., CIFERSONS A., GERBETH G., GUNDRUM T., STEFANI F., CHRISTEN M. and WILL G., *Phys. Rev. Lett.*, **86** (2001) 3024.

[5] PÉTRÉLIS F., PhD Thesis, Université Paris 6 (2002); FAUVE S. and PÉTRÉLIS F., *Peyresq Lectures on Nonlinear Phenomena*, edited by SEPULCHRE J.-A., Vol. **II** (World Scientific, Singapore) 2003, pp. 1-66.
[6] LAVAL J.-P., BLAINEAU P., LEPROVOST N., DUBRULLE B. and DAVIAUD F., *Phys. Rev. Lett.*, **96** (2006) 204503.
[7] LANDAU L. D. and LIFSHITZ E. M., *Fluid Mechanics*, Vol. **6** (Butterworth Heinemann) 1987, pp. 129-134.
[8] ROBERTS G. O., *Philos. Trans. R. Soc. London. Ser. A*, **266** (1970) 535.
[9] MOFFATT H. K., *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge University Press, Cambridge) 1978.
[10] COURVOISIER A., HUGHES D. W. and TOBIAS S. M., *Phys. Rev. Lett.*, **96** (2006) 034503.
[11] NORMAND C., *Phys. Fluids*, **15** (2003) 1606.
[12] GALLOWAY D. J. and PROCTOR M. R. E., *Nature*, **356** (1992) 691.
[13] PONTY Y., MININNI P. D., MONTGOMERY D. C., PINTON J.-F., POLITANO H. and POUQUET A., *Phys. Rev. Lett.*, **94** (2005) 164502; PONTY Y., MININNI P. D., PINTON J.-F., POLITANO H. and POUQUET A., submitted to *Phys. Rev. Lett.* (2006).
[14] SCHEKOCHIHIN A., COWLEY S. C., MARON J. L. and MCWILLIAMS J. C., *Phys. Rev. Lett.*, **92** (2004) 054502.