Surface Instability Driven by Dipole-Dipole Interactions in a Granular Layer

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In a layer of grains that interact through a tunable dipolar force, we show that an instability occurs due to the competition between gravity and the interaction between grains. Above a critical intensity of the interaction, the flat surface of the layer is unstable and peaks are generated. Fluctuations are shown to have conflicting effects. On the one hand, they favor the instability process and for large enough fluctuations the peak amplitude behaves similarly to the order parameter of a supercritical instability. On the other hand, the spatial organization of the peaks is affected by fluctuations that postpone the appearance of spatial ordering.

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Since Faraday [1], the formation of structures at the surface of granular layers has driven a large number of studies [2]. Vertical shaking of a granular layer leads to an instability that forms intriguing patterns [3]. Structures also appear when a granular medium is subject to the flow of a fluid, for instance at the bottom of oceans and rivers [4]. In most granular media, interactions between grains are dominated by contact events: either dissipative collisions in diluted granular media or friction in denser ones. However, there are systems for which the grains interact at large distance. For instance grains in a liquid can interact through the flow they create; grains can also interact when they carry electric charges [5]. Recently, some experiments investigated the behavior of magnetic granular media. Most of them concern the properties of monolayers. When each grain carries a permanent magnet, pattern formation [6] and segregation [7] have been described. Grains with a large magnetic permeability and driven electrostatically have displayed the formation of a rich variety of structures: chains, clusters, rings, and networks [8]. In a magnetic granular gas subject to a magnetic field, the interactions between grains modify the velocity statistics [9]. Interacting grains dispersed in a turbulent flow have been shown to display an oscillatory instability [10]. Here we consider the regime of large compacity and report on the behavior of a packed layer of magnetic grains subject to both a vertical shaking and a vertical magnetic field. This system can be described as an ensemble of dissipative particles subject to interactions mediated by the magnetic field. We show that an instability takes place when the interaction is large enough. We point out the conflicting roles of fluctuations and discuss some effects of the size of the grains.

Our system is a layer of stainless steel spheres. Unless otherwise stated, we consider spheres of monodispersed diameter $d = 1$ mm. We present measurements performed in a rectangular cell of size 26×70 mm^2, subject to a sinusoidal vertical acceleration with frequency f from 40 to 80 Hz and with peak-peak amplitude Γ ranging from 0 to 10 g, where g is the acceleration of gravity. We apply a vertical magnetic field using a pair of Helmholtz coils. The magnetic field B is tuned between -125 and 125 G and is uniform within 4% on the size of the layer. The spheres are ferromagnetic and acquire a magnetic dipolar moment that is proportional to the applied magnetic field. This leads the spheres to interact through dipole-dipole interactions whose amplitude may be tuned by the applied field. For an acceleration $\Gamma = 6.13$ g at frequency 60 Hz, we display in Fig. 1 a snapshot of the layer at four different magnetic fields. Above some critical value of the magnetic field, the flat layer is unstable and peaks appear at the surface. The height of the peaks increases with the magnetic field and can be larger than the layer thickness at rest. We measure the height of the peaks as follows. We perform up to $N = 300$ pictures of the system and determine h the maximal height of the layer on a slab of width 1.5 cm. We then compute H the average of h. In Fig. 2, we plot H for two amplitudes of the vertical acceleration. For the largest amplitude, the curve displays a plateau for $B \leq 30$ G and increases for larger B. This is, of course, associated with the formation of peaks presented in Fig. 1. Within experimental accuracy, there is no hysteresis and the results are not changed by changing B into $-B$. For the smaller vibration, the behavior is different: an increase of the layer thickness takes place but does not reach values higher than two or three sphere diameters. The evolution depends on various parameters including the initial configuration of the grains and a strong hysteresis takes place when decreasing the magnetic field. In other words, a vibrated layer of magnetic granular spheres displays an instability when plunged into a vertical magnetic field. This instability leads to the formation of peaks. The nature of the instability is sensitive to the amount of vibration: hysteretic and reaching low peak amplitude if the vibration is small, without hysteresis and reaching large peak amplitude for large vibration.

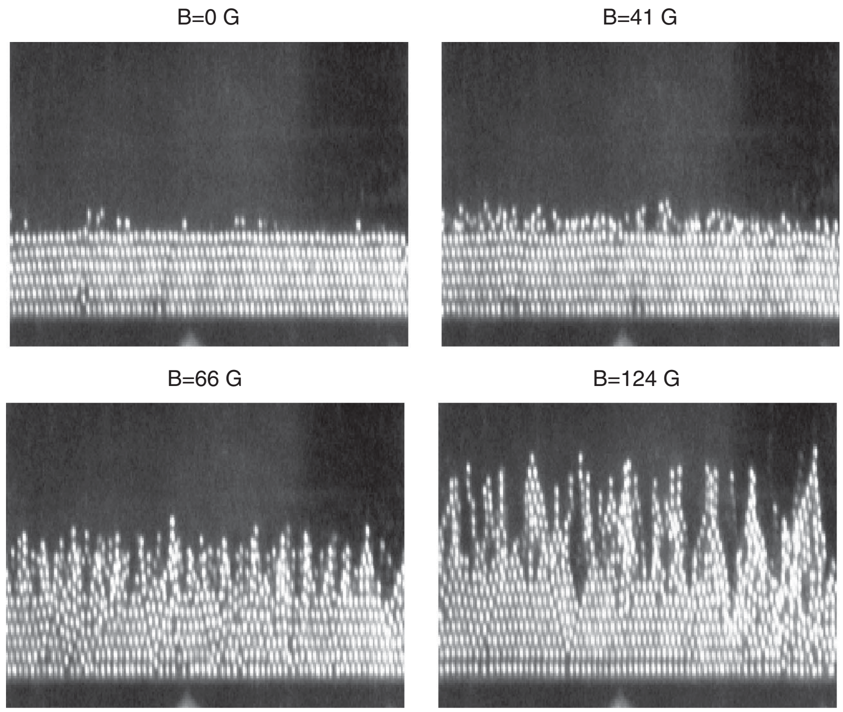

FIG. 1. Pictures of the granular layer subject to an acceleration $\Gamma = 6.13$ g at frequency $f = 60$ Hz and to a magnetic field (from left to right and top to bottom): $B = 0$, 41, 66, and 124 G.

We now focus on the regime of large vibration where the instability is continuous with no hysteresis. We determine the nature of the instability by plotting the square of the variation of the layer thickness $(H(B) - H_0)^2$ as a function of the magnetic field in Fig. 3 [11]. We notice that for this range of acceleration: f between 40 and 80 Hz and Γ between 5 and 10 g, the peak amplitude does not depend on the acceleration or on the layer thickness at rest that was varied between 7 and 11 mm. In addition, above the critical magnetic field $B_c = 33 \pm 2$ G, the instability is supercritical since the peak amplitude $H(B) - H_0$ scales as the square root of the distance from onset $\sqrt{B - B_c}$. The mechanism for the peak formation is the interaction between the magnetic dipoles induced in each sphere by the applied magnetic field. Two spheres minimize their magnetic energy by lining up vertically. This competes with the related increase in gravitational potential energy. We note that a balance between the magnetic energy $B^2 d^3/\mu_0$ and the gravitational energy $\rho g d^4$ leads to a characteristic magnetic field $B \simeq 100$ G in order of magnitude agreement

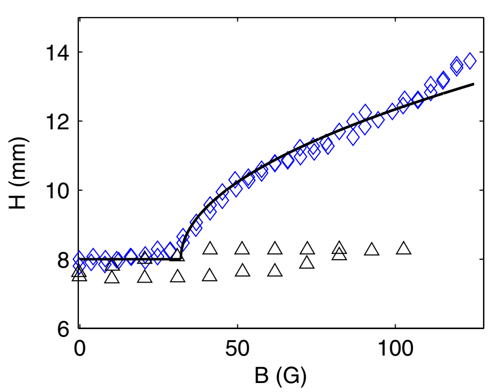

FIG. 2 (color online). Average height of the layer H as a function of the applied magnetic field B. The vibration frequency is $f = 60$ Hz and the acceleration is ($\Diamond$): $\Gamma = 7.4$ g; ($\triangle$): $\Gamma = 0.65$ g. The standard deviation on the measurement of H is 0.6 mm. The continuous line is a fit $H - H_o \propto \sqrt{B - B_c}$.

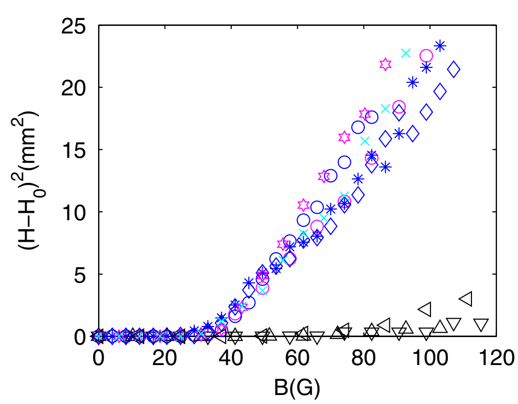

FIG. 3 (color online). Square of the peak amplitude $(H - H_0)^2$ as a function of the magnetic field B. For a system made of 10 layers, the vibration frequency is $f = 60$ Hz and the acceleration ($\bigcirc$) : $\Gamma = 9.7$ g; ($\Diamond$) : $\Gamma = 7.4$ g; (*): $\Gamma = 5.5$ g; ($\bigtriangledown$): $\Gamma = 1.8$ g; ($\triangle$): $\Gamma = 0.65$ g. For a system made of 7 layers, (six-pointed star): $f = 40$ Hz, $\Gamma = 6.15$ g; ($\times$): $f = 80$ Hz, $\Gamma = 5.5$ g; ($\triangleleft$): $f = 60$ Hz, $\Gamma = 1.8$ g.

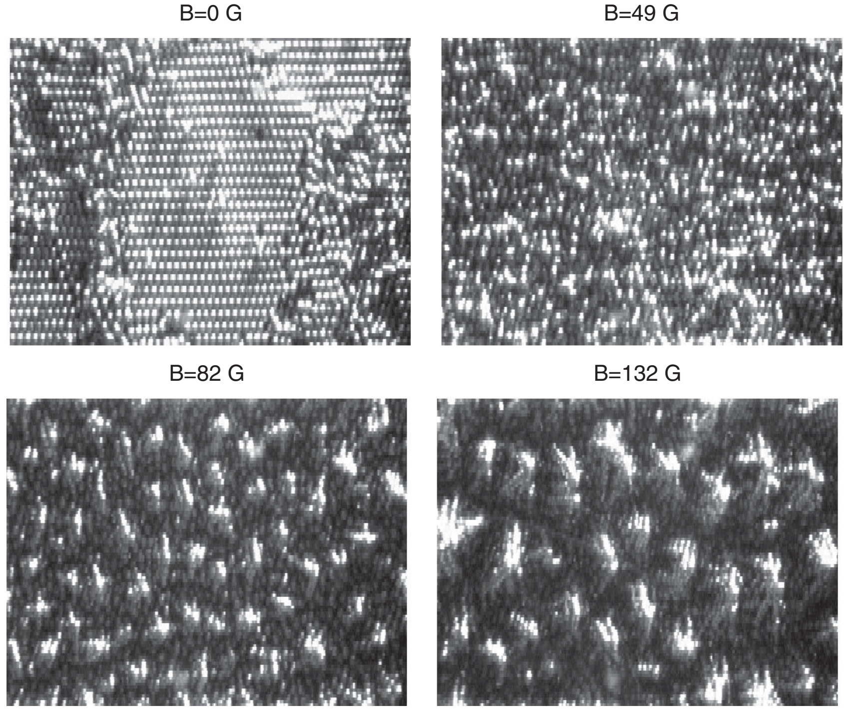

FIG. 4. Snapshots of the surface seen from above. The vibration frequency is $f = 60$ Hz and its amplitude $\Gamma = 9.7$ g. The magnetic field is from top to bottom: $B = 0$, 49, 82, and 132 G.

with the magnetic field at the onset of peak formation. The role of the vibration is more complex. One could expect that it destroys the peaks and thus reduces their amplitude. In the regime under study, we see that a large enough vibration is necessary for the peaks to develop, and then the peak amplitude does not depend on the vibration. Indeed if the vibration is strong enough, some spheres jump above the layer and can explore positions favored by the magnetic field, for instance at the top of a peak. If the vibration is low, the spheres do not have enough kinetic energy to cross the barrier of gravitational energy and reach the positions that minimize the magnetic energy; in this regime, the layer thickness does not increase much, even for large magnetic field. In some sense, the system is frozen in a high energy state because fluctuations are inefficient to explore states that are energetically favored.

We now discuss the spatial organization of the peaks. Snapshots of the layer seen from above are displayed in Fig. 4. The pattern is roughly hexagonal but visual inspection shows that the peaks move randomly inducing a strong disorder. Therefore most peaks have six neighboring peaks but there is no long range spatial periodicity. To evaluate the distance between the peaks, we perform up to $N = 100$ snapshots of the layer surface, calculate the autocorrelation function of the image $F(r, \theta)$ and integrate on θ to obtain the radial autocorrelation function $G(r)$. It displays a maximum for $r = 0$ and a secondary maximum for a larger value of r. At small magnetic field, the secondary maximum is the distance between the neighboring spheres whereas for large magnetic field, it is the distance between the peaks. We plot in Fig. 5 the average distance λ extracted from this analysis. In the regime of large vibration, the distance between the peaks increases from roughly three sphere diameters to six for a magnetic field varying from $B_{cp} = 50$ to 120 G. Indeed, the pattern evolves when the peak height increases and this leads to an increase of the pattern wavelength λ. Two mechanisms are involved in the interaction between the peaks that leads to the organization of the pattern. Geometrical constraints can force the peaks to order. Moreover, a peak amounts to a magnetic dipole and two parallel magnetic dipoles located in a plane perpendicular to their axis repel each other. It is likely that both mechanisms act when the organization of the pattern takes place.

We note also that the wavelength λ is determined only above $B_{cp} = 50$ G, a value higher than the onset of peak formation $B_c = 33$ G. We point out that this is not linked to our method of measurement: visual inspection of the pattern (compare for instance Fig. 1 for $B = 41$ G and Fig. 4 for $B = 49$ G) shows that for a magnetic field between B_c and B_{cp} peaks are formed but evolve randomly and do not order. Therefore, in this regime, only the order of the neighboring spheres appears in our analysis. The spatial organization of the peaks is thus delayed compared to the appearance of the peaks. Even though fluctuations favor the instability and are necessary for the peaks to reach large amplitude, they have a limiting effect for what concerns the spatial organization of the pattern that

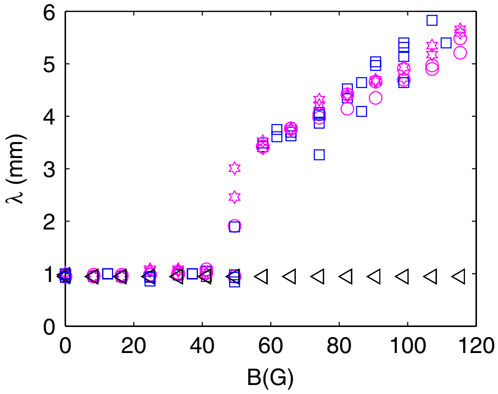

FIG. 5 (color online). Length at the secondary maximum of the autocorrelation function λ(mm) as a function of the applied magnetic field B(G). The system is made of 7 layers. The vibration frequency is (◁): $f = 60$ Hz, and its acceleration $\Gamma = 1.8$ g; (□): $f = 60$ Hz, $\Gamma = 6.15$ g; (six-pointed star): $f = 40$ Hz, $\Gamma = 6.15$ g; (○): $f = 60$ Hz, $\Gamma = 8.9$ g.

is destructed by the fluctuations of the peak positions induced by the vibration.

We now summarize the results obtained with spheres of larger diameter $d = 1.5$ mm. The onset of instability is $B_c = 44 \pm 4$ G. It is larger than the one obtained with smaller spheres and, within error bars, the difference is in agreement with the $d^{1/2}$ prediction discussed above. The spatial organization is also postponed compared to the appearance of the peaks. Because the spheres are bigger, a wavelength of the pattern contains less grains and the system is thus more geometrically constrained. Even in the regime of large vibration, this has two effects. First, peaks of smaller amplitudes are formed. In addition, the distribution of peak amplitudes h is modified. For spheres of diameter 1 mm, the distribution has only one maximum which value is roughly the same as the averaged height H. For larger spheres, the distribution can have two maxima. In contrast with the averaged height that increases continuously with the magnetic field, the maxima of the distribution correspond to specific values associated to one or two or three sphere diameters. This quantification of the maxima of the distribution results from the geometrical constraints that rigidify the layer and do not allow enough spatial reordering within a peak. The height of a given peak thus increases discontinuously when the magnetic field is increased and only the height averaged over the peaks increases continuously.

This instability is similar to the Rosensweig instability that occurs in ferrofluids subject to a vertical magnetic field. This is not surprising since ferrofluids are suspensions of ferromagnetic particles of micrometric size in a viscous fluid [12]. Although the mechanism of instability is similar in both systems, we emphasize several differences. For the Rosensweig instability, surface tension is important and selects the wavelength at onset. Besides, vibrations are not necessary for the peaks to appear. When the applied magnetic field is increased, a slightly subcritical instability takes place and the pattern appears at onset [13]. For the granular layer, the wavelength is of the order of the particle size. When vibration is large enough, the instability is supercritical and the spatial ordering of the pattern takes place above a critical magnetic field larger than the onset of peak formation.

Future work will explore the regime of smaller spheres in which this granular system should resemble more to a ferrofluid. Other field configurations and higher vibrations will also be tested to gain understanding in this dissipative system of interacting particles. We conclude by stressing that the influence of dipolar interactions on granular behavior can be tested in several situations. A few questions that come to mind are the following. Are usual granular flows or instabilities modified? In a dense granular medium, how do the networks of forces and contacts evolve? Is the propagation of a local perturbation changed? Are global properties of the medium varied such as transport coefficients, packing properties, sound velocity, and attenuation?

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