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Oscillatory instability of interacting grains in a turbulent flow

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Abstract – We present an instability occuring in a turbulent flow containing grains that interact via a dipole-dipole force. Above a critical intensity of the interaction, the local density of grains displays a noisy oscillation. This instability takes place in a spatially and temporally fluctuating medium. We identify the control parameter of the instability and study the statistical properties of the local density. In spite of the turbulent fluctuations, measurements performed at different positions show that a spatially coherent mode is generated.

The interplay of instability processes and fluctuations has been studied in various experimental systems. Early ones considered electronic circuits [1]. Electro-convection subject to random fluctuations of the applied voltage is described in [2]. The effects of different noises on the Faraday instability at the surface of a liquid layer have been investigated in [3,4]. In all these experiments, the fluctuations depend only on time either because the system has no spatial degree of freedom or because the applied fluctuations are spatially homogeneous.

In contrast, some instabilities are subject to spatio-temporal fluctuations. Thermal fluctuations close to the onset of convection have been considered in [5] and drive a fluctuation-induced first-order transition. In most cases, thermal effects can be neglected because the energy associated to thermal fluctuations is small compared to the caracteristic energies of the deterministic processes. For out of equilibrium systems, non thermal fluctuations can play a role since there is no constraint on the their intensity and spectral properties. For instance, this is the case of the dynamo effect, an instability that converts kinetic energy into magnetic energy. This instability, which generates the magnetic field of astrophysical objects such as the Earth or the Sun, has been recently observed in a turbulent flow of liquid sodium in which fluctuations of the velocity field occur at all scales [6]. The effect of turbulent spatio-temporal fluctuations on the amplitude of the unstable mode still involves several open problems [7]. Compared to instabilities that occur in a non fluctuating medium, one can wonder whether an onset of instability can still be defined, whether the critical behavior is modified and whether turbulent fluctuations affect the spatial coherence of the unstable mode...

These questions are of interest out of the dynamo context. Here, we study an instability that occurs in an ensemble of grains subject to an interacting force. The grains are fluidized by a turbulent flow so that the instability is generated in a medium that fluctuates both spatially and temporally. We first describe the characteristic features of this instability and then focus on the effect of turbulent fluctuations on the statistical properties of the unstable mode.

Experimental set-up. – The system under study is an ensemble of stainless-steel spherical particles of diameter $d = 1$ mm. They are located in a vertical cylindrical vessel of diameter $D = 96$ mm and height $H = 85$ mm that is filled with water, see fig. 1. Typical experiments involve a mass $m = 100$ g of grains, roughly $N \simeq 2 \times 10^4$ spheres corresponding to a volume fraction of 2%. A flow is generated by the rotation of a propeller of radius $R = 32$ mm located 1 cm above the bottom of the cylinder. The propeller is put in motion by a PARVEX motor RS420. It is rotated at constant frequency and the applied torque Γ is measured via the current delivered to the motor.

We consider the situation where the sense of rotation of the propeller generates a flow with axial component directed downward in the middle of the cylinder and upward close to the lateral boundaries. The flow has also a strong azimuthal component. Four vertical cylinders of

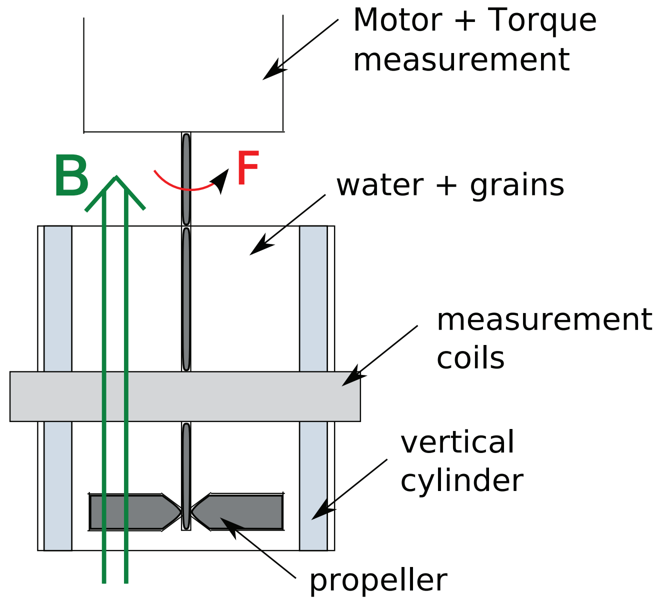

Fig. 1: (Colour on-line) Sketch of the experiment. A cylinder of diameter $D=96$ mm and height $H=85$ mm is filled with water and stainless-steel spheres. The flow is driven by the rotation of a propeller and the whole experiment is submitted to an externally applied magnetic field.

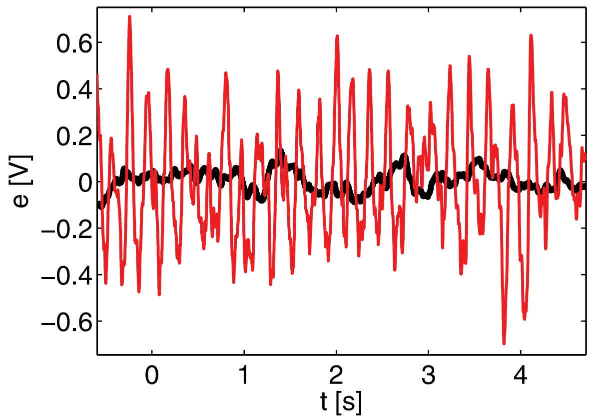

Fig. 2: (Colour on-line) Time series of the fluctuations of density $e=V-\langle V\rangle$, for $F=25$ Hz and thick line: $B=0$ G, thin line: $B=190$ G.

height 85 mm and diameter 8 mm are fixed close to the lateral boundaries in order to break solid body rotation of the flow. The kinetic Reynolds number $Re=2\pi R^2 F/\nu$, where F is the propeller rotation rate and ν the kinematic viscosity of water, is of order 10^5 so that the flow is strongly turbulent. We have checked by visual inspection that for $F \geqslant 20$ Hz, a roughly homogeneous suspension of spheres is obtained by turbulent mixing.

The experiment is plunged into a vertical magnetic field of amplitude B up to 250 G. The spheres are made of stainless steel so that they can be magnetized and acquire a magnetic dipole moment the amplitude of which is proportional to the applied magnetic field. The grains form an ensemble of particules that interact by dipole-dipole interaction. The intensity of the interaction can be tuned by changing the applied magnetic field.

Two closely coupled coils of diameter 96 mm are fixed around the cylinder, at an height h above its bottom. Unless otherwise stated, h is 3 cm. Following the method presented in [8], an a.c. voltage with frequency $f=3$ kHz is applied to one of the coils. The homodyne detection between the applied voltage and the a.c. voltage induced at the second coil gives a signal $V(t)$ which depends on the local density of spheres within the coils. For moderate variations in the density of spheres, we consider that $e=V-\langle V\rangle$, where $\langle V\rangle$ is the time-average of V, is proportional to the fluctuations of sphere density close to the plane of the measurement coils.

Experimental results. – Time series of the fluctuations of density e are displayed in fig. 2 for two values of the imposed magnetic field. For low magnetic fields, the density displays small fluctuations. These random fluctuations are associated to the fluctuations of the sphere density due to turbulent mixing. For larger magnetic fields, e displays a noisy periodic oscillation with large amplitude. This change of behavior from small fluctuations to large noisy oscillations is associated to a bifurcation in a turbulent medium.

From the time series of e, we compute its root mean square fluctuation $\langle e^2\rangle^{1/2}$ and present it in fig. 3 as a function of the applied magnetic field for three values of the propeller rotation rate. For small magnetic fields, $\langle e^2\rangle^{1/2}$ is roughly independent of the applied magnetic field B. For larger magnetic fields, it increases with B. Fluctuations are reduced if the rotation rate is increased.

The mechanism responsible for the density oscillation is the attacting interaction between the magnetic dipoles carried by each grain. Two dipoles tend to line up vertically and this favors the formation of packed ensembles of spheres. Visual inspection shows that moving clusters of spheres are formed and destroyed inside the cylinder. The motion of these clusters is responsible for the oscillation of the density. The amplitude of the oscillation depends also on the flow velocity. Indeed, the attacting interaction is in competition with the turbulent mixing that tends to separate the grains. A dimensionless control parameter can be constructed balancing the magnetic energy of two induced dipoles $B^2\, d^3/\mu_0$ with the kinetic energy of a grain of mass m_p moving at the flow velocity $m_p R^2(2\pi F)^2$. We thus define $N=B/(\sqrt{\rho_p\mu_0}R2\pi F)$, where ρ_p is the density of a grain ($\rho_p \simeq 8\times 10^3\,\mathrm{kg\,m^{-3}}$ for stainless steel) and μ_0 is the magnetic permeability of vacuum. The sphere density fluctuations, $\langle e^2\rangle^{1/2}$, are represented as a function of N in fig. 3. The collapse of the curves for different rotation rates shows that the instability is controlled by the competition between magnetic interaction and turbulent dispersion.

Even far above onset, the density signal is only roughly periodic. The probability density functions (PDF) of e

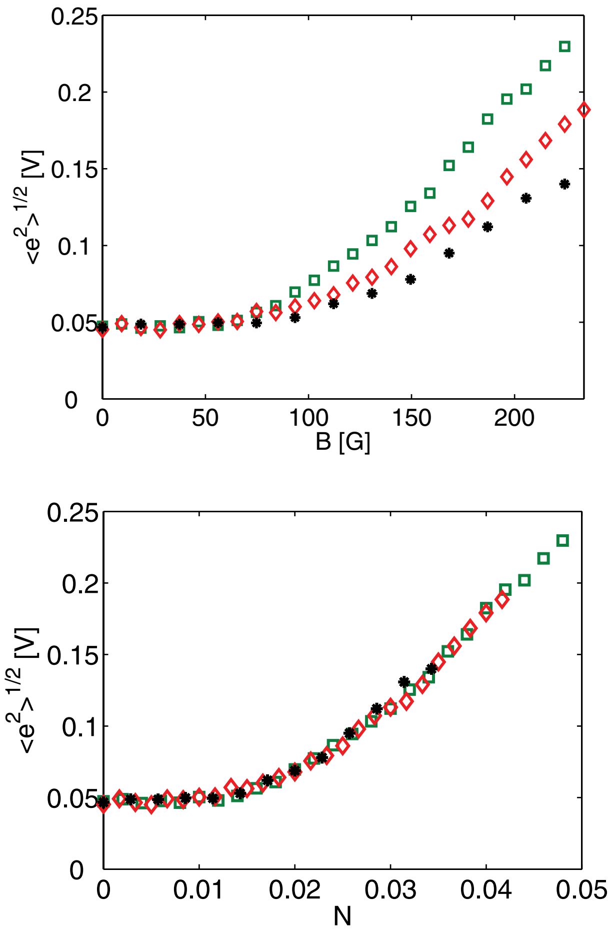

Fig. 3: (Colour on-line) R.m.s. fluctuations of the density $\langle e^2\rangle^{1/2}$. Top: as a function of the magnetic field B. Bottom: as a function of the dimensionless parameter $N = B/(\sqrt{\rho_p\mu_0}R2\pi F)$. The propeller rotation rate is ($\square$) $F = 25$ Hz, ($\diamond$) $F = 30$ Hz, ($*$) $F = 35$ Hz.

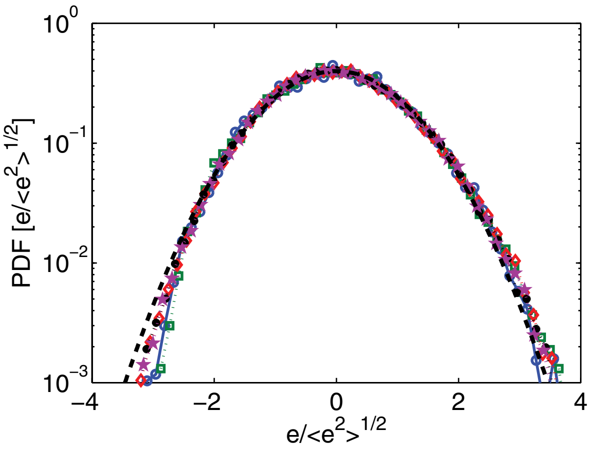

Fig. 4: (Colour on-line) Probability density function (PDF) of the sphere density e for a propeller rotation rate $F = 25$ Hz and different values of the magnetic field $B = 0$, 94, 141, 188 and 235 G. The Gaussian distribution of unit width is represented by the dotted line.

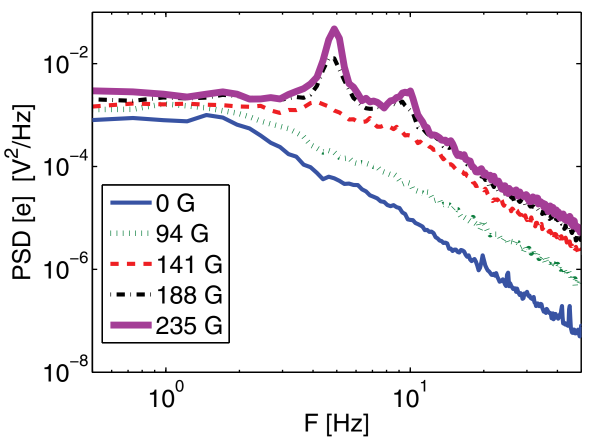

Fig. 5: (Colour on-line) Power spectrum density of e for a propeller rotation rate $F = 25$ Hz and different values of the magnetic field $B = 0$, 94, 141, 188 and 235 G.

are displayed in fig. 4. They display only one maximum for $e = 0$. Even in regimes where the signal seems to be periodic, its distribution remains Gaussian and is different from the expected distribution of a periodic signal (that displays maxima for the extremal values of the signal). This is due to the fluctuations. The signal is an oscillation of fluctuating amplitude and the central limit theorem implies that if the fluctuations are increased, the PDF tends to a Gaussian. The power spectrum density (PSD) of e is presented in fig. 5. For low magnetic fields, the spectra contain a plateau at low frequency and decrease at high frequency. An increase in the magnetic field amplitude initially results in a global increase in the whole spectrum. However, for large magnetic fields, the spectrum displays a peak around a frequency $f = 4.7$ Hz and its first harmonic. This peak is related to the oscillation observed in the time series of fig. 2. We note that this peak is broad. Its width at half maximum is $\Delta f \simeq 1$ Hz. Moreover, its amplitude is not very large compared to the amplitude of the spectrum at low frequencies. These observations are a consequence of the instability being generated in a turbulent flow which leads the oscillation to display large variations in amplitude.

The frequency of the oscillation is $f \simeq 4.7$ Hz and does not strongly depend on N. Other measurements (not detailed here) performed with different masses of grains show a small decrease of f with m. More precisely, f evolves from 5.2 Hz to 4.2 Hz for m varying from 50 g to 200 g. A variation of less than 20% is thus observed when the mass is multiplied by 4. In other measurements in a cylinder of smaller height $H = 5.5$ cm, a higher frequency $f \simeq 6$ Hz is obtained. The period is of the order of magnitude of a free fall time T based on H and the gravity g, $T = \sqrt{H/g}$. It is not surprizing that part of the dynamics is controlled by gravity: the clusters of grains are heavier and are therefore likely to fall toward the bottom of the cylinder where more intense

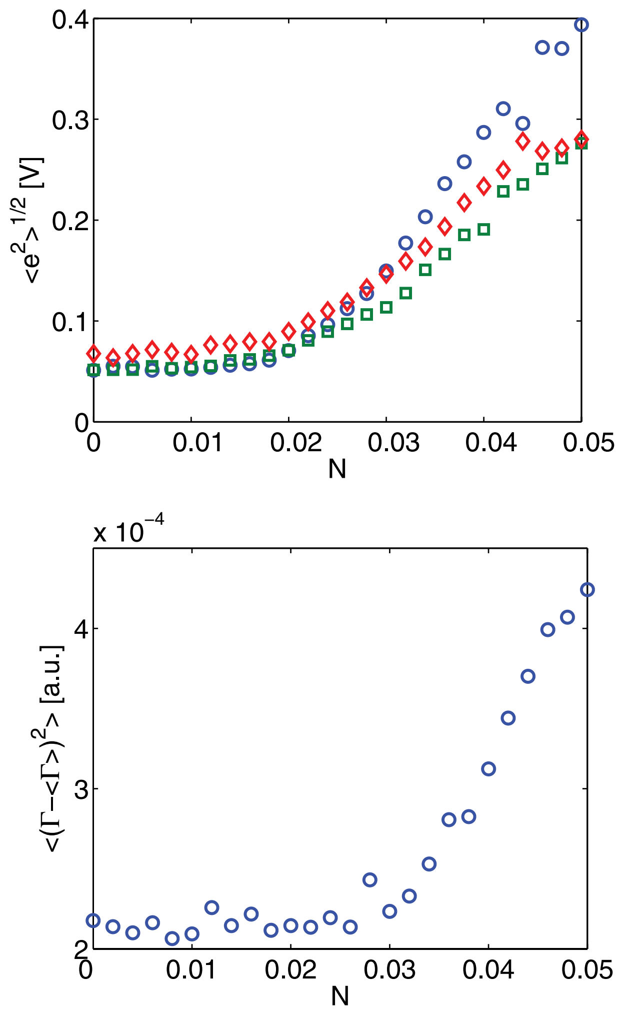

Fig. 6: (Colour on-line) For a propeller rotation rate $F = 25\,\text{Hz}$, top: r.m.s. fluctuations of the density $\langle e^2\rangle^{1/2}$ as a function of N for three different heights, ($\circ$) $h = 3\,\text{cm}$, ($\square$) $h = 5.7\,\text{cm}$ and ($\diamond$) $h = 8.1\,\text{cm}$; bottom: fluctuations of the torque $\langle(\Gamma - \langle\Gamma\rangle)^2\rangle$ as a function of N.

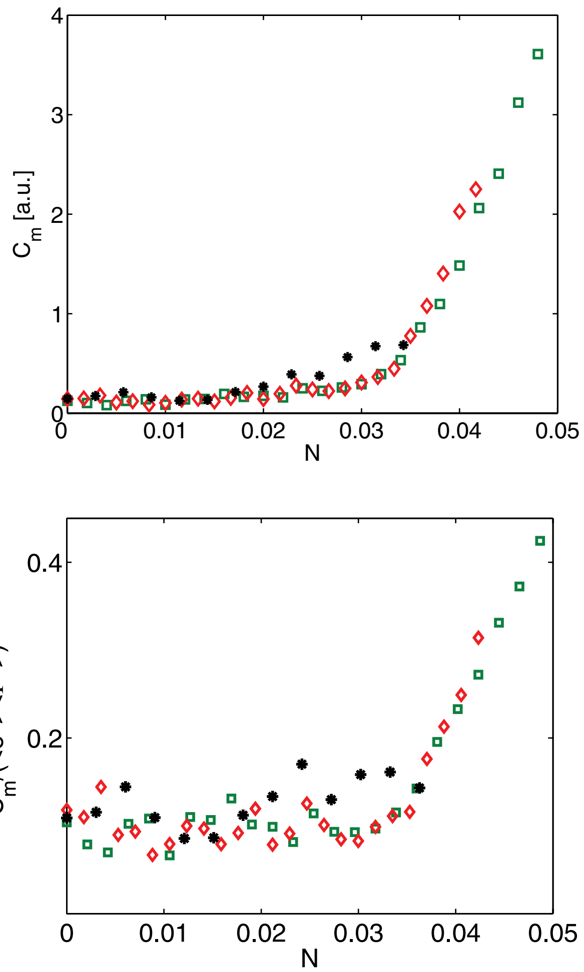

Fig. 7: (Colour on-line) Top: C_m as a function of N. The mass of grains is $m = 100\,\text{g}$ and the propeller rotation rate is ($\square$) $F = 25\,\text{Hz}$, ($\diamond$) $F = 30\,\text{Hz}$, ($*$) $F = 35\,\text{Hz}$. Bottom: normalized correlation $C_m/\sqrt{\langle e^2\rangle\langle(\Gamma - \langle\Gamma\rangle)^2\rangle}$.

mixing takes place. Nevertheless, we note that the system remains roughly homogeneous and the formation and destruction of clusters of grain can be observed everywhere in the cell, not only at its bottom. It is important to realize that the frequency of the oscillation is quite smaller than the rotation frequency of the propeller and is not simply related to a caracteristic time scale of the flow.

We now discuss measurements performed at different locations in order to evaluate how spatially coherent the unstable mode is. In fig. 6, the dependence of $\langle e^2\rangle^{1/2}$ on N is shown for density measurements performed at three different heights. The three curves have similar shapes which indicates that the unstable mode is not spatially localized and that its statistical properties are roughly independent of the position.

We have also measured the torque Γ applied by the motor. Time series of the torque also display an oscillation. Indeed, when the sphere density in the vicinity of the propeller changes, it modifies the torque that the motor must apply in order to maintain a constant propeller speed. Standard deviation of the torque is displayed as a function of N in fig. 6. It also bifurcates from a constant level above a critical value of N. We note that the critical value extracted from the torque measurements ($N_c \simeq 0.03$) is larger than the one extracted from the density measurements ($N_c \simeq 0.02$). This may be related to the higher sensitivity of the density measurements which display less noise than the torque ones.

Simultaneous measurements of the torque and density allow to estimate how spatially coherent the unstable mode is on the size of the experiment. To wit, we calculate their cross-correlation $C(\Delta T) = \langle\Gamma(t)e(t + \Delta T)\rangle$. This correlation is maximum for a value ΔT that depends on the height of the coils since a cluster of spheres is first measured by the coils before it falls down and modifies the torque. For a given height, we calculate C_m the maximum of C. In some sense, this procedure amounts to the homodyne detection of the periodic part of the signal from two probes. Indeed, the local incoherent noises on the two measurements do not contribute to the correlation function of the two signals, so that C_m represents only the part of the oscillation that is coherent when measured at two different positions in the experiment. In fig. 7, we plot the correlation C_m and the normalized correlation $C_m/\sqrt{\langle e^2\rangle\langle(\Gamma-\langle\Gamma\rangle)^2\rangle}$. A correlation of about 0.5 is reached which proves that the instability generates a spatially coherent mode on the scale of the experiment despite the intense turbulent fluctuations.

Conclusion. – We have identified an instability that occurs when an ensemble of fluidized grains are subject to a dipole-dipole interaction. When the interaction is strong enough, clusters of grains are formed and destroyed in the turbulent flow. The motion of these clusters results in a roughly periodic oscillation of the grain density. The control parameter of the instability is the ratio between the magnetic energy of two grains and their kinetic energy. Fluctuations are involved in the dynamics of the unstable mode. The frequency spectrum of the density of grains displays a peak at the frequency of the oscillation together with a plateau at low frequencies. Its probability density function remains maximum only at zero. Despite the importance of the fluctuations, the unstable mode has homogeneous statistical properties. We have introduced the square root of the maximum correlation between two spatially distant measurements (density and torque) as a measure of the spatial coherence of the unstable mode. It shows that the oscillation is coherent on the whole experiment.

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