

Transport of Magnetic Field by a Turbulent Flow of Liquid Sodium

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We study the effect of a turbulent flow of liquid sodium generated in the von Kármán geometry, on the localized field of a magnet placed close to the frontier of the flow. We observe that the field can be transported by the flow on distances larger than its integral length scale. In the most turbulent configurations, the mean value of the field advected at large distance vanishes. However, the rms value of the fluctuations increases linearly with the magnetic Reynolds number. The advected field is strongly intermittent.

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Transport of a magnetic field by an electrically conducting fluid plays a central role in various astrophysical processes [1] as well as in laboratory plasmas. Magnetic fields induced by flows of liquid metals in the presence of an externally applied field have also been observed in laboratory experiments: the generation of a toroidal field from an axial one by differential rotation (the “ ω effect”) [2–4], the generation of an axial field by an external one applied transversally to the axis of a swirling flow [the “Parker mechanism” [5]] [6]. These induction effects are the key mechanisms of most astrophysical and geophysical dynamo models [5,7–9]. However, magnetic eigenmodes generated by dynamo mechanisms are usually strongly localized in space whereas induction effects have been studied to date with externally applied uniform magnetic fields [except in Ref. [10]]. Geophysical or astrophysical flows generally involve regions of strong differential rotation or strong helicity which are not located in the same part of the flow but are both believed to be necessary for dynamo action. It is thus important to understand how the magnetic field induced in one region is transported to another by strongly turbulent flows. In addition, below the dynamo threshold and when the externally applied magnetic field is weak, such that it does not affect the flow, the induced magnetic field behaves as a passive vector. Transport of a passive vector by turbulence is a problem at an intermediate level of complexity between advection of a passive scalar (a pollutant for instance) and advection of vorticity [11]. We report here the experimental observation of a magnetic field advected by a turbulent flow and study its statistical properties.

We have measured the induced magnetic field $\vec{B}$ generated by a turbulent von Kármán swirling flow of liquid sodium (VKS2) away from a localized external magnetic field $\vec{B}_0(\vec{r})$ (see Fig. 1). The experimental setup is similar to the previously described one (VKS1) [4], but involves the

following modifications: the flow volume (150 l) and the driving power (300 kW) have been doubled and a temperature regulation facility has been implemented. The flow is generated by rotating two disks of radius $R = 154.5$ mm, 371 mm apart in a cylindrical vessel, 578 mm in inner diameter, and 604 mm in length. The disks are fitted with 8 curved blades of height $h = 41.2$ mm driven at a rotation frequency up to $\Omega/2\pi = 20$ Hz. A turbulent swirling flow with an integral Reynolds number, $Re = R^2\Omega/\nu$, up to 410^6 is thus generated. The corresponding magnetic Reynolds number is $R_m = \mu_0\sigma R^2\Omega \approx 30$, where μ_0 is the magnetic permeability of vacuum and σ is the electrical conductivity of sodium. The mean flow has the following characteristics: the fluid is ejected radially outward by the disks; this drives an axial flow toward the disks along their axis and a recirculation in the opposite direction along the cylinder lateral boundary [12]. In the case of counter-rotating disks, the azimuthal flow involves a strong shear in

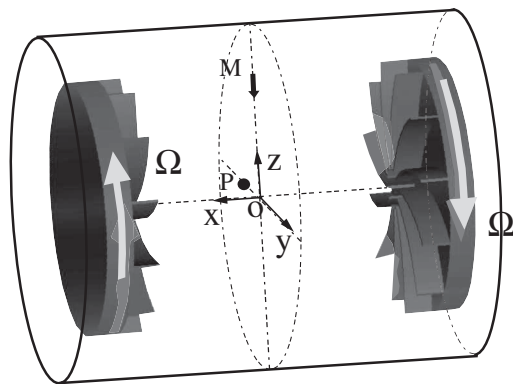


FIG. 1. Geometry of the experimental setup. Location of the magnet (M) and of the Hall probe (P). The magnet can be put in the bulk of the flow if the probe P is removed.

the vicinity of the midplane between the disks, which drives large turbulent fluctuations [13].

A localized magnetic field $\vec{B}_0(\vec{r})$ is generated by a NdFeB cylindrical magnet, 22 mm in diameter and 10 mm in height, placed close to the lateral boundary, 195 mm away from the center along the z axis (M in Fig. 1). The maximum value of the field at contact is about 0.1 T but it decays to less than 1 mT, at a distance 100 mm away from the magnet. The three components of the field induced when the flow is set into motion are measured with an inhouse linear array of 10 Hall transducers (Sentron 2SA-10). They are separated by a distance equal to 28 mm, the first probe being 45 mm away from the center (P in Fig. 1). Thus, the spatial profiles of magnetic induction along the y axis are recorded. The magnet is 200 to 360 mm away from the probes such that its field, measured in the absence of flow motion, is comparable or less than the Earth magnetic field. Velocity fluctuations are dominant with respect to the mean flow in the vicinity of the midplane. Their rms value is typically 3 m/s for $\Omega/2\pi = 10$ Hz. Thus, the ratio of the Lorentz force to the inertial one is of order one close to the magnet but is smaller than 10^{-6} in the measurement region. The ratio of the energy density of the magnetic field fluctuations to the one of the kinetic energy is also smaller than 10^{-6} in the measurement region. The field is dynamically weak in most of the flow volume.

The time recordings of the fluctuations of the three components of the induced field B_i are displayed in Fig. 2. We observe an intermittent signal with the occurrence of bursts with a few Gauss amplitude. There is no coherence between different components. The mean value $\langle \vec{B}(\vec{r}) \rangle$ of the induced field depends very weakly on the rotation frequency, as shown in Fig. 3 where the components $\langle B_i \rangle(\Omega/2\pi) - \langle B_i \rangle(\Omega/2\pi = 8 \text{ Hz})$ are plotted. We observe a slight scatter of the points measured for higher frequencies. Within error bars, this can be ascribed to the

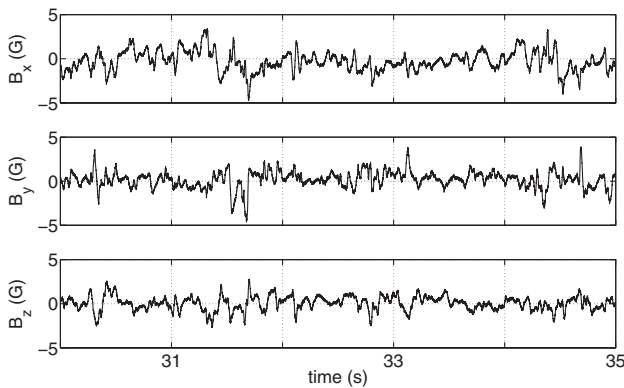


FIG. 2. Direct time recordings of the fluctuations of the induced magnetic field components measured by the first probe (P in Fig. 1), 200 mm away from the magnet. Disk rotation frequency, $\Omega/2\pi = 15$ Hz.

mean induction related to the Earth magnetic field. Consequently, we consider that the time averaged magnetic field $\langle \vec{B}(\vec{r}) \rangle$ induced by the turbulent flow submitted to $\vec{B}_0$ roughly vanishes. We thus concentrate on the properties of its fluctuations. The standard deviation of the fluctuations of each component is an order of magnitude larger than the one obtained without the magnet (i.e., when the Earth magnetic field is acting alone) and increases linearly with the rotation frequency (see Fig. 3). Despite that $\vec{B}_0$ is larger along the z axis, we observe the same level of the fluctuations on the y and z components. The larger fluctuations of B_x are related to the elongational mean flow around the center O , which is directed along the axis of the disks.

Using a linear array of probes, we can study the decay of the amplitude of rms fluctuations of the induced field when the distance to the magnet increases. The results are displayed in Fig. 4. We observe that the decay can be fitted by an exponential. The typical decay length is about 100 mm which is comparable to the integral scale of the flow. This is not much larger than the magnetic diffusive scale $R/R_m^{3/4}$ or the skin depth $R/\sqrt{R_m}$. However, measurements at different rotation frequencies from 8 to 16 Hz have not displayed significant variations of the slopes in Fig. 4. Thus, the typical decay length does not seem to depend on Re or R_m .

Power spectra of the fluctuations are shown in Fig. 5. As in the presence of a uniform applied field [4], they have a much steeper slope above the rotation frequency $\Omega/2\pi$ than below. The inset of Fig. 5 shows that their high frequency part scales with Ω . The intensities are the same for the three components above the rotation frequency. The x component is larger at low frequency, as expected if, as noted above, this results from the elongational geometry of the mean flow in the vicinity of the measurement point. Another argument in favor of this

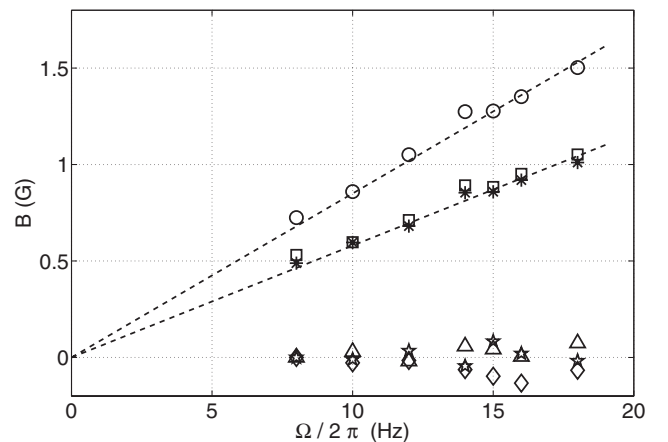


FIG. 3. Evolution with the rotation frequency of the increments of the mean values of components $\langle B_i \rangle(\Omega/2\pi) - \langle B_i \rangle(\Omega/2\pi = 8 \text{ Hz})$, $\langle B_x \rangle$ ($\diamond$), $\langle B_y \rangle$ ($\triangle$), $\langle B_z \rangle$ ($\star$), and standard deviations, B_{xrms} ($\circ$), B_{yrms} ($\square$), B_{zrms} ($\ast$). Linear fits of the standard deviations with dashed lines.

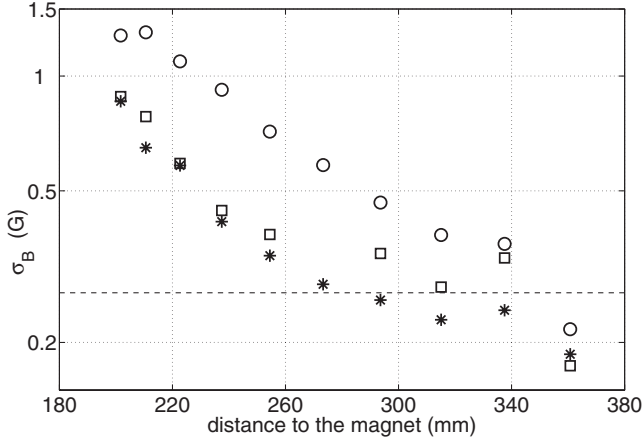


FIG. 4. Spatial decay of the induced magnetic field rms fluctuations in space. B_{xrms} ($\circ$), B_{yrms} ($\square$), B_{zrms} (*). Disk rotation frequency, $\Omega/2\pi = 15$ Hz. The horizontal dashed line corresponds to 0.26 G, i.e., twice the standard deviation of the fluctuations induced by the Earth magnetic field alone.

interpretation is that spectra with the same intensity at all frequencies for the three components of the field are observed at other locations away from the midplane.

The probability density functions (PDF) of the fluctuations of the induced magnetic field B_i are shown in Fig. 6. As already noticed, the x component displays larger fluctuations than the two others. In addition, we observe that the shape of its PDF is also qualitatively different from the one of the y and z components that display roughly exponential tails. At low rotation frequency or far enough from the magnet, the histograms become less stretched. When measurements are performed out of the midplane, the PDF are strongly asymmetric and the asymmetry depends on the location of the probe as well as on the sign of the rotation frequency. We note that roughly Gaussian histograms were observed in the presence of a uniform externally applied field [4].

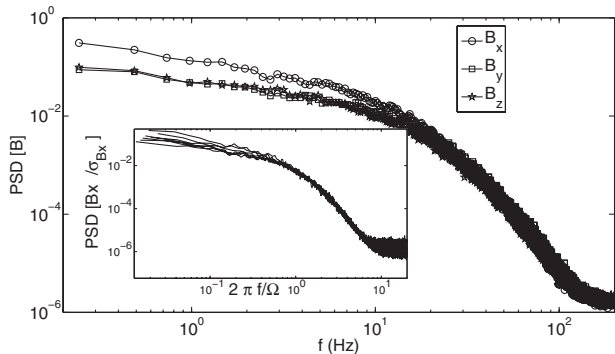


FIG. 5. Power spectra of the fluctuations of each component of the induced magnetic field for $\Omega/2\pi = 15$ Hz. B_x ($\circ$), B_{yrms} ($\square$), B_{zrms} (*). Inset: power spectra of the fluctuations of B_x normalized by its standard deviation σ_{Bx} vs $2\pi f/\Omega$ for $\Omega/2\pi = 8, 10, 12, 14, 15, 16, 18$ Hz.

These results show that magnetic induction by a turbulent flow of an electrically conducting fluid, measured away from an applied localized magnetic field, differ significantly from induction due to a uniform applied field. When the applied field is uniform, the mean induced field has been found larger than its fluctuations (except for the components that must vanish at some locations because of symmetry constraints) [4,6]. On the contrary, away from an applied localized field, rms fluctuations are much larger than the mean induced field. Direct time recordings show very intermittent signals with bursts of magnetic field (see Fig. 2). This is the picture one expects if intense eddies move randomly from the neighborhood of the magnet to the probe, transporting magnetic field. The PDF of the transported field (Fig. 6) confirm the intermittent character of the fluctuations. For R_m large enough, they display exponential tails similar to some flow configurations involving the random advection of a scalar field [14] although the values of the respective Prandtl numbers strongly differ.

The mean transported magnetic field measured by the probe almost averages to zero because its orientation is randomly distributed. This is expected in homogeneous isotropic turbulence. The smallness of the mean values measured in the present experiment show that fluctuations are dominant compared to the mean flow in the transport process of the field, at least as far as its orientation is concerned. In order to check our hypothesis concerning the dominant effect of fluctuations compared to the mean flow, we have also measured the induced field with only one rotating disk. The flow is then much less turbulent. We have recorded the induced field transported from the magnet placed in the bulk (P in Fig. 1) to a Hall probe placed close to the rotating disk. Both the mean and rms values of the induced field components vary linearly with R_m , the slope of the mean being larger. Thus, when the effect of the

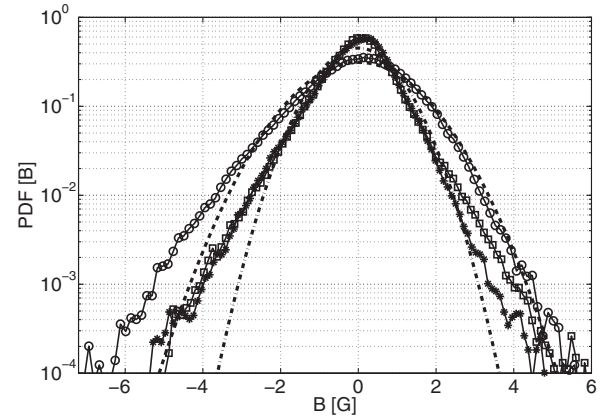


FIG. 6. PDF of the fluctuations of the induced magnetic field: B_x ($\circ$), B_y ($\square$), B_z (*). Disk rotation frequency, $\Omega/2\pi = 15$ Hz. The dashed (dash-dotted) curve is a Gaussian with standard deviation equal to the one of B_x (B_y or B_z).

mean flow is large enough, the mean transported field does not average to zero and can be larger than the fluctuations. In intermediate cases where mean flow and fluctuations have comparable effects, a mean transported field is mostly observed in geometries for which mean flow lines connect the magnet to the probe. The study of the spatial decorrelation of the induced field as the level of fluctuations is increased will be reported elsewhere [15].

Finally, it is worth discussing some consequences of the present results on simple dynamo processes. Most dynamo models consist of a feedback loop involving several steps in which induction processes occur in different regions of the flow. For instance, in the case of an alpha-omega dynamo, toroidal field is created in regions with strong differential rotation from the poloidal component and poloidal field is regenerated from the toroidal one in regions containing cyclonic eddies with the same helicity. In this mechanism, as well as in others, it is crucial that the mean field component generated in some region is transported coherently to another region. We show that the mean field orientation is lost if turbulent fluctuations are large [16].

These observations question the validity of dynamo models based only on the geometry of the mean flow, thus neglecting the effect of large scale turbulent fluctuations. The problem is to estimate to which extent the dynamo threshold computed as if the mean flow were acting alone, is shifted by turbulent fluctuations. This question has been addressed only recently [17] and should not be confused with dynamo generated by random flows with zero mean [18]. It has been shown that weak turbulent fluctuations do not shift the dynamo threshold of the mean flow at first order. In addition, in the case of small scale fluctuations, there is no shift at second order either if the fluctuations have no helicity. This explains why the observed dynamo threshold in Karlsruhe and Riga experiments has been found in good agreement with the one computed as if the mean flow were acting alone [19]. Recent numerical simulations have also displayed a shift in threshold only at second order with small fluctuations but a more complex bifurcation structure for larger ones [20]. Thus, the problem is still open in the parameter range of the sodium experiments with unconstrained flows that involve very large fluctuations [4,21].

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