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Mechanisms for magnetic field reversals

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We present a review of the different models that have been proposed to explain reversals of the magnetic field generated by a turbulent flow of an electrically conducting fluid (fluid dynamos). We then describe a simple mechanism that explains several features observed in palaeomagnetic records of the Earth's magnetic field, in numerical simulations and in a recent dynamo experiment. A similar model can also be used to understand reversals of large-scale flows that often develop on a turbulent background.

Keywords: dynamo theory; oscillations; magnetic field reversals

1. Introduction

It has been known since the work of Brunhes (1906) that the Earth's magnetic field remains roughly parallel to its rotational axis for long durations, but from time to time it flips, with the poles reversing sign. Polarity reversals are also observed for the magnetic field of the Sun, but they occur nearly periodically. It is strongly believed that the magnetic fields of planets and stars are generated by the dynamo effect, i.e. the amplification of electric currents by the motion of an electrically conducting fluid (Moffatt 1978). Flows in the interiors of planets or stars have huge kinetic Reynolds numbers, $Re = VL/\nu$, where V is the typical velocity, L is the integral length scale and ν is the kinematic viscosity. For instance, $Re \sim 10^9$ in the Earth's liquid core or $Re \sim 10^{15}$ in the convective zone of the Sun. These flows being strongly turbulent, we would expect them to advect and distort the magnetic field lines in a very complicated way. Thus, it is puzzling that the generated magnetic fields display a large-scale coherent component with rather simple dynamics.

Reversals of the magnetic field generated by a turbulent swirling flow of liquid sodium (the von Kármán sodium or VKS experiment) have been observed only recently in laboratory experiments (Berhanu *et al.* 2007). Although $Re \sim 5 \times 10^6$ for these flows, it has been observed that the large-scale dynamics of the magnetic field result from the interactions of a few modes, and that the low-dimensional nature of these dynamics is not smeared out by strong turbulent fluctuations of the flow (Ravelet *et al.* 2008).

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One contribution of 13 to a Theme Issue 'Turbulent mixing and beyond'.

This type of behaviour also occurs in purely hydrodynamical turbulent flows, where the largest scales sometimes seem to be governed by low-dimensional dynamics. In dynamo experiments or to some extent for the geodynamo, this can result from the proximity of the dynamo threshold given by a critical value of the magnetic Reynolds number, $R_m = \mu_0 \sigma VL$, where μ_0 is the magnetic permeability of vacuum and σ is the electrical conductivity of the fluid. This is unlikely for the solar magnetic field, for which R_m is huge. However, the magnetic Prandtl number, $Pr_m = R_m/Re = \mu_0 \sigma \nu$, is very small for all planetary and stellar dynamos as well as in the present experiments ($Pr_m < 10^{-5}$), meaning that the magnetic diffusion time scale is much smaller than the hydrodynamic one. This scale separation can explain the low-dimensional nature of the dynamics of the magnetic field despite strong turbulent fluctuations.

2. Phenomenological models and numerical simulations

(a) *Disc dynamos and truncations of the magnetohydrodynamic equations*

The first simple models of field reversals considered coupled rotor disc dynamos (Rikitake 1958) or a Bullard disc dynamo when a shunt is added (Robbins 1977). The equations for the currents are of the same type as for the Lorenz model (Lorenz 1963). When two solutions related to one another by the $B \rightarrow -B$ symmetry are unstable and chaotic regimes occur, these systems stay for a while in the vicinity of one solution and then flip to the neighbourhood of the other. These transitions occur in a random fashion and this can be considered as reversal dynamics. However, both the shape of the transitions displayed by direct recordings as well as their statistical properties differ from the experimental observations of field reversals and from palaeomagnetic records. In addition, equations governing disc dynamos strongly differ from full magnetohydrodynamic (MHD) equations and cannot be obtained from them in any consistent approximation. When truncating the full MHD equations by keeping two magnetic modes of the diffusion operator and one velocity mode, Nozières (1978) found equations similar (but not identical) to those of Rikitake (1958). He then described reversals as a relaxation limit cycle between two quasi-stationary states related by the $B \rightarrow -B$ symmetry.

(b) *Normal forms*

A different class of models, also involving a few coupled differential equations, is based on the assumption that several magnetic eigenmodes are competing above the dynamo threshold. These models have mostly been used to describe the solar cycle. Tobias *et al.* (1995) take into account two magnetic modes (a poloidal and a toroidal one) undergoing a Hopf bifurcation. They assume that the velocity field generating the magnetic field is close to a saddle-node bifurcation and couple the marginal velocity mode to the magnetic modes in order to obtain a third-order system that can display periodic, quasi-periodic and chaotic behaviours. Wilmot-Smith *et al.* (2005) obtain similar results but with a coupling term that does not break the $B \rightarrow -B$ symmetry.

Knobloch & Landsberg (1996) consider a different model that involves not marginal velocity modes but two magnetic modes, a dipolar and a quadrupolar one, both generated through a Hopf bifurcation. Taking into account 1:1

resonant coupling terms, they find aperiodic regimes that can also represent the modulation of the cyclic activity of the solar magnetic field. Finally, Knobloch *et al.* (1998) assume the existence of two velocity modes, symmetric and antisymmetric with respect to the equatorial plane, and couple them to the dipolar and quadrupolar magnetic modes of the previous model. They show that two different types of modulation of the cyclic activity can be described.

In the framework of normal forms, it has been proposed to relate reversals to trajectories close to heteroclinic cycles that connect unstable fixed points $\pm\mathbf{B}$ (Armbruster *et al.* 2001; Chossat & Armbruster 2003). Heteroclinic cycles provide a simple framework to describe separation of time scales between rapid reversals that connect quasi-steady states with a given polarity. The latter is related to saddle points in the vicinity of which the system slows down. Melbourne *et al.* (2001) tried to describe the dynamics of the Earth's magnetic field by writing amplitude equations for an equatorial dipole coupled to an axial dipole and quadrupole. This model has heteroclinic cycles but no connection of states with opposite polarities except when additional coupling terms that break the symmetries are taken into account. Strictly speaking, a stable heteroclinic cycle connecting $\pm\mathbf{B}$ cannot describe reversals because the period goes to infinity as the trajectory is attracted on the cycle. However, an arbitrary amount of noise is enough to kick the system away from the saddle points and to generate random reversals with a finite mean period (Stone & Holmes 1990).

(c) *Metastable states in the presence of external noise*

Other models rely on external noise in a stronger way. They start from a dipolar magnetic mode with amplitude $D(t)$ that bifurcates supercritically and model the effect of hydrodynamic turbulence through random fluctuations of the coefficients of the dynamical system governing D and the amplitudes of the stable modes in the vicinity of the bifurcation threshold. Fluctuations only in the amplitude equation, $\dot{D} = \mu D - D^3$, i.e. a growth rate μ that involves a noisy component, do not lead to reversals between the two stationary solutions $D = \pm\sqrt{\mu}$. However, taking into account that D is coupled with the stable modes, which are also excited by fluctuations, can lead to reversals (Schmitt *et al.* 2001). D behaves as the position of a strongly damped particle driven by random noise in a two-well potential. The crucial role of damped modes has been emphasized further by Hoyng & Duistermaat (2004). The reversals are triggered by large fluctuations of damped modes driven by noise. These modes act on D as an effective additive noise.

Recent numerical simulations have modelled hydrodynamic fluctuations with a noisy α -effect (Giesecke *et al.* 2005; Stefani & Gerbeth 2005; Stefani *et al.* 2007). The deterministic part of this model can generate periodic relaxation oscillations with the system slowing down in the vicinity of two states with opposite polarities $\pm\mathbf{B}$. In this respect, it belongs to the class of systems described by Nozières (1978). The addition of external noise is thus crucial to generate random reversals. It is likely that the phenomenology of this model is related to the proximity of a codimension-two point that results from two interacting modes with different radial structures.

(d) Hydrodynamic mechanisms and direct numerical simulations

The above descriptions of reversals assume the existence of some large-scale dominant modes of the magnetic field. The random dynamics of reversals are either of deterministic nature (low-dimensional chaos) or result from the addition of external noise that traces back to hydrodynamic fluctuations.

A different approach, initiated by Parker (1969), consists in trying to identify the nature of the fluctuations of the velocity field that is required to generate a reversal. In the case of the Earth, it is believed that the magnetic field is generated through an α - ω mechanism, ω being related to differential rotation and α resulting from the existence of cyclonic convective cells in the Earth's core, which fluctuate both in number and in position. When strong enough, these fluctuations can reverse the magnetic field (Parker 1969; Levy 1972).

Another mechanism has also been proposed by Parker (1979). It follows from the observation by Roberts (1972) that a meridional circulation favours stationary dipolar α - ω dynamos in spherical geometries. Parker (1979) suggested that, if the meridional circulation is altered for a while, an oscillatory magnetic mode may become dominant and generates a reversal of the magnetic field. It was later claimed that this mechanism is suggested from palaeomagnetic data (McFadden & Merrill 1995). Numerical simulations of the MHD equations in a rotating sphere have displayed this in a clear-cut way: it has been shown by Sarson & Jones (1999) and Sarson (2000) that the random emission of poleward light plumes, or 'buoyancy surge', generates fluctuations of the meridional flow that can trigger a reversal. They also found that this mechanism is not affected much by the back reaction of the magnetic field on the flow and does result from the proximity in parameter space of stationary and time-periodic dynamo modes, depending on the intensity of the meridional flow. A process also related to convective plumes has been observed by Wicht & Olson (2004). They found that a magnetic field with an opposite polarity is produced locally in the convective plumes and that the transport of this reversed flux can generate a reversal. They also showed that the observed reversals are almost unchanged when the Lorentz force is removed from their numerical code. Other advection processes of the magnetic field by the flow have been studied in detail by Aubert *et al.* (2008). It should be noted that all these numerical simulations have been performed with large values of Pr_m ($1 < Pr_m < 20$). Local modifications of the magnetic field by the flow are likely to play a less important role for small values of Pr_m because of strong ohmic diffusion.

Since 1995 (Glatzmaier & Roberts 1995), a lot of three-dimensional numerical simulations of the MHD equations in a rotating sphere have been able to simulate a self-consistent magnetic field that displays reversals (see the reviews by Dormy *et al.* (2000) and Roberts & Glatzmaier (2000)). However, it has been emphasized that most relevant dimensionless parameters that can be achieved in direct simulations are orders of magnitude away from their value in the Earth's core or laboratory experiments. Even in the limited range accessible to direct simulations, it has been shown that the geometry of the generated magnetic field and the properties of field reversals can strongly depend on the values of the relevant dimensionless numbers (Kutzner & Christensen 2002; Busse & Simitev 2006). Thus, one may conclude as in Coe *et al.* (2000) that 'each reversal in the simulations has its own unique character, which can differ greatly in

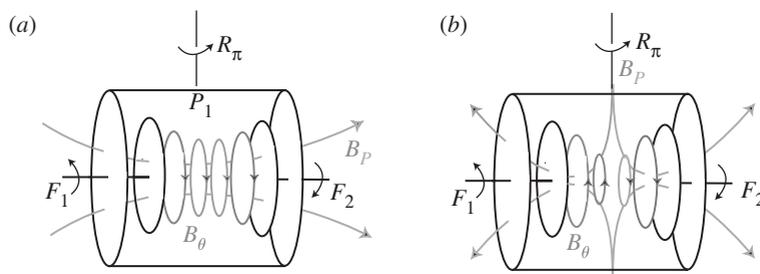


Figure 1. Possible eigenmodes of the VKS experiment. The two discs counter-rotate with frequencies F_1 and F_2 . (a) Magnetic dipolar mode. (b) Magnetic quadrupolar mode. Poloidal, B_P , and toroidal, B_θ , components are sketched.

various aspects from others'. However, we emphasize that a lot of these numerical simulations also display similar properties at a more global level, if one considers how the symmetries of the flow and the magnetic field evolve during a reversal.

(e) *Dipole–quadrupole interaction and equatorial symmetry*

A dipole–quadrupole interaction is clearly visible in the first reversals simulated by Glatzmaier & Roberts (1995). They note that ‘the toroidal field is asymmetric with respect to the Equator before and after the reversal but is symmetric midway through the transition’, thus has a quadrupolar symmetry at the transition (see their fig. 2). This has been confirmed by the simulations of Sarson & Jones (1999), who find that reversals rely ‘heavily upon the interaction between dipole and quadrupole symmetries’ and that they are triggered by the random emission of poleward light plumes, i.e. events that break the equatorial symmetry of the flow. Similar features have been observed by Wicht & Olson (2004). Li *et al.* (2002) also emphasize that ‘the dipole polarity can reverse only ... where the north–south symmetry of the convection pattern is broken’ and that ‘the quadrupole mode grows ... before the reversal’. It has also been shown that, if the flow or the magnetic field is forced to remain equatorially symmetric, then reversals do not occur (Nishikawa & Kusano 2008).

3. Reversals of the magnetic field in laboratory experiments

Reversals of the magnetic field generated by a turbulent swirling flow of liquid sodium (VKS experiment) have been observed only recently in laboratory experiments (Berhanu *et al.* 2007). The VKS experiment involves a turbulent swirling flow of liquid sodium, generated by two impellers, counter-rotating at frequency F_1 (respectively, F_2) in an inner copper cylinder, as sketched in figure 1. When the discs counter-rotate with the same frequency F , a statistically stationary magnetic field is generated when F is large enough. Its mean value involves a dominant poloidal dipolar component \mathbf{B}_P along the axis of rotation, together with a related azimuthal component \mathbf{B}_θ , as displayed in figure 1a. When the rotation frequencies are different, the magnetic field can display periodic or random reversals as well as random bursts. Although the kinetic Reynolds number

is large, $Re \sim 5 \times 10^6$, for these flows, it has been observed that the dynamics of the magnetic field are not smeared out by strong turbulent fluctuations. In particular, all reversals involve the same transitional field morphology: the amplitude of the dipolar field first decreases. If it changes polarity, the amplitude increases on a faster time scale and then displays an overshoot before reaching its statistically stationary state. Otherwise, the magnetic field grows again with its direction unchanged. The above features are also observed in recordings of the Earth's magnetic field, the aborted reversals often being called excursions (Valet *et al.* 2005).

The most striking feature of the VKS experiment is that time-dependent magnetic fields are generated only when the impellers rotate at different frequencies (Berhanu *et al.* 2007; Ravelet *et al.* 2008). We have shown in Pétrélis & Fauve (2008) that this is related to the broken invariance under \mathcal{R}_π when $F_1 \neq F_2$ (rotation of an angle π along any axis in the mid-plane). In that case, symmetric and antisymmetric modes (under \mathcal{R}_π) are coupled. Such modes are displayed in figure 1: a dipolar mode is changed to its opposite by \mathcal{R}_π , whereas a quadrupolar mode is unchanged.

4. A generic mechanism for reversals

Although the flow in the VKS experiment strongly differs from the one in the Earth's core, dipolar and quadrupolar modes can be defined in both cases (using different symmetries). We assume that the magnetic field is the sum of a dipolar component with an amplitude D and a quadrupolar one, Q . We define $A = D + iQ$ and we assume that an expansion in powers of A and its complex conjugate \bar{A} is pertinent close to threshold in order to obtain an evolution equation for both modes. Taking into account the invariance $\mathbf{B} \rightarrow -\mathbf{B}$, i.e. $A \rightarrow -A$, we obtain

$$\dot{A} = \mu A + \nu \bar{A} + \beta_1 A^3 + \beta_2 A^2 \bar{A} + \beta_3 A \bar{A}^2 + \beta_4 \bar{A}^3, \quad (4.1)$$

where we limit the expansion to the lowest-order nonlinearities. In the general case, the coefficients are complex and depend on the experimental parameters.

Symmetry of the experiment with respect to \mathcal{R}_π amounts to constraints on the coefficients. Applying the transformation \mathcal{R}_π changes D and Q in different ways: $D \rightarrow -D$, $Q \rightarrow Q$, thus $A \rightarrow -\bar{A}$. We conclude that, in the case of exact counter-rotation, all the coefficients are real. More generally, the real parts are even and the imaginary parts are odd functions of the frequency difference $f = F_1 - F_2$.

The coefficients of equation (4.1) can be chosen such that it has two stable dipolar solutions $\pm D$ and two unstable quadrupolar solutions $\pm Q$ when $f = 0$ (Pétrélis & Fauve 2008). When f is increased, these solutions become more and more mixed due to the increase of the strength of the coupling terms between dipolar and quadrupolar modes. Dipolar (respectively, quadrupolar) solutions get a quadrupolar (respectively, dipolar) component and give rise to the stable solutions $\pm B_s$ (respectively, unstable solutions $\pm B_u$) displayed in figure 2. When f is increased further, a saddle-node bifurcation can occur, i.e. the stable and unstable solutions collide by pairs and disappear (Pétrélis & Fauve 2008). This generates a limit cycle that connects the collision point with its opposite. This result can be understood as follows. The solution $B = 0$ is unstable with respect to the two different fixed points, and their opposite. It is an unstable point,

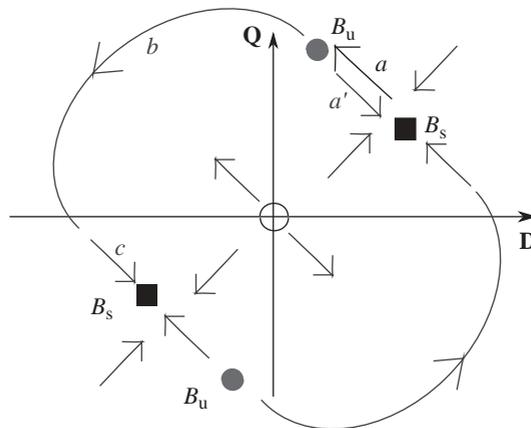


Figure 2. A generic saddle-node bifurcation in a system with the $\mathbf{B} \rightarrow -\mathbf{B}$ invariance: below threshold, fluctuations can drive the system against its deterministic dynamics (phase a). If the effect of fluctuations is large enough, this generates a reversal (phases b and c). Otherwise, an excursion occurs (phase a').

whereas one of the two bifurcating solutions is a stable point, a node, and the other is a saddle. If the saddle and the node collide, say at B_c , what happens to initial conditions located close to these points? They cannot be attracted by $B = 0$, which is unstable, and they cannot reach other fixed points, since they just disappeared. Therefore, the trajectories describe a cycle. The associated orbit contains $B = 0$, since, for a planar problem, in any orbit, there is a fixed point. Suppose that the orbit created from B_c is different from the one created by $-B_c$. These orbits being images by the transformation $\mathbf{B} \rightarrow -\mathbf{B}$, they must intersect at some point. Of course, this is not possible for a planar system because it would violate the uniqueness of the solutions. Therefore, there is only one cycle that connects points close to B_c and $-B_c$.

This provides an elementary mechanism for field reversals in the vicinity of a saddle-node bifurcation. First, in the absence of fluctuations, the limit cycle generated at the saddle-node bifurcation connects $\pm B_c$. This corresponds to periodic reversals. Slightly above the bifurcation threshold, the system spends most of the time close to the two states of opposite polarity $\pm B_c$. Second, in the presence of fluctuations, random reversals can be obtained slightly below the saddle-node bifurcation. B_u being very close to B_s , even a fluctuation of small intensity can drive the system to B_u , from which it can be attracted by $-B_s$, thus generating a reversal.

The effect of turbulent fluctuations on the dynamics of the two magnetic modes governed by equation (4.1) can be easily modelled by adding some noisy component to the coefficients (Pétreliis & Fauve 2008). Random reversals are displayed in figure 3*a*. The system spends most of the time close to the stable fixed points $\pm B_s$. We observe in figure 3*b* that a reversal consists of two phases. In the first phase, the system evolves from the stable point B_s to the unstable point B_u (in the phase space sketched in figure 2). The deterministic part of the dynamics acts against this evolution and the fluctuations are the motor of the

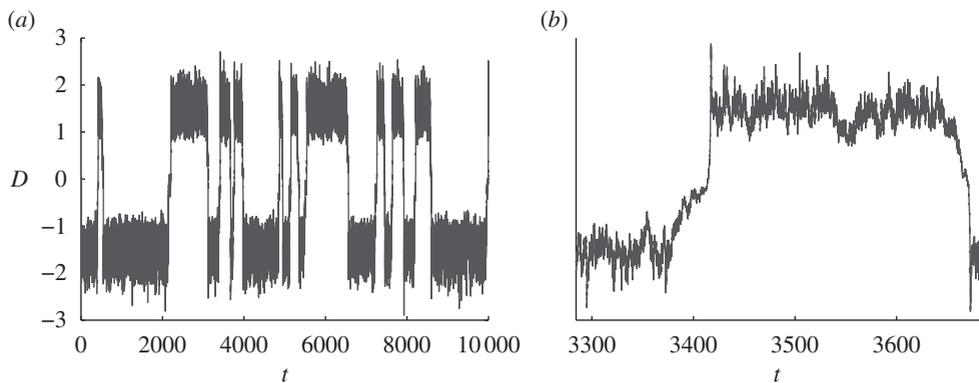


Figure 3. Reversals of the magnetic field modelled by equation (4.1) with noisy coefficients: (a) time recording of the magnetic dipolar mode, and (b) zoom of one of the reversals shown in (a).

dynamics. That phase is thus slow. In the second phase, the system evolves from B_u to $-B_s$, the deterministic part of the dynamics drives the system and this phase is faster.

The behaviour of the system close to B_s depends on the local flow in the phase space. Close to the saddle-node bifurcation, the positions of B_s and B_u define the slow direction of the dynamics. If a component of B_u is smaller than the corresponding one of B_s , that component displays an overshoot at the end of a reversal. In the opposite case, that component will increase at the beginning of a reversal. For instance, in the phase space sketched in figure 2, the component D decreases at the end of a reversal and the signal displays an overshoot. The component Q increases just before a reversal.

For some fluctuations, the second phase connects B_u not to $-B_s$ but to B_s . It is an aborted reversal or an excursion in the context of the geodynamo. Note that, during the initial phase, a reversal and an excursion are identical. In the second phase, the approaches to the stationary phase differ because the trajectory that links B_u and B_s is different from the trajectory that links B_u and $-B_s$. In particular, if the reversals display an overshoot, this will not be the case for the excursion (see figure 3b and the sketch of the cycle in figure 2).

5. A simple model for the Earth's magnetic field reversals

Although the symmetries of the flow in the Earth's core strongly differ from the ones of the VKS experiment, dipolar and quadrupolar modes can be defined with respect to equatorial symmetry such that model (4.1) can be transposed for the geodynamo. Pétrélis *et al.* (2009) have shown that this explains many intriguing features of the reversals of the Earth's magnetic field. The most significant output is that the mechanism predicts specific characteristics of the field obtained from palaeomagnetic records (Valet *et al.* 2005), in particular their asymmetry: the Earth's dipole decays on a slower time scale than it recovers after a reversal. In addition, it displays an overshoot that immediately follows the reversals. Other characteristic features such as excursions as well as the existence of superchrons are understood in the same framework. In addition, we obtain an interesting

prediction about the liquid core in that case: if reversals involve a coupling of the Earth's dipole with a quadrupolar mode (McFadden *et al.* 1991), then this requires that the flow in the core has broken mirror symmetry. In contrast, another scenario has been proposed in which the Earth's dipole is coupled to an octupole, i.e. another mode with a dipolar symmetry (Clement 2004). This does not require an additional constraint on the flow in the core in the framework of our model with two interacting modes. In any case, the existence of two coupled modes allows the system to evolve along a path that avoids $\mathbf{B} = \mathbf{0}$. In physical space, this means that the total magnetic field does not vanish during a reversal but that its spatial structure changes.

6. Conclusion

We have studied dynamical regimes that can arise when two axisymmetric magnetic eigenmodes are coupled. Symmetry considerations allow us to identify properties of the magnetic modes and, in some cases, put constraints on the coupling between the modes. We have shown that, when a discrete symmetry is broken by the flow that generates the magnetic field, the coupling between an odd and an even magnetic mode (with respect to the symmetry) can generate a bifurcation from a stationary state to a periodic state. This behaviour is generic when a saddle-node bifurcation occurs in a system that is invariant under $\mathbf{B} \rightarrow -\mathbf{B}$. Close to the bifurcation threshold, fluctuations drive the system into a state of random reversals that connect a solution B_s to its opposite $-B_s$. This scenario provides a simple explanation for many features of the dynamics of the magnetic field observed in the VKS experiment: alternation of stationary and time-dependent regimes when a control parameter is varied, continuous transition from random reversals to time-periodic ones, characteristic shapes of the time recordings of reversals versus excursions.

Although the discrete symmetry involved for the flow in the Earth's core is different from the one of the VKS experiment, a similar analysis can be performed for the geodynamo (Pétrélis *et al.* 2009).

We emphasize that the above scenario is generic and not restricted to the equation considered here. Limit cycles generated by saddle-node bifurcations that result from the coupling between two modes occur in Rayleigh–Bénard convection (Tuckerman & Barkley 1988; Siggers 2003). A similar mechanism can explain reversals of the large-scale flow generated over a turbulent background in thermal convection (Krishnamurti & Howard 1981; Liu & Zhang 2008) or in periodically driven flows (Sommeria 1986). A model analogue to the present one explains how this large-scale field can reverse without the need for a very energetic turbulent fluctuation acting coherently in the whole flow volume.

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