RESEARCH SUMMARY (2013) .................................................. Denis BERNARD

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Topics:
Over the years, my researches were mainly focused on:

Conformal field theories (CFT) and integrable systems find applications to 2d phase transitions,
to critical quantum systems, and they are closely related to string theories. I have been involved
in the development of the CFT methodological tools. Part of my work relies on deciphering and
using quantum symmetries, others have a more mathematical flavour related either to algebraic
structures and to geometrical aspects of Riemann surfaces.

— Turbulent systems, Turbulent Transports.
Turbulent phenomena are ubiquitous in many everyday phenomena, but still lack a complete
theoretical understanding. I participated to the physical and mathematical collective understanding
of intermittence phenomena in the (up-to-now) unique solvable model of turbulent transport, and
to enlighten traces of conformal invariance in two dimensional turbulence.

— Random Geometry, Random Spatial Processes.
Understanding random fractal patterns is at the core of the comprehension of many physical phe-
omena or mathematical structures, and the brownian motion is a historical example of such struc-
tures. I participated to the understanding of newly constructed planar random curves or interfaces
(called SLE), a theme which fits into random geometry. I developed bridges linking probabilistic
approaches from mathematicians with those of physicists based on field theories.

Experimental progresses in controlling quantum systems gave new impetus to study unexplored
territory of quantum dynamics, and simultaneously to answer old questions of quantum mechanics.
My recent research aims at studying quantum stochastic processes, their mathematical structures
and their applications to the physics of open quantum systems.

— Motivated by curiosity, I also considered problems related to 2d disordered systems, to locali-
sation phenomena, to topological matters, and to facets of semi-classical gravity or to random or
 genetic networks.

Publications:
See below for a complete list. Publications posterior to 1991 (refs.[34,84] are missing) can be found
at: http://arxiv.org/a/bernard_d_1.atom

Collaborators:
O. Babelon (Lpthe, Paris), M. Bauer (Spht, Saclay), G. Boﬀetta (Turin), A. Celani (Nice), B. Doyon
(King’s College), G. Felder (Eth–Zürich), G. Falkovich (Wiezmann Inst.), K. Gawedzki (Ihes, Ens-
Lyon), D. Haldane (Princeton Univ.), A. Kupiaimen (Helsinki Univ.), K. Kytola (Helsinki Univ.),
A. LeClair (Cornell Univ.), P. LeDoussal (Lpt-Ens), V. Pasquier (Spht, Saclay), F. Smirnov (Lpthe,
Paris), and J. Thierry-Mieg (Montpellier), etc.
MAIN RESEARCH ACHIEVEMENTS

Conformal field theories: structures and applications (1986-1995):
— Construction of modular invariant partition functions of WZW models using Kac-Moody algebras and their automorphisms. This relation between automorphisms and partition functions has since then be generalised to other operator algebras, and the list of partition functions obtained in this way was almost exhaustive. Ref.[7].
— Formulation of the WZW models on the torus and on higher genus Riemann surfaces. It required introducing new degrees of freedom coupled to the moduli space of flat fiber bundles on these surfaces. It lead me to write the now called KZB equations (series of differential equations on the moduli space of flat fiber bundles) which had some echoes in other branches of mathematical physics (integrable systems, geometric quantification, quantum groups,...). Refs.[10,11].
— A mathematically precise study of free field representations of the su(2) WZW models, which can be used to exactly solve these models. It required resolving some cohomology defined over representation spaces of Kac-Moody algebras. These methods have since then be extended to other classes of CFTs. Ref.[15].
— Applications of conformal methods to disordered systems and to 2d localisation problems (with attempts to include the quantum Hall transition). Refs.[45,47].

Beyond critical points, integrable systems and applications (1989-1997):
— Highlighting of non-local symmetries in 2d quantum field theories reflecting quantum group symmetries (Yangians, quantum affine algebras), and their use to solve integrable perturbations of CFTs. Semi-classical interpretation of these symmetries and their relations with Lie-Poisson groups. These kinds of non-local symmetries have recently reappeared in super-symmetric gauge theories and in connection with Smirnov’s holomorphic currents. Refs.[17-19,20,23].
— Exact solution of long range interaction spin models (Calogero and Haldane-Shastry models) whose excitations are spinons with fractional statistics. Discovery of their Yangian symmetries and consequences of those. It lead to a new representation of CFTs in terms of quasi-particles with fractional statistics which had some echoes. Ref.[34,36,40].
— Results on integrable systems, on integrable field theories, and on quantum groups and their representations. Formulation of the first algebraic framework for the dynamical Yang-Baxter equations (we introduced a notion of ‘twisted cocycle’ which has been re-used in the theory of quasi-Hopf algebra). Refs.[29,48,49,51].

Turbulent systems (1996-2001, then by intermittence-2008):
— Comprehension of the physical origin of the deviations from mean field and of the multi-fractality in Kraichann’s model of turbulent transports. Beyond computing anomalous dimensions, we were the first to provide an interpretation of these deviations in terms of statistically conserved quantities. A ‘News &Views’, Nature 409 (2001), mentioned “In recent years there has been a fundamental shift in the theoretical approach to such characteristics of turbulence” quoting [55]. Refs.[50,55].
— Study of the influence of friction on the direct cascade of 2d turbulence. We showed that anomalous dimensions are friction dependent, and this explained differences in some experimental results. Formulation of a conjecture concerning the absence of anomalous enstrophy dissipation in presence of friction, a conjecture which has recently been proved. Ref.[61].
— Discovery of traces of conformal symmetry in the inverse cascade of 2d turbulence obtained by studying statistical properties of iso-vorticity lines. This study constituted the first statistical analysis of non-local structures pointing toward the existence of emergent symmetries in turbulence; cf. ‘News & Views’, Nature Physics 2 (2006). Ref.[85].
Curves, domains and random geometry (2002-2012):
— Comprehension and formulation of a relation between spatial random processes, the so-called SLE, and conformal field theories (CFT). We were the first to establish the link between two intrinsically different approaches, one probabilistic (SLE) and the other algebraic (CFT). Refs.[74,75,77].
— Definition of the dipolar SLE processes. Generalisation of SLEs to n-SLEs describing multiple random curves and their relations with statistical mechanics. This extension lead us to reveal the connection between SLE processes and partition functions, a point of view now shared by quite a few probabilists. Refs.[81,83].
— Extension of the domain of applicability of SLEs to disordered or out-of-equilibrium physical systems (turbulence, spin glasses). Refs.[87,89,93].

Open quantum systems and quantum noises (2011-present):
— A rigourous proof of the collapse of probability distribution on repeated non-demolition measurements and an estimation of the collapse rate in terms of certain entropy. Ref.[96].
— A new universal formula for the heat transport in 1d critical systems and a step toward formulating non-equilibrium CFTs. Ref.[97].

A few connexions with mathematics:
Some of my works possess a mathematical flavour or resemble mathematical studies: on Kac-Moody algebras (vertex operator representations [5], cohomology on certain modules of affine algebras [15], KZB equations [10,11]); on quantum groups (Lie-Poisson groups [29], Yangian representations and Dunkl’s operators [36], quasi-Hopf algebras and the dynamical Yang-Baxter equations [48]); on transport and certain stochastic equations (explosive solutions [55]); on Schramm-Loewner processes (space of martingales and Virasoro algebra representations [75,76], SLE-dipolar et n-SLE [81,83]).