Engineering and Probing Topological Bloch Bands in Optical Lattices

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Thomas Reimann, Alexander Keesling, Christian Gross

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Outline

Realizing Artificial Gauge Fields

1. Realizing the Hoftstadter & Quantum Spin Hall Hamiltonian
2. 'Meissner'-currents in bosonic flux ladders

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Probing Topological Features of Bloch Bands

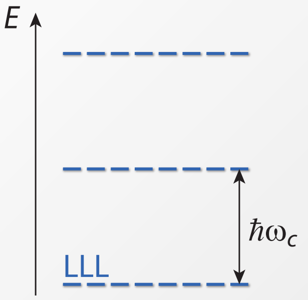
3. Probing Zak Phases in Topological Bands
4. An 'Aharonov Bohm' Interferometer for measuring Berry curvature

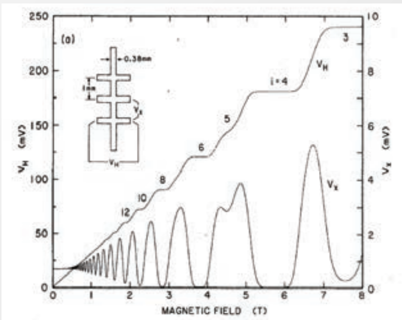
Realizing Artificial Gauge Fields in Optical Lattices

Gauge Fields

Quantum Hall Effect in 2D Electron Gases

Integer Quantum Hall Effect



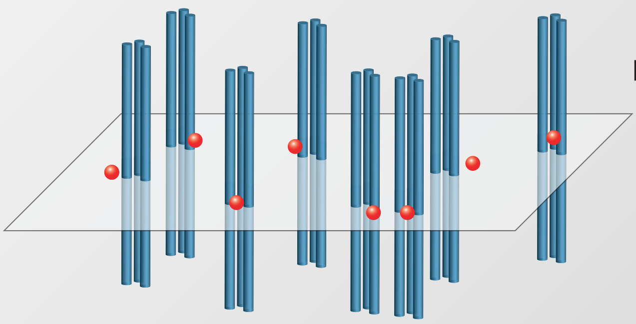


$$\sigma_{xy} = \nu e^2/h$$

ν Integer

Chern Insulators
(w/o magnetic field
see D. Haldane 1988)

Fractional Quantum Hall Effect

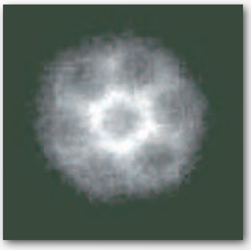


Laughlin state at $\nu = 1/3$

flux quantum ϕ_0=h/ec
electron

State of the Art

1) Rotation

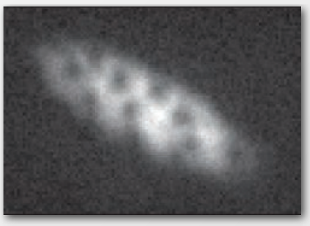


In rapidly rotating gases, ***Coriolis force*** is equivalent to ***Lorentz force***.

$$\mathbf{F}_{\mathrm{L}} = q\mathbf{v}\times\mathbf{B} \Longleftrightarrow \mathbf{F}_{\mathrm{C}} = 2m\mathbf{v}\times\Omega_{\mathrm{rot}}$$

K. Madison *et al.*, PRL (2000)
J.R. Abo-Shaeer *et al.* Science (2001)

2) Raman Induced Gauge Fields



Spatially dependent optical couplings lead to a **Berry phase** analoguous to the **Aharonov-Bohm phase**

Y. Lin *et al.*, Nature (2009)

Gauge Fields

State of the Art

1) Rotation

In rapidly rotating gases, ***Coriolis force*** is equivalent to ***Lorentz force***.

$\mathbf{v}\times\Omega_{\mathrm{rot}}$

t al., PRL (2000)
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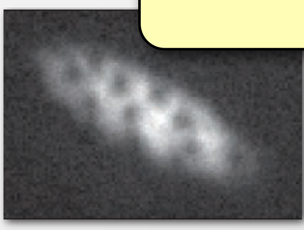
2) Raman

Problem in both cases: small B-fields (large ν>1000 for now), heating...



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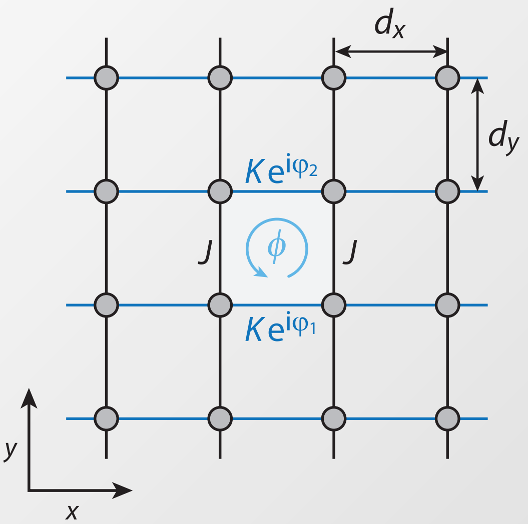
Sunday 22 June 14

Artificial B-Fields with Ultracold Atoms

Controlling atom tunneling along x with Raman lasers leads to effective tunnel coupling with spatially-dependent Peierls phase $\varphi(\mathbf{R})$

$$\hat{H} = -\sum_{\mathbf{R}} \left(K e^{i\varphi(\mathbf{R})} \, \hat{a}^\dagger_{\mathbf{R}} \hat{a}_{\mathbf{R}+\mathbf{d}_x} + J \, \hat{a}^\dagger_{\mathbf{R}} \hat{a}_{\mathbf{R}+\mathbf{d}_y} \right) + \text{h.c.}$$



Magnetic flux through a plaquette:

$$\boxed{\phi = \int_{\blacksquare} B \, \mathrm{d}S = \varphi_1 - \varphi_2}$$

D. Jaksch & P. Zoller, New J. Phys. (2003)
F. Gerbier & J. Dalibard, New J. Phys. (2010)
N. Cooper, PRL (2011)
E. Mueller, Phys. Rev. A (2004)
L.-K. Lim et al. Phys. Rev. A (2010)
A. Kolovsky, Europhys. Lett. (2011)

see also: lattice shaking
E. Arimondo, PRL(2007) , K. Sengstock, Science (2011),
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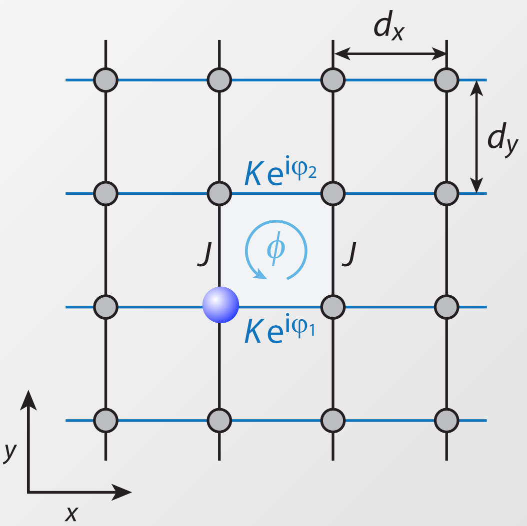

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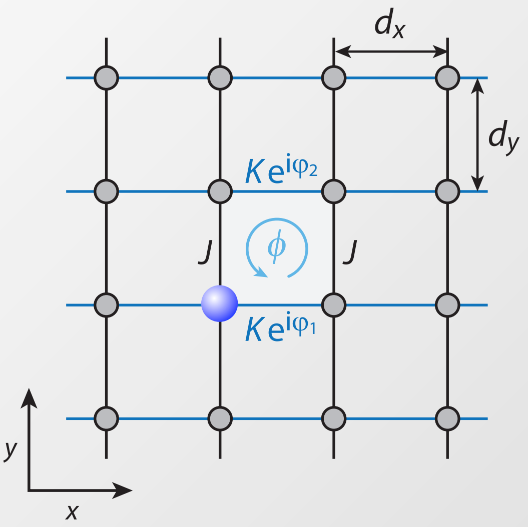

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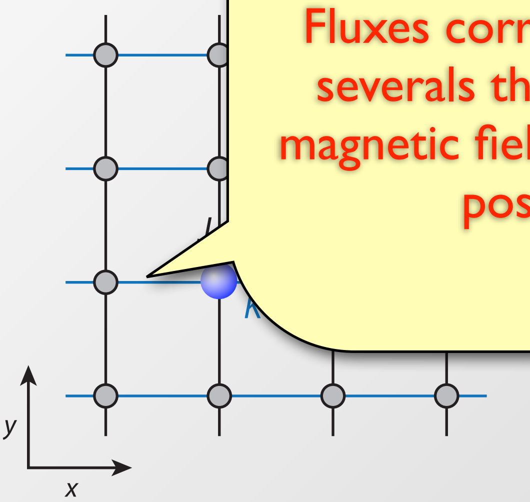
Controlling atom tunneling along *x* with Raman lasers leads to effective tunnel coupling with spatially-dependent Peierls phase $\varphi(\mathbf{R})$

$\hat{H} = -\sum_{\mathbf{R}} ($ … $)$ h.c.



Fluxes corresponding to severals thousand Tesla magnetic field strength are possible!

…ough a plaquette:

$$\ldots S = \varphi_1 - \varphi_2$$

…h & P. Zoller, New J. Phys. (2003)
… & J. Dalibard, New J. Phys. (2010)
N. Cooper, PRL (2011)
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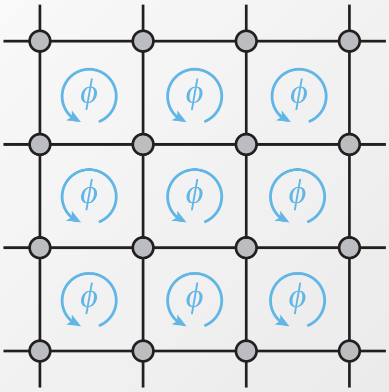
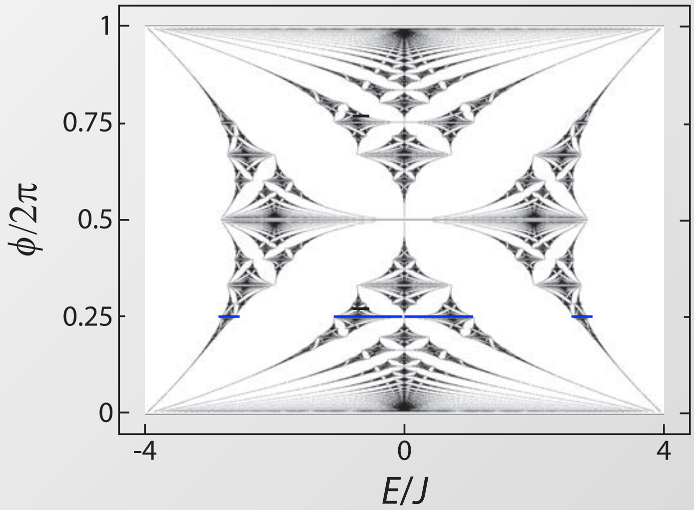
Gauge Fields

Harper Hamiltonian and Hofstadter Butterfly

Harper Hamiltonian: J=K and ϕ uniform.





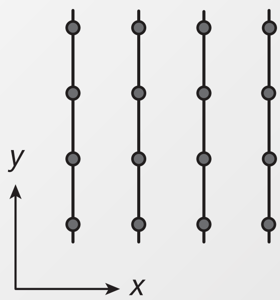
The lowest band is topologically equivalent to the lowest Landau level.

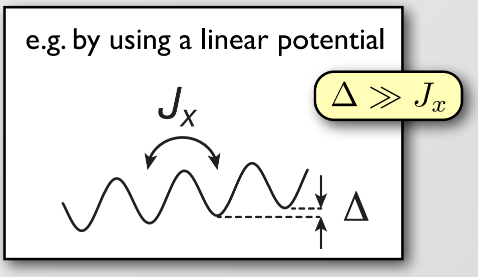
D.R. Hofstadter, Phys. Rev. B**14**, 2239 (1976)
see alo Y. Avron, D. Osadchy, R. Seiler, Physics Today 38, 2003

Artificial magnetic fields

Experimental method

- Atoms in a 2D lattice
- Tunneling inhibited along one direction using energy offsets





M. Aidelsburger *et al.*, PRL (2011); M. Aidelsburger *et al.*, Appl. Phys. B (2013)

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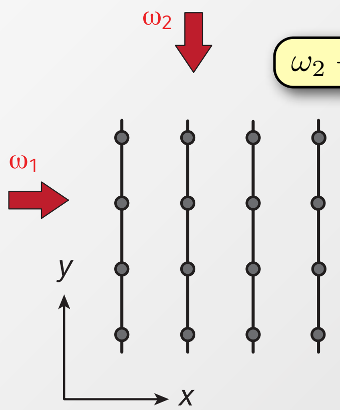
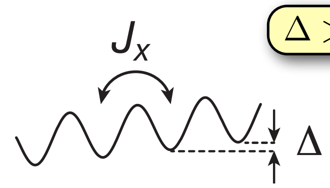


$\omega_2 - \omega_1 = \Delta/\hbar$

e.g. by using a linear potential

$\Delta \gg J_x$

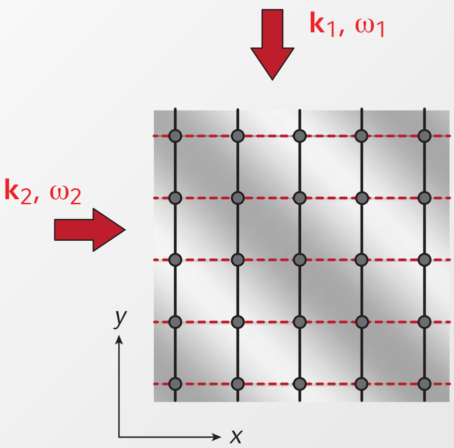


- Induce resonant tunneling with a pair of ***far-detuned*** running-wave beams
  → ***Reduced heating*** due to spontaneous emission compared to Raman-assisted tunneling!
  → ***Independent*** of the internal structure of the atom

M. Aidelsburger *et al.*, PRL (2011); M. Aidelsburger *et al.*, Appl. Phys. B (2013)

Experimental method

- Interference creates a running-wave that ***modulates*** the lattice
- The ***phase of the modulation*** depends on the position in the lattice



Lattice modulation:

$V_K^0 \cos(\omega t + \phi(\mathbf{r}))$

with ***spatial-dependent*** phase

$\phi(\mathbf{r}) = \delta\mathbf{k} \cdot \mathbf{r}$

$\delta\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$

$\omega = \omega_2 - \omega_1$

- Realization of ***time-dependent*** Hamiltonian, where tunneling is restored
- Discretization of the phase due to underlying lattice → $\phi_{m,n}$

M. Aidelsburger *et al.*, PRL (2011); M. Aidelsburger *et al.*, Appl. Phys. B (2013)

Experimental method

- Time-dependent Hamiltonian:

$$\hat{H}(t) = \sum_{m,n} \left(-J_x\ \hat{a}^\dagger_{m+1,n}\hat{a}_{m,n} - J_y\ \hat{a}^\dagger_{m,n+1}\hat{a}_{m,n} + \text{h.c.}\right)$$

$$+ \sum_{m,n} \left[m\Delta + V_K^0 \cos(\omega t + \phi_{m,n})\right] \hat{n}_{m,n}$$

Artificial magnetic fields

Experimental method

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$$+ \sum_{m,n} \left[m\Delta + V_K^0 \cos(\omega t + \phi_{m,n})\right] \hat{n}_{m,n}$$

- Can be mapped on an effective time-averaged time-independent Hamiltonian for $\hbar\omega \gg J_x, J_y, U$

$$\hat{H}_{eff} = \sum_{m,n} \left(-Ke^{i\phi_{m,n}}\ \hat{a}^\dagger_{m+1,n}\hat{a}_{m,n} - J\ \hat{a}^\dagger_{m,n+1}\hat{a}_{m,n} + \text{h.c.}\right)$$

- To avoid excitations to higher bands $\hbar\omega$ has to be smaller than the band gap

F. Grossmann and P. Hänggi, EPL (1992)
M. Holthaus, PRL (1992)
A. Kolovsky, EPL (2011); A. Eckardt, PRL (2005)
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- Can be mapped on an e ... nian
  for $\hbar\omega \gg J_x, J_y, U$

Note: Corrections could be important!
see e.g. N. Goldman & J. Dalibard arXiv:1404.4373
& related work A. Polkovnikov

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Effective coupling strength

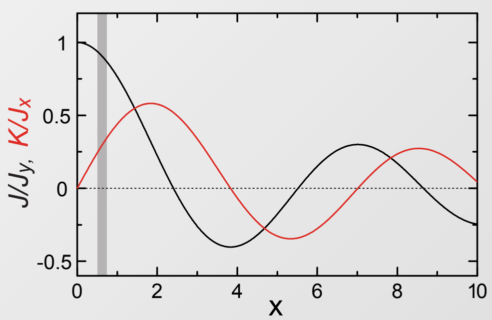
Effective coupling strength:

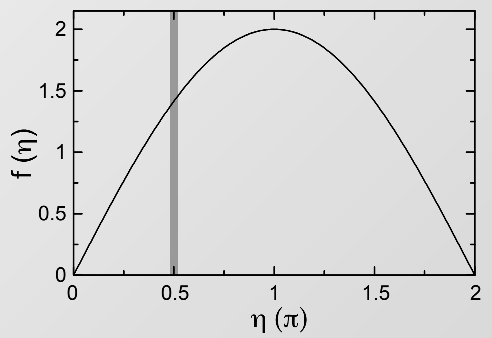
$$K = J_x \mathcal{J}_1(x)$$
$$J = J_y \mathcal{J}_0(x)$$

$\mathcal{J}_\nu(x)$: Bessel-functions of the first kind

and $x = \dfrac{f(\eta) V_K^0}{\Delta}$

η : Phase difference of the modulation between neighboring bonds

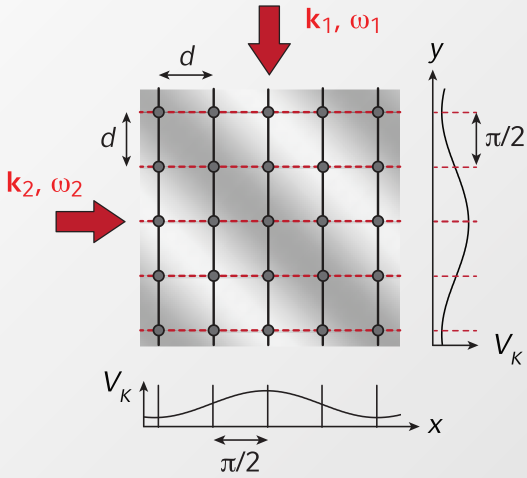




see also: H. Lignier et al. PRL (2007)

Experimental setup

Experimental parameters:



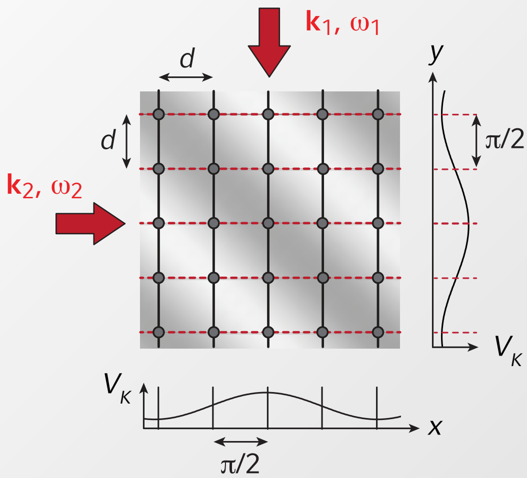
$$|\mathbf{k}_1| \simeq |\mathbf{k}_2| = \frac{\pi}{2d}$$

$$\Rightarrow \phi_{m,n} = \frac{\pi}{2}(m+n)$$

Artificial magnetic fields

Experimental setup

Experimental parameters:



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Flux through one unit cell:

$$\Phi = \phi_{m,n+1} - \phi_{m,n} = \frac{\pi}{2}$$

depends only on phase difference along y!

Experimental setup

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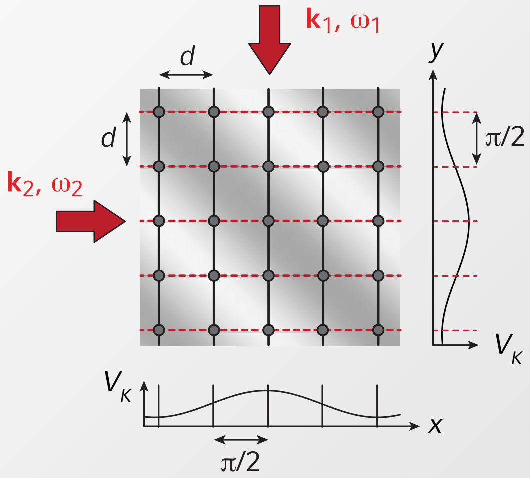

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Flux through one unit cell:

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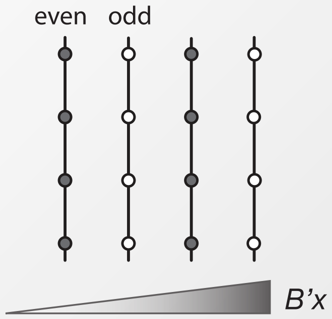
depends only on phase difference along y!

The value of the flux is ***fully tunable*** by changing the geometry of the driving-beams!

Laser-assisted tunneling

Study laser-assisted tunneling in the presence of a magnet field gradient

- Initial state: atoms (^{87}Rb) in 3D lattice only populate even sites



M. Aidelsburger *et al.*, PRL **111**, 185301 (2013)
similar work: H. Miyake *et al.*, PRL **111**, 185302 (2013)

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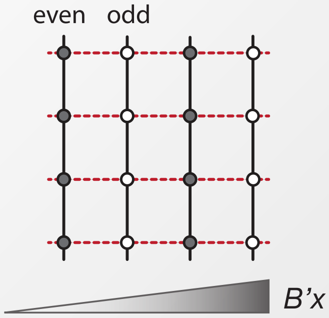

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Uniform flux

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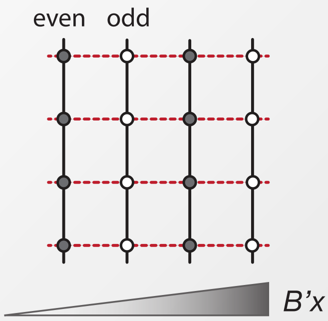
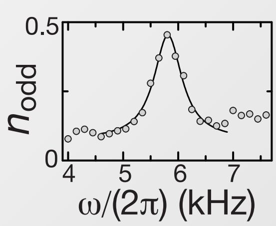


- Atom population in odd sites vs. modulation frequency



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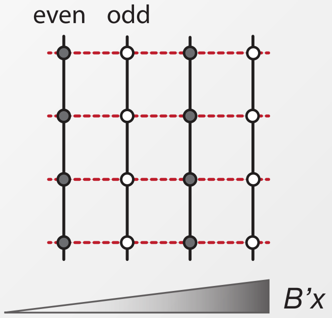

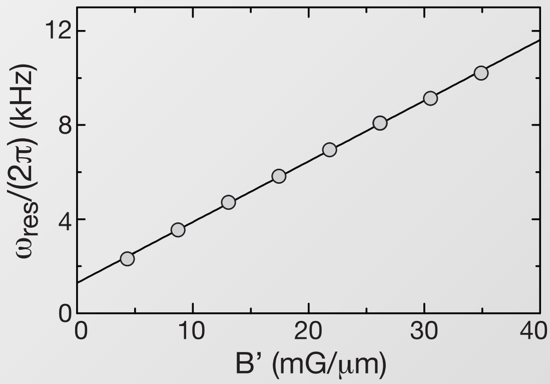

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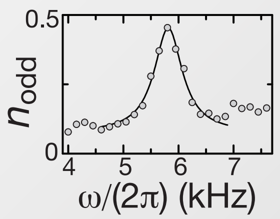


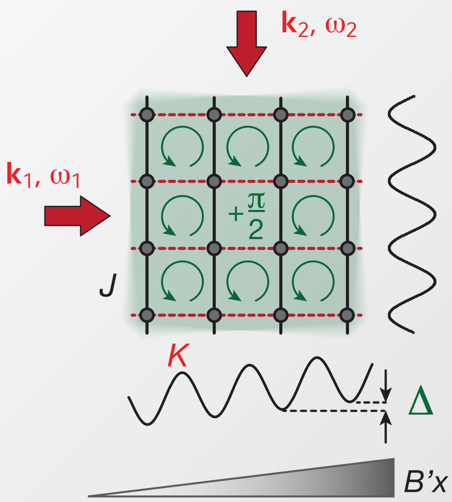
Large range of values accessible!

M. Aidelsburger *et al.*, PRL **111**, 185301 (2013)
similar work: H. Miyake *et al.*, PRL **111**, 185302 (2013)

Uniform flux — Hamiltonian

Realization of the Hofstadter-Harper Hamiltonian

$$\hat{H} = -\sum_{m,n} \left(K e^{i\phi_{m,n}} \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} + J \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} \right) + \text{h.c.}$$



Scheme allows for the realization of an effective uniform flux of

$$\boxed{\Phi = \pi/2}$$

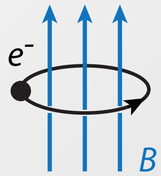
M. Aidelsburger *et al.*, PRL **111**, 185301 (2013)
similar work: H. Miyake *et al.*, PRL **111**, 185302 (2013)

Uniform flux

Local observable

- **Classical:**

  Charged particle in magnetic field



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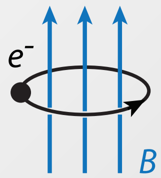
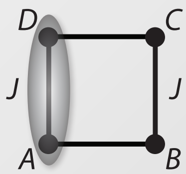


- **Quantum Analogue:**
  - Initial State:
  - Single Atom in the ground state of a tilted plaquette.

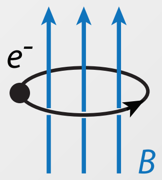


$$|\psi_0\rangle = \frac{|A\rangle + |D\rangle}{\sqrt{2}}$$

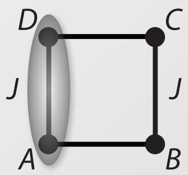
Local observable

- **Classical:**

  Charged particle in magnetic field

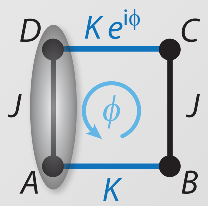


- **Quantum Analogue:**
  - Initial State:
  - Single Atom in the ground state of a tilted plaquette.



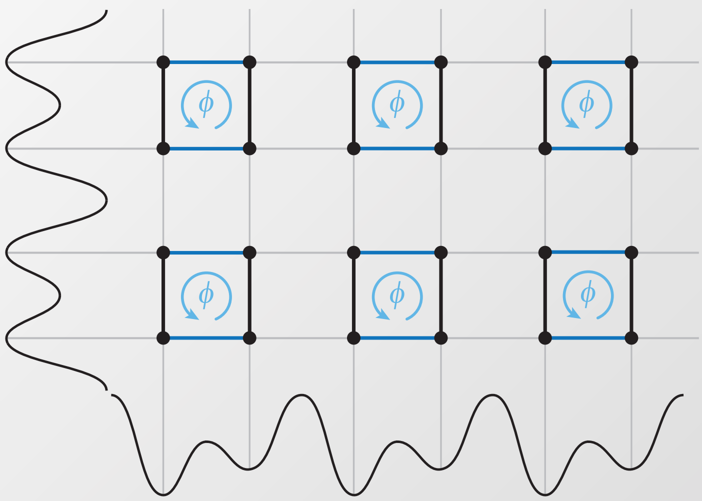
$$|\psi_0\rangle = \frac{|A\rangle + |D\rangle}{\sqrt{2}}$$

  - Switch on running-wave to induce tunneling



A Lattice of Plaquettes

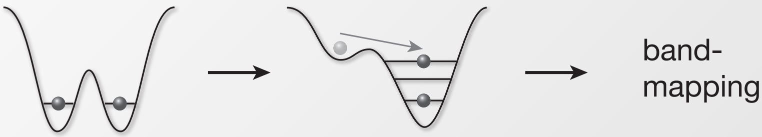
Using two superlattices, we realize a lattice whose elementary cell is a 4-site plaquette.



Uniform flux

Mean atom position

- Site resolved detection along one direction



S. Fölling *et al.*, Nature (2007); J. Sebby-Strabley *et al.*, PRL (2007)

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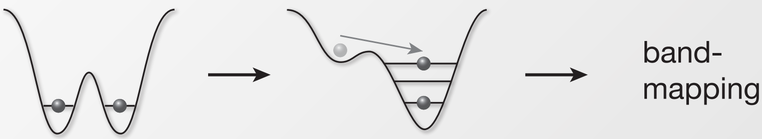
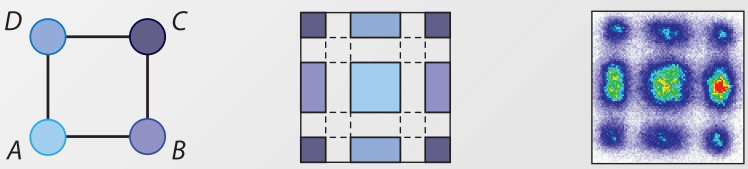


S. Fölling *et al.*, Nature (2007); J. Sebby-Strabley *et al.*, PRL (2007)

- Site resolved detection in plaquettes



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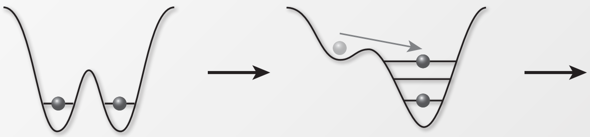

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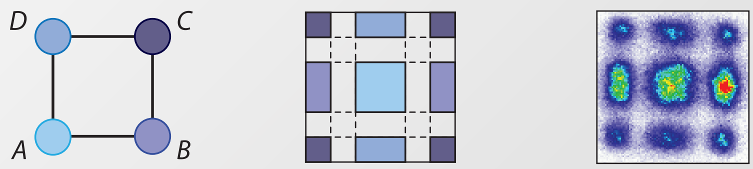
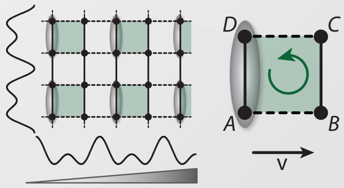


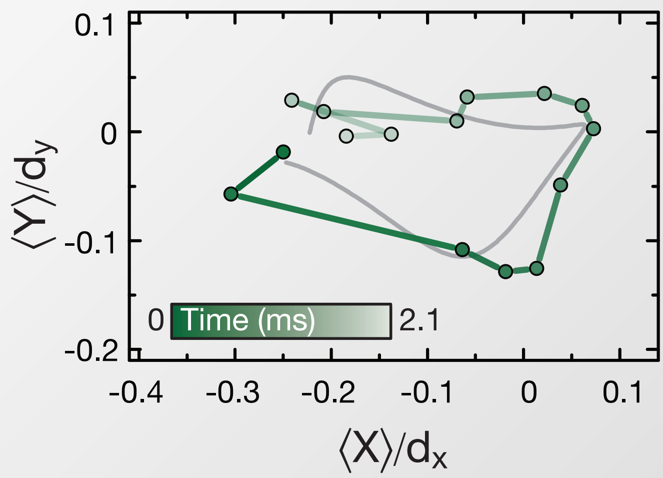
- Mean atom position along *x* and *y*

$$\frac{\langle X\rangle}{d_x} = \frac{-N_A + N_B + N_C - N_D}{2N} \quad \text{and} \quad \frac{\langle Y\rangle}{d_y} = \frac{-N_A - N_B + N_C + N_D}{2N}$$

Cyclotron orbit

Quantum analogue of cyclotron orbit





Parameters:

$$J/h = 0.5\,\mathrm{kHz}$$
$$K/h = 0.3\,\mathrm{kHz}$$
$$\Delta/h = 4.5\,\mathrm{kHz}$$

Uniformity of the flux

Observation of the uniformity of the effective flux:

- Superlattice potential shifted by one lattice constant

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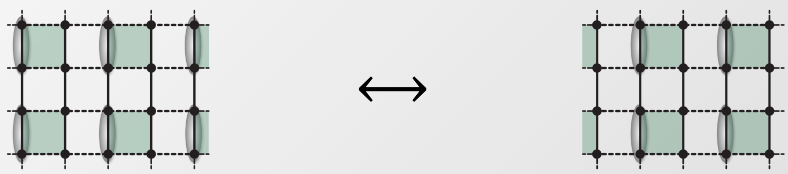

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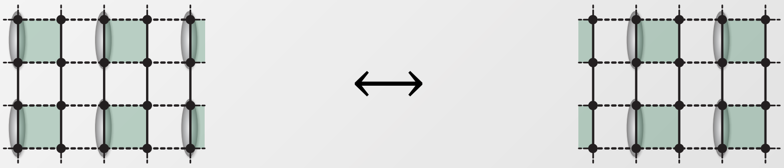

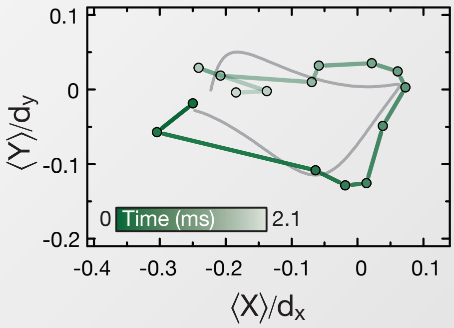

Uniform flux

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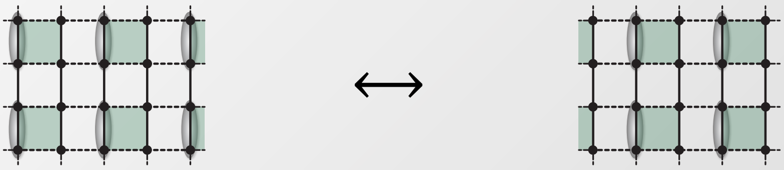

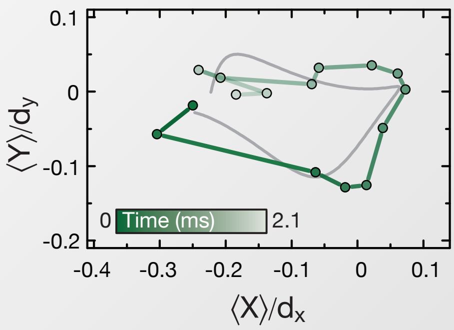

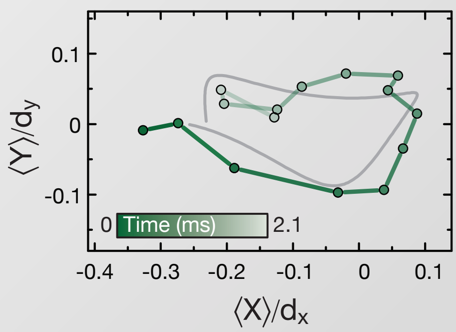

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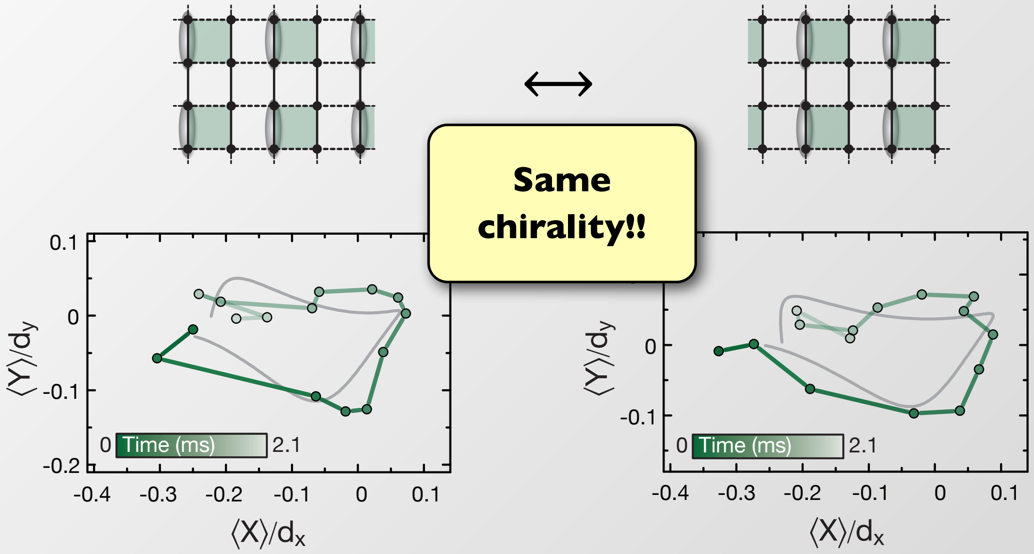
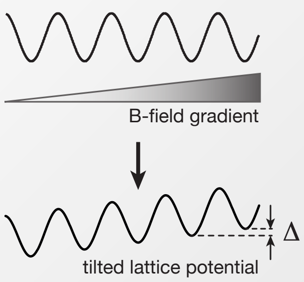


Quantum Spin Hall Hamiltonian

Value of the flux depends on the internal state of the atom

- $|{\uparrow}\rangle = |F = 1, m_F = -1\rangle$



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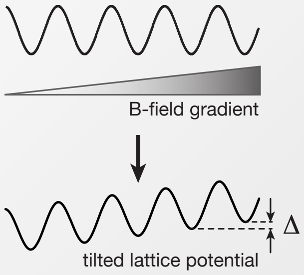
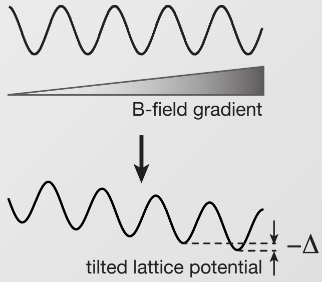


- $|\downarrow\rangle = |F = 2, m_F = -1\rangle$



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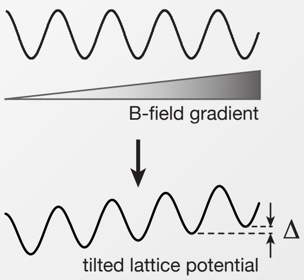

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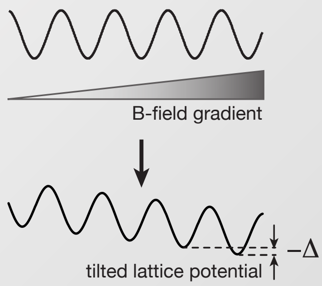

Spin-dependent optical potential: $\Delta \Longleftrightarrow -\Delta$

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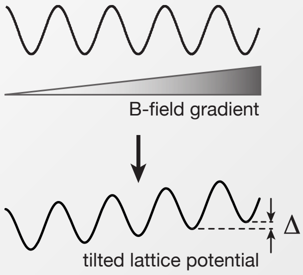

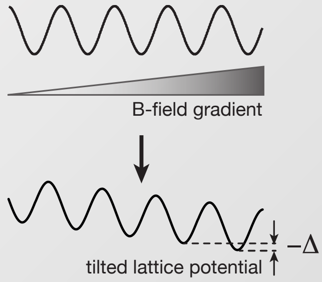

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Spin-dependent complex tunneling amplitudes: $Ke^{i\phi_{mn}} \Longleftrightarrow Ke^{-i\phi_{mn}}$

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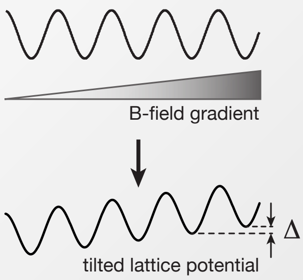

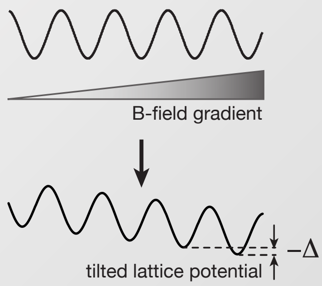

Spin-dependent optical potential: $\Delta \Longleftrightarrow -\Delta$

Spin-dependent complex tunneling amplitudes: $Ke^{i\phi_{mn}} \Longleftrightarrow Ke^{-i\phi_{mn}}$

Spin-dependent effective magnetic field: $\Phi = \pi/2 \Longleftrightarrow \Phi = -\pi/2$

Uniform flux

Quantum Spin Hall Hamiltonian

Time-reversal-symmetric quantum spin Hall Hamiltonian:

$$\hat{H}_{\uparrow,\downarrow} = -\sum_{m,n} \left(K e^{\pm i \phi_{m,n}} \hat{a}^{\dagger}_{m+1,n} \hat{a}_{m,n} + J \hat{a}^{\dagger}_{m,n+1} \hat{a}_{m,n} \right) + \text{h.c.}$$

Bernevig and Zhang, PRL **96**, 106802 (2006); N. Goldman *et al.*, PRL (2010)

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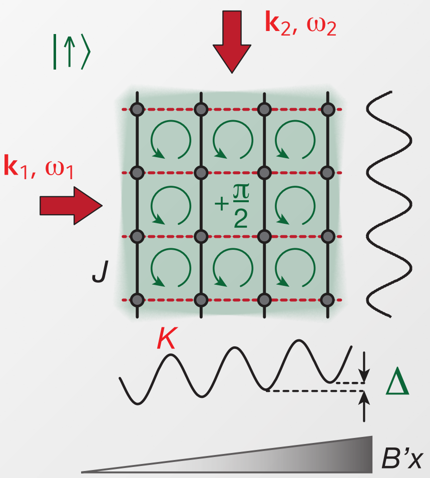

$$\boxed{\Phi_{\uparrow} = \pi/2}$$

Bernevig and Zhang, PRL **96**, 106802 (2006); N. Goldman *et al.*, PRL (2010)

Sunday 22 June 14

Uniform flux

Quantum Spin Hall Hamiltonian

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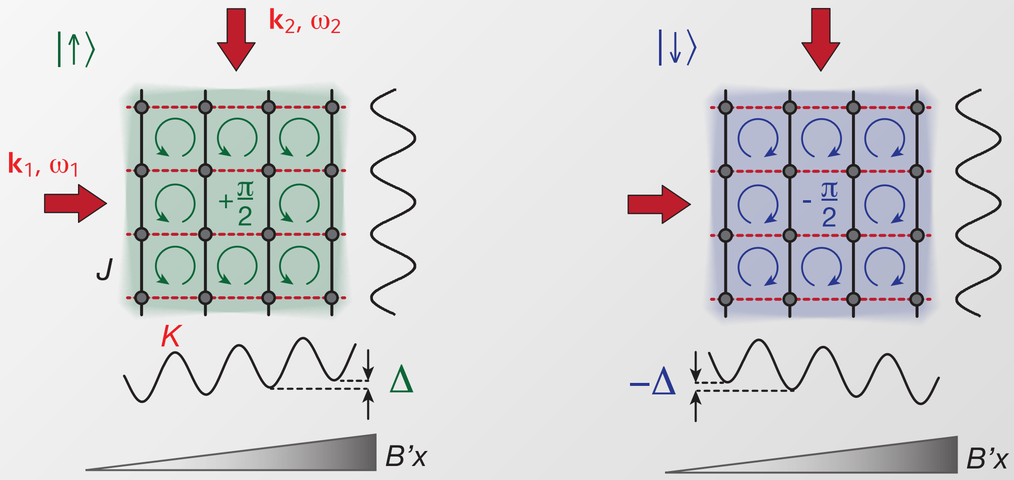

$$\boxed{\Phi_{\uparrow} = \pi/2}$$

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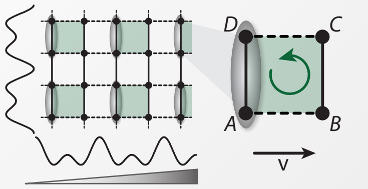
Bernevig and Zhang, PRL **96**, 106802 (2006); N. Goldman *et al.*, PRL (2010)

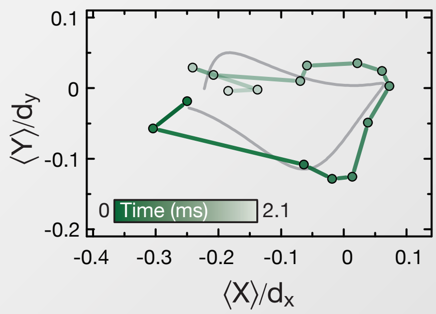
Uniform flux

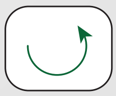
Spin-dependent cyclotron orbit

- Spin up:



$$|\Psi_{\uparrow}\rangle = (|A\rangle + |D\rangle)/\sqrt{2}$$





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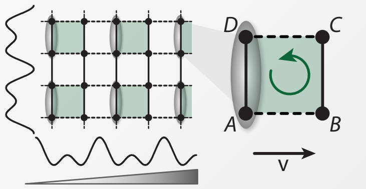

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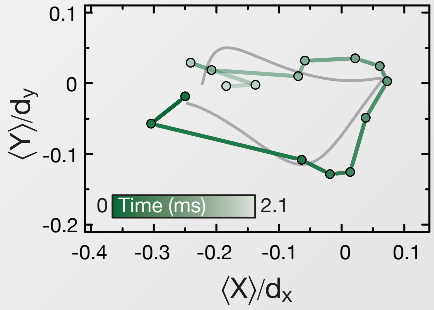
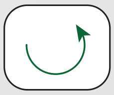
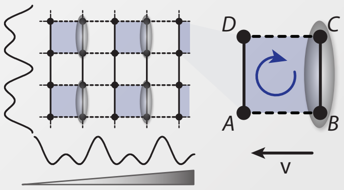




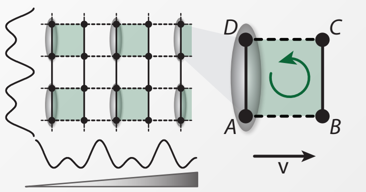
- Spin down:



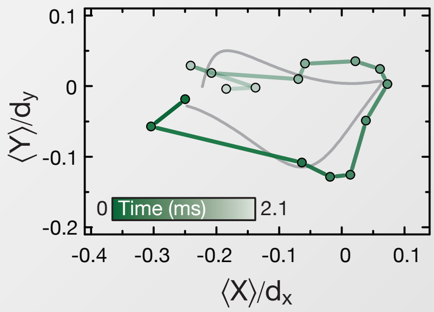
$$|\Psi_{\downarrow}\rangle = (|B\rangle + |C\rangle)/\sqrt{2}$$

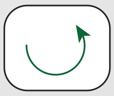
Spin-dependent cyclotron orbit

- Spin up:

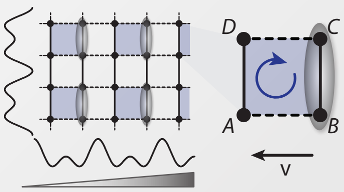


$$|\Psi_{\uparrow}\rangle = (|A\rangle + |D\rangle)/\sqrt{2}$$





- Spin down:



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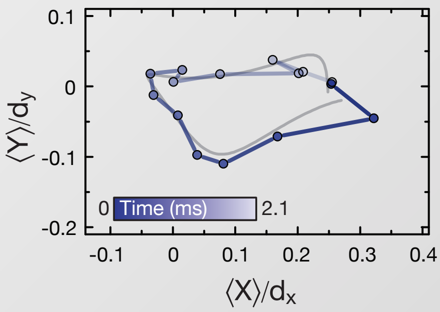
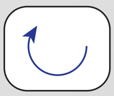
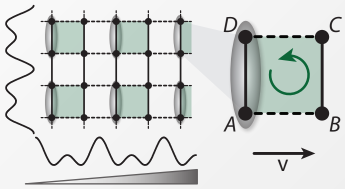


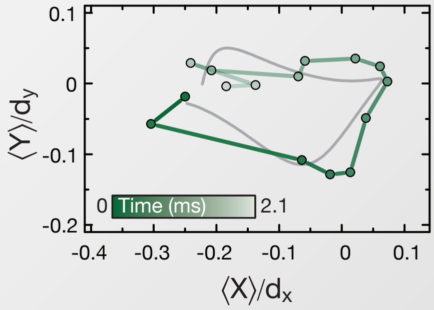


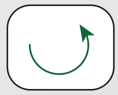
Spin-dependent cyclotron orbit

- Spin up:



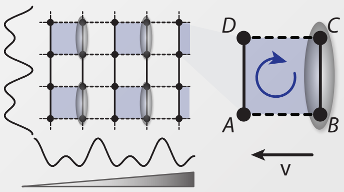
$$|\Psi_{\uparrow}\rangle = (|A\rangle + |D\rangle)/\sqrt{2}$$





Opposite chirality!

- Spin down:



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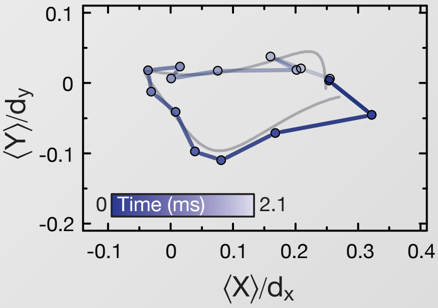
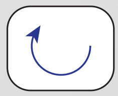
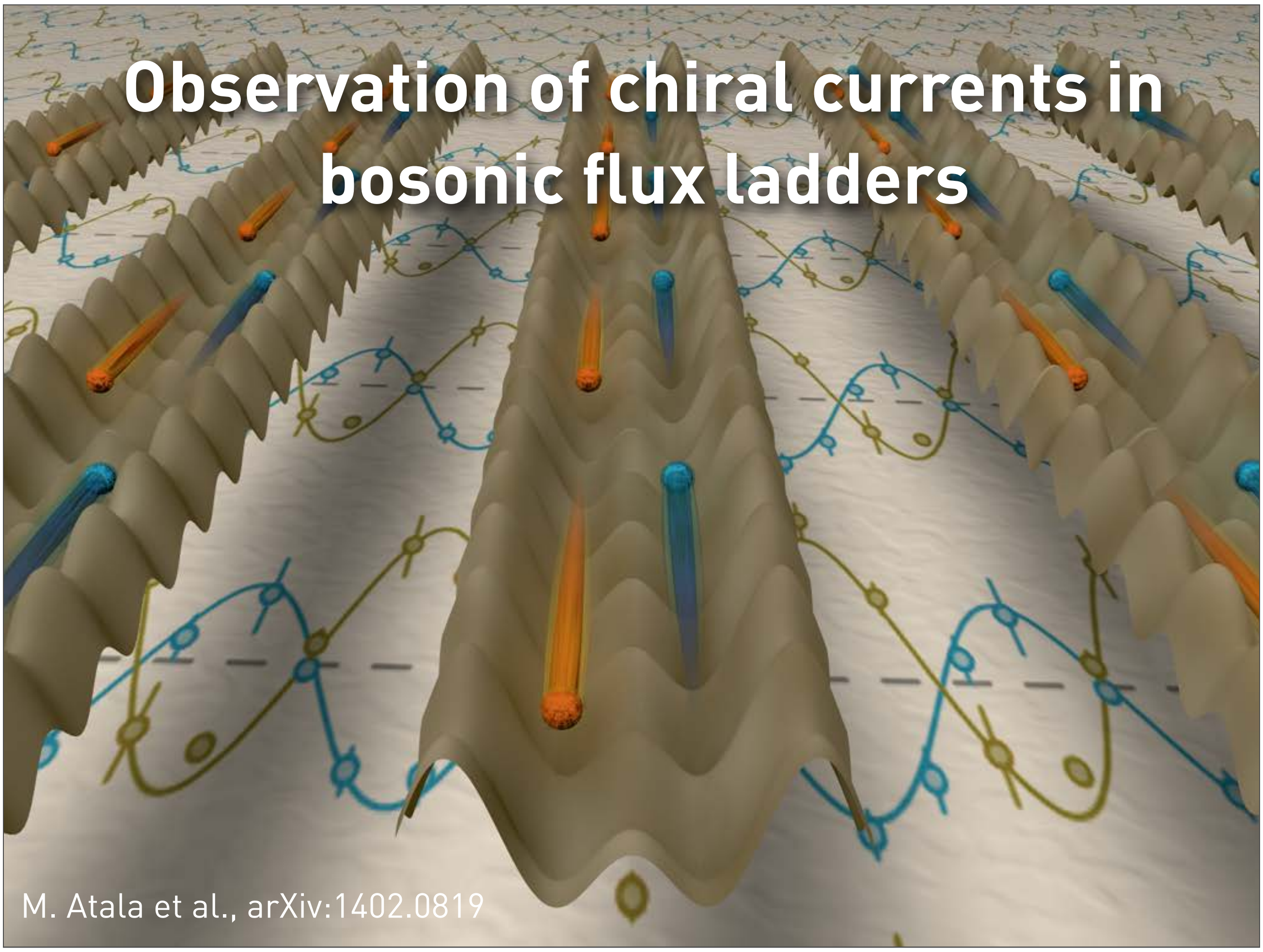
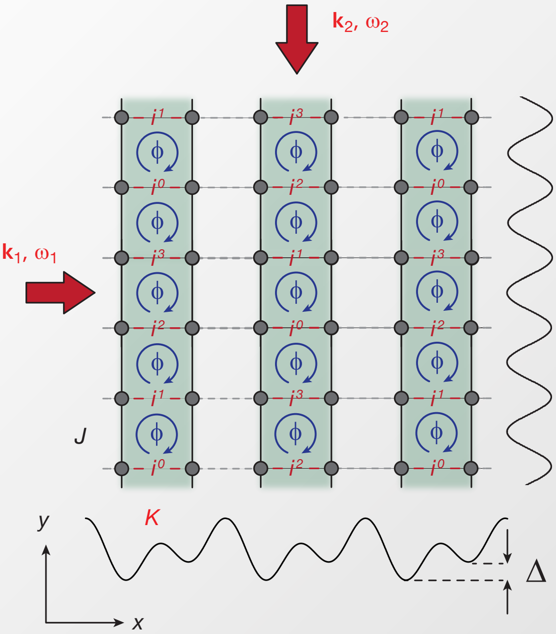






Flux ladder: experimental realization



- resonant laser-assisted tunneling:

$$\omega_1 - \omega_2 = \Delta/\hbar$$

- Spatial dependent phase factors

$$\phi_n = n \cdot \pi/2$$

- Uniform flux

$$\boxed{\Phi = \pi/2}$$

Experiment: M. Atala et al., arXiv:1402.0819 (2014)
Theory: D. Hügel, B. Paredes, PRA 89, 023619 (2014)
E. Orignac & T. Giamarchi PRB 64, 144515 (2001)
A. Tokuno & A. Georges arXiv:1403.0413
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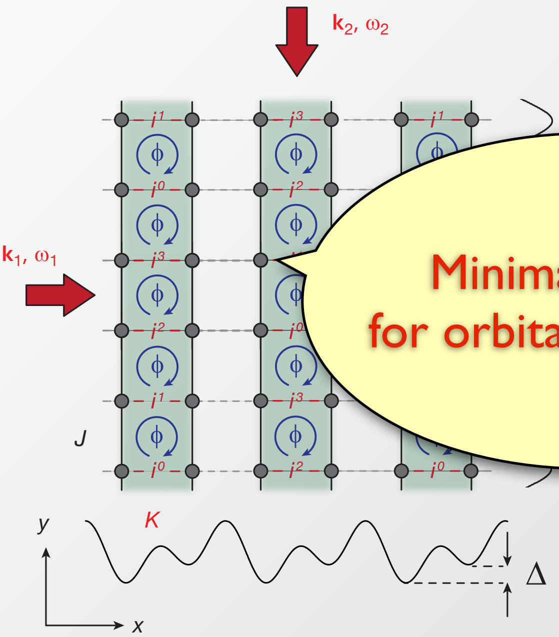

- resonant laser-assisted tunneling:

$$\omega_1 - \omega_2 = \Delta/\hbar$$

Minimal System
for orbital magnetism

dent phase factors

$$\cdot \pi/2$$

ux

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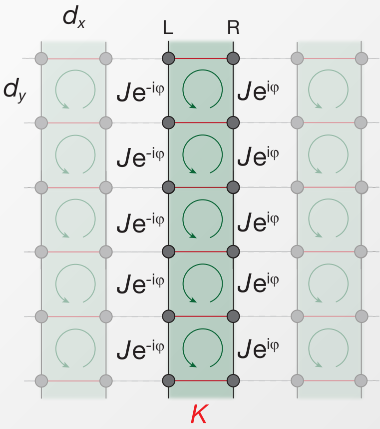
Flux Ladder Hamiltonian

Hamiltonian of the system written in a simpler theory gauge



$$H = -J\sum_\ell \left(e^{-i\ell\varphi}\hat{a}^\dagger_{\ell+1;L}\hat{a}_{\ell;L} + e^{i\ell\varphi}\hat{a}^\dagger_{\ell+1;R}\hat{a}_{\ell;R}\right)$$
$$- K\sum_\ell \left(\hat{a}^\dagger_{\ell;L}\hat{a}_{\ell;R}\right) + \text{h.c.}$$

Flux: $\boxed{\phi = 2\varphi}$

Flux Ladder

Flux Ladder Hamiltonian

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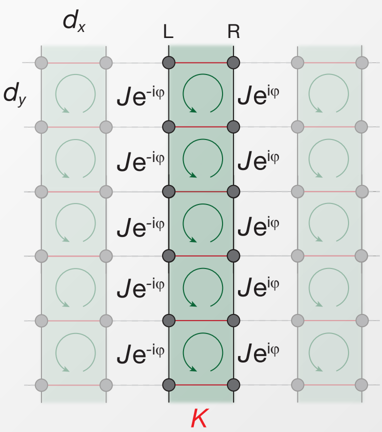

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Flux: $\boxed{\phi = 2\varphi}$

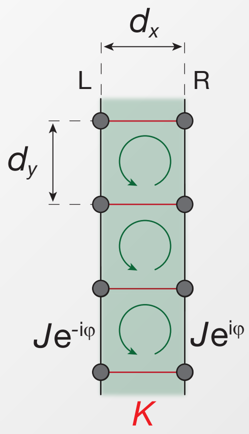
Define: $$\hat{a}_{q;\mu} = \sum_\ell e^{iq\ell}\hat{a}_{\ell;\mu},$$

and solve for the ansatz $$|\psi_q\rangle = (\alpha_q \hat{a}^\dagger_{q;L} + \beta_q \hat{a}^\dagger_{q;R})\,|0\rangle$$

Sunday 22 June 14

Ladder Band Structure

$$\epsilon_q = 2J\cos(q)\cos(\varphi) \pm \sqrt{K^2 - 4J^2\sin^2(\varphi)\sin^2(q)}$$



Flux Ladder

Ladder Band Structure

Two energy bands

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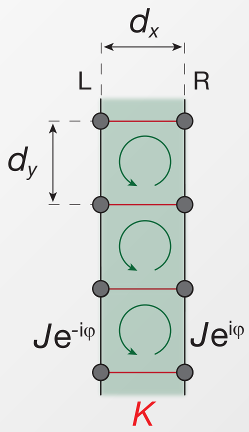

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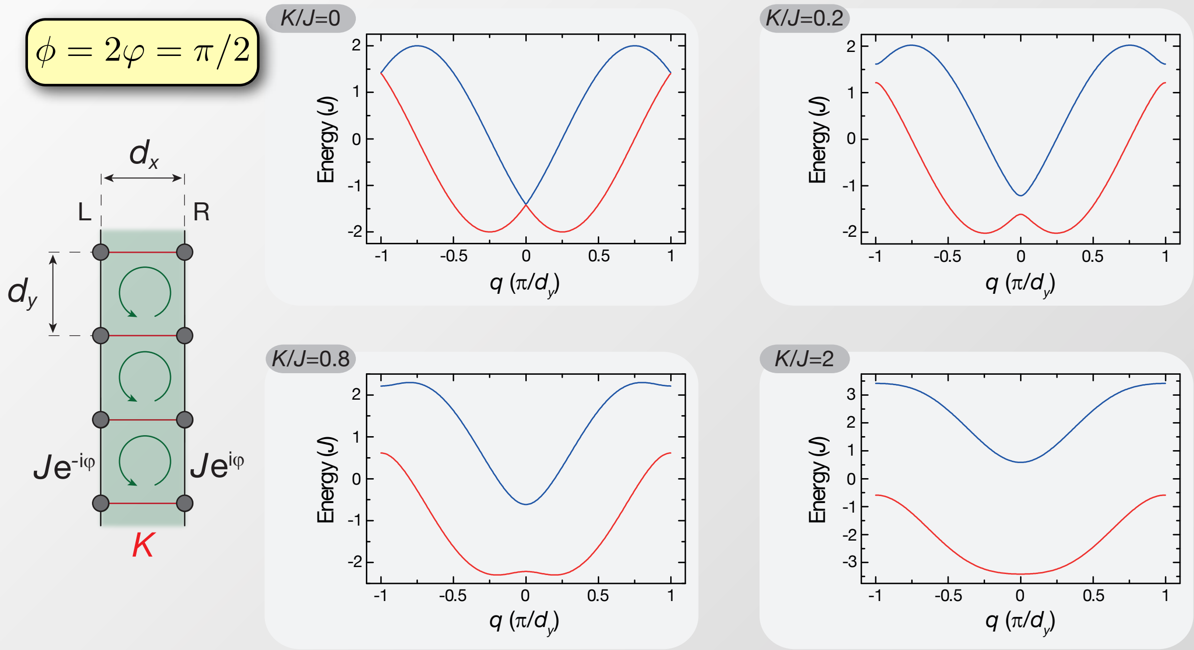

Flux Ladder

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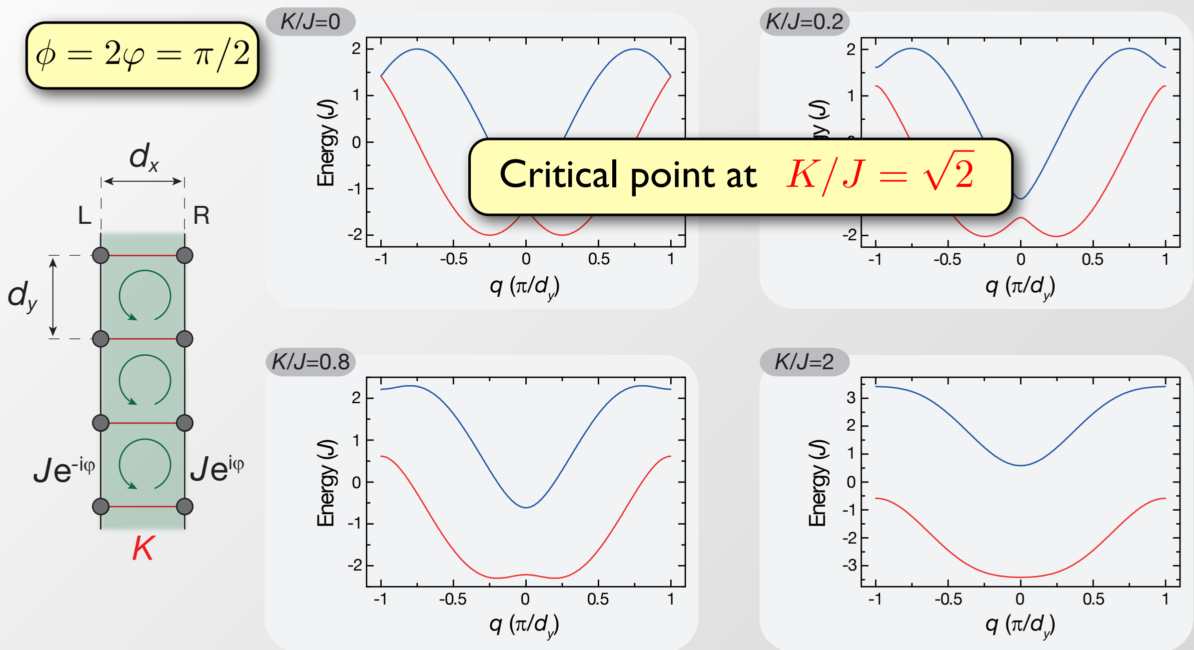
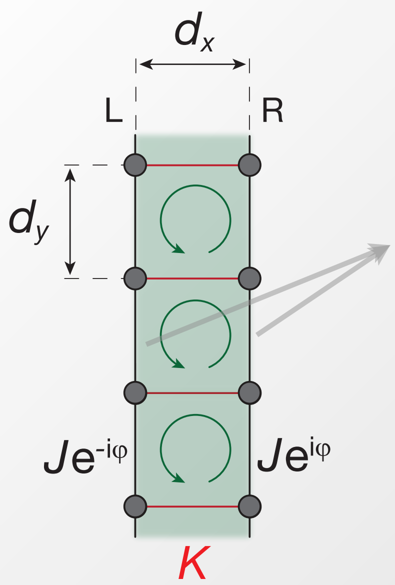


Critical point at $K/J = \sqrt{2}$

Probability Currents in Ladder



Current along the legs:

$$\hat{\mathbf{j}}^y_{\ell;\mu} = -\frac{i}{\hbar}\left(\hat{a}^\dagger_{\ell+1;\mu}\hat{a}_{\ell;\mu}H_{\ell\to\ell+1;\mu} - \mathrm{h.c}\right)$$

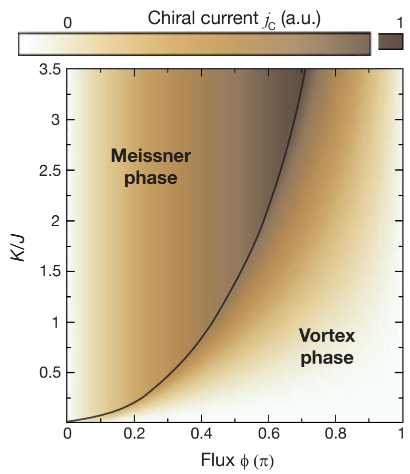
$$\mu = (\mathrm{L{=}left, R{=}right})$$

In the experiment ***total current*** is measured

$$\mathbf{j}_\mathrm{L} = N_{leg}^{-1}\sum_l \mathbf{j}^y_{l;L}$$

Chiral current: $\mathbf{j}_\mathrm{C} = \mathbf{j}_\mathrm{L} - \mathbf{j}_\mathrm{R}$

Phase Diagram



see E. Orignac & T. Giamarchi PRB 64, 144515 (2001)

Flux Ladder

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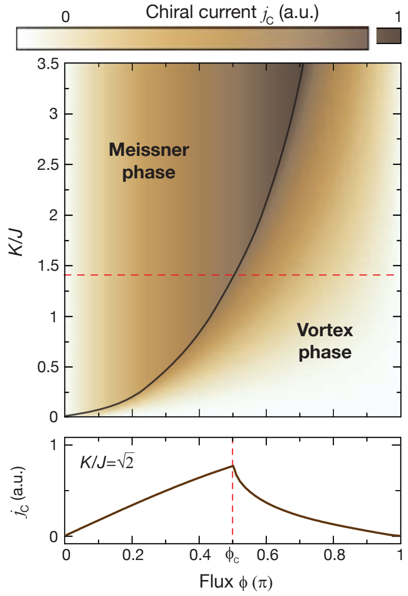

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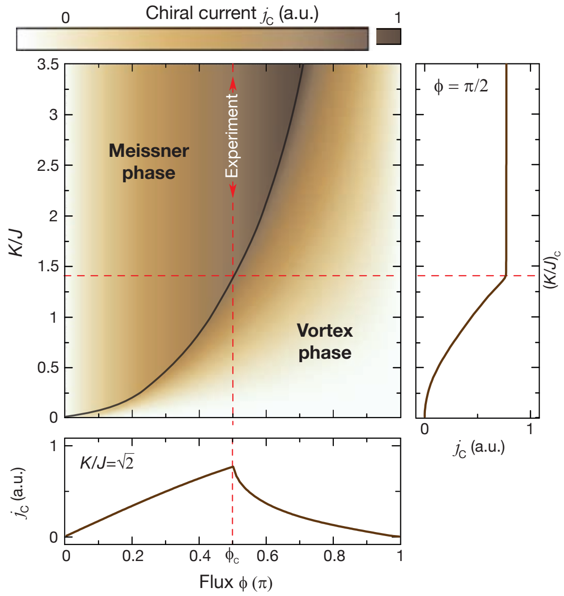

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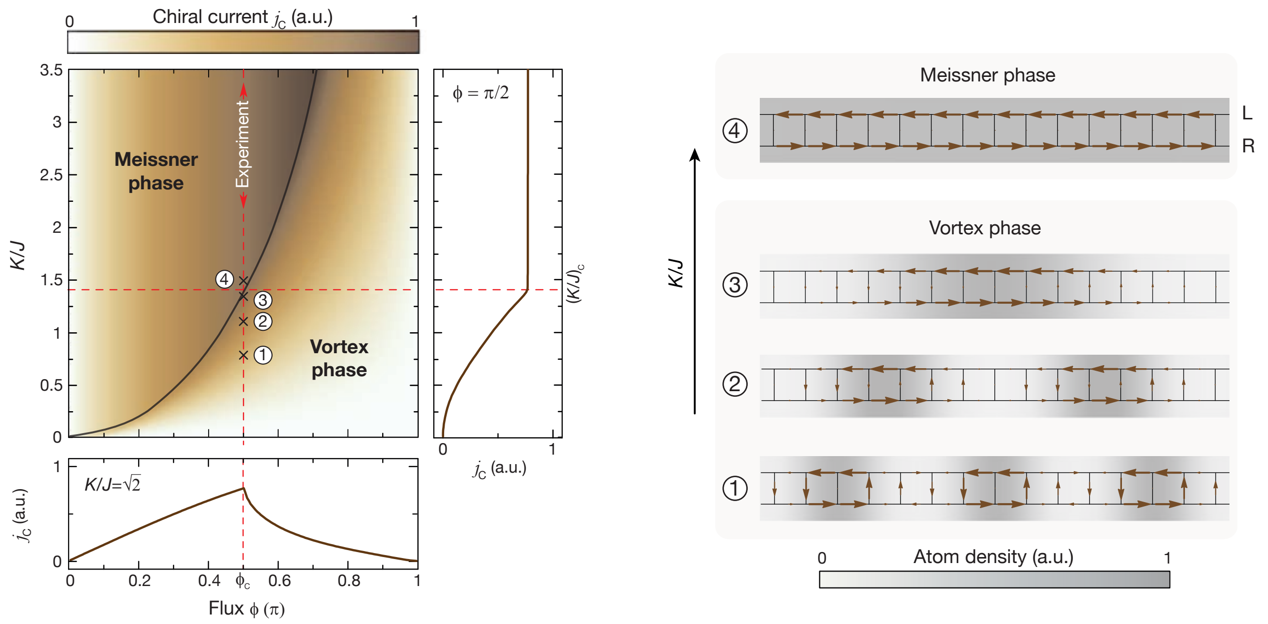

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Flux ladder

Spin-orbit coupling - short digression

- The flux ladder Hamiltonian can be mapped into a spin-orbit coupled system
- Left right legs are mapped into pseudo-spins:

$$\hat{a}_{\ell;R} \to \hat{a}_{\ell;\downarrow} \qquad \hat{a}_{\ell;L} \to \hat{a}_{\ell;\uparrow}$$

Spin-Momentum locking: D. Hügel, B. Paredes, PRA 89, 023619 (2014) **Continuum:** I. B. Spielman Nature 471, 83 (2011)

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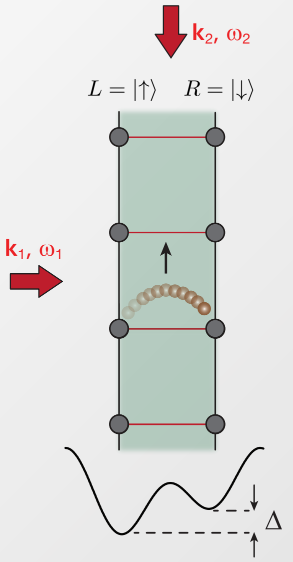

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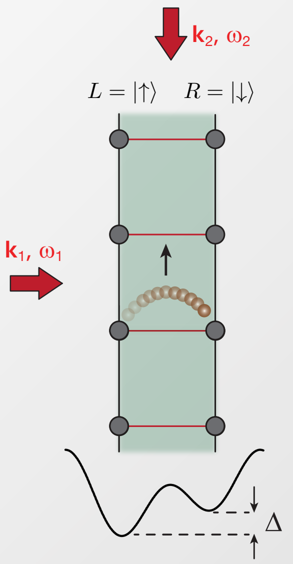

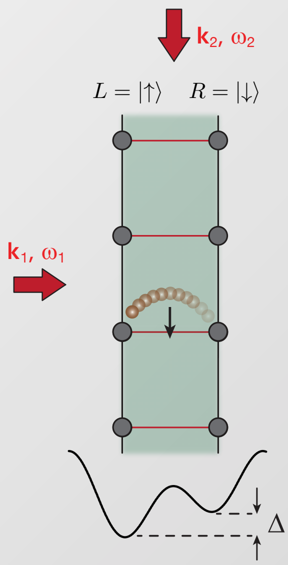

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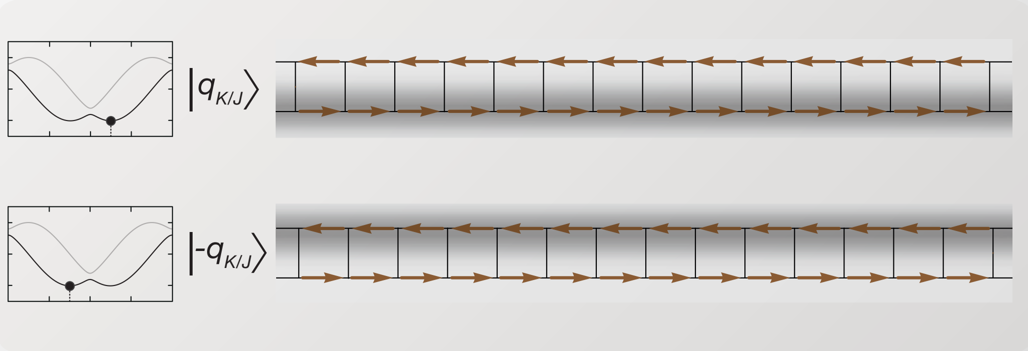

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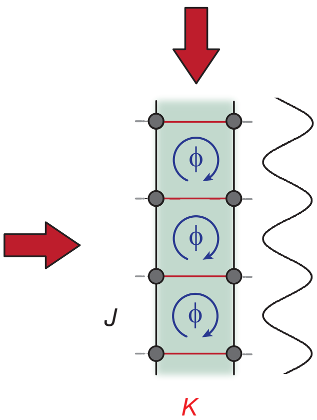
Flux Ladder

Current Measurements: Sequence

How to measure currents in our setup?

→ *project the state into isolated double wells*

S. Trotzky *et al.* Nature Physics 8, 325 (2012)
S. Kessler & F. Marquardt, arXiv:1309.3890 (2012)



Groundstate

Josephson oscillations in double wells

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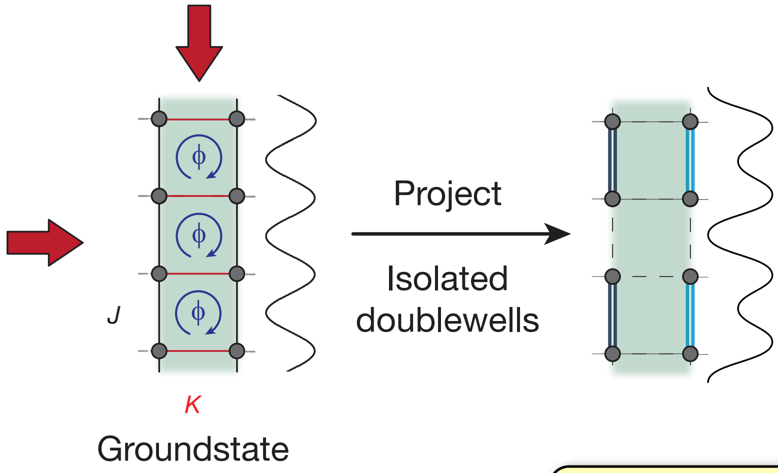

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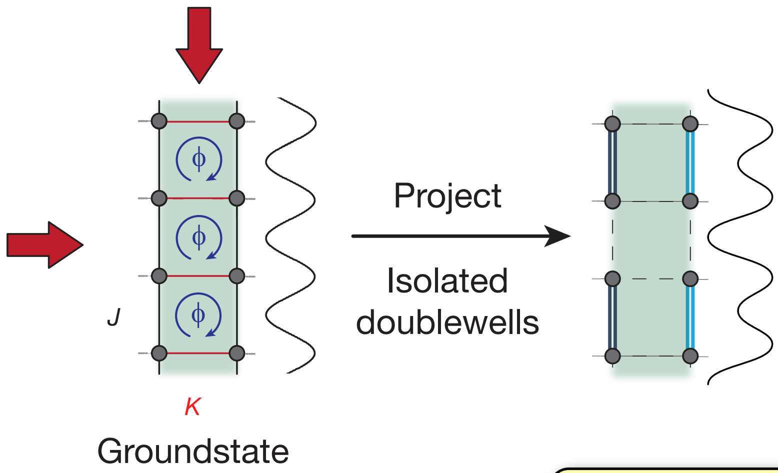

- Hold in double wells for a time *t*

Josephson oscillations in double wells

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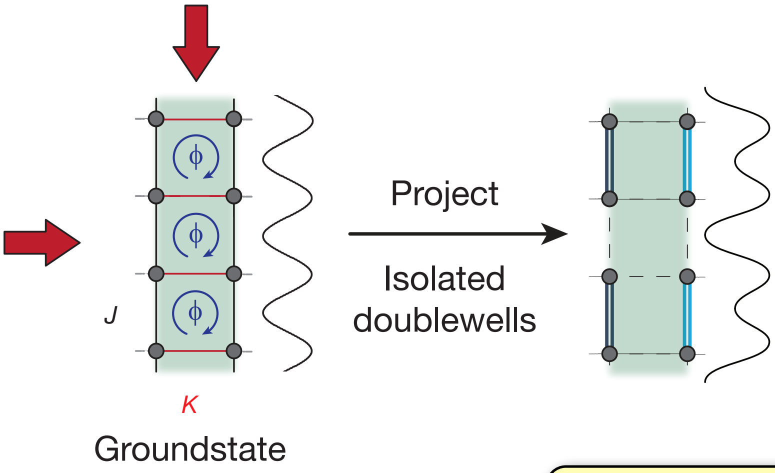


- Hold in double wells for a time *t*
- Measure *even-odd population* in double well

Josephson oscillations in double wells

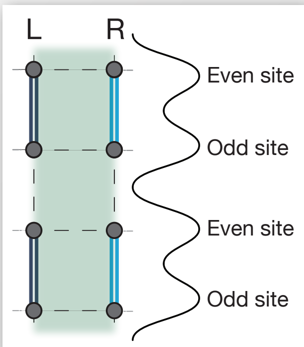
Sunday 22 June 14

Flux Ladder

Double well oscillations - currents

In the experiment we measure the average of all the oscillations on either side of the ladder:

$$n_{\mathrm{even};\mu}(t) = \frac{1}{2}[1 + (n_{\mathrm{even};\mu}(0) - n_{\mathrm{odd};\mu}(0))\cos(2\omega t) - \frac{j_\mu}{J/\hbar}\sin(2\omega t)]$$



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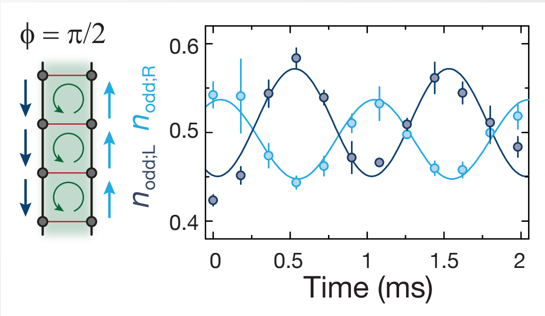
Oscillations in double wells

• Prepare ground state of the flux ladder with K/J=2 and project into isolated double wells

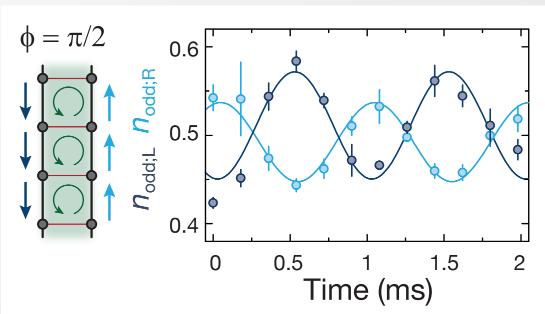


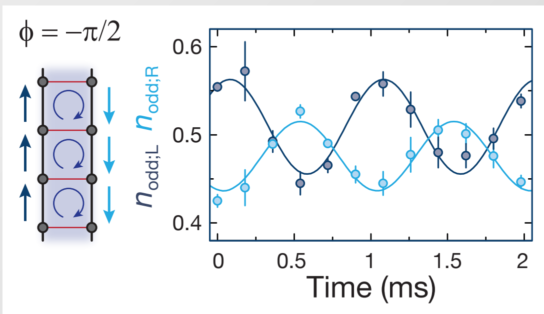
Flux Ladder

Oscillations in double wells

• Prepare ground state of the flux ladder with K/J=2 and project into isolated double wells

Reversed flux

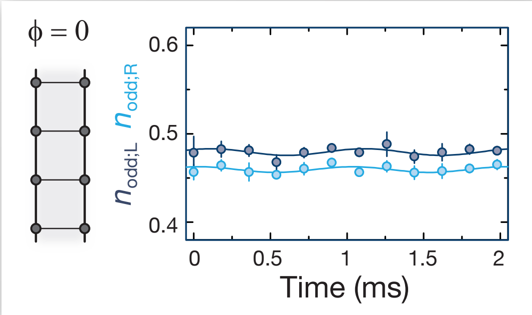




When inverting the flux the current gets reversed

Zero flux ladders

- Prepare ground state of the ladder with zero flux
- project into isolated double wells



Flux Ladder

Extracting the Chiral current

The chiral current can be reliably calculated by

$$n_{\mathrm{even};\mu}(t) = \frac{1}{2}[1 + (n_{\mathrm{even};\mu}(0) - n_{\mathrm{odd};\mu}(0))\cos(2\omega t) - \frac{j_\mu}{J/\hbar}\sin(2\omega t)]$$

$$\boxed{n_{\mathrm{even;L}}(t) - n_{\mathrm{even;R}}(t) = \frac{\mathbf{j}_C}{J/\hbar}\sin(2\omega t)}$$

Flux Ladder

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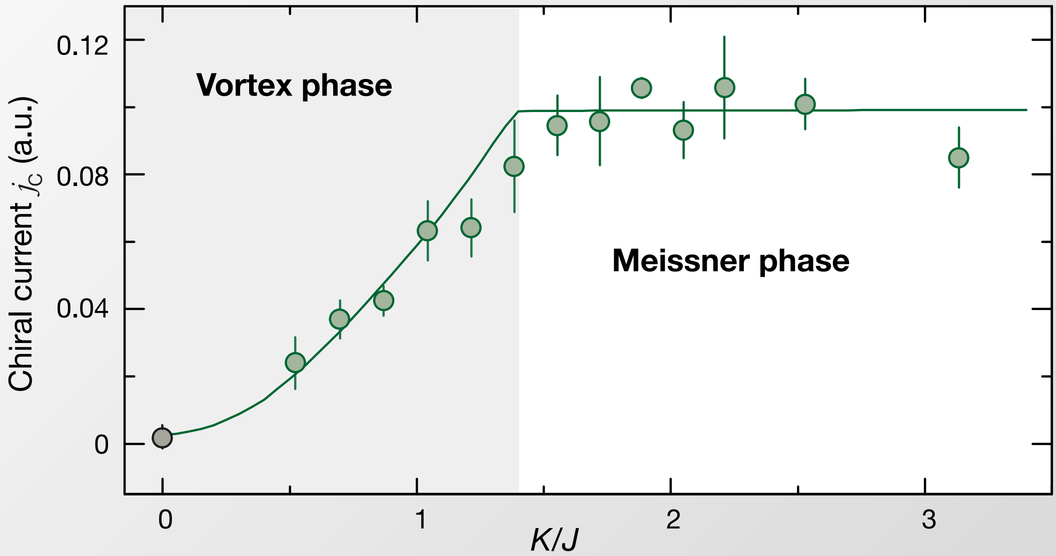
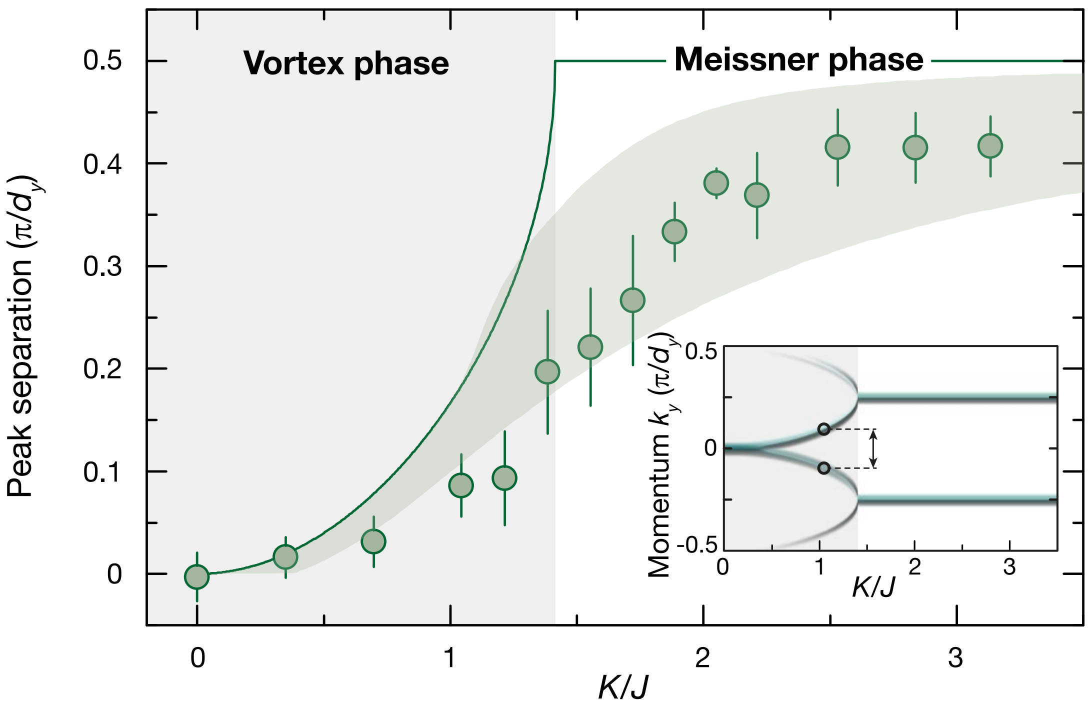


Flux Ladder

Experimental Results - Momentum Distribution



Summary and Outlook

- ***New detection method*** for probability currents
- Measurement of ***Chiral Edge States*** in Ladders
- Identification of ***Meissner-like effect in bosonic ladder***

Outlook:

- Entering the strongly correlated regime
- Chiral Mott Insulators
- Spin Meissner effect
- Connection of chiral ladder states to Hoftstadter model edge states
- Spin-Orbit Coupling in 1D

E. Orignac & T. Giamarchi PRB 64, 144515 (2001)
Dhar, A *et al.,* PRA 85, 041602 (2012)
Petrescu, A. & Le Hur, K. PRL 111, 150601 (2013)
A Tokuno & A Georges, arXiv:1403.0413

Probing Band Topology

Measuring the Zak-Berry's Phase of Topological Bands

M. Atala et al., Nature Physics (2013)

www.quantum-munich.de

Berry Phase in Quantum Mechanics

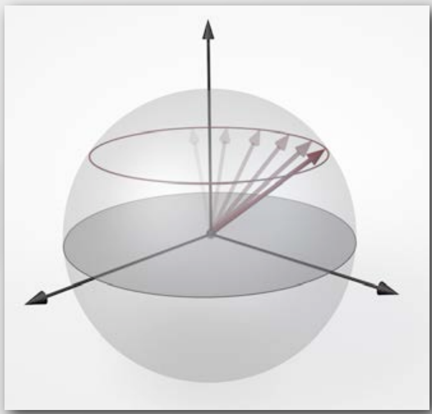
$$\Psi(R) \rightarrow e^{i(\varphi_{\text{Berry}}+\varphi_{\text{dyn}})}\ \Psi(R)$$

Adiabatic evolution through closed loop

$$\varphi_{\text{Berry}} = \oint_{\mathcal{C}} A_n(R)dR = i \oint_{\mathcal{C}} \langle n(R)|\nabla_R|n(R)\rangle\, dR$$

$$\varphi_{\text{Berry}} = \oint_{\mathcal{A}} \Omega_n(R)\, dA$$ **Berry Phase**

M.V. Berry, Proc. R. Soc. A (1984)



Example: Spin-1/2 particle in magnetic field

Berry connection

$$A_n(R) = i\langle n(R)|\nabla_R|n(R)\rangle$$

Berry curvature

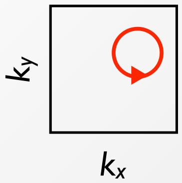
$$\Omega_{n,\mu\nu}(R) = \frac{\partial}{\partial R^\mu}A_{n,\nu} - \frac{\partial}{\partial R^\nu}A_{n,\mu}$$

LMU

Berry Phase for Periodic Potentials

$\Psi_k(\mathbf{r}) = e^{i\mathbf{kr}} u_k(\mathbf{r})$ Bloch wave in periodic potential

Adiabatic motion in momentum space generates Berry phase!



Mention Problem with going on a line is generally **NOT A LOOP IN PARAMETER SPACE!**

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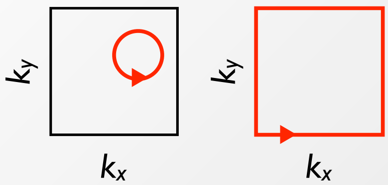

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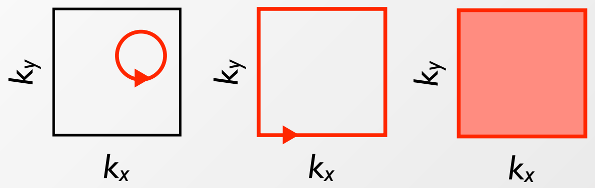

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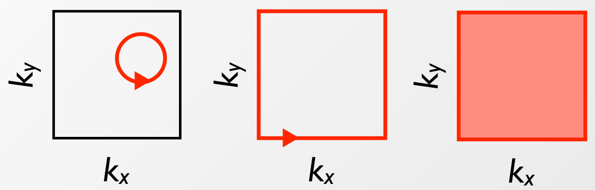

Berry phase is fundamental to characterize topology of energy bands

$$n_{\mathrm{Chern}} = \frac{1}{2\pi}\oint_{BZ} A_k dk = \frac{1}{2\pi}\int_{BZ} \Omega_k \, d^2k \iff \sigma_{xy} = n_{\mathrm{Chern}}\, e^2/h$$

Chern Number (Topological Invariant) **Quantized Hall Conductance**

Thouless, Kohmoto, den Nijs, and Nightingale (TKNN), PRL 1982
Kohmoto Ann. of Phys. 1985

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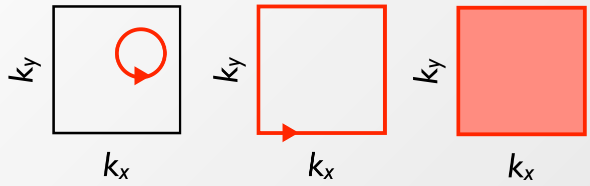

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What is the extension to 1D?

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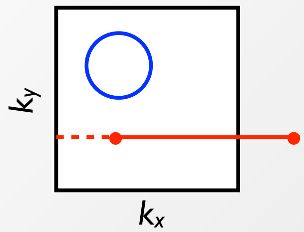
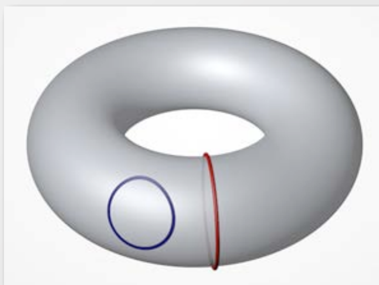
Zak Phase

2D Brillouin Zone



going straight means going around!



Band structure has torus topology!

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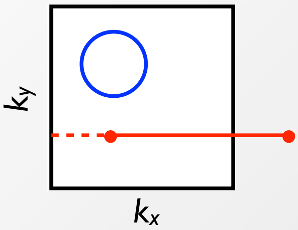

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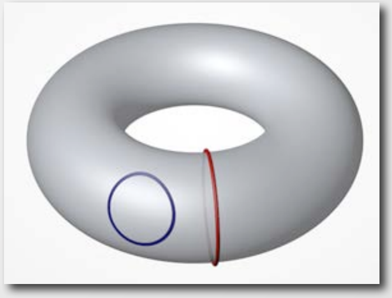

Band structure has torus topology!

$$\varphi_{Zak} = i \int_{k_0}^{k_0+G} \langle u_k | \partial_k | u_k \rangle \; dk$$

Zak Phase -
the 1D Berry Phase

J. Zak, Phys. Rev. Lett. **62**, 2747 (1989)

Zak Phase

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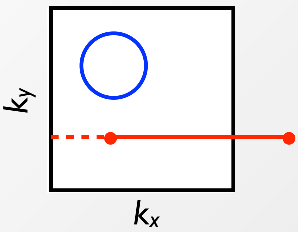

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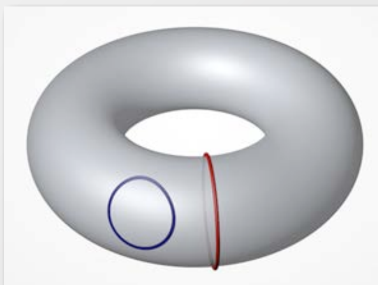

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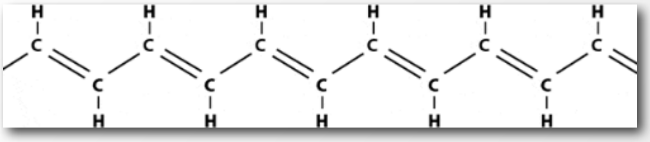
J. Zak, Phys. Rev. Lett. **62**, 2747 (1989)

Non-trivial Zak phase:

- Topological Band
- Edge States (for finite system)
- Domain walls with fractional quantum numbers

R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976)
J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47, 986 (1981)

Su-Shrieffer-Heeger Model (SSH)



Polyacetylene

W. P. Su, J. R. Schrieffer & A. J. Heeger
Phys. Rev. Lett. 42, 1698 (1979).

$$H_{SSH} = -\sum_n \{J\hat{a}_n^\dagger \hat{b}_n + J'\hat{a}_n^\dagger \hat{b}_{n-1} + \text{h.c.}\}$$

Berry Phase

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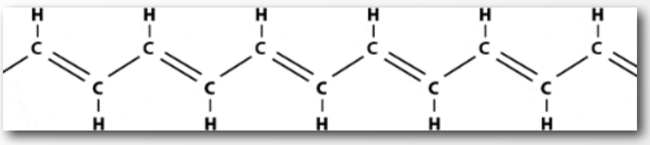

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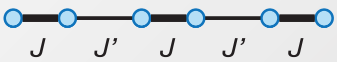
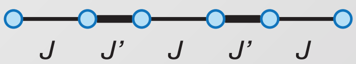
Two topologically distinct phases:

D1: $J > J'$



D2: $J' > J$



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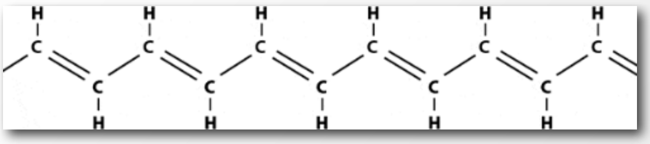


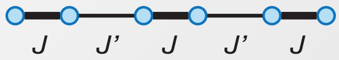
Polyacetylene

W. P. Su, J. R. Schrieffer & A. J. Heeger
Phys. Rev. Lett. 42, 1698 (1979).

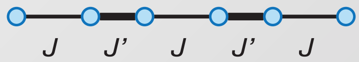
$$H_{SSH} = -\sum_n \{J\hat{a}_n^\dagger \hat{b}_n + J' \hat{a}_n^\dagger \hat{b}_{n-1} + \text{h.c.}\}$$

Two topologically distinct phases:

D1: $J > J'$



D2: $J' > J$

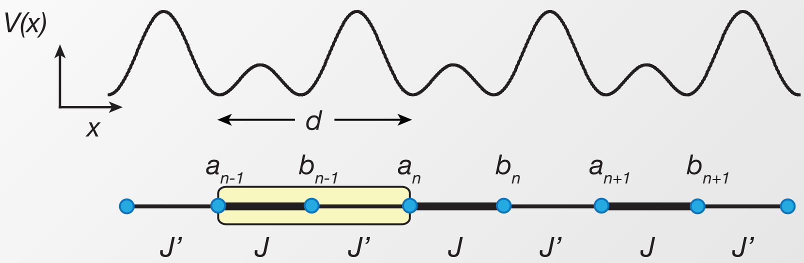


$$\delta\varphi_{Zak} = \varphi_{Zak}^{D1} - \varphi_{Zak}^{D2} = \pi$$

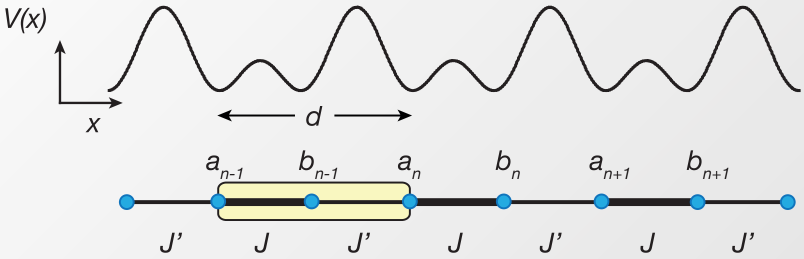
Topological properties:
domain wall features fractionalized excitations

Zak phase difference $\delta\varphi_{Zak}$ is gauge-invariant

SSH Energy Bands - Eigenstates



SSH Energy Bands - Eigenstates

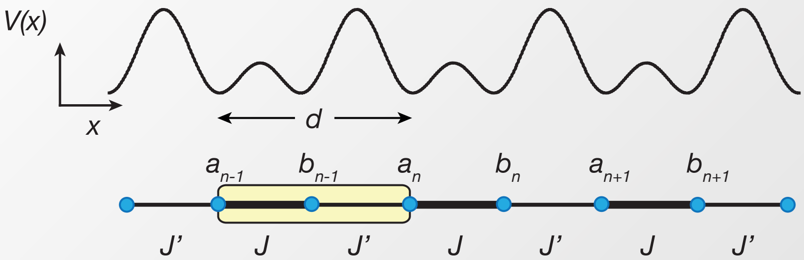


...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$

Berry Phase

SSH Energy Bands - Eigenstates



...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$

2x2 Hamiltonian:

$$\boxed{\begin{bmatrix} 0 & -\rho_k \\ -\rho_k^* & 0 \end{bmatrix} \begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \tilde{\epsilon}_k \begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix}}$$

with $\quad \rho_k = J e^{ikd/2} + J' e^{-ikd/2} = |\epsilon_k| e^{i\theta_k}$

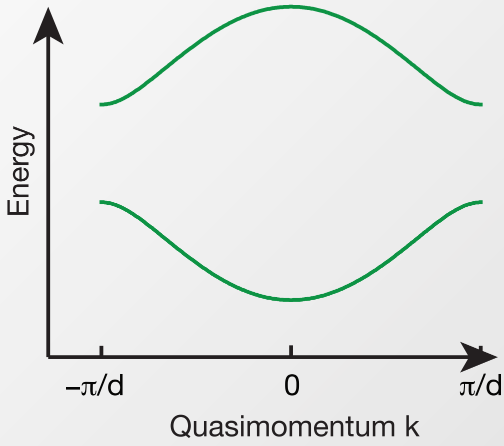
SSH Energy Bands - Eigenstates

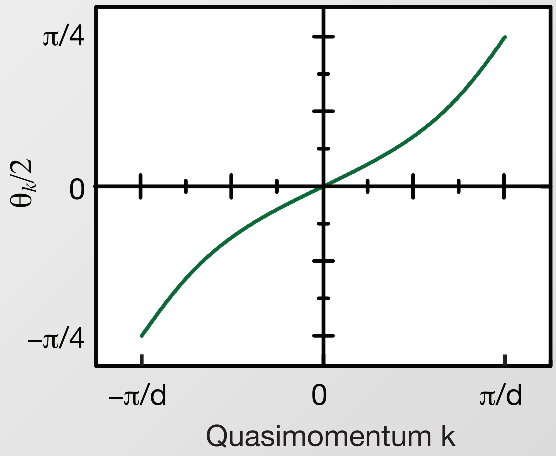
...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$

Eigenstates

$$\begin{pmatrix} \alpha_{k,\mp} \\ \beta_{k,\mp} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{-i\theta_k} \end{pmatrix}$$





SSH Energy Bands - Eigenstates

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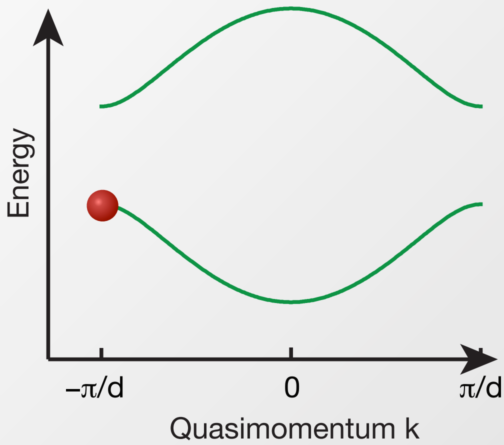

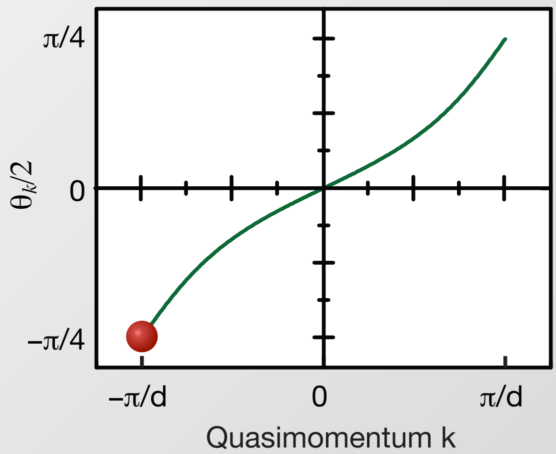

Adiabatic evolution in momentum space

SSH Energy Bands - Eigenstates

...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$

Eigenstates

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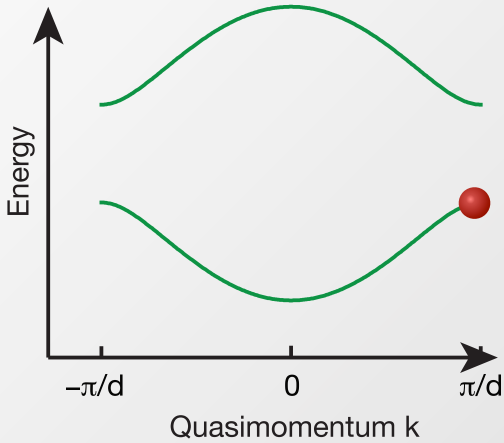
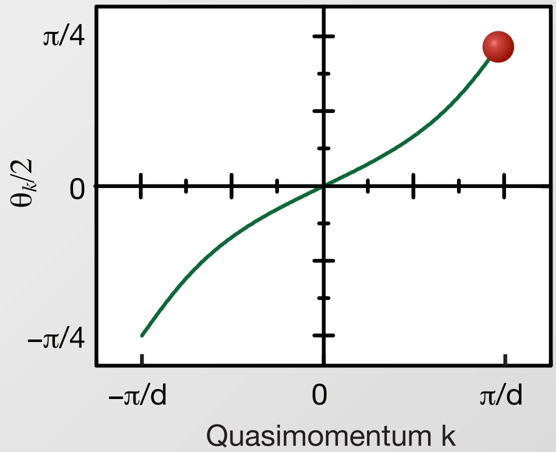




Adiabatic evolution in momentum space

Berry Phase

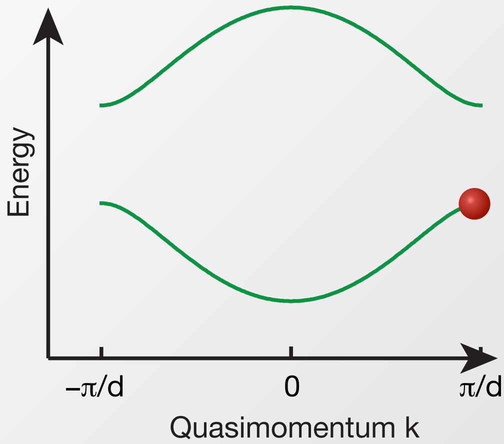
SSH Energy Bands - Eigenstates

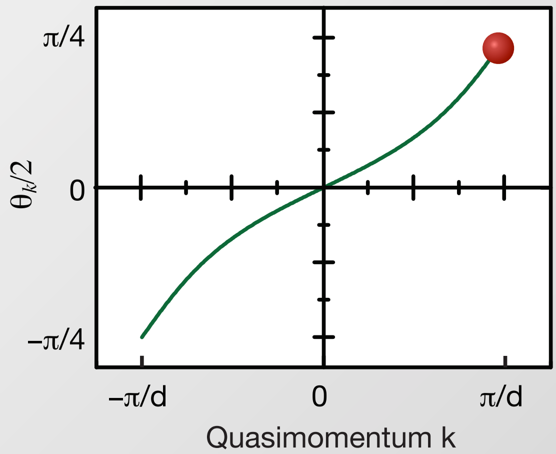
...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$

Eigenstates

$$\begin{pmatrix} \alpha_{k,\mp} \\ \beta_{k,\mp} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{-i\theta_k} \end{pmatrix}$$





$$\boxed{\varphi_{Zak} = i \int_{k_0}^{k_0+G} \left(\alpha_k^* \partial_k \alpha_k + \beta_k^* \partial_k \beta_k \right) \; dk}$$

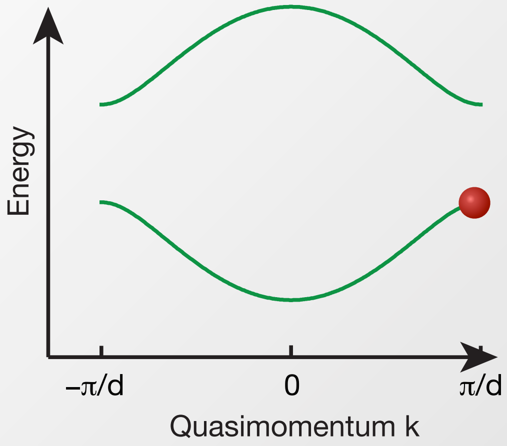
Zak Phase
SSH Model

Berry Phase

SSH Energy Bands - Eigenstates

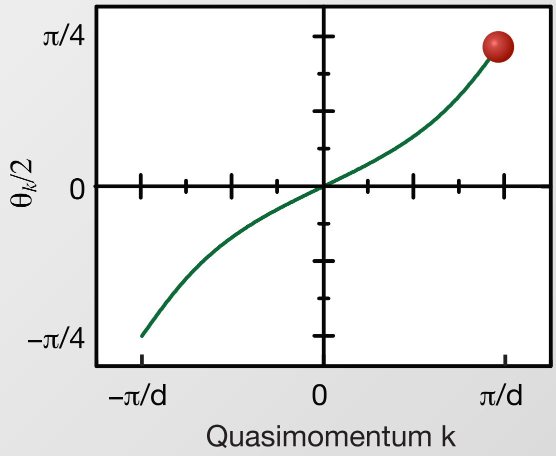
...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$



Eigenstates

$$\begin{pmatrix} \alpha_{k,\mp} \\ \beta_{k,\mp} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{-i\theta_k} \end{pmatrix}$$



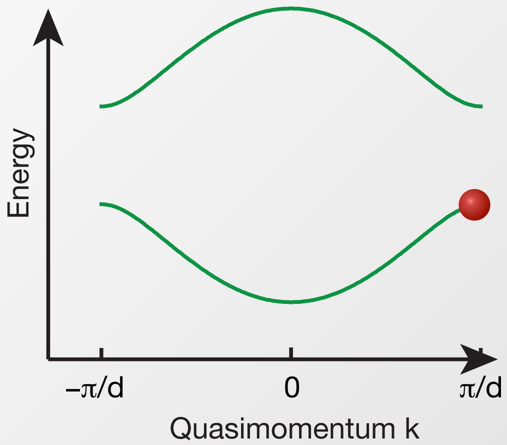
$$\boxed{\varphi_{Zak} = \frac{1}{2} \int_{k_0}^{G+k_0} \partial_k \theta_k \; dk}$$

Berry Phase

SSH Energy Bands - Eigenstates

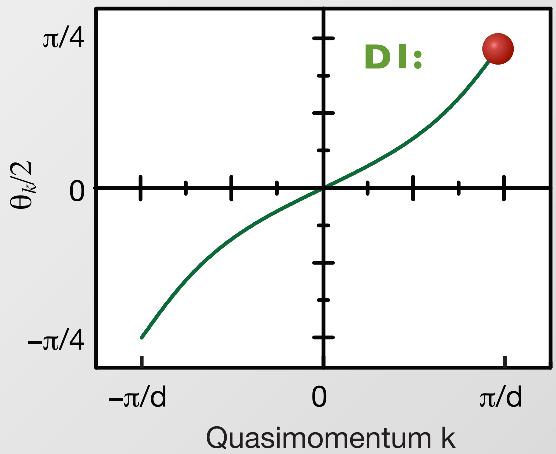
...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$



Eigenstates

$$\begin{pmatrix} \alpha_{k,\mp} \\ \beta_{k,\mp} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{-i\theta_k} \end{pmatrix}$$



DI: $J > J'$ $\boxed{\varphi^{D1}_{Zak} = \frac{\pi}{2}}$

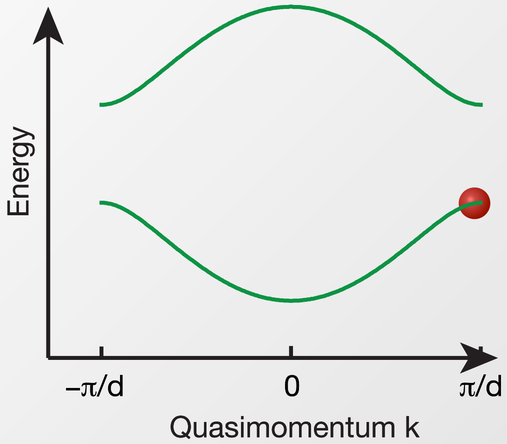
SSH Energy Bands - Eigenstates

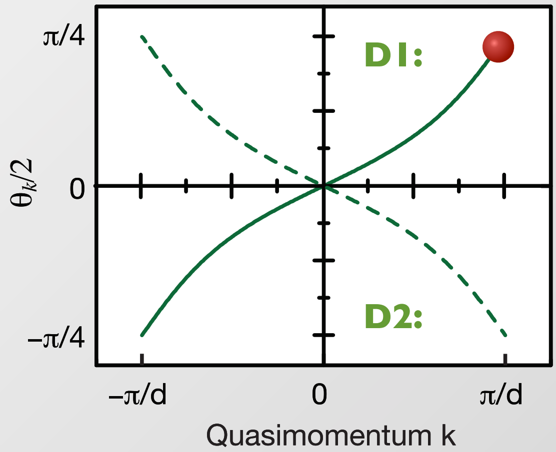
...ABABA... Lattice Structure....

$$\sum_x \Psi_x = \sum_x e^{ikx} \times \begin{cases} \alpha_k \\ \beta_k \; e^{ikd/2} \end{cases}$$

Eigenstates

$$\begin{pmatrix} \alpha_{k,\mp} \\ \beta_{k,\mp} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{-i\theta_k} \end{pmatrix}$$





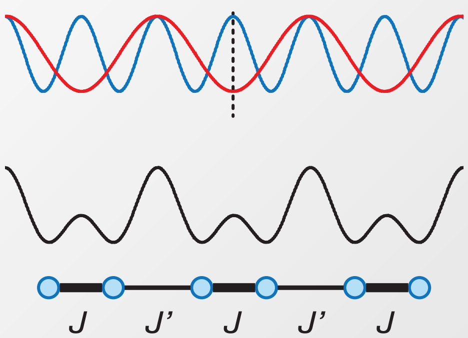
D1: $J > J'$ $\boxed{\varphi^{D1}_{Zak} = \frac{\pi}{2}}$ **D2:** $J' > J$ $\boxed{\varphi^{D2}_{Zak} = -\frac{\pi}{2}}$

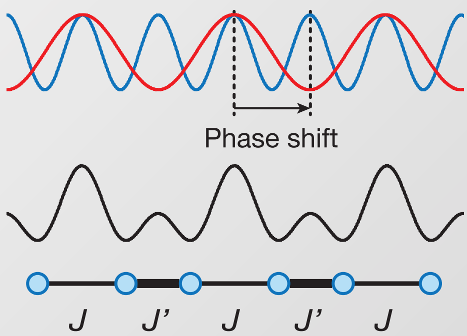
Realization with ultracold atoms

$$H_{\mathrm{SSH}} = -\sum_n \{J a_n^\dagger b_n + J' a_n^\dagger b_{n-1} + \mathrm{h.c.}\}$$

767 nm
1534 nm

D1: $J > J'$ **D2:** $J' > J$

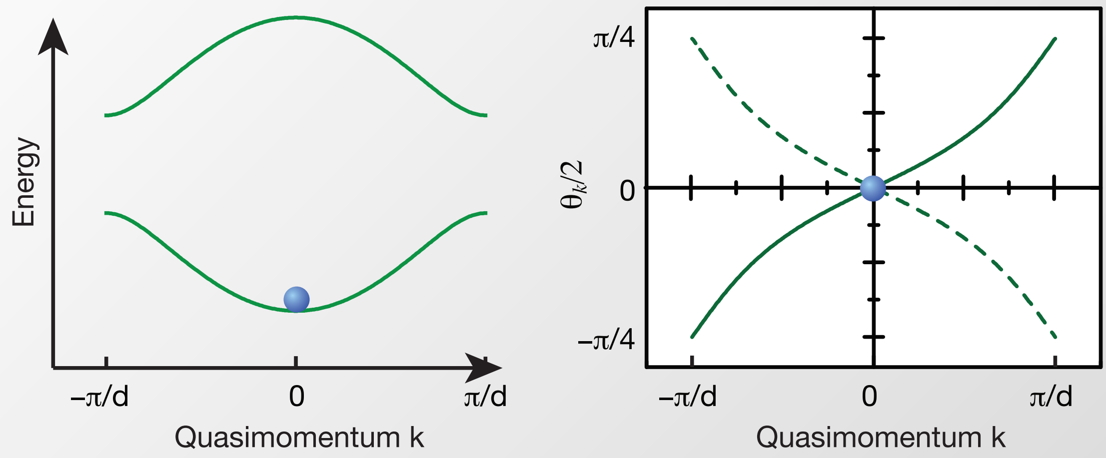




$$\boxed{\delta\varphi_{Zak} = \varphi^{D1}_{Zak} - \varphi^{D2}_{Zak} = \pi}$$

Measuring the Berry-Zak Phase (SSH Model)

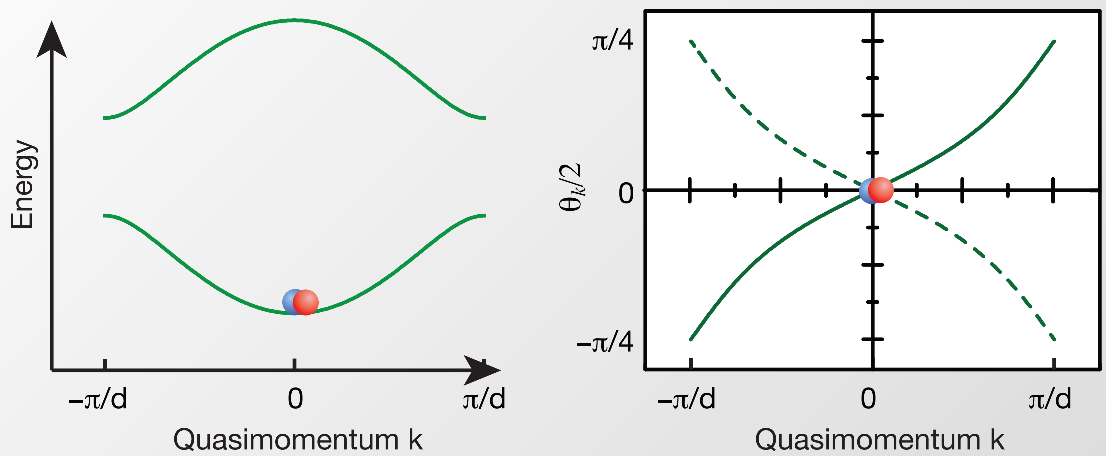
DI: $J > J'$ Spin-dependent Bloch oscillations + Ramsey interferometry

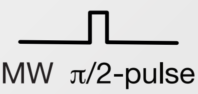


Prepare BEC in state $|\sigma, k\rangle = |\downarrow, 0\rangle$, with $\sigma = \uparrow, \downarrow$

Measuring the Berry-Zak Phase (SSH Model)

DI: $J > J'$

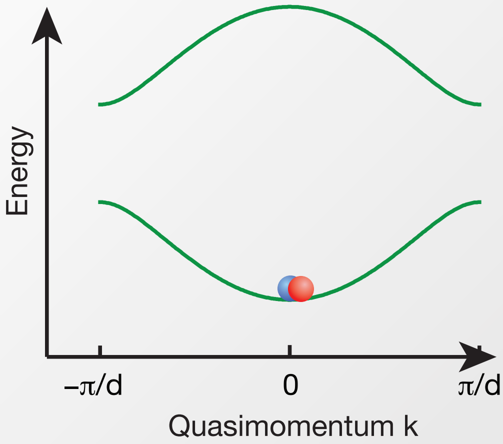


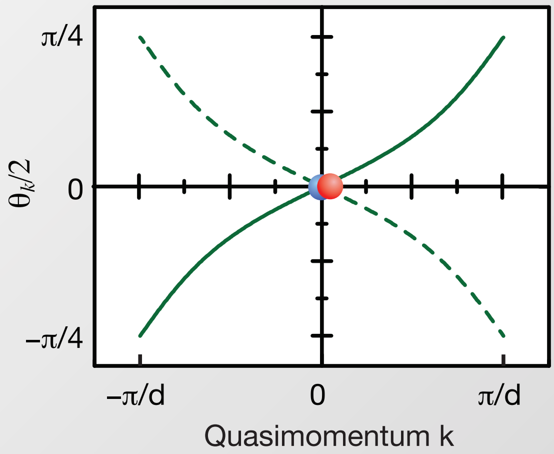


Create coherent superposition $\frac{1}{\sqrt{2}}\left(|\uparrow, 0\rangle + |\downarrow, 0\rangle\right)$

Measuring the Berry-Zak Phase (SSH Model)

DI: $J > J'$





MW π/2-pulse

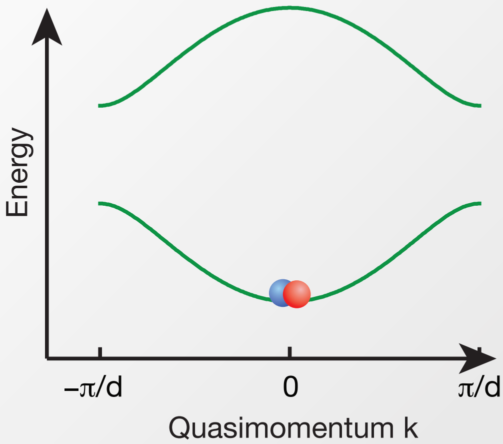
Spin components with opposite magnetic moments!

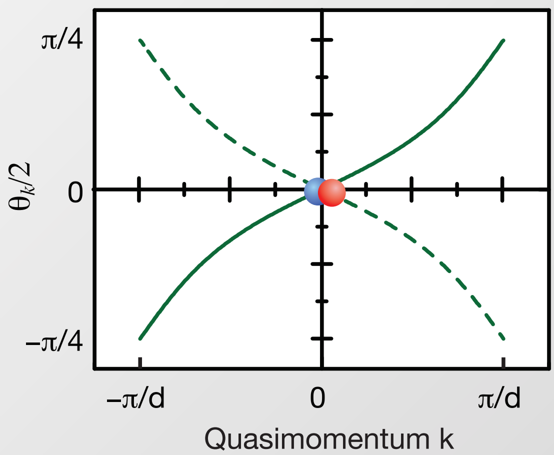
Create coherent superposition $\frac{1}{\sqrt{2}}\left(|\uparrow, 0\rangle + |\downarrow, 0\rangle\right)$

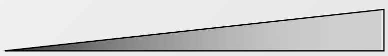
Berry Phase

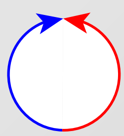
Measuring the Berry-Zak Phase (SSH Model)

DI: $J > J'$







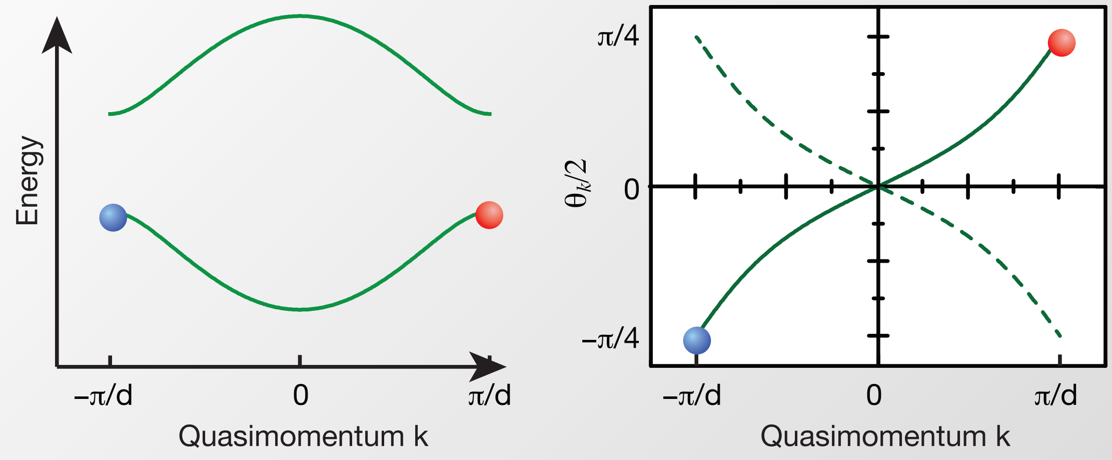


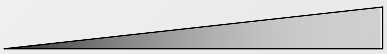
Closed Loop in *k*-Space

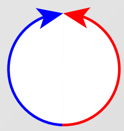
Apply magnetic field gradient $\longrightarrow$ adiabatic evolution in momentum space

Measuring the Berry-Zak Phase (SSH Model)

DI: $J > J'$







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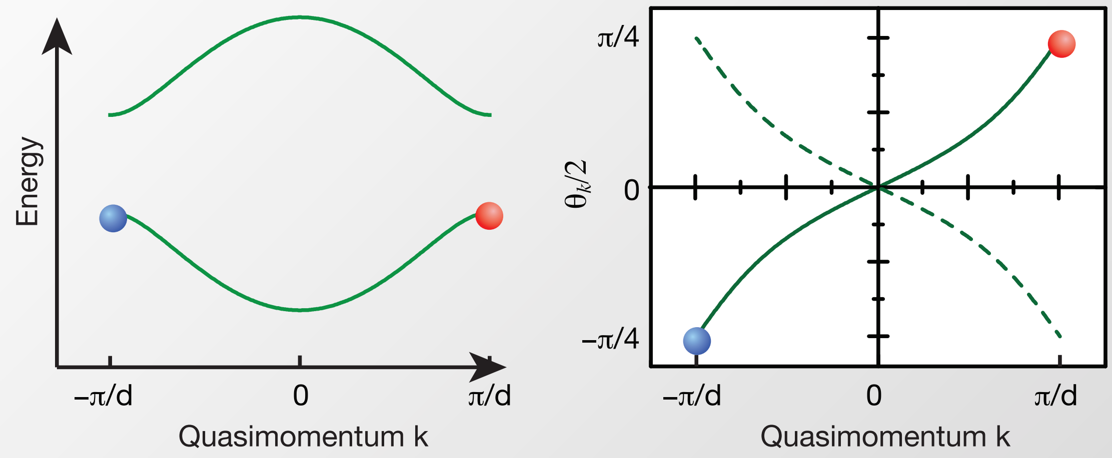
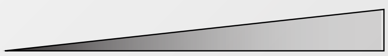
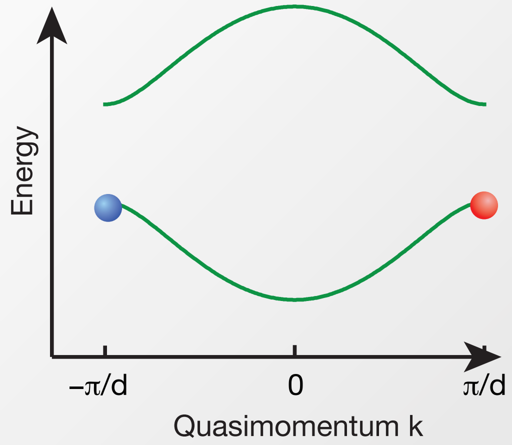


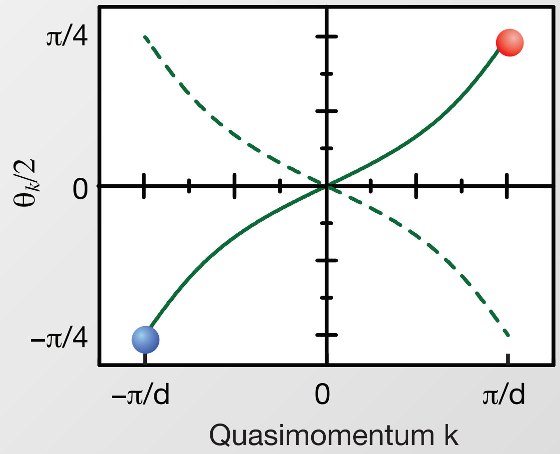


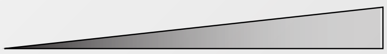
$$\delta\varphi_{Zak} = \varphi_{Zak}^{D1} + \varphi_{dyn} + \varphi_{Zeeman}$$

Measuring the Berry-Zak Phase (SSH Model)

D1: $J > J'$







$$\delta\varphi_{Zak} = \varphi_{Zak}^{D1} + \cancel{\varphi_{dyn}} + \varphi_{Zeeman}$$

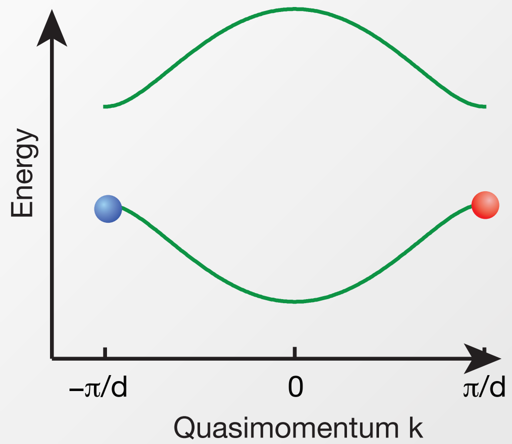
$$\varphi_{dyn} = \int E(t)/\hbar \; \mathrm{dt}$$

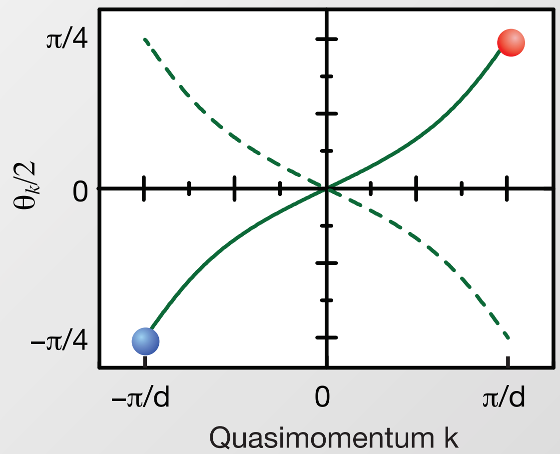
$$E(k) = E(-k)$$

Berry Phase

Measuring the Berry-Zak Phase (SSH Model)

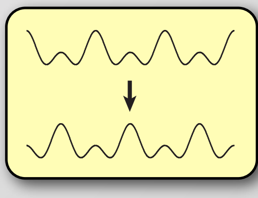
D1: $J > J'$ → **D2:** $J' > J$





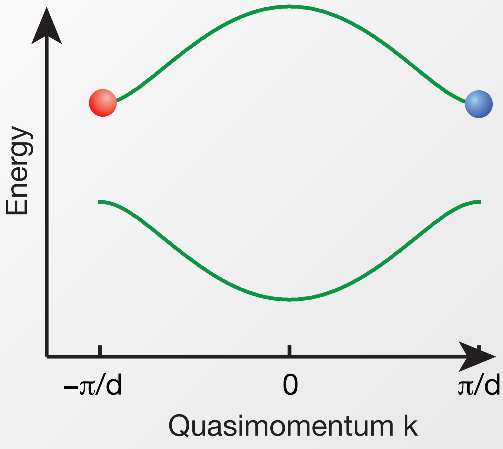
MW π-pulse

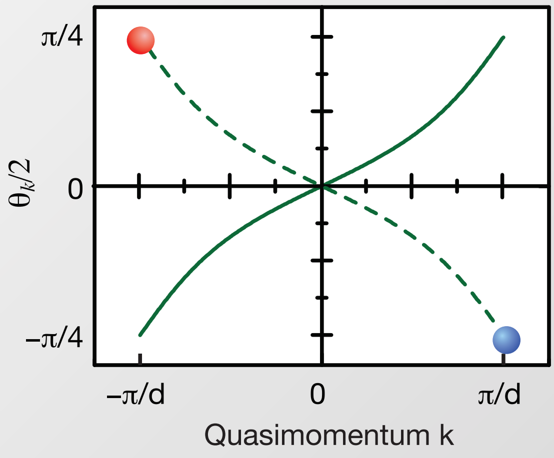
Apply Spin-Echo pulse + dimerization change



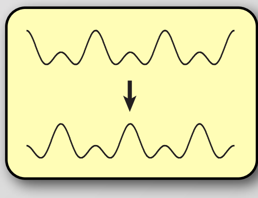
Measuring the Berry-Zak Phase (SSH Model)

D1: $J > J'$ $\rightarrow$ **D2:** $J' > J$





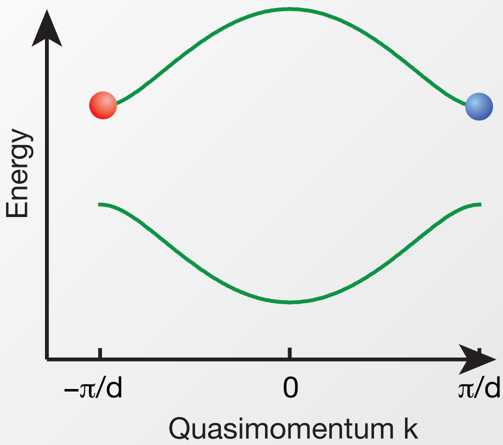
Apply Spin-Echo pulse + dimerization change

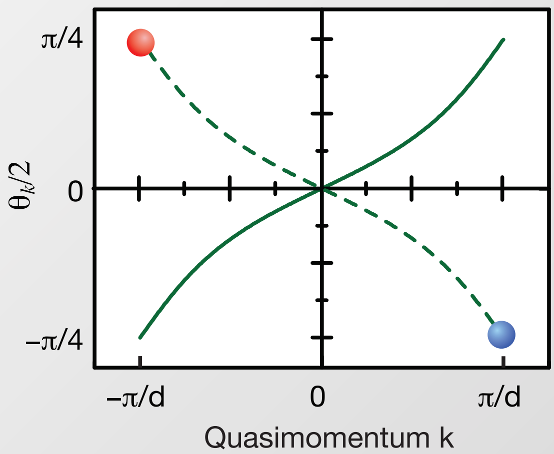


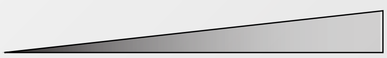
Berry Phase

Measuring the Berry-Zak Phase (SSH Model)

D2: $J' > J$



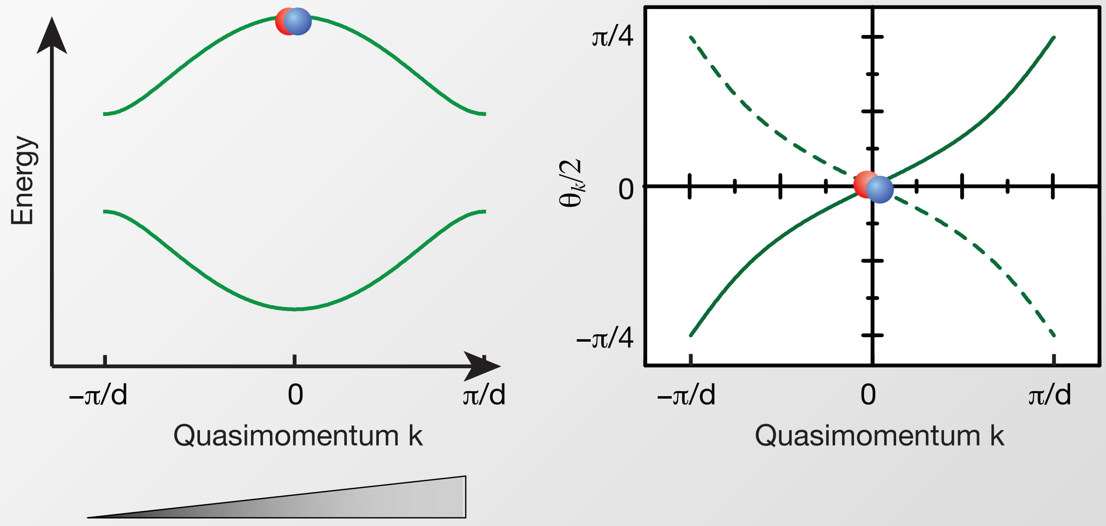




Apply magnetic field gradient $\longrightarrow$ adiabatic evolution in momentum space

Berry Phase Measuring the Berry-Zak Phase (SSH Model)

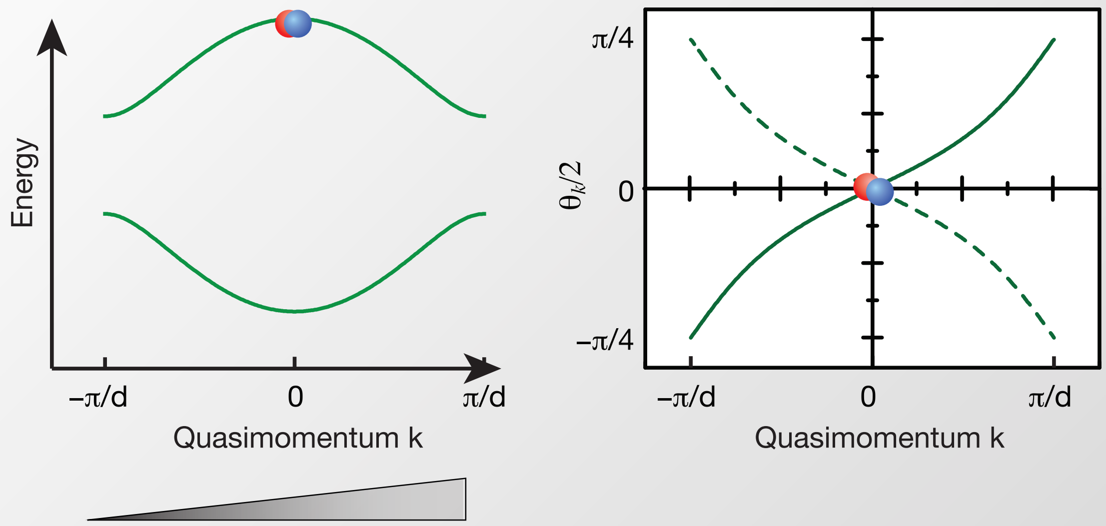
D2: $J' > J$



Apply magnetic field gradient $\longrightarrow$ adiabatic evolution in momentum space

Berry Phase Measuring the Berry-Zak Phase (SSH Model)

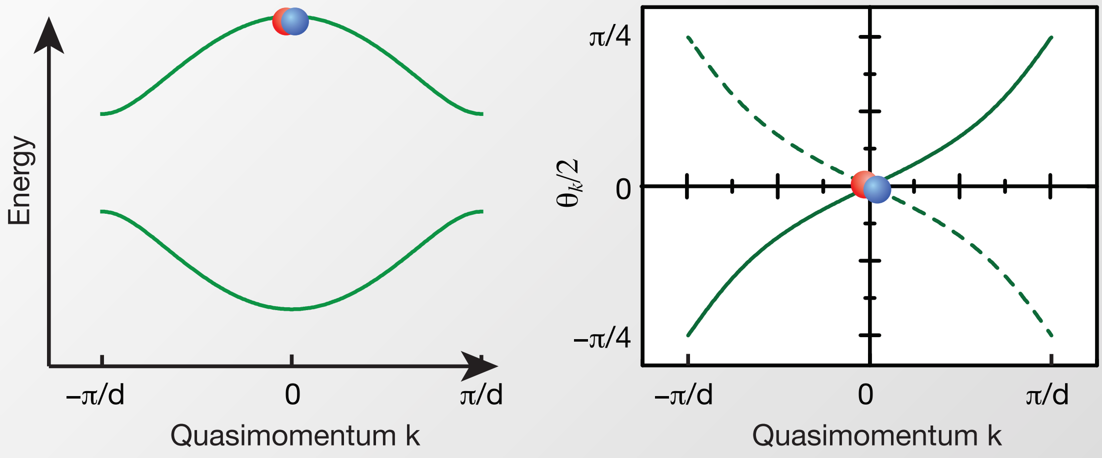
D2: $J' > J$

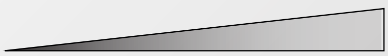


$$\delta\varphi_{Zak} = \varphi^{D1}_{Zak} - \varphi^{D2}_{Zak} + \varphi_{Zeeman}$$

Measuring the Berry-Zak Phase (SSH Model)

D2: $J' > J$



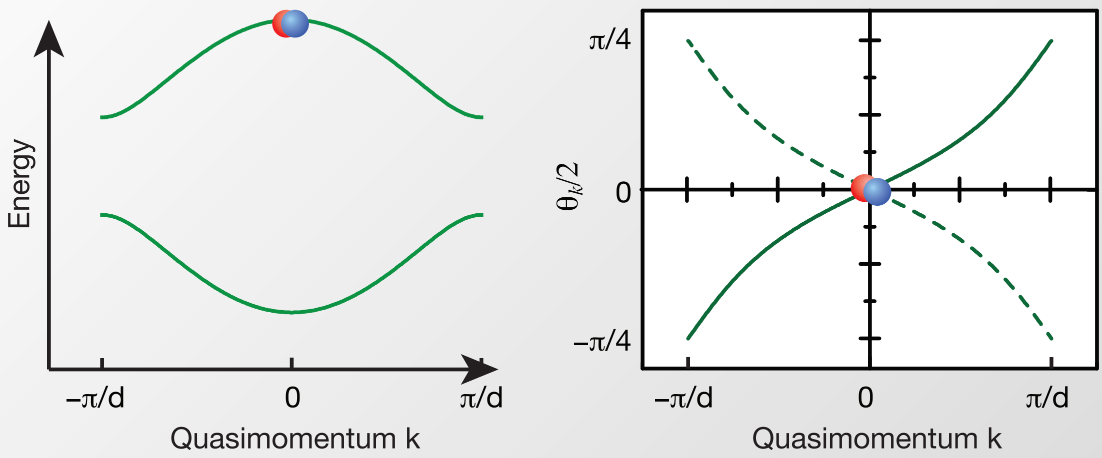


Spin-Echo pulse

$$\delta\varphi_{Zak} = \varphi^{D1}_{Zak} - \varphi^{D2}_{Zak} + \cancel{\varphi_{Zeeman}}$$

Measuring the Berry-Zak Phase (SSH Model)

D2: $J' > J$



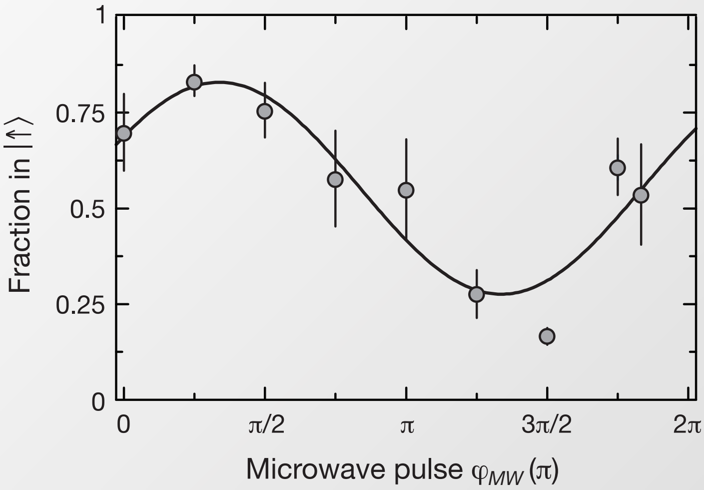
MW $\pi/2$-pulse, with phase φ_{MW}

Detect phase difference
with Ramsey interferometry

$$\delta\varphi_{Zak} = \varphi^{D1}_{Zak} - \varphi^{D2}_{Zak}$$

Reference measurement

Phase of reference fringe:



$\delta\varphi \neq 0$

Average of five individual measurements

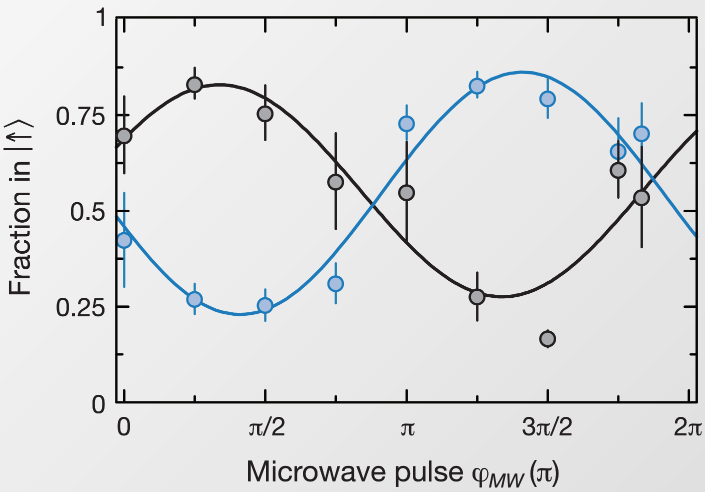
Exp. imperfections: - Small detuning of the MW-pulse

- Magnetic field drifts

Measuring the Zak Phase (SSH Model)

Measured Topological invariant:
Zak phase difference

$$\varphi_{Zak}^{D1} - \varphi_{Zak}^{D2} = \pi$$



$$\delta\varphi_{Zak} = 0.97(2)\pi$$

obtained from 14 independent measurements

Measuring the Zak Phase (SSH Model)

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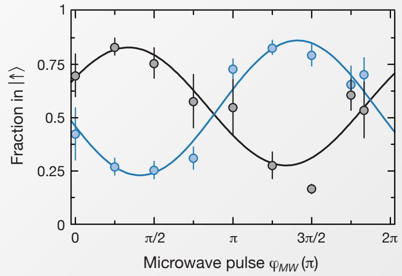
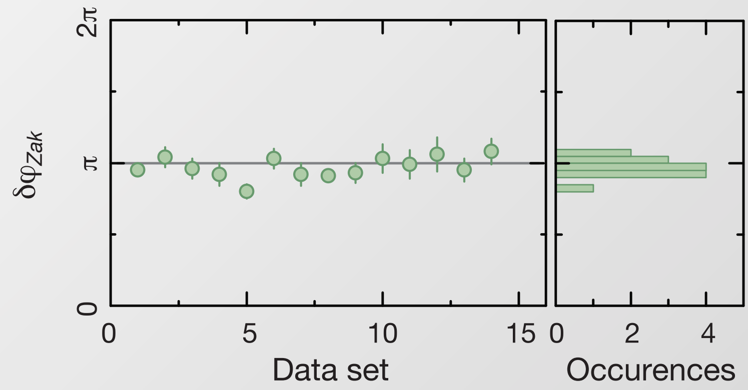
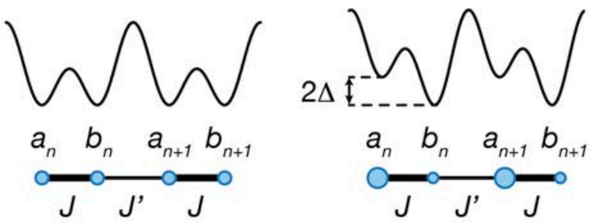


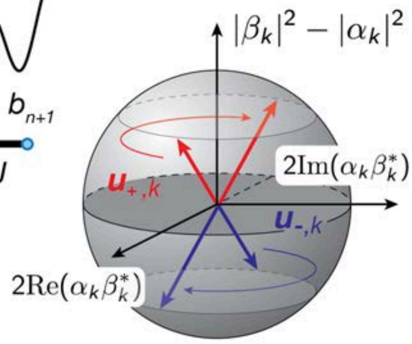


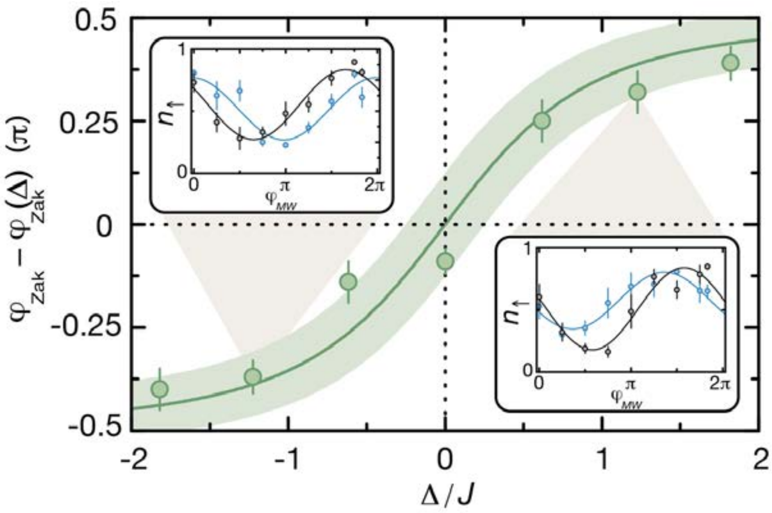
$$\delta\varphi_{Zak} = 0.97(2)\pi$$

obtained from 14
independent measurements

Fractional Zak Phase



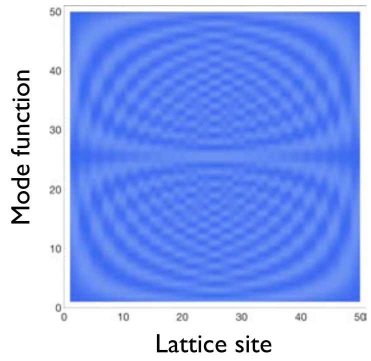


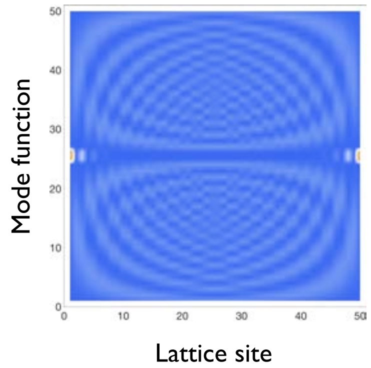


Zak Phase becomes **fractional** for heteropolar dimerization!

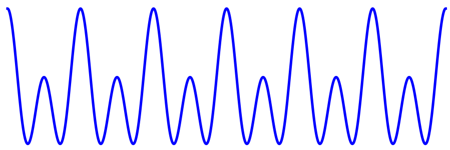
Measuring Fractional Charge

Probability Density of Eigenstates

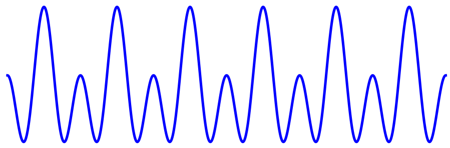




Lattice Topology

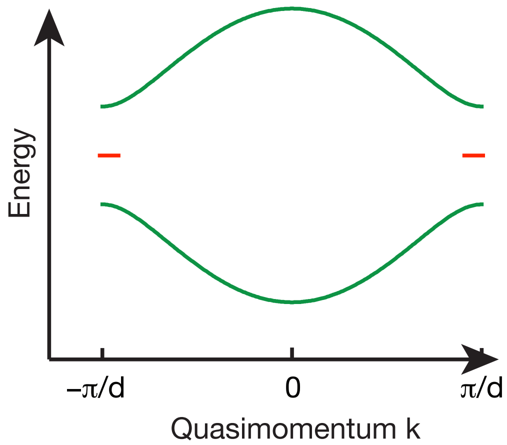


Topologically Trivial



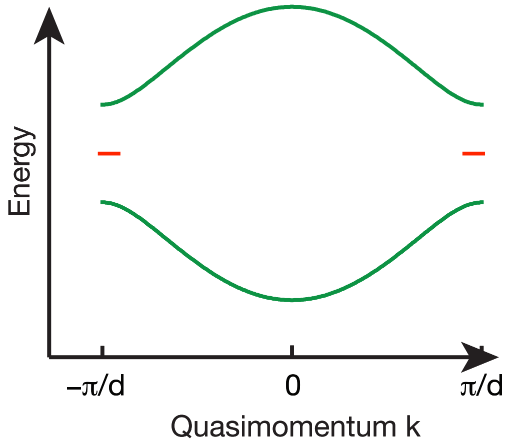
Topologically Non-Trivial

Fractional Charge



R. Rajaraman & J. Bell, Phys. Lett B 1982, Nucl. Phys. B 1983

Fractional Charge



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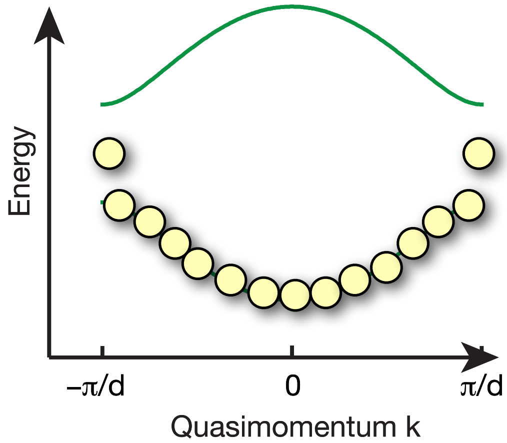
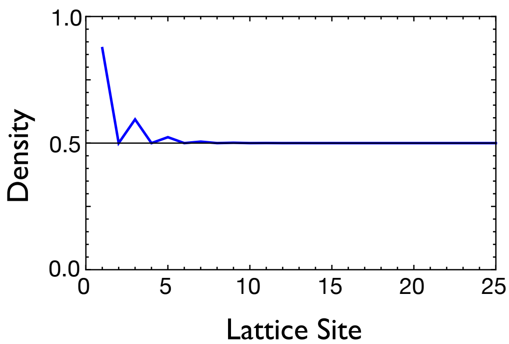
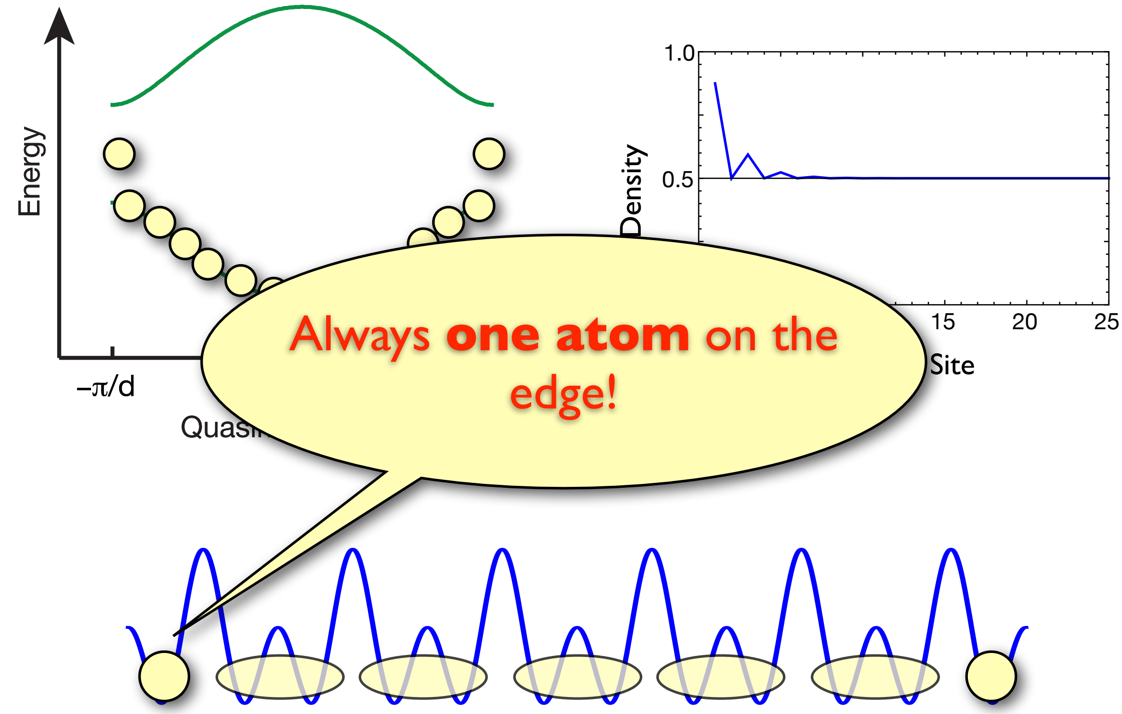
Fractional Charge





R. Rajaraman & J. Bell, Phys. Lett B 1982, Nucl. Phys. B 1983

Fractional Charge



R. Rajaraman & J. Bell, Phys. Lett B 1982, Nucl. Phys. B 1983

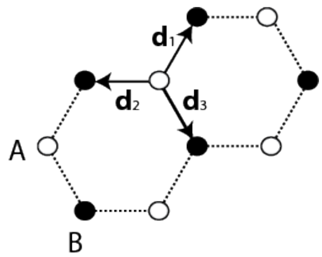
'Aharonov-Bohm' Interferometer for Measuring Berry Curvature

Hexagonal Lattices

Lattice: A and B degenerate sublattices

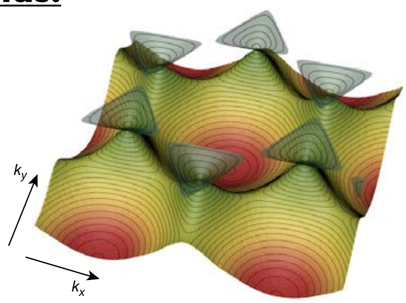
$$H = H_0 - J\sum_{\mathbf{R}}\sum_{i=1}^{3}\left(\hat{a}_{\mathbf{R}}\hat{b}^{\dagger}_{\mathbf{R}+\mathbf{d}_i} + \mathrm{h.c.}\right)$$

Real Space

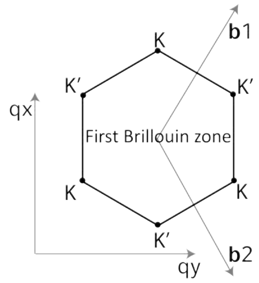


Lowest energy bands:

Dirac points at the corners of the first BZ



Reciprocal Space



A. Castro Neto et al., Rev. Mod. Phys. **81**, 109 (2009)
cold atoms: hexagonal - K. Sengtsock (Hamburg), brick wall - T. Esslinger (Zürich)

Scalar & Geometric Features

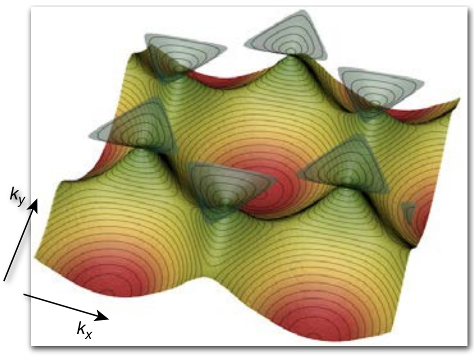
Band structure characterized by **scalar** & **geometric** features!

Eigenstates: Bloch waves $\boxed{\psi_{\mathbf{q},n}(\mathbf{r}) = e^{i\mathbf{q}\mathbf{r}}u_{\mathbf{q},n}(\mathbf{r})}$

Scalar Features

Dispersion relation

$E_{\mathbf{q},n}$



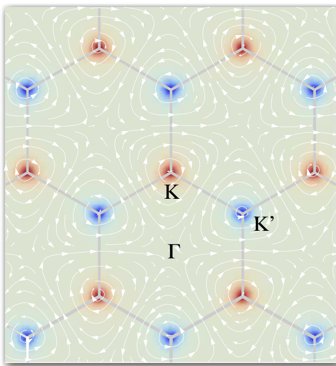
Geometric Features

Berry connection

$\mathbf{A}_n(\mathbf{q}) = i\langle u_{\mathbf{q},n}|\nabla_{\mathbf{q}}|u_{\mathbf{q},n}\rangle$

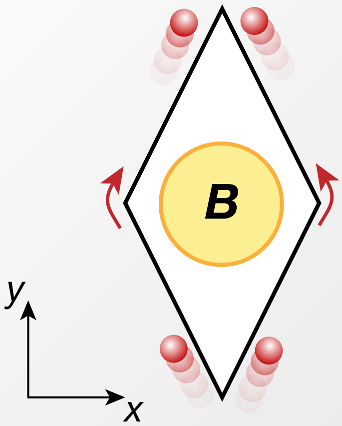
Berry curvature

$\Omega_n(\mathbf{q}) = \nabla_{\mathbf{q}} \times \mathbf{A}_{\mathbf{n}}(\mathbf{q}) \cdot \mathbf{e}_z$



Band Topolgy **'Aharonov Bohm' Interferometer in Momentum Space**

Real Space



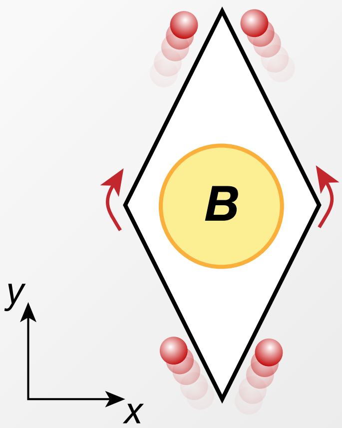
$$\varphi_{AB} = \frac{q}{\hbar}\oint_C \mathbf{A}(\mathbf{r})\,d\mathbf{r} = \frac{q}{\hbar}\int_S \nabla\times\mathbf{A}(\mathbf{r})\,d^2r$$

$$\boxed{\varphi_{AB} = \frac{q}{\hbar}\int \mathbf{B}\,d\mathbf{S} = 2\pi\Phi/\Phi_0}$$

Aharonov-Bohm Phase

Band Topolgy **'Aharonov Bohm' Interferometer in Momentum Space**

Real Space

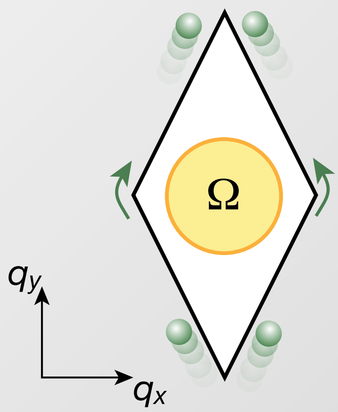


$$\varphi_{AB} = \frac{q}{\hbar}\oint_C \mathbf{A}(\mathbf{r})\,d\mathbf{r} = \frac{q}{\hbar}\int_S \nabla\times\mathbf{A}(\mathbf{r})\,d^2r$$

$$\boxed{\varphi_{AB} = \frac{q}{\hbar}\int \mathbf{B}\,d\mathbf{S} = 2\pi\Phi/\Phi_0}$$

Aharonov-Bohm Phase

Momentum Space



$$\varphi_{\text{Berry}} = \oint_C \mathbf{A}_n(\mathbf{q})\,d\mathbf{q} = \int_{S_q} \nabla\times\mathbf{A}_n(\mathbf{r})\,d\mathbf{S}_q$$

$$\boxed{\varphi_{\text{Berry}} = \int \Omega_n(\mathbf{q})\,d\mathbf{S}_q}$$

Berry Phase

Berry Curvature in Hexagonal Lattices

Berry curvature **concentrated to Dirac cones, alternating in sign**!

Breaking time reversal or inversion symmetry gaps Dirac cones and spreads Berry curvature out

Hexagonal Lattice Hamiltonian

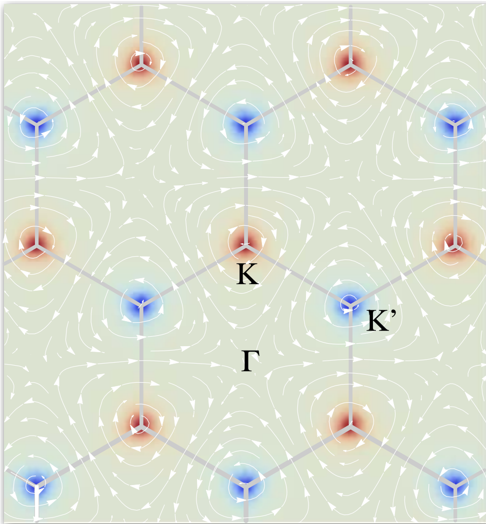
$$H(\mathbf{q}) = \begin{pmatrix} \Delta & f(\mathbf{q}) \\ f(\mathbf{q}) & -\Delta \end{pmatrix}$$

Expanding momenta close to K Dirac point

$$H(\tilde{\mathbf{q}}) = \begin{pmatrix} 0 & \tilde{q}_x + i\tilde{q}_y \\ \tilde{q}_x - i\tilde{q}_y & 0 \end{pmatrix}$$

Eigenstates

$$u^{\pm}_{\mathbf{K},\tilde{\mathbf{q}}} = \frac{1}{2}\left(e^{i\theta(\mathbf{q})/2} \pm e^{-i\theta(\mathbf{q})/2}\right)$$



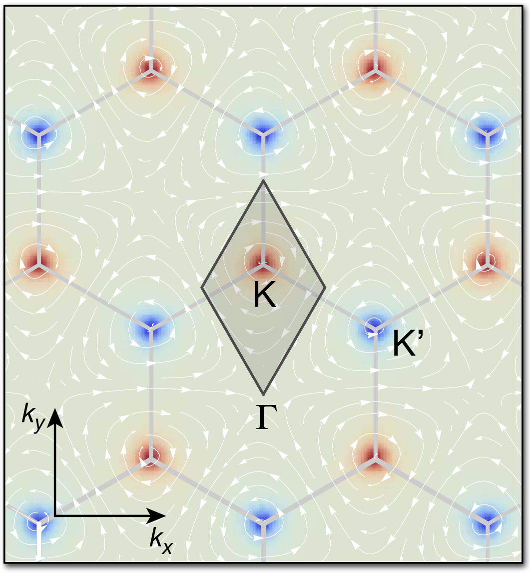
Berry Phases in Graphene

Berry Phase around K-Dirac cone

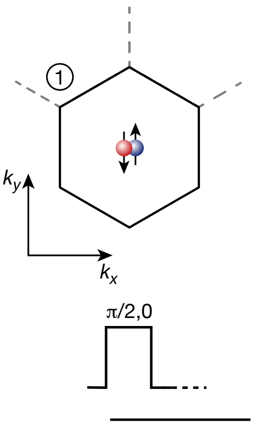
$$\boxed{\varphi_{\text{Berry},\mathbf{K}} = \oint_C \mathbf{A}(\mathbf{q})\, d\mathbf{q} = \pi}$$

Berry Phase around K'-Dirac cone

$$\boxed{\varphi_{\text{Berry},\mathbf{K}'} = -\pi}$$



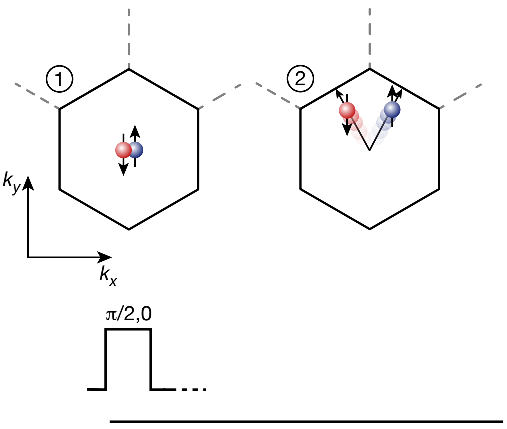
The Interferometer



Forces applied by lattice acceleration and magnetic gradients!

Band Topology

The Interferometer



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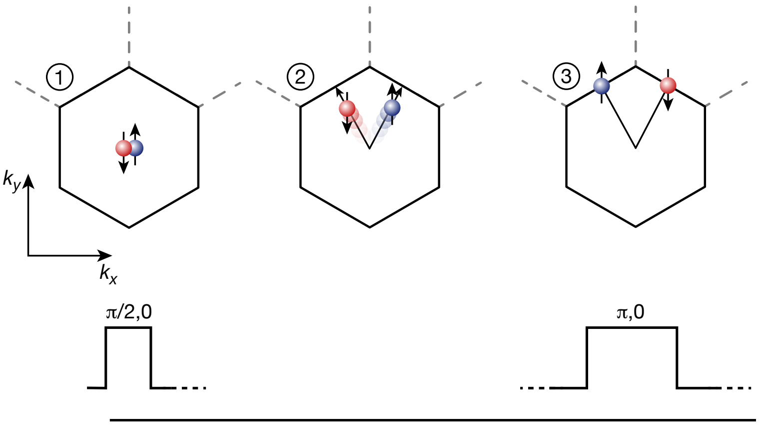
The Interferometer



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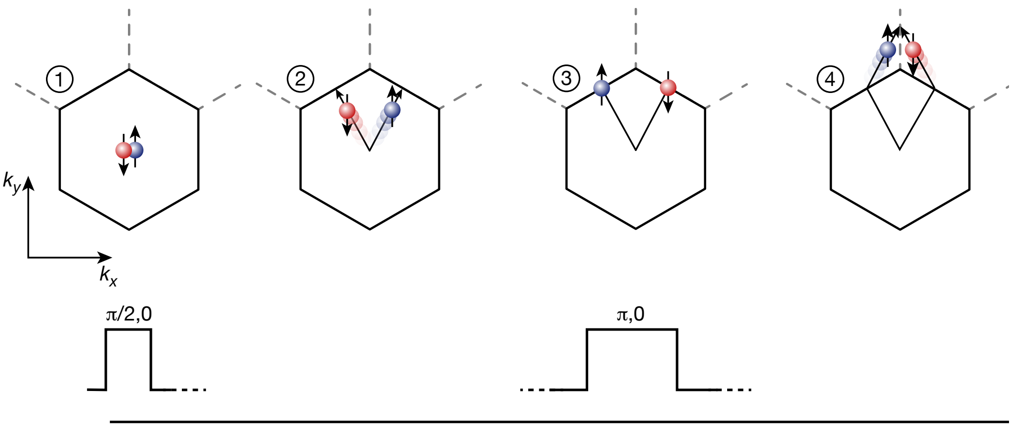
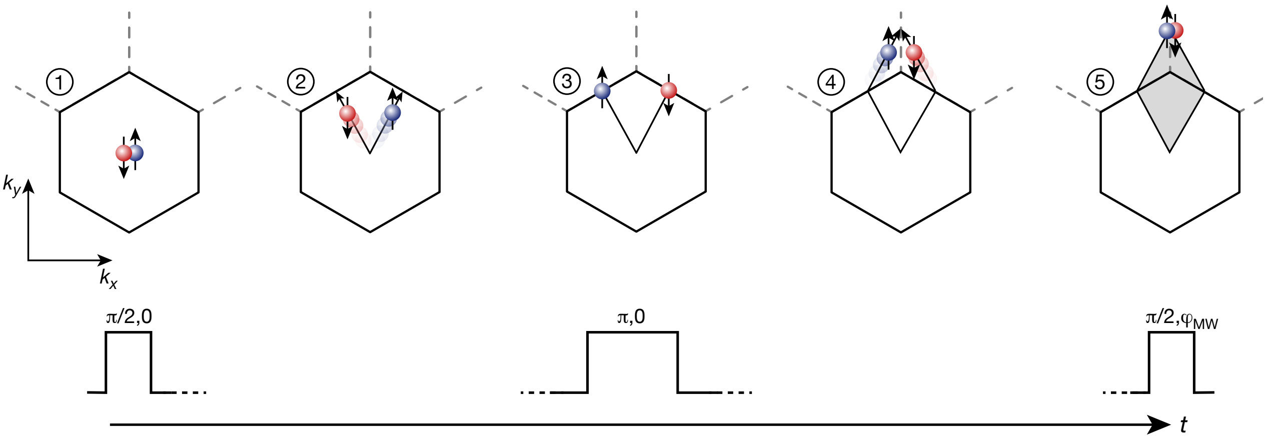
The Interferometer



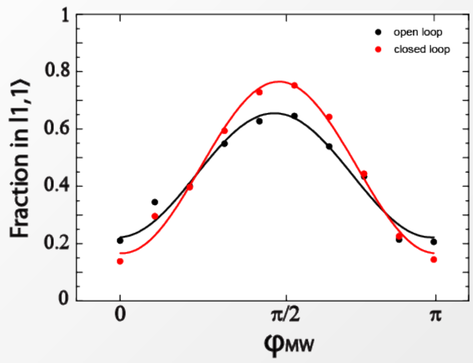
Forces applied by lattice acceleration and magnetic gradients!

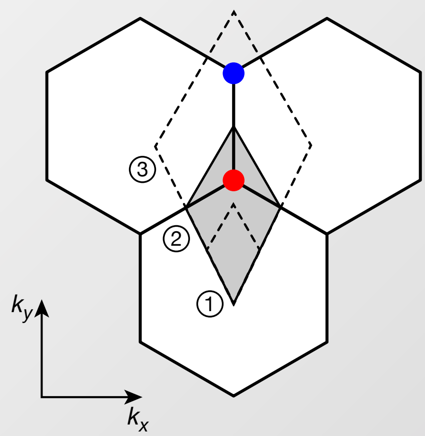
The Interferometer



Forces applied by **lattice acceleration** and **magnetic gradients**!

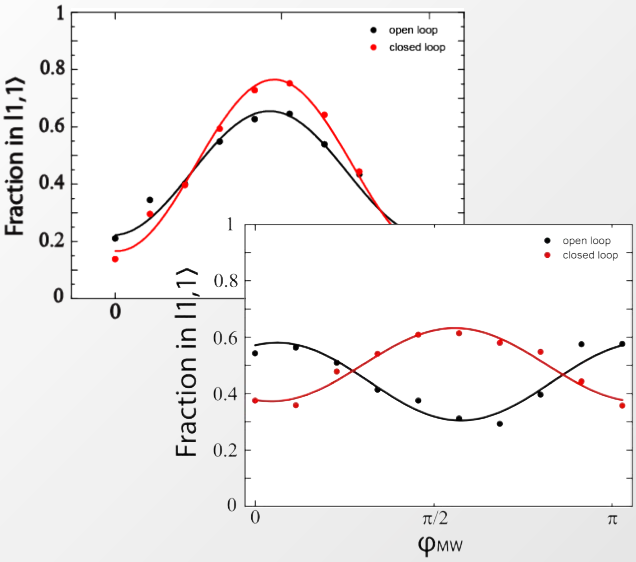
Interferometry Results

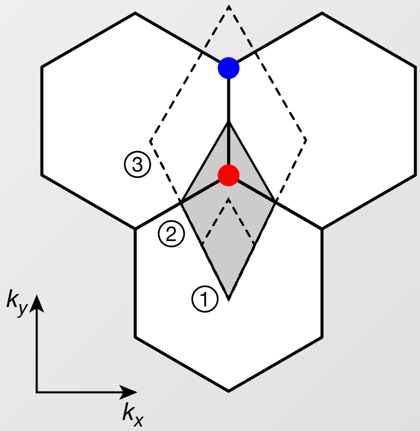




Band Topology

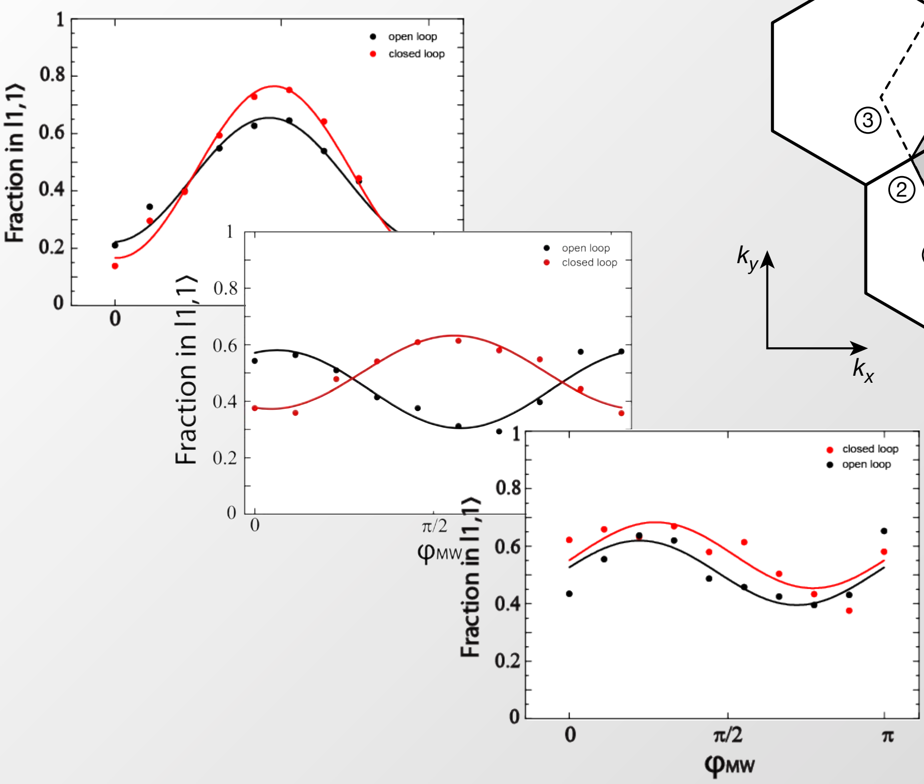
Interferometry Results

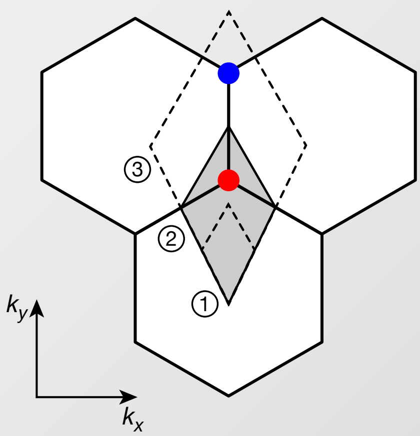




Band Topology

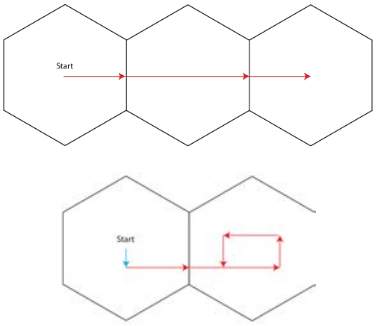
Interferometry Results

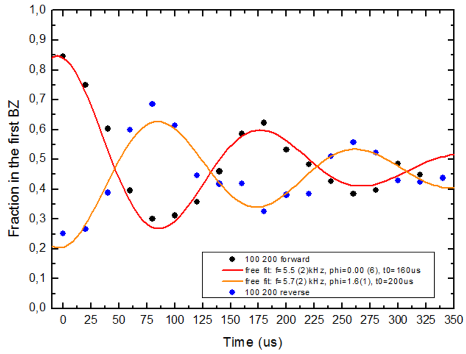


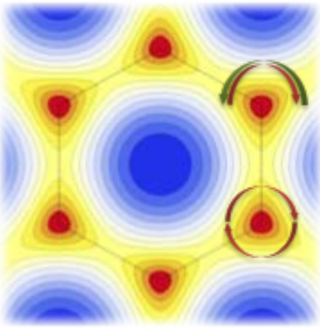


Stückelberg Interferometry

Lattice acceleration allows for arbitrary path choice

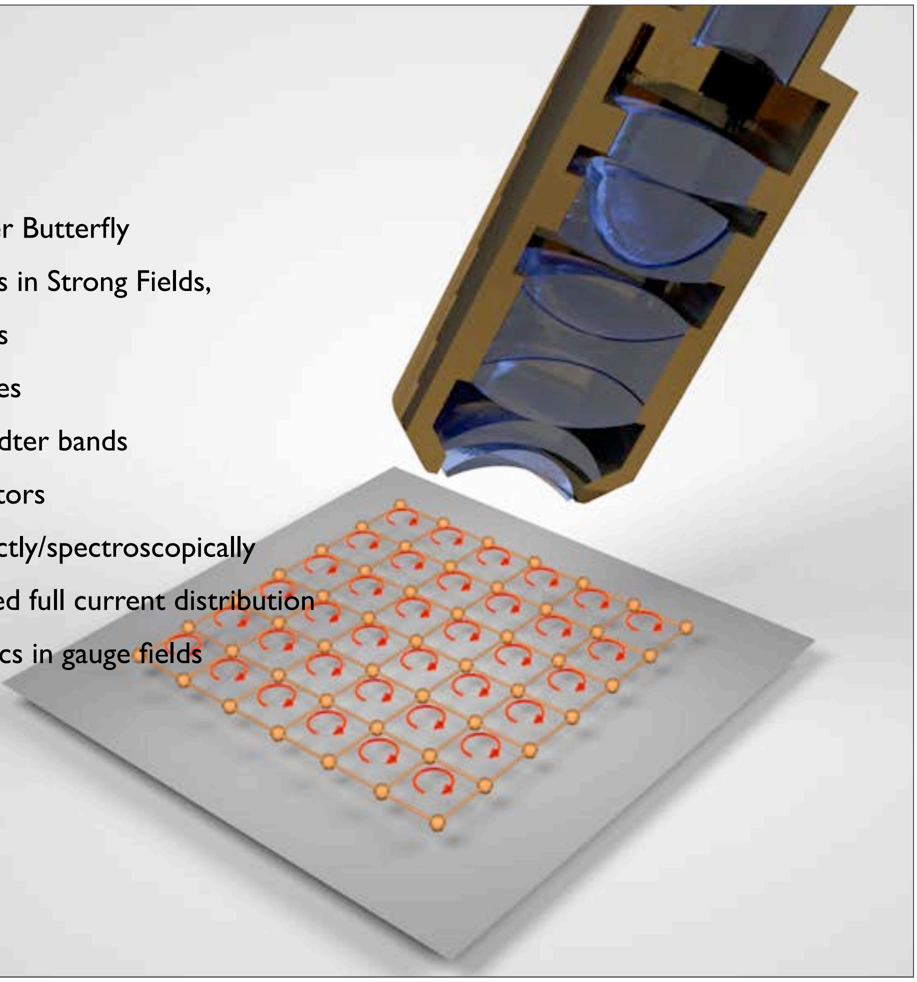






Has allowed us to detect **off-diagonal Berry connection through Wilson loops**!

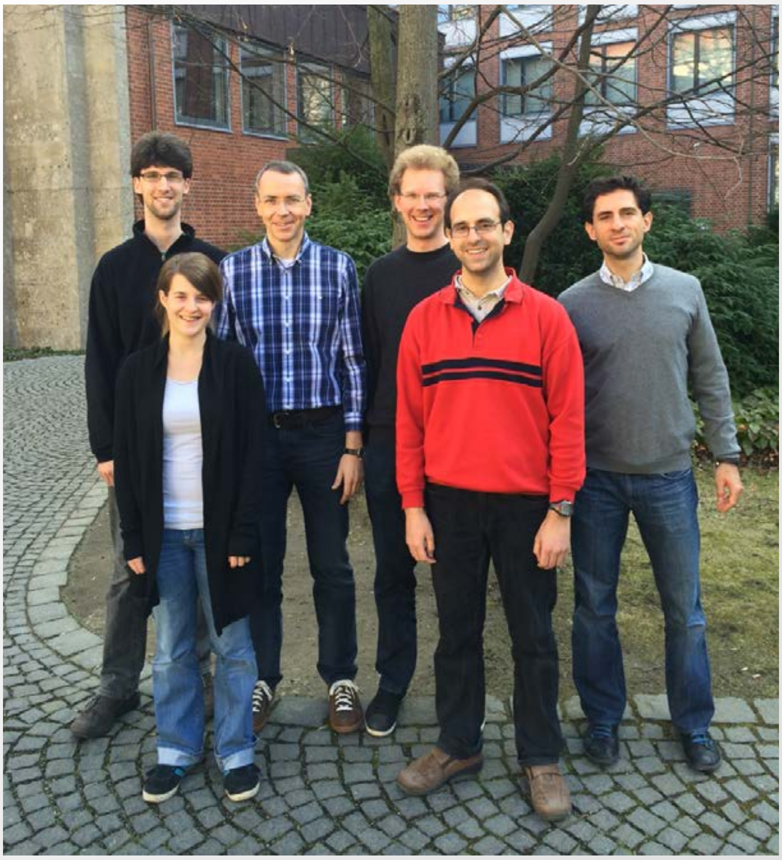
Outlook



- Rectified Flux, Hofstadter Butterfly
- Novel Correlated Phases in Strong Fields, Transport Measurements
- Adiabatic loading schemes
- Spectroscopy of Hoftstadter bands
- Novel Topological Insulators
- Image Edge States - directly/spectroscopically
- Measure spatially resolved full current distribution
- Non-equilibrium dynamics in gauge fields
- Thermalization?

⋮

Gauge Field Team

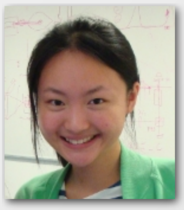


From left to right:

Christian Schweizer
Monika Aidelsburger
I.B.
Michael Lohse
Marcos Atala
Julio Barreiro

2D Berry Curvature Interferometer Team

Lucia Duca



Tracy Li

Martin Reitter

Monika Schleier-Smith

IB

Ulrich Schneider


www.quantum-munich.de