Dipolar quantum gases

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Outline

- Introduction
- Dipole moment
- Progress in creating dipolar gases
- Scattering problem. Ultracold limit
- Dipolar BEC. Uniform case
- Trapped dipolar BEC
- Stability problem

Collaborations: P. Pedri (Orsay), L. Santos (Hannover), M. A. Baranov (Amsterdam), M. Lewenstein (Barcelona)

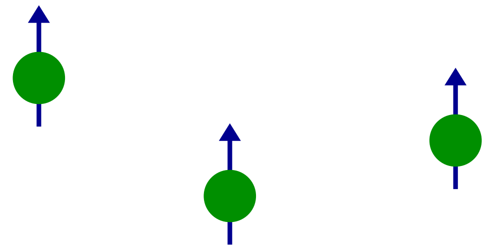
Dipolar quantum gases

- Roton-maxon spectrum. Prehistory
- Roton-maxon spectrum of a dipolar BEC
- Thermal fluctuation
- Roton instability and collapse

- Dipolar Feri gas. Interaction problem
- Superfluid pairing in a single-component dipolar Fermi gas
- Interesting problem to solve

Collaborations: P. Pedri (Orsay), L. Santos (Hannover), M. A. Baranov (Amsterdam), M. Lewenstein (Barcelona)

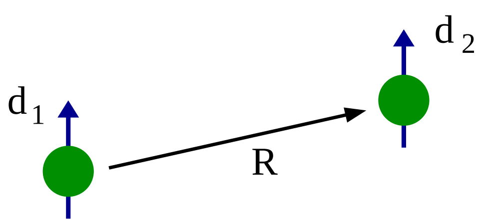
Dipolar gas



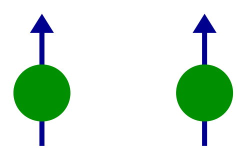
Polar molecules, atoms with a large magnetic moment

Dipole-dipole interaction

$$V_d = \frac{\vec{d}_1 \vec{d}_2 R^2 - 3(\vec{d}_1 \vec{R})(\vec{d}_2 \vec{R})}{R^5} \sim \frac{1}{R^3}$$



long-range, anisotropic



repulsion

attraction

Different physics compared to ordinary atomic ultracold gases

Dipole moment

Polar Molecules

CO	$0.1\mathcal{D} \approx 0.05au$
CsCl	$10\mathcal{D} \approx 5au$

Atoms with a large magnetic moment

$$\mathrm{Cr} \Rightarrow 6\mu_B$$

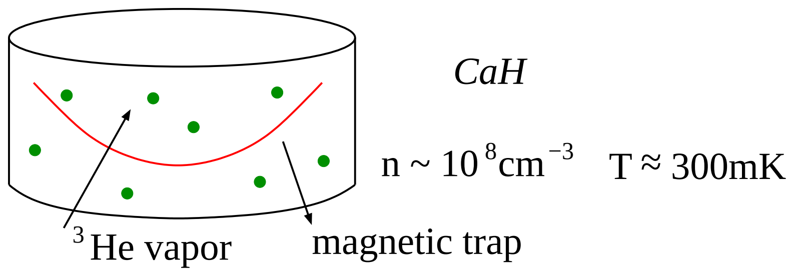
equivalent to $d = 0.048\mathcal{D} = 0.022au$
Presently a quatum gas of Cr bosons is created and Bose-condensed (T. Pfau, Stuttgart)

$\mathrm{Cr} \Rightarrow V_d \sim \frac{d^2}{r^3} \sim 10\mu K$ at $r \sim 200a_0$

and already exceeds the Van der Waals interaction

Polar molecules

Buffer-gas cooling (Doyle, Harvard)

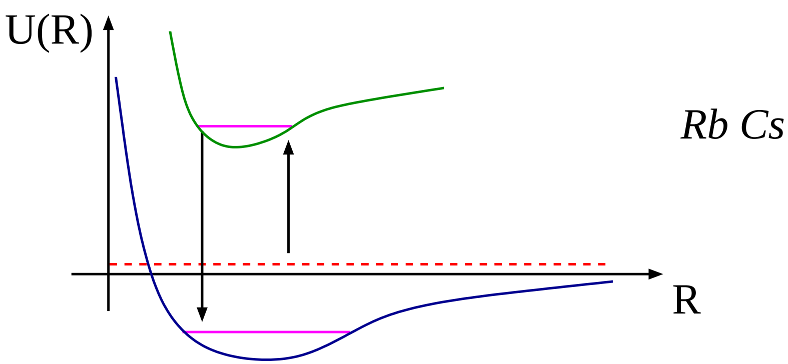


Stark deceleration (Meijer, Berlin)

H_3, ND_3, CO, etc. $n \sim 10^8 cm^{-3}$; $T \sim 10 mK$

Photoassociation

(many labs; De Mille, Yalle)



small n, T $100\mu K$

Theoretical studies of dipolar gases

Bose gas

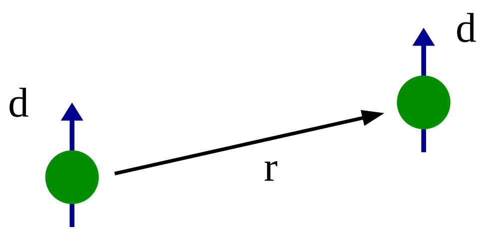
L. Santos, M. Lewenstein, P. Zoller, G. S.,
C. Eberlein, S. Giovanazzi, O'Dell,
K. Góral, M. Brewekyk, K. Rzatenski,
S. Yi and L. You,
S. Giovanazzi, A. Gorliz, T. Pfau.

Fermi gas
M. A. Baranov, M. Lewenstein, et al.
K. Góral, K. Rzatenski, M. Brewekyk

Bosons/Fermions in optical lattices

(L. Santos)...

Dipole-diple scattering



$$V_d = \frac{d^2}{r^3}\underbrace{\left(1 - 3\cos^2\theta_{rd}\right)}_{\sim Y_{20}(\theta_{rd})}$$

Wave function of the relative motion

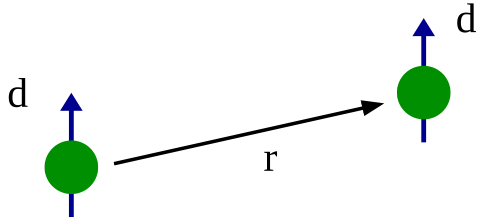
$$\psi_{in} \rightarrow \sum_{l,m} \psi_{kl}(r) i^l Y^*_{lm}(\theta_{kd}, \varphi_{kd}) Y_{lm}(\theta_{rd}, \varphi_{rd})$$

$$\psi_{out} \rightarrow \sum_{l',m'} \psi_{kl'}(r) i^{l'} Y^*_{l'm'}(\theta_{kd}, \varphi_{kd}) Y_{l'm'}(\theta_{rd}, \varphi_{rd})$$

Scattering matrix $\sim \int Y^*_{lm} Y_{l'm'} Y_{20} d\Omega_{rd}$

V_d couples all even l, and all odd l, but even and odd l are decoupled from each other

Radius of the dipole-dipole interaction



$$\left(-\frac{\hbar^2}{m}\Delta + V_d(\vec{r})\right)\psi(\vec{r}) = \frac{\hbar^2 k^2}{m}\psi(\vec{r})$$

$$\frac{\hbar^2}{m r_*^2} = \frac{d^2}{r_*^3} \Rightarrow r_* \approx \frac{md^2}{\hbar^2}$$

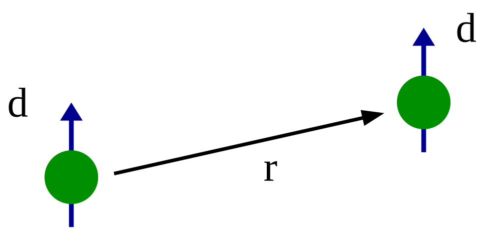
$r \gg r_*$ $\quad \rightarrow$ free relative motion

$r_* \sim 10^6 \div 10^2 a_0$ polar molecules

$r_* \approx 50 a_0 \rightarrow$ chromium atoms

$$kr_* \ll 1 \qquad \rightarrow \underbrace{\text{Ultracold limit}}_{T \ll 1mK \text{ for Cr}}$$

Scattering amplitude I



$$V(\vec{r}) = \mathcal{U}(\vec{r}) + V_d(\vec{r})$$
$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3r$$

Ultracold limit $kr_* \ll 1$

$$V_d = 0 \Rightarrow f = g = \frac{4\pi\hbar^2}{m} a$$

What V_d does?

$$k = 0 \rightarrow g = \int \psi^*_0(\vec{r})(\mathcal{U}(\vec{r}) + V_d(\vec{r}))d^3r = \text{const}; \qquad r \lesssim r_*$$

g may depend on d and comes from all even l

Scattering amplitude II

$$k \neq 0$$

$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3 r$$

$$r \lesssim r_* \rightarrow \text{put } k = 0 \rightarrow g$$

$$r \gg r_* \rightarrow \psi_{k_i} = e^{i\vec{k}_i \vec{r}}$$

$$f = \int V_d(\vec{r}) e^{i\vec{q}\vec{r}} d^3 r \longrightarrow \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1);\ \vec{q} = \vec{k}_f - \vec{k}_i$$

$$f = g + \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1)$$

Dipolar BEC I

uniform gas

$$H = \int d^3 \Big[\psi^\dagger(\vec{r}) \left(-\frac{\hbar^2}{2m}\Delta\right) \psi(\vec{r}) + \frac{1}{2} g \psi^\dagger(\vec{r})\psi^\dagger(\vec{r})\psi(\vec{r})\psi(\vec{r})$$
$$+ \frac{1}{2} \int d^3 r' \psi^\dagger(\vec{r})\psi^\dagger(\vec{r}')V_d(\vec{r}-\vec{r}')\psi(\vec{r}')\psi(\vec{r})\Big]$$

Bogoliubov approach $\psi = \psi_0 + \delta\Psi \rightarrow$ biliniear hamiltonian

$$H_B = \frac{N^2}{2V} g + \sum_k \Big[\frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + n\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right) a_k^\dagger a_k$$
$$\frac{n}{2}\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right)\left(a_k^\dagger a_{-k}^\dagger + a_k a_{-k}\right)\Big]$$

Dipolar BEC II

Excitation spectrum

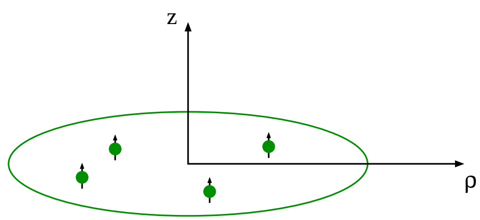
$$\epsilon_k = \sqrt{E_k^2 + 2E_k n\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right)}$$

$g > \frac{4\pi d^2}{3} \rightarrow$ dynamically stable BEC

$g < \frac{4\pi d^2}{3} \rightarrow$ complex frequencies at small k

and $\cos^2\theta_k < \frac{1}{3} \rightarrow$ collapse

Trapped dipolar BEC



Cylindrical trap

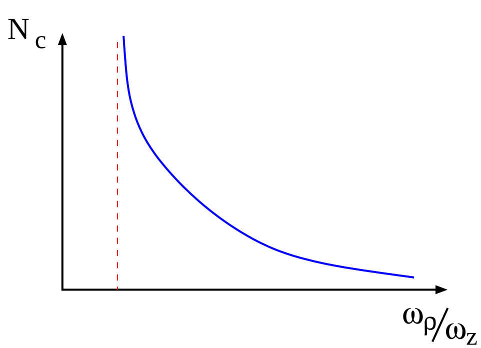
$$V_h = \frac{m}{2}\left(\omega_\rho^2 \rho^2 + \omega_z^2 z^2\right)$$

Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\Delta + V_h(\vec{r}) + g\psi_0^2 + \int \psi_0(\vec{r}')^2 V_d(\vec{r}-\vec{r}')d^3r'\right]\psi_0(\vec{r}) = \mu\psi_0(\vec{r})$$

Important quantity

$$V_{eff} = g\int \psi_0^4(\vec{r})d^3r + \int \psi_0^2(\vec{r}')V_d(\vec{r}-\vec{r}')\psi_0^2(\vec{r})d^3r d^3r'$$



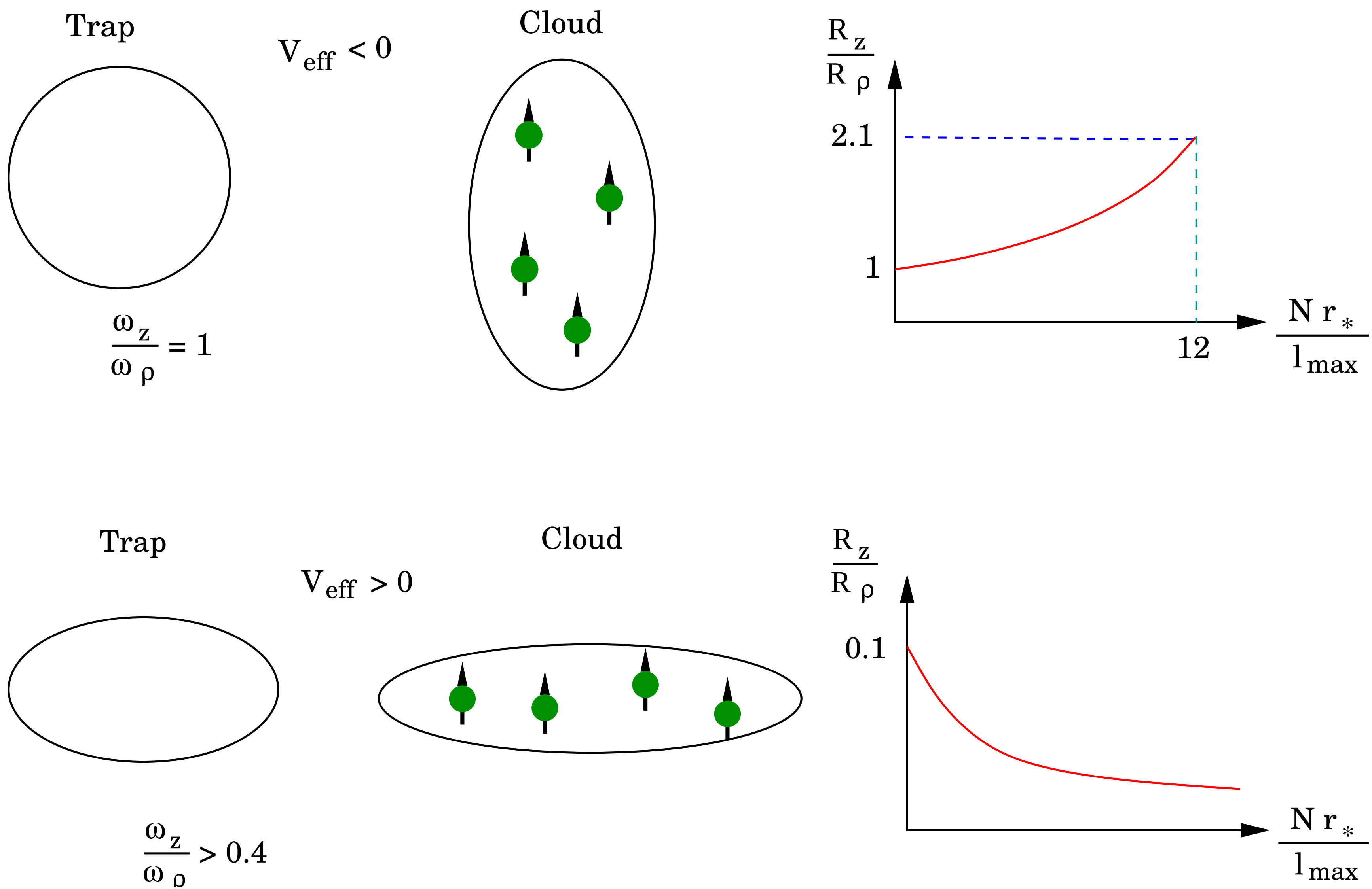
$V_{eff} > 0$ or $V_{eff} < 0$ and $|V| < \hbar\omega$

$g = 0 \to N < N_c \to$ suppressed

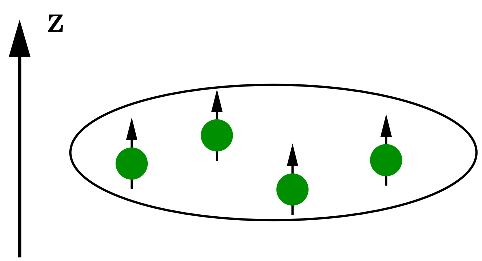
low k instability

(Santos et.al, 2000)

It is sufficient?



Stability problem



Dipolar BEC

$$\langle V_d \rangle = \int n_0(\vec{r}')V_d(\vec{r}' - \vec{r})d^3r = -d^2 \frac{\partial^2}{\partial z^2} \int \frac{n_0(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r$$

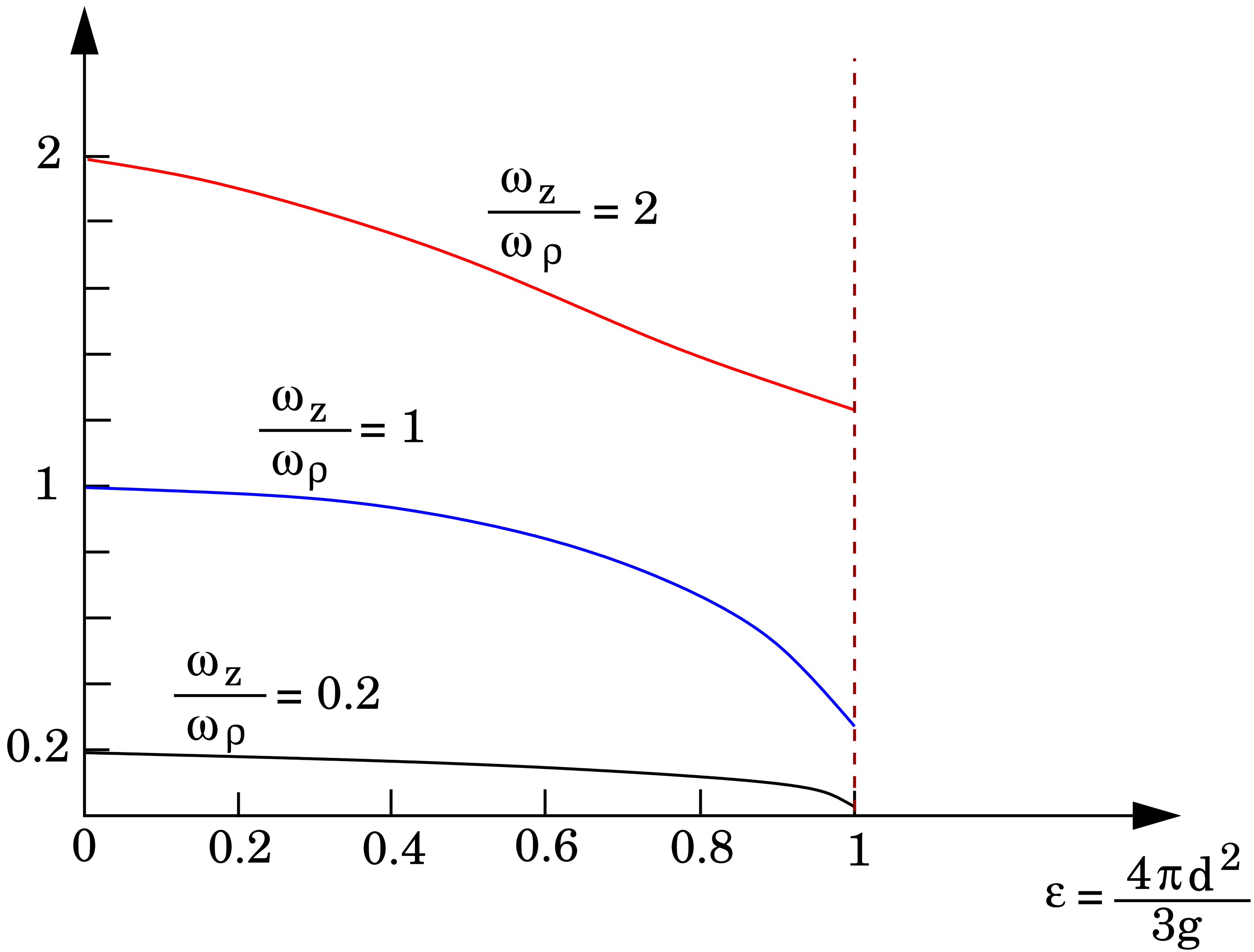
$$V_d = -d^2 \frac{\partial^2}{\partial z^2} \frac{1}{|\vec{r}-\vec{r}'|} - \frac{4\pi d^2}{3} \delta(\vec{r} - \vec{r}')$$

Large $N \Rightarrow$ Thomas-Fermi BEC

$$n_0 = n_{0\,max} \left(1 - \frac{z^2}{R_z^2} - \frac{\rho^2}{R_\rho^2}\right)$$ Eberlein et. al (2005)

$g > \frac{4\pi d^2}{3} \rightarrow$ stable at any N

Example



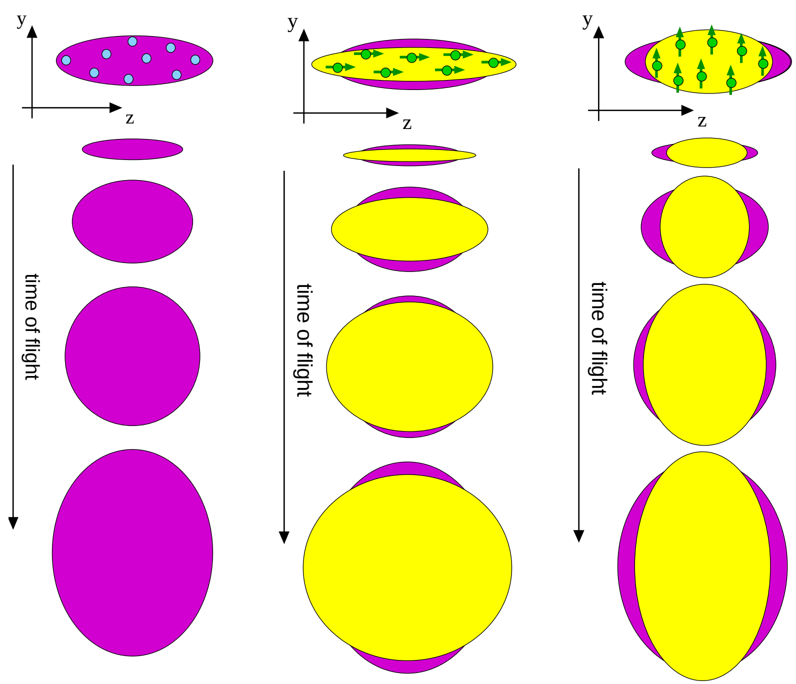
Experiment with Cr

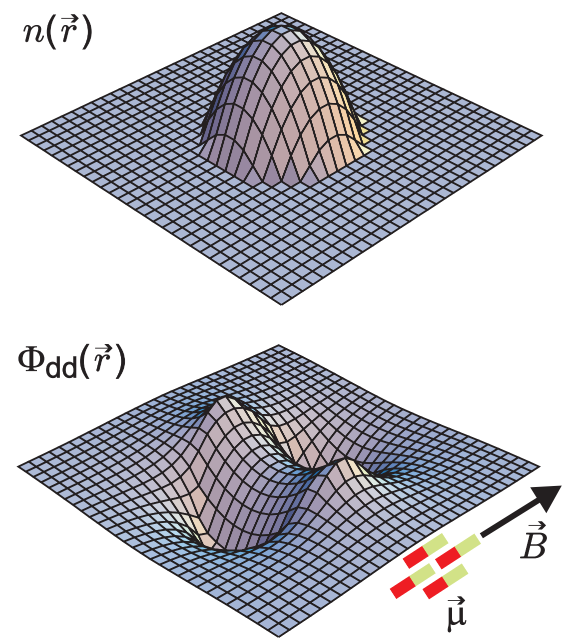
$$g > \frac{4\pi d^2}{3}$$

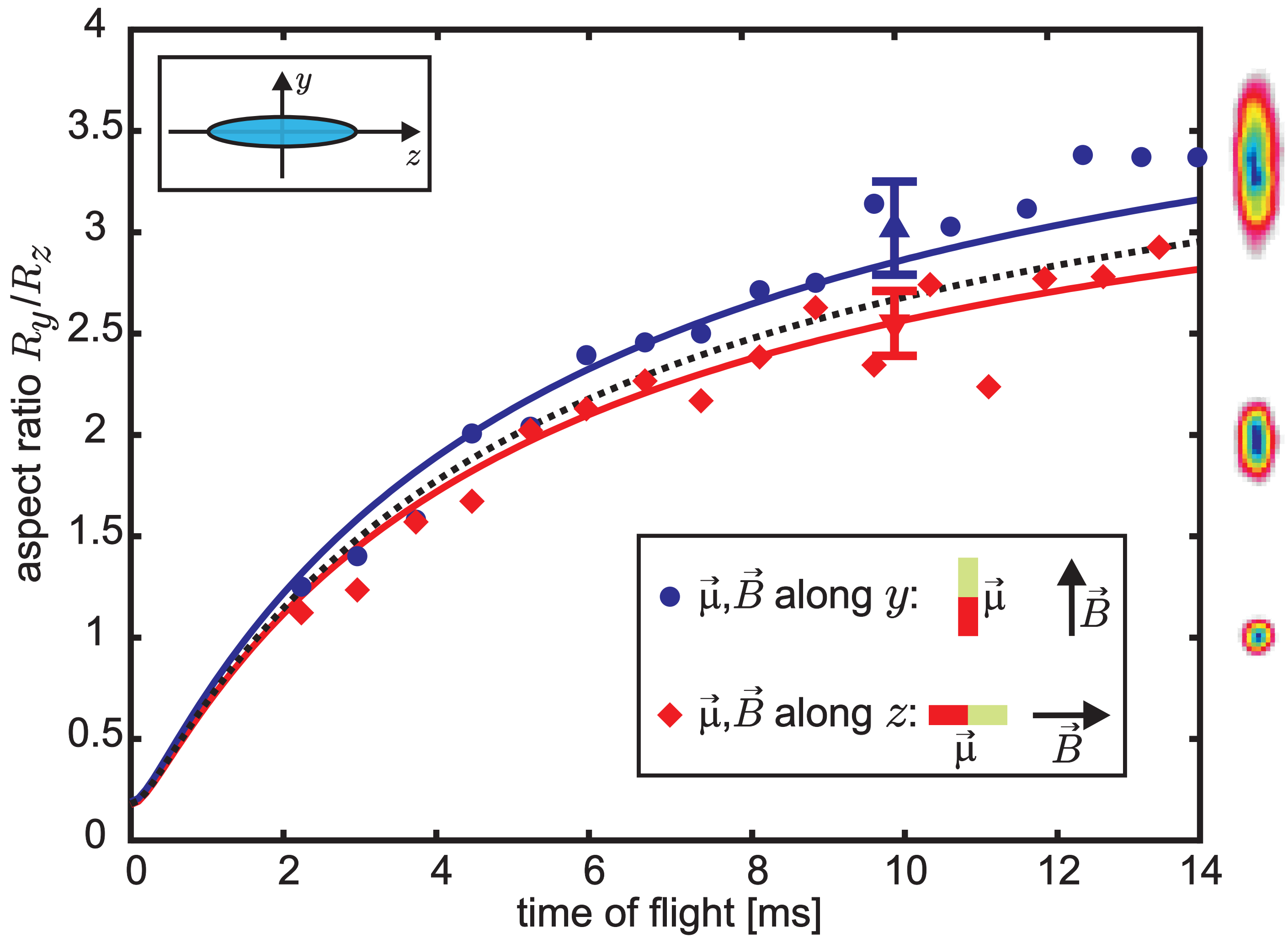
$(\mu = 6\mu_B!)$

(T. Pfau, Stuttgart) BEC ($n \sim 10^{14} cm^{-3}$)

effect of the dipole-dipole interaction (small)







aspect ratio R_y/R_z
time of flight [ms]
0
0.5
1
1.5
2
2.5
3
3.5
4
0
2
4
6
8
10
12
14
y
z
$\vec{\mu},\vec{B}$ along y:
$\vec{\mu}$
$\vec{B}$
$\vec{\mu},\vec{B}$ along z:
$\vec{\mu}$
$\vec{B}$

Question

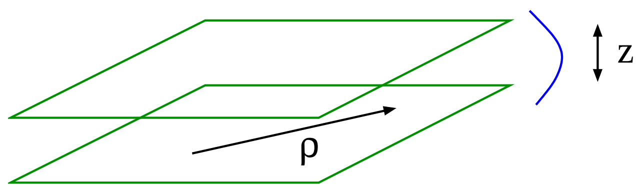
What happens at

$$g < g_d = \frac{4\pi d^2}{3} \quad ?$$

Pancake dipolar BEC

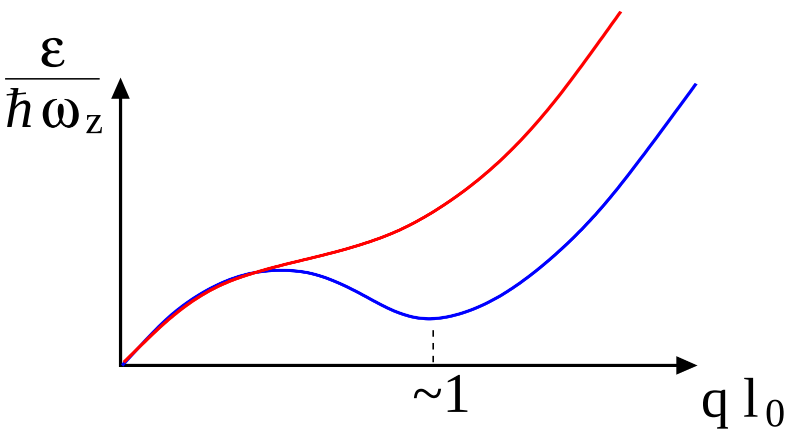
Thomas-Fermi in the z direction

Extreme pancake ($\omega_\rho = 0$)



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$V_d + g$ (short-range) $\quad g > 0$



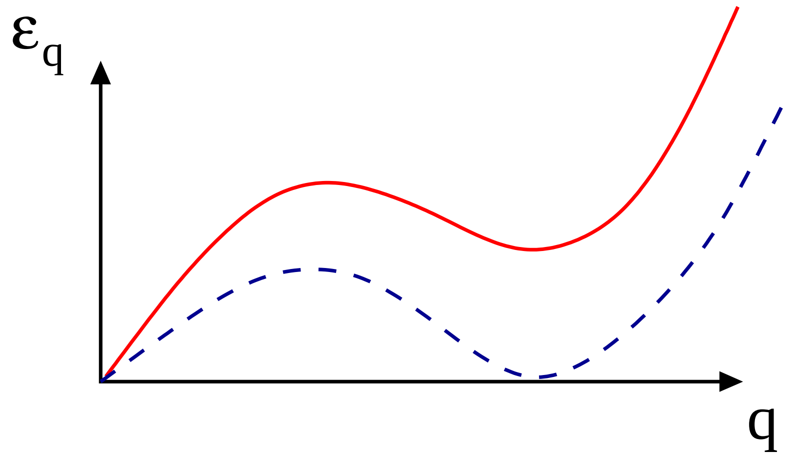
$g/g_d = 1.06 \qquad \mu/\hbar\omega_z = 46$

$g/g_d = 0.94 \qquad \mu/\hbar\omega_z = 53$

The reason for the roton structure

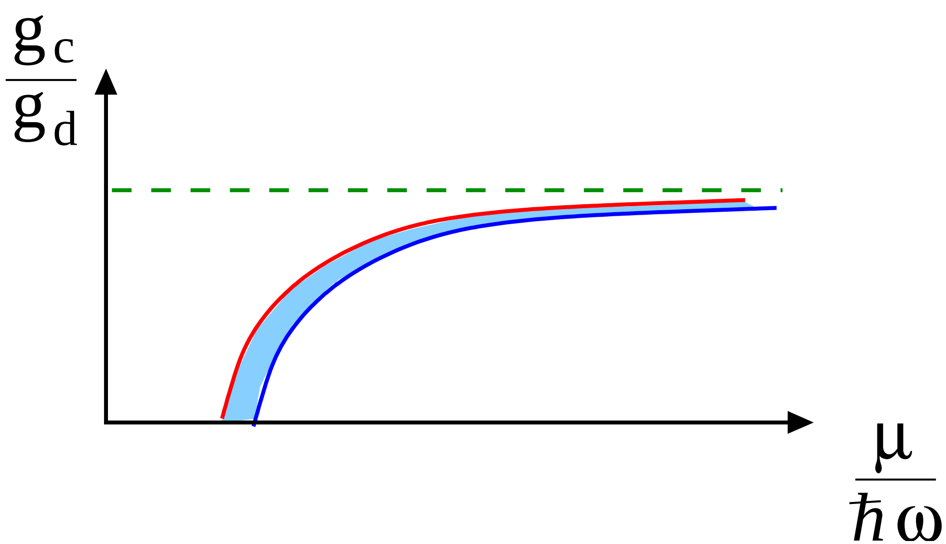
$\Rightarrow$ decrease of the interaction amplitude

Roton-maxon structure



Roton minimum can be but at zero

Instability!



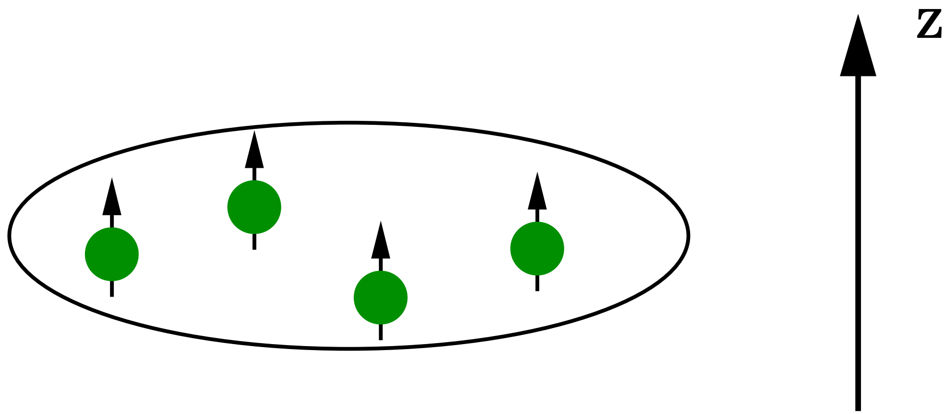
rotonization

instability

(L. Santos et al., 2003)

Problem

Consider a Thomas-Fermi dipolar BEC, with $g > 0$



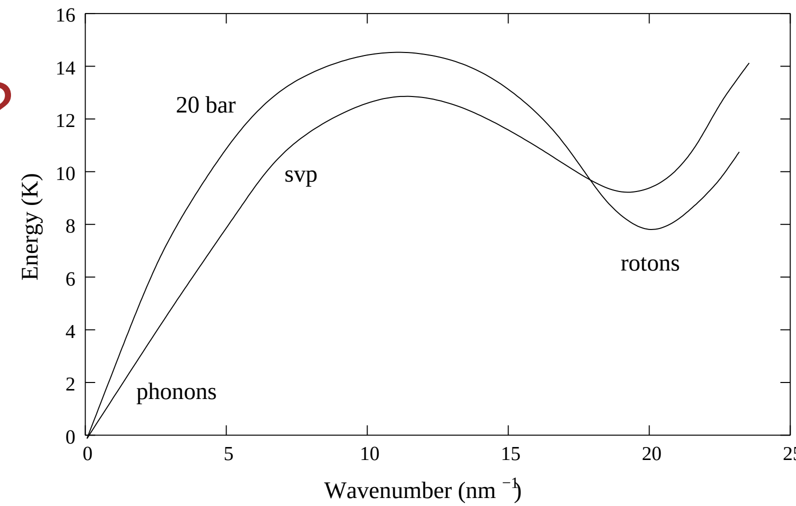
Find the TF density profile and the Bogoliubov excitations.

Obtain the stability diagram.

Ideas from superfluid ^4He

Why the roton-maxon structure?
R. Feynman

$$\epsilon_q = \frac{q^2}{2mS(q,\epsilon)}$$



How to put the roton minimum higher (*) or lower (**)?

What happens? (S.Balibar, P. Nozieres, L. Pitaevskii)

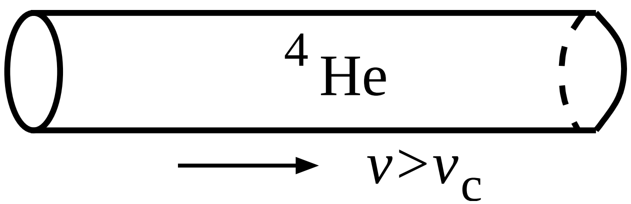
(*) negative pressure (acoustic pulses)
(**) increase the pressure

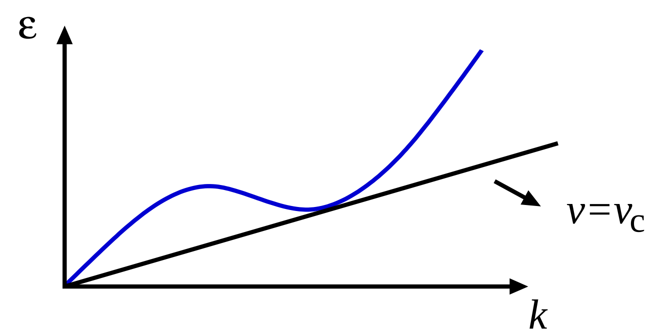
Metastable liquid

(**) $\Rightarrow$ supersolid (density wave), loss of superfluidity, or?

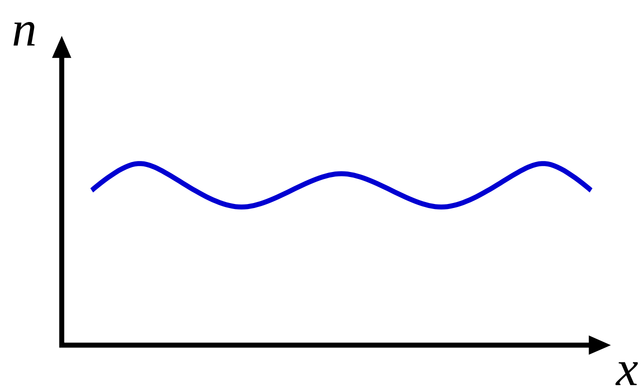
Prehistory

L.P. Pitaevskii(1981)



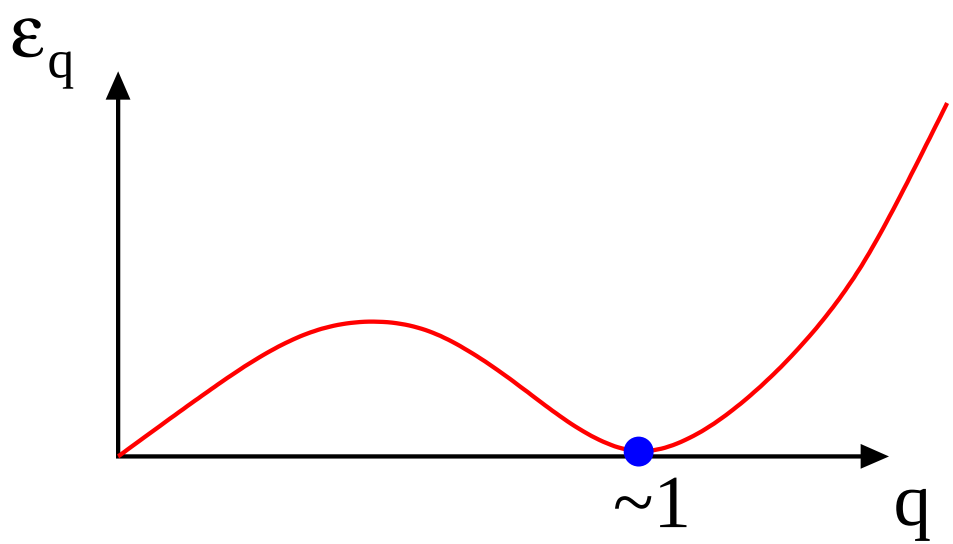


$v > v_c \quad \Rightarrow$ Density wave



Roton Instability in dipolar BEC

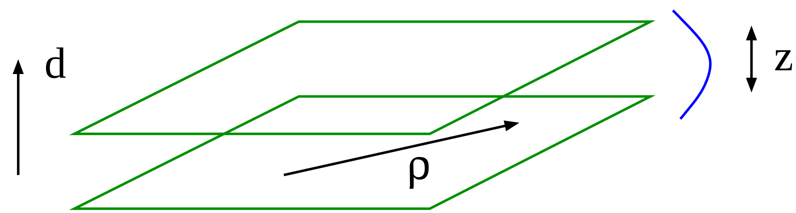
Extreme pancake BEC in the TF regime along z



Supersolid state with a period $q_r^{-1} \sim l_0 \ll L_z$ is collapsing

The main reason is the formation of local collapsing small-size bubbles

Toy 2D model at T=0



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{\frac{-z^2}{2l_0^2}\right\}$$

short range interaction (g) + dipole-dipole

Consider $0 < g \ll \dfrac{4\pi d^2}{3}$. Then, for $ql_0 \ll 1$

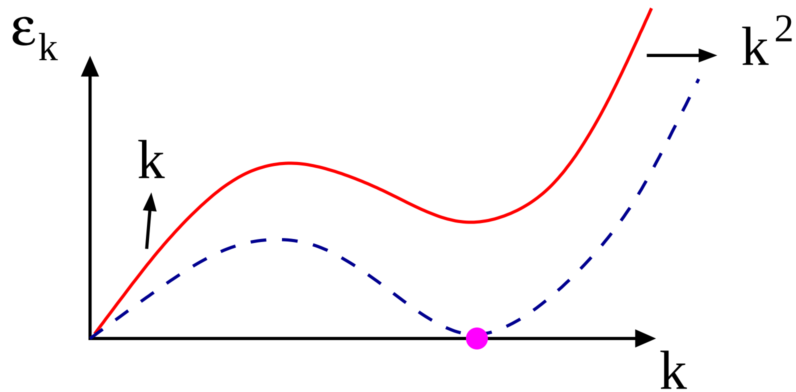
$$V_{\vec{q}\vec{p}} = g(1 - C|\vec{q} - \vec{p}|)$$

where

$$C = \frac{2\pi d^2}{g l_0}$$

Spectrum

$$\hat{H} = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a^\dagger_{\vec{k}} a_{\vec{k}} + \frac{g}{2} \sum_{\vec{k},\vec{q},\vec{p}} (1 - C|\vec{q} - \vec{p}| l_0) a^\dagger_{\vec{k}+\vec{q}} a^\dagger_{\vec{k}-\vec{q}} a_{\vec{k}+\vec{p}} a_{\vec{k}-\vec{p}}$$



$$\epsilon_k^2 = E_k^2 + 2\mu E_k (1 - Ckl)$$ consider $kl \ll 1$

$$k_{r,m} = \left[\frac{3}{2} C \frac{\mu}{\hbar\omega_0} \left\{ 1 \pm \sqrt{1 - \frac{8}{9} \left(\frac{\hbar\omega_0}{\mu} \right)^2 \frac{1}{C^2}} \right\} \right] \frac{1}{l_0}$$

Rotonization

$$\left(\frac{\hbar\omega_0}{\mu}\right)^{1/2} \geq C \geq \frac{\sqrt{8}}{3}\left(\frac{\hbar\omega_0}{\mu}\right)^{1/2}$$

The roton minimum touches zero for

$$C = \left(\frac{\hbar\omega_0}{\mu}\right)^{1/2} \gg 1$$

$$k_{r*} = 2\left(\frac{\mu}{\hbar\omega_0}\right)^{1/2}\frac{1}{l_0} = \left(\frac{m\mu}{\hbar^2}\right)^{1/2} \ll \frac{1}{l_0}$$

(twice the inverse healing length)

Cooper, Komineas

Pedri, Shlyapnikov.

Instability. Supersolid state ?

$$C > \left(\frac{\hbar\omega_0}{\mu}\right)^{1/2} \Rightarrow \text{new ground state}$$

Supersolid Ψ_0

It is iportant that the expected period
of the supersolid state is $\sim l_* \gg l_0$

$$\psi_0 = A_0 + \sum_i A_i \exp\left\{i\vec{k}_i\vec{\rho}\right\}$$

standing wave
triangular lattice
quadratic lattice

Imaginary time evolution $\Rightarrow$ Collapse!

Fluctuations I

Assume we are still in the regime of a uniform BEC

$$C < \left(\frac{\hbar\omega_0}{\mu}\right)^{1/2}$$

Density fluctuations

Quantum fluctuations $\Rightarrow$ important only for the roton energy ϵ_r extremely close to zero

Thermal fluctuations at $T \gg \epsilon_r$

$$\left(\frac{\delta n}{n}\right)^2 = \int \frac{1}{n} \frac{d^2k}{(2\pi)^2} \frac{E_k}{\epsilon_k} \frac{1}{\exp(\epsilon_k/T) - 1} \approx \underbrace{\frac{T}{\epsilon_r}}_{\text{large}} \; \underbrace{\frac{2}{n\xi^2}}_{\text{small}}$$

Fluctuations II

$$\frac{\hbar^2}{m\xi^2} = \mu$$

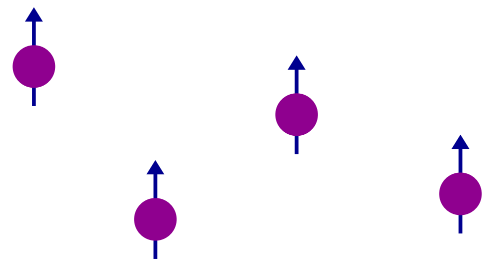
$$\Downarrow$$

$$\left(\frac{\delta n}{n_0}\right)^2 \approx \frac{T}{\varepsilon_r} \underbrace{\frac{mg}{\hbar^2}}_{\text{small parameter for a 2D weakly interacting gas}}$$

The bosons can remain non-condensed even at T much smaller than T_{KT}. Even at $T < \mu$

What kind of state we get?

Dipolar Fermi gas



What does th dipole-dipole interaction do in a Fermi gas?

- Single component gas

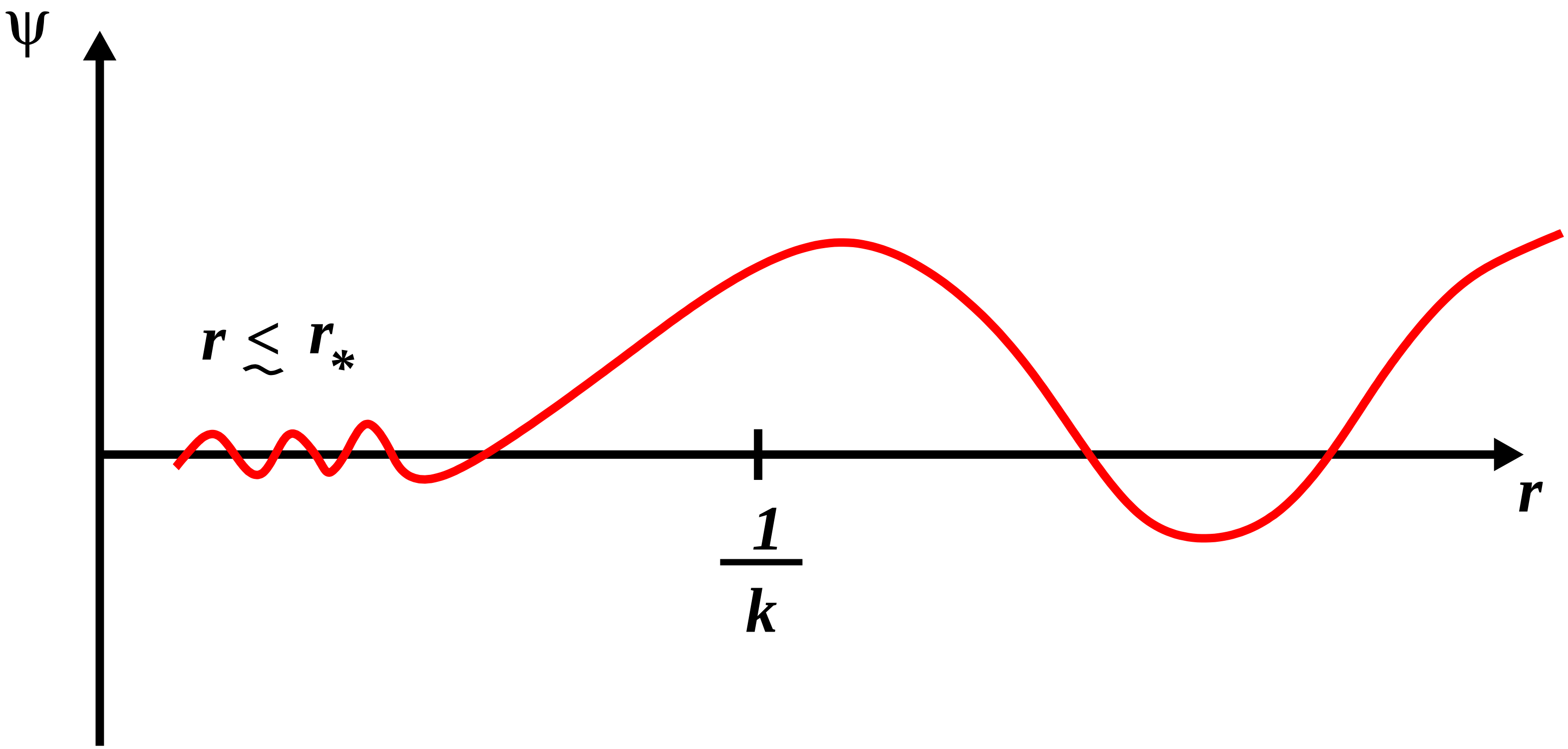
The dipole-dipole scattering amplitude is independent of $|k|$ at any orbital angular momenta alowed by the selection rules.

$$\text{Long-range cotribution} \sim d^2$$

$$\underbrace{\text{Short-range cotribution} \sim k}_{\text{omit}}$$

Universal result for f

Physical picture



Odd-l scattering amplitude

$$f = \frac{1}{2} \int \left(e^{i\vec{k}_i \vec{r}} - e^{-i\vec{k}_i \vec{r}} \right) V_d(\vec{r}) \left(e^{-i\vec{k}_f \vec{r}} - e^{i\vec{k}_f \vec{r}} \right) d^3 r =$$
$$= 4\pi d^2 (\cos^2 \theta_{dq_-} - \cos^2 \theta_{dq_+})$$

$$\vec{q}_\pm = \vec{k}_i - \vec{k}_f$$

$$e^{i\vec{k}\vec{r}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} j_l(kr) i^l Y_{lm}(\hat{r}) Y_{lm}^*(\hat{k})$$

Partial amplitude $\quad f(l_i m_i; l_f m_f)$

$$f(10;10) = -\frac{6d^2}{5} \cos\theta_{dk_i} \cos\theta_{dk_f}$$

Superfluid p-wave pairing

$$nd^2 \ll E_F$$

Analog of $a \to \sim \frac{md^2}{\hbar^2}$ $(r_*!)$

$$\Delta = g\langle\psi\psi\rangle \sim E_F \exp\left(-\frac{1}{\lambda}\right); \ \lambda \sim \frac{1}{k_F r_*};$$

$$\Delta \propto E_F \exp\left\{-\frac{\pi E_F}{12nd^2}\right\}$$

Cooper pairs are superposition

of all odd angular momenta (for $m_l = 0$)

Transition temperature

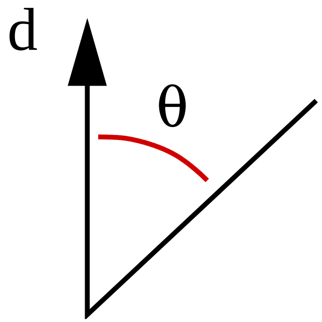
$$nd^2 \ll E_F$$

$$T_c = 1.44 E_F \exp\left\{-\frac{\pi E_F}{12nd^2}\right\}$$

Baranov et.al (2002)

GM correction included

$\Delta \rightarrow$ anisotropic $\propto \sin\left(\frac{\pi}{2}\cos\theta\right)$



- Maximum in the direction of the dipoles.
- Vanishes in the direction perpendicular to the dipoles

Distiguished features

Anisotropic gap $\rightarrow$ gapless excitations

in the direction perpendicular to

the dipoles. Damping rates etc.

Anisotropy $\rightarrow$ different from p-wave

superfluid B and A phases of ^{3}He. In B

Δ is isotropic, and in A it vanishes only at

2 points on the Fermi shpere ($\theta = 0$ and $\theta = \pi$)

Value of T_c

$$d \sim 0.1 \div 1 \ \mathcal{D}$$

$ND_3 \Rightarrow$ $d = 1.5\mathcal{D}$ equivalent to $a = -1450\mathring{A}$

$\frac{T_c}{E_F} \to 0.1$ at $n \to 10^{12}\mathrm{cm}^{-3}$

($T_c \to 100nK$)

HCN $\Rightarrow$ $d \approx 3\mathcal{D}$ equivalent to $a = -7400\mathring{A}$

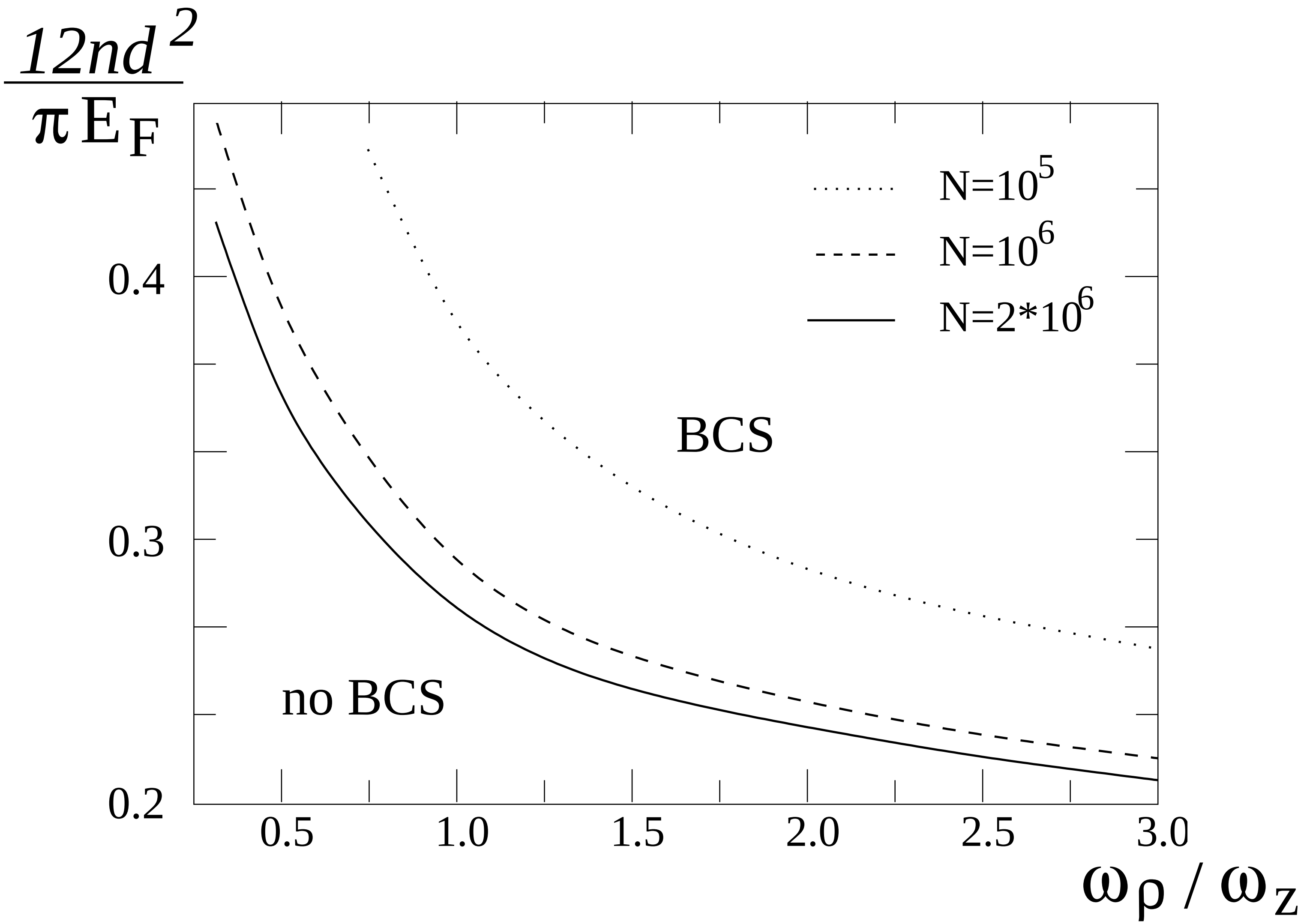
Problem

Consider a two-component dipolar Fermi gas $\uparrow$ and $\downarrow$

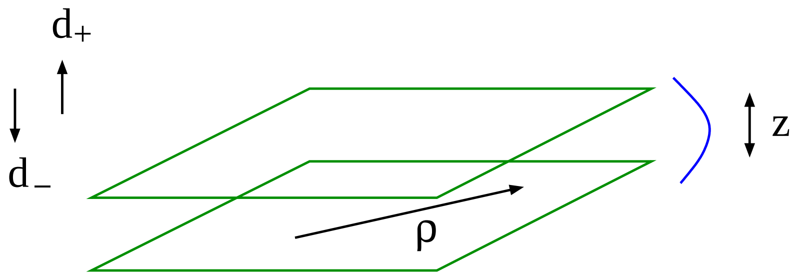
Discuss competition between s-wave and p-wave pairing

Construct the phase diagram

Trapped Fermi gas



Quasi 2D dipolar Fermi gas



$$l_0 = \left(\frac{\hbar}{m\omega_z} \right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{ \frac{-z^2}{2l_0^2} \right\}$$

short range $\tilde{g} < 0$

$$H = \sum_k \frac{\hbar^2 k^2}{2m} \left(a^\dagger_{k\downarrow} a_{k\downarrow} + a^\dagger_{k\uparrow} a_{k\uparrow} \right) +$$
$$+ |g| \sum_{k,p,q} \left(-1 + C|\vec{q} - \vec{p}| \right) a^\dagger_{k+q\uparrow} a^\dagger_{k-q\downarrow} a_{k-p\downarrow} a_{k+p\uparrow}$$

$$C = \frac{2\pi d^2}{|g| l_0}$$

Interesting problem to solve

Interaction at the Fermi surface

$$|g|(-1 + Ck_F l_0)$$

$k_F l_0 < \frac{1}{C} \rightarrow 2\pi d^2 k_F < |g| \rightarrow$ superfluidity

$k_F l_0 > \frac{1}{C} \rightarrow 2\pi d^2 k_F > |g| \rightarrow$ no superfluidity

Quantum transition to a normal state with increasing density