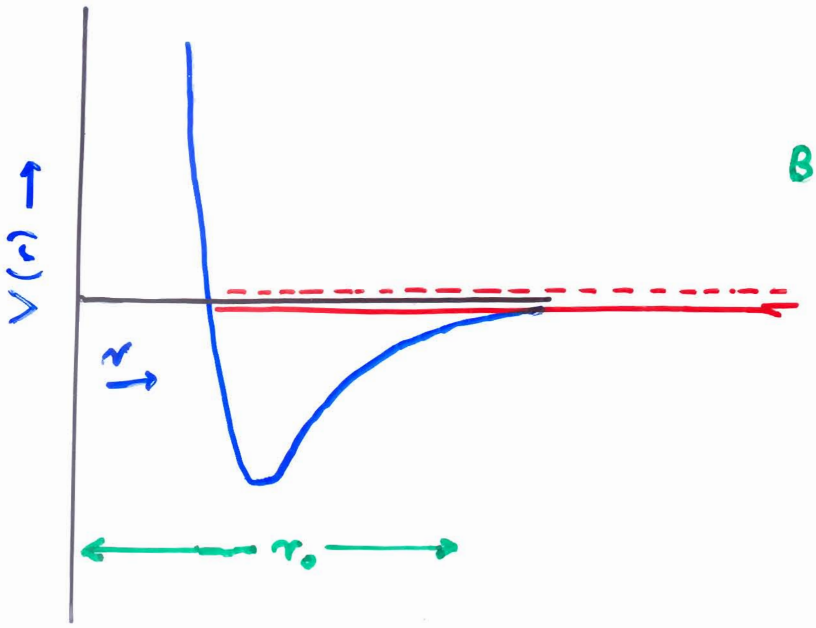
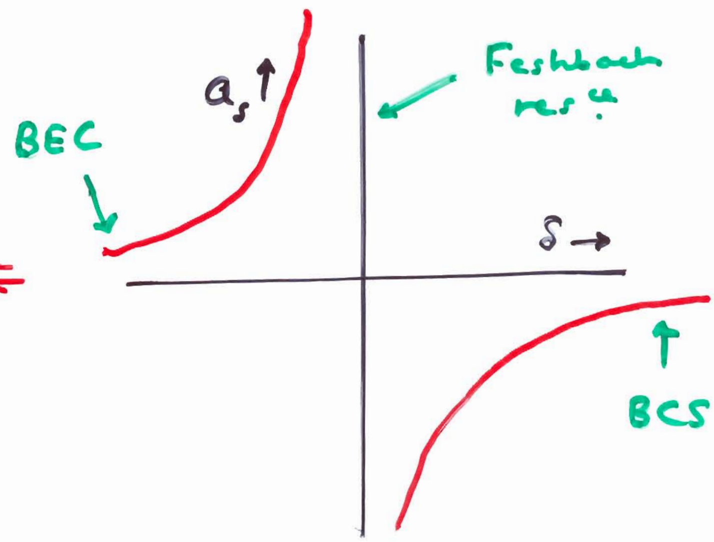
Degenerate ($T \ll T_F$) Fermions with (Partially) Attractive Interaction:





Equal populations of 2 (pseudo) spin states, total no. = N. Df. $k_F \equiv (3\pi^2 N/V)^{1/3}$, $\epsilon_F \equiv \hbar^2 k_F^2/2m$, $T_F \equiv \epsilon_F/k_B$.

"Action" in BEC-BCS Xover takes place in regime $k_F|a_s| \gtrsim 1$. By df., "intrinsic" width δ_c of F.R. is value of δ at which $\hbar^2/ma_s^2 \sim \delta$; hence, if $\epsilon_F \ll \delta_c$ ("broad" resonance, eg ^{6}Li at 822 G) then all action is at $\delta \ll \delta_c \Rightarrow$ can neglect closed channel: effective single-channel problem with $a_s = a_s(\delta)$.

BEC limit

Tightly bound (but still open-channel) diatomic spin singlet molecules:

$$\Psi_{mol}(\underbrace{\mathbf{r}}_{\text{relative}} : \underbrace{\mathbf{R}}_{\text{COM}}) \sim \exp i\mathbf{K}\cdot\mathbf{R}\ \frac{\exp - r/\overbrace{a_s}^{\gg r_0}}{r} \times \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

BEC limit (cont.)

expect molecules to form (nearly) noninteracting Bose gas,

$$T_c \cong 2\pi k_B^{-1} \left(\frac{\overset{N/2V}{n_B}}{\zeta(3/2)}\right)^{2/3} \frac{\hbar^2}{\underset{2m}{m_B}} \cong 0.21\, T_F$$

MB wave function ($T=0$) in terms of fermions:

$$\Psi(r_1 r_2 .. r_N : \sigma_1 \sigma_2 \ldots \sigma_N) \sim \underset{\text{norm}^n}{\mathcal{N}} \cdot \underset{\text{antisymmetrizer}}{\mathcal{A}} \cdot \chi_0(\underset{\sim}{r}_1 - \underset{\sim}{r}_2 : \sigma_1\sigma_2) \cdot \chi_0(\underset{\sim}{r}_3 - \underset{\sim}{r}_4 : \sigma_3\sigma_4) \ldots \chi_0(\underset{\sim}{r}_{N-1} - \underset{\sim}{r}_N : \sigma_{N-1}\sigma_N)$$

$$\chi_0(\underset{\sim}{r}_i - \underset{\sim}{r}_j : \sigma_i\sigma_j) \equiv \frac{1}{\sqrt{2}}(\uparrow_i\downarrow_j - \downarrow_i\uparrow_j)\, \frac{\exp - |\underline{r}_i - \underline{r}_j|/a_s}{|\underline{r}_i - \underline{r}_j|}$$

Why can we neglect effects of antisymmetriz^n ?

In single molecule in vol. V: $\Delta k \sim a_s^{-1}$, 3D DOS in $\underline{k}$-space $\sim V$. $\Rightarrow \langle n_k \rangle \sim a_s^3 V$. For N molecules in volume V,

$$\langle n_k \rangle \sim (N/V) a_s^3 \equiv n a_s^3$$

$\Rightarrow$ for $n a_s^3 \ll 1$ (i.e. $k_F a_s \ll 1$) $\langle n_k \rangle \ll 1 \Rightarrow$ effect of antisymmetriz^n between different molecules negligible.

Single-boson DM has single eigenvalue $= N$

Single-fermion " " all eigenvalues ≤ 1 (Pauli)

What about 2-fermion DM ?

Df. of 2-fermion density matrix:

$$\rho_2(\underset{\sim}{r}_1\sigma_1, \underset{\sim}{r}_2\sigma_2 : \underset{\sim}{r}'_1\sigma'_1, \underset{\sim}{r}'_2\sigma'_2 : t) \equiv \sum_{\sigma_3 \dots \sigma_N} \int d\underline{r}_3 \dots \int d\underline{r}_N \cdot N(N-1)$$

$$\Psi^*_N(\underset{\sim}{r}_1\sigma_1, \underset{\sim}{r}_2\sigma_2, \underset{\sim}{r}_3\sigma_3, \dots \underset{\sim}{r}_N\sigma_N : t)\,\Psi_N(\underset{\sim}{r}'_1\sigma'_1, \underset{\sim}{r}'_2\sigma'_2, \underset{\sim}{r}_3\sigma_3 \dots \underset{\sim}{r}_N\sigma_N : t)$$

$\cong$ "behavior of pair of particles averaged over remaining $N-2$" (mixture: $\rho_2 = \sum_s p_s \rho_2^{(s)}$)

ρ_2 is Hermitian $\Rightarrow$ can be diagonalized:

$$\rho_2(\underset{\sim}{r}_1\sigma_1, \underset{\sim}{r}_2\sigma_2 : \underset{\sim}{r}'_1\sigma'_1, \underset{\sim}{r}'_2\sigma'_2 : t) = \sum_i n_i(t)\,\chi^*_i(\underset{\sim}{r}_1\sigma_1, \underset{\sim}{r}_2\sigma_2 : t) \cdot$$

$$\chi_i(\underset{\sim}{r}'_1\sigma'_1, \underset{\sim}{r}'_2\sigma'_2 : t)$$

In noninteracting Fermi sea ($\Psi_N = \prod_{\substack{k<k_F \\ \sigma}} a^+_{k\sigma} |vac\rangle$)

all eigenvalues of ρ_2 are 0 or 1: for $n_i = 1$, can take (e.g.)

$$\chi_{singlet}(\underline{r}_1\sigma_1 \underline{r}_2\sigma_2) = \frac{1}{\sqrt{2}V} \exp i\underline{K}\cdot\underline{R} \cos \underline{q}\cdot\underline{r} \cdot \frac{1}{\sqrt{2}}(\delta_{\sigma_1+}\delta_{\sigma_2-} - \delta_{\sigma_1-}\delta_{\sigma_2+})$$

$$\chi_{triplet}(\underline{r}_1\sigma_1 \underline{r}_2\sigma_2) = \frac{1}{\sqrt{2}}V \exp i\underline{K}\cdot\underline{R} \sin \underline{q}\cdot\underline{r} \begin{cases} \delta_{\sigma_1+}\delta_{\sigma_2+} \\ \dots \\ \dots \end{cases}$$

$$\left(|(\underline{K}+\underline{q})/2|, |(\underline{K}-\underline{q})/2| < k_F \right)$$

But, in BEC limit:

ρ_2 has $O(N^2)$ eigenvalues $O(1)$,

" one eigenvalue $= N$

Eigenfunction associated with this special eigenvalue is just $\chi_0(\underline{r}_1 - \underline{r}_2 : \sigma_1\sigma_2)$, the "molecular" wave function!

Definition of "condensation" in Fermi system (in general case):

ρ_2 has one and only one eigenvalue $O(N)$

In this case, if relevant value of i is labelled 0,

$N_0(t) \equiv$ "condensate number"

$\chi_0(\underset{\sim}{r}_1\sigma_1, \underset{\sim}{r}_2\sigma_2 : t) \equiv$ "condensate wave function"

(in BCS limit, conventional notation is

$\chi_0(\underset{\sim}{r}_1\sigma_1 \underset{\sim}{r}_2\sigma_2 : t) \Rightarrow F(\underset{\sim}{r}\sigma, \underset{\sim}{r}'\sigma' : t) \equiv$ "Cooper-pair wave function" : GL $\Psi(\underline{R}t) = F(r\uparrow, r'\downarrow : t)|_{\underline{r}=\underline{r}'=\underline{R}}$
$= |\Psi(\underline{R}t)| \exp i\varphi(\underline{R}t)$, $\underline{v}_s = \hbar/2m \nabla_R \varphi$)

Digression: Why only One Macroscopic Eigenvalue?

Partial answer:

consider spin-1 condensate (eg ^{3}He-A). Crudely, can have as extremes:

$$\Psi_{Fock} = (\uparrow\uparrow)^{N/4}(\downarrow\downarrow)^{N/4} \qquad S_z^2 = 0, \langle\Delta\varphi\rangle = 0$$

$$\Psi_{CP}^{(\Delta\varphi)} = ((\uparrow\uparrow) + e^{i\Delta\varphi}(\downarrow\downarrow))^{N/2} \qquad S_z^2 \sim N, \ \langle\Delta\varphi\rangle \neq 0$$

$$(\Psi_{Fock} \sim \int d(\Delta\varphi) \exp -i\Delta\varphi \, \Psi_{CP}(\Delta\varphi)$$

$\Rightarrow S_z, \Delta\varphi$ are conjugate variables)

$$E = \frac{S_z^2}{2\chi} - g_D \cos\Delta\varphi \qquad \text{but: } \chi \sim N, \ g_D \sim N$$

$\Rightarrow$ in thermodynamic limit ($N \to \infty$)

GP state always wins!

Dependence of 2-Particle Quantities on Eigenfunctions and Eigenvalues of ρ_2:

Quite generally, any quantity of the form

$$\langle V \rangle \equiv \langle \tfrac{1}{2} \sum_{ij} V(r_i - r_j, \sigma_i \sigma_j) \rangle$$

(e.g. (true) potential energy)

can be written in form

$$\langle V \rangle = \sum_i n_i \int d\underset{\sim}{R}\; V(\underset{\sim}{r}) |\chi_i(\underset{\sim}{r}, \underset{\sim}{R})|^2$$

(COM: $\underset{\sim}{R}$; rel^{ve}: $\underset{\sim}{r}$)

(Plausible) thesis:

In limit $r_0 \ll a_s, n^{-1/3}, \ldots$ the dependence of all $\chi_i(\underline{r}, \underline{R})$ on $\underset{\sim}{r}$ for $|\underset{\sim}{r}| \ll a_s, n^{-1/3}, \ldots$ is just that of the 2-body problem (for the given l)

Since in almost all cases of interest the range of $V(r)$ is $\lesssim r_0$, it follows that

$$\langle V \rangle = \text{const} \sum_i^{(l=0)} n_i p_i$$

where p_i is simply the normaliz^n of χ_i (i.e. the value of $|\chi_i(\underline{r})|^2$ for $r_0 \ll r \ll a_s, n^{-1/3}$.)

What is p_i?

For unbound eigenfunctions, $p_i \sim V^{-1}$! But there are $O(N^2)$ such, so total contr^n to $\langle V \rangle$ is $O(N)$

("Hartree" term)

For a bound state with radius $\sim a$ (not nec^y $\sim a_s$)

$p_i \sim a^{-1}$

e.g. in BEC limit, $p_0 \sim a_s^{-1}$, $N_0 \sim N$ so $\langle V \rangle \sim N a^{-1}$

In general, $\sum_{i \neq 0}^{bs} n_i p_i$ may be comparable to $N_0 p_0$!

THE "NAIVE" (MEAN-FIELD, GENERALIZED BCS) ANSATZ ($T=0$)

In extreme BEC limit, v. plausible that GS is approx.

$$\Psi_N \sim \mathcal{N} \cdot \mathcal{A} \prod_i \chi_0(\underset{\sim}{r}_i - \underset{\sim}{r}_{i+1}, \sigma_i, \sigma_{i+1})$$

with χ_0 the 2-body bound state wf. In 2^{nd}-quantized language this is, up to normaliz$^{\text{n}}$:

$$\Psi_N = \Big(\sum_k c_k a^+_{k\uparrow} a^+_{-k\downarrow}\Big)^{N/2} |\text{vac}\rangle \qquad (*)$$

$c_k \equiv$ Fourier component of $\chi_0(r)$.

The naive ansatz is simply that $(*)$ holds, but with a variationally determined c_k. Note that the ideal Fermi gas is a special case of $(*)$, with $c_k = \Theta(k - k_F)$.

In BCS (particle-nonconserving) form this is

$$\Psi_N \sim \exp \sum_k c_k a^+_{k\uparrow} a^+_{-k\downarrow} |\text{vac}\rangle = \prod_k (u_k + v_k a^+_{k\uparrow} a^+_{-k\downarrow}) |\text{vac}\rangle$$

$$\text{with } v_k/u_k = c_k, \quad |u_k|^2 + |v_k|^2 = 1.$$

However, let's try to do things in a fixed-N representation: write

$$a^+_{k\uparrow} a^+_{-k\downarrow} \equiv b^+_k$$

Then, the KE terms are of the form $\sum_{k\sigma} \epsilon_k n_{k\sigma}$ (+ const) $\quad \epsilon_k = \frac{\hbar^2 k^2}{2m} - \mu$

and the potential terms are

(a) Hartree, $\frac{1}{2} V(0) N^2$

(b) Fock, $-\sum_{kk'} V_{kk'} \langle n_{k\sigma} n_{k'\sigma} \rangle$

(c) pairing, $\sum_k V_{kk'} \langle b^+_k b_{k'} \rangle$

MINIMIZATION OF ENERGY IN NAIVE ANSATZ

(norm'd) $\Psi_N = \prod_k (1+|c_k|^2)^{-1/2} \left(\sum_k c_k b_k^\dagger\right)^N |vac\rangle$ $\quad (b_k^\dagger \equiv a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow})$

can rewrite in forms

$$\Psi_N \equiv (1+|c_k|^2)^{-1/2}\left(\Psi'_N + c_k b_k^\dagger \Psi'_{N-1}\right)$$

$$\equiv (1+|c_k|^2)^{-1/2}(1+|c_{k'}|^2)^{-1/2}\left(\Psi''_N + (c_k b_k^\dagger + c_{k'} b_{k'}^\dagger)\Psi''_{N-1} + c_k c_{k'} b_k^\dagger b_{k'}^\dagger \Psi''_{N-2}\right)$$

($\Psi'_M \equiv 2M$-particle state missing k, $\Psi''_M = 2M$-particle state missing k and k')

From 1st eqn.,

$$\langle n_{k\sigma}\rangle = \frac{|c_k|^2}{1+|c_k|^2}$$

From 2nd eqn.,

$$\langle b_k b_{k'}^\dagger\rangle = \frac{c_k c_{k'}^*}{(1+|c_k|^2)(1+|c_{k'}|^2)} \equiv F_k F_{k'}^*$$

$$F_k \equiv \frac{c_k}{1+|c_k|^2} \Rightarrow \langle n_{k\sigma}\rangle = \tfrac{1}{2}\left(1 \pm \sqrt{1-4|F_k|^2}\right)$$

(from energy considerations, sgn $\surd$ = $-$ sgn ϵ_k)

Hence in approx^n of neglect of Fock term,

$$E = \sum_{k\sigma} \epsilon_k \langle n_{k\sigma}\rangle + \sum_{kk'} V_{kk'} \langle b_k b_{k'}^\dagger\rangle$$

$$= \sum_k |\epsilon_k| \cdot \left(1 - \sqrt{1-4|F_k|^2}\right) + \sum_{kk'} V_{kk'} F_k F_{k'}^*$$

$$\left(\epsilon_k \equiv \frac{\hbar^2 k^2}{2m} - \mu\right)$$

Groundstate Energy as Function of F_k in Naive Ansatz (Recap):

$$E = \sum_k |\epsilon_k| \left(1 - \sqrt{1 - 4|F_k|^2}\right) + \sum_{kk'} V_{kk'} F_k F_{k'}^*$$

note: in limit that all $F_k \ll 1$,

$$E = \sum_k 2|\epsilon_k||F_k|^2 + \sum_{kk'} V_{kk'} F_k F_{k'}^*$$

$\Rightarrow$ 2-particle problem ($|\epsilon_k| \to \epsilon_k \equiv \frac{\hbar^2 k^2}{2m} - \mu$, $\mu \to E$, $F_k \to \psi_k$)

(next term: $\frac{+}{-}$ const. $|F_k|^4$)

Minimize wrt F_k^*:

$$\frac{2|\epsilon_k| F_k}{\sqrt{1 - 4|F_k|^2}} + \sum_{k'} V_{kk'} F_{k'} = 0 \qquad (*)$$

Introduce:

$$E_k \equiv \frac{|\epsilon_k|}{\sqrt{1 - 4|F_k|^2_{eq}}}.$$

$$\Delta_k \equiv (F_k/|F_k|)_{eq} (E_k^2 - \epsilon_k^2)^{1/2}$$

(*) becomes

$$\Delta_k = -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \qquad \text{BCS gap eqn.}$$

or:

$$F_k = -\frac{1}{2E_k} \sum_{k'} V_{kk'} F_{k'}$$

cf zero-en. SE:

$$\psi_k = -\frac{1}{2\xi_k} \sum_{k'} V_{kk'} \psi_{k'} \qquad (\xi_k \equiv \hbar^2 k^2/2m)$$

$\Rightarrow$ in region where $E_k \sim \xi_k$ (typically, for $k \sim r_0^{-1} \gg a_s^{-1}, k_F$)

F_k is prop. to ψ_k (2-body wf)

(special case of general conjecture)

CRUCIAL QUALITATIVE CONCLUSION:

at distances $\ll a_s, k_F^{-1}$ (but possibly $\sim r_0$), form of Cooper-pair (condensate) wave function is identical to that of zero-energy 2-body w.f.

$\Rightarrow$ possibility of renormalizn procedure for gap eqn:

$$\sum_k \left(\xi_k^{-1} - E_k^{-1}\right) = m/2\pi\hbar^2 a_s$$

$\xi_k = \frac{\hbar^2 k^2}{2m}$, $E_k = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + |\Delta|^2}$

$(\Delta \equiv \lim_{k\to 0} \Delta_k)$

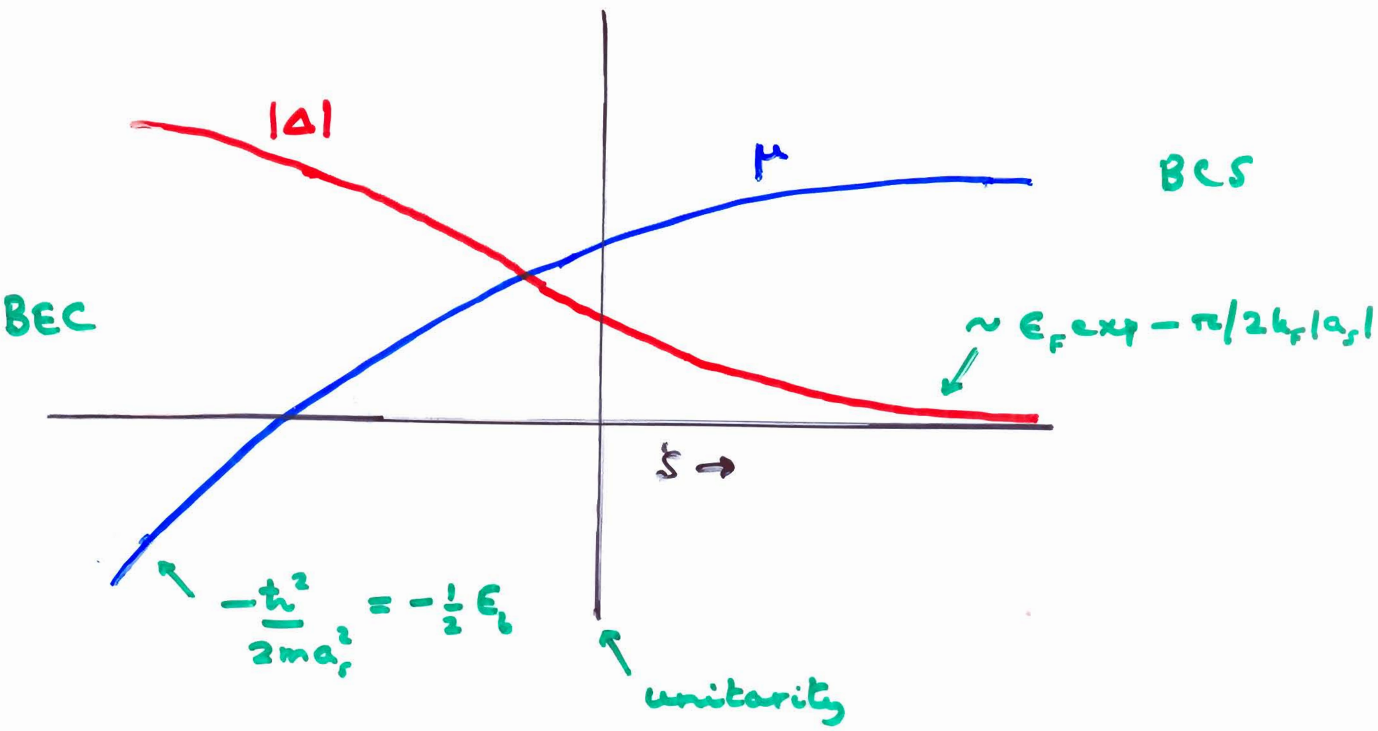
one eqn. for 2 unknowns (μ, Δ): need also number conservation eqn.

$$\sum_k \left(1 - \frac{\xi_k - \mu}{E_k}\right) = k_F^3/3\pi^2 \quad (= N)$$

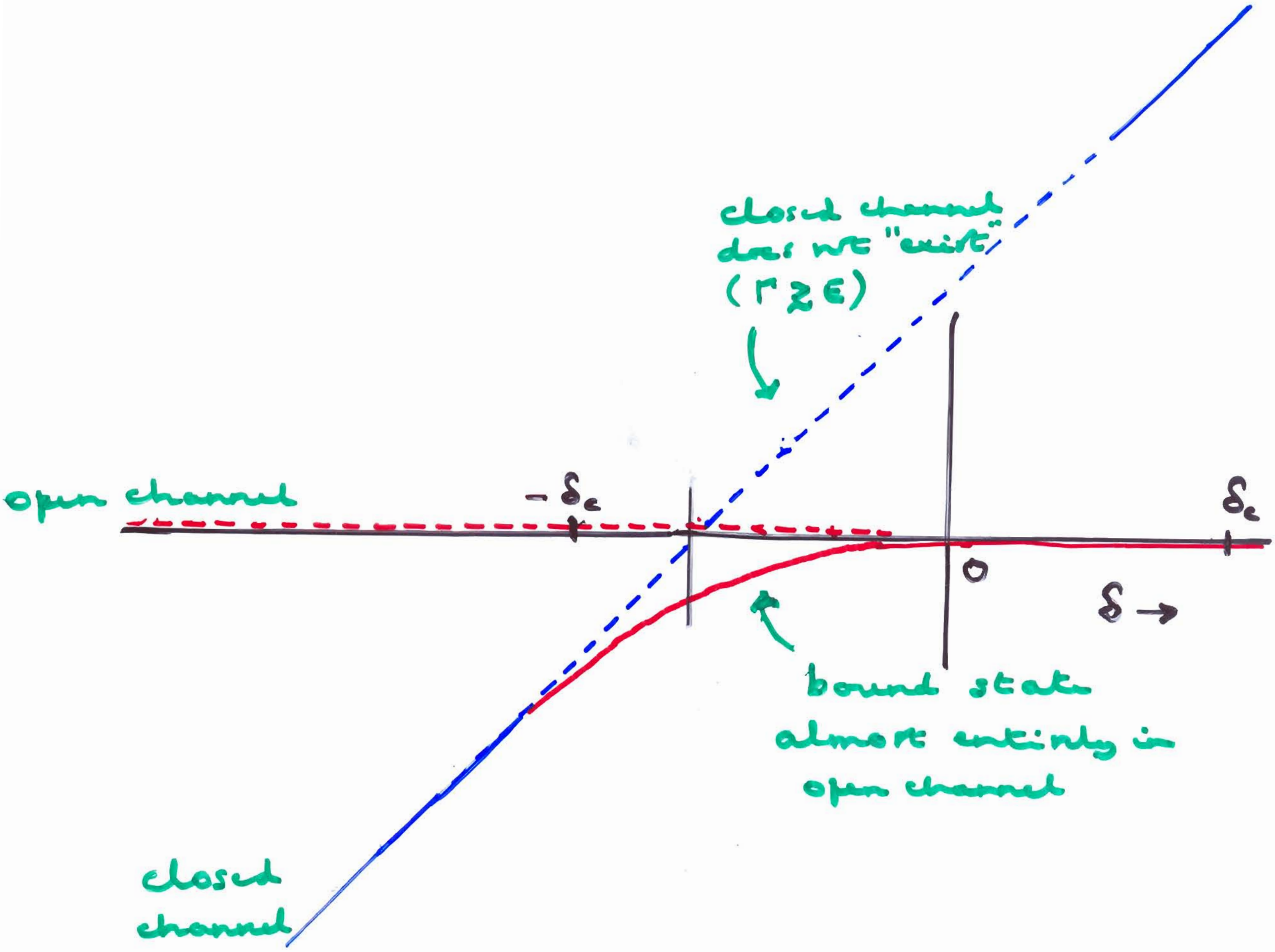
Evidently

$$\Delta = \epsilon_F f(\zeta)$$
$$\mu = \epsilon_F g(\zeta)$$

$\zeta \equiv -1/k_F a_s(\delta)$



Feshback Resonances: Broad vs Narrow



The "intrinsic width" of the resonance is given by

$$\delta_c = \frac{m}{\hbar^2} \lim_{\delta \to 0} \left(\frac{d a_s^{-1}}{d\delta} \right)^2$$

(nb. no explicit reference to a_{bg}!)

If $\epsilon_F \ll \delta_c$, all the interesting many-body action ($k_F|a_s| \gtrsim 1$) takes place in region where 2-body wf is overwhelmingly in open channel.

If $\epsilon_F \gtrsim \delta_c$ ("narrow" ("saturated") resonance) must take closed channel into account explicitly.

Two different cases:

	O	C
"2-channel"	a, b	c, d
"1.5-channel"	a, b	c, a

[nb: dist^{n}: unimportant in 2-particle problem]

A. "2-channel" case (qualitatively like ^3He-A)

obvious generalization of naive ansatz is (in BCS formalism)

$$\Psi_N \sim \prod_k (u_k^o + v_k^o a^+_{ka} a^+_{-kb}) \cdot (u_k^c + v_k^c a^+_{kc} a^+_{-kd}) |vac\rangle$$

$\Rightarrow$ coupled gap eqns. for Δ_o, Δ_c + single no. consn. eqn.,

$$\sum_k (n_k^o + n_k^c) = N.$$

Provided "typical" k of C channel (usu. $\sim r_o^{-1}$) is $\gg k_F$ (usual "diluteness" condn. $nr_o^3 \ll 1$), then $F(r)$ in both closed channel and $r \lesssim r_o$ segment of open channel is identical to 2-body wf $\psi_o, \psi_c(r)$ $\rightarrow a_s(\delta)$ still given by 2-body formula. So, main effect comes from "sidelining" of particles by C channel $\Rightarrow$ effective reduction of $k_F(\delta)$.

B. "1.5-channel" case (somewhat like ^3He-B)

Now must write generalizn. of naive ansatz in form

$$\Psi_N \sim \prod_k (u_k + v_k^o a^+_{ka} a^+_{-kb} + v_k^c a^+_{kc} a^+_{-ka}) |vac\rangle$$

$\Rightarrow$ generalizn. of gap eqn. (PE has "obvious" terms in $F_k^o F_{k'}^{o*}$, $F_k^c F_{k'}^{c*}$, $(F_k^o F_{k'}^{*c} + c.c.)$, but KE is of form

$$\langle T \rangle = \sum_k |\epsilon_k| \sqrt{1 - 4(|F_k^o|^2 + |F_k^c|^2)}$$

When does this matter? Only if $|F_k^c|^2 \not\ll 1$, which only happens when $nr_o^3 \not\ll 1$. (assuming CC state not anomalously close to threshold)

2. Alternative Forms of "BCS" Ansatz

1. Original BCS (particle-nonconserving)

$$\Psi_{(N)} = \prod_k (u_k + v_k a^+_{k\uparrow} a^+_{-k\downarrow})|vac\rangle$$

$$|u_k|^2 + |v_k|^2 = 1.$$

$$\langle F_k \rangle = u_k v_k^*$$

superposition of different N → some ambiguities in calculating single-particle properties.

2. Particle-conserving on vacuum:

$$\Psi_N = \mathcal{N} \cdot \left(\sum_k c_k a^+_{k\uparrow} a^+_{-k\downarrow}\right)^{N/2} |vac\rangle$$

$$c_k = v_k/u_k.$$

$$F_k = \frac{c_k}{1+|c_k|^2} \qquad (\equiv u_k v_k^*)$$

3. Particle-conserving on normal Fermi sea:

1st shot: $|FS\rangle = \left(\sum_{k<k_F} a^+_{k\uparrow} a^+_{-k\downarrow}\right)^{N/2} |vac\rangle$

Try:

$$\Psi_N = \left(\sum_{k>k_F} \tilde{c}_k a^+_{k\uparrow} a^+_{-k\downarrow}\right)^{N_+} \left(\sum_{k<k_F} \tilde{d}_k a_{-k\downarrow} a_{k\uparrow}\right)^{N_-} |FS\rangle$$

$$\equiv \Psi_N(N_+)$$

where in extreme BCS limit

$N_+ = N_-$. (so N unchanged from FS).

for normalization,

$$N_+ = N_- = \sum_{k>k_F} \frac{\tilde{c}_k}{1+|\tilde{c}_k|^2}$$

Actually, this does not allow scattering of pairs from above Fermi surface to below, so must modify it somewhat:

$$\Psi_N = \sum_{N_+} c(N_+)\Psi(N_+).$$

$c(N_+)$ slowly varying function of N_+.

Critical question: in

$$\Psi(N_+) \equiv \sum_{k>k_F} (\tilde{c}_k a^+_{k\uparrow} a^+_{-k\downarrow})^{N_+} (\tilde{d}_k a_{-k\downarrow} a_{k\uparrow})^{N_-} |FS\rangle$$

What is relation of $\tilde{d}_k$ & $\tilde{c}_k$?

Principle: must obtain same value of F_k as in methods (1) and (2)

$\Rightarrow$ $\tilde{c}_k$ is "original" c_k of method 2 ($\equiv v_k/u_k$).

but

$$\tilde{d}_k = c_k^{-1}$$

This has essentially no consequences for s-wave pairing.

But: what about $\underbrace{\text{anisotropic}}_{\text{by method (2)}}$ pairing (eg d-wave)?

If we calculate $\langle L_z \rangle$ for (eg) $c_k = |c_k| \exp 2i\varphi_k$.

we get exactly $N\hbar$ (ie $2\hbar$ per pair)

If we calculate it using method (3), we get in BCS limit exact cancellation of particle + hole contrn:

$$\langle L_z \rangle = 2\hbar(N_+ - N_-) = 0 \;!$$

(? : calcn may be meaningless in absence of specificn of geometry + boundary conditions).

3. UNIVERSALITY OF 2-BODY QUANTITIES IN BEC-BCS XOVER

RECAP: Various measurable quantities (e.g. total potential energy, rf spectroscopy sum rule, weight of closed-channel component....) can be related to the two-particle density matrix and hence to its eigenvalues and eigenfunctions:

$$\langle Y\rangle = \int d\underset{\sim}{R} \int d\underset{\sim}{r} \sum_{\sigma_1\sigma_2} Y(\underset{\sim}{r},\sigma_1,\sigma_2)\left\{\sum_i^{(l=0)} n_i |\chi_i(r,R:\sigma_1\sigma_2)|^2\right\}$$

typical range $\sim r_0$ eigenvalue eigenfunction

Thesis (plausible, not rigorously demonstrated):

In limit $r \ll a_s$, k_F^{-1} etc. (but possibly $r \sim r_0$)

all eigenfunctions $\chi_i(r,\sigma_1\sigma_2)$ have same dependence on r, σ_1, σ_2 as 2-particle zero-energy scattering state.

If this is correct, we can write normaliz^n of χ_i

$$\langle Y\rangle = \Phi_Y \left(\sum_i^{(l=0)} n_i p_i\right) \equiv \Phi_Y\, A(k_F a_s, T\ldots)$$

$$\Phi_Y \equiv \int d\underset{\sim}{R} \int d\underset{\sim}{r}\, |\psi(\underset{\sim}{r},\sigma_1\sigma_2)|^2\, Y(\underset{\sim}{r},\sigma_1,\sigma_2)$$

= calculable from 2-particle problem.

Thus, predict:

ALL (short-range) 2-body quantities should correspond to same value of $A(k_F a_s, T\ldots)$

should be v. useful check on mutual consistency of various expts!