Slide number zero to check beamer !

(You still have enough time to get a coffee outside)

FQHE states in bosonic and fermionic systems

A cold atom perspective

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<u>Today is a bit special for atomic physicists :</u>

NO Lasers

NO MOT, TOF whatever

NO Evaporative cooling ...

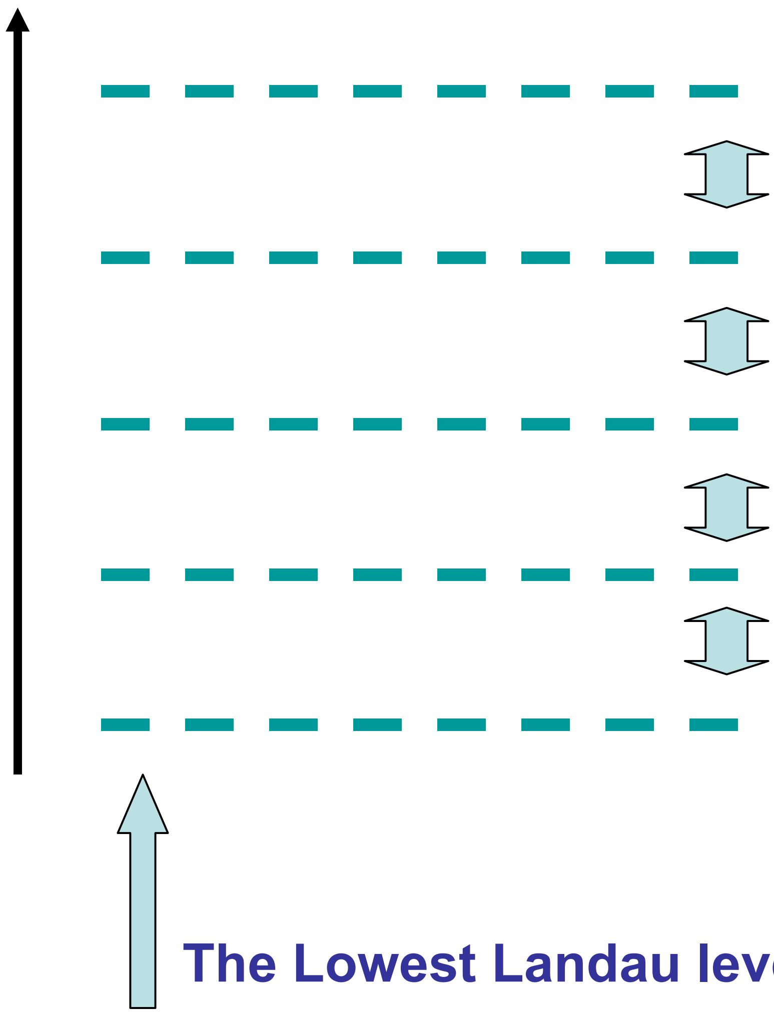
Don't even think to ask about experiments !

Outline :

- Landau levels in two dimensions
- Electron systems : many FQHE liquids, open questions
- General strategy for FQHE states
- Composite fermions for electrons and bosons
- CFT approach for Laughlin state
- Pfaffian state
- Parafermions
-

2D Landau levels

- **Charged particles in a plane and a perpendicular field**



$$\boxed{\nu = \frac{nh}{qB}}$$

$$\hbar\omega_c = \frac{eB\hbar}{m}$$

The Lowest Landau level

·One-body eigenstates : $z^m \, \mathrm{e}^{-|z|^2/4\ell^2}$

Physics in the rotating frame

$$\mathcal{H} \rightarrow \mathcal{H} - \omega L_z$$

$$\mathcal{H} = \sum_{i=1}^{N} \frac{1}{2m}(\mathbf{p}_i - m\omega\hat{\mathbf{z}} \times \mathbf{r}_i)^2 + \frac{1}{2}m(\omega_0^2 - \omega^2)(x_i^2 + y_i^2)$$
$$+ \frac{1}{2}m\omega_z^2 z_i^2 + \sum_{i<j}^{N} V(\mathbf{r}_i - \mathbf{r}_j).$$

$\mathbf{B} = (2m\omega/e)\hat{\mathbf{z}}$	$\ell = \sqrt{\hbar/(2m\omega)}$	$\omega_c = 2\omega$

Bosons in the LLL: may have filling factor higher than one

ultracold bosons have s-wave scattering contact interaction

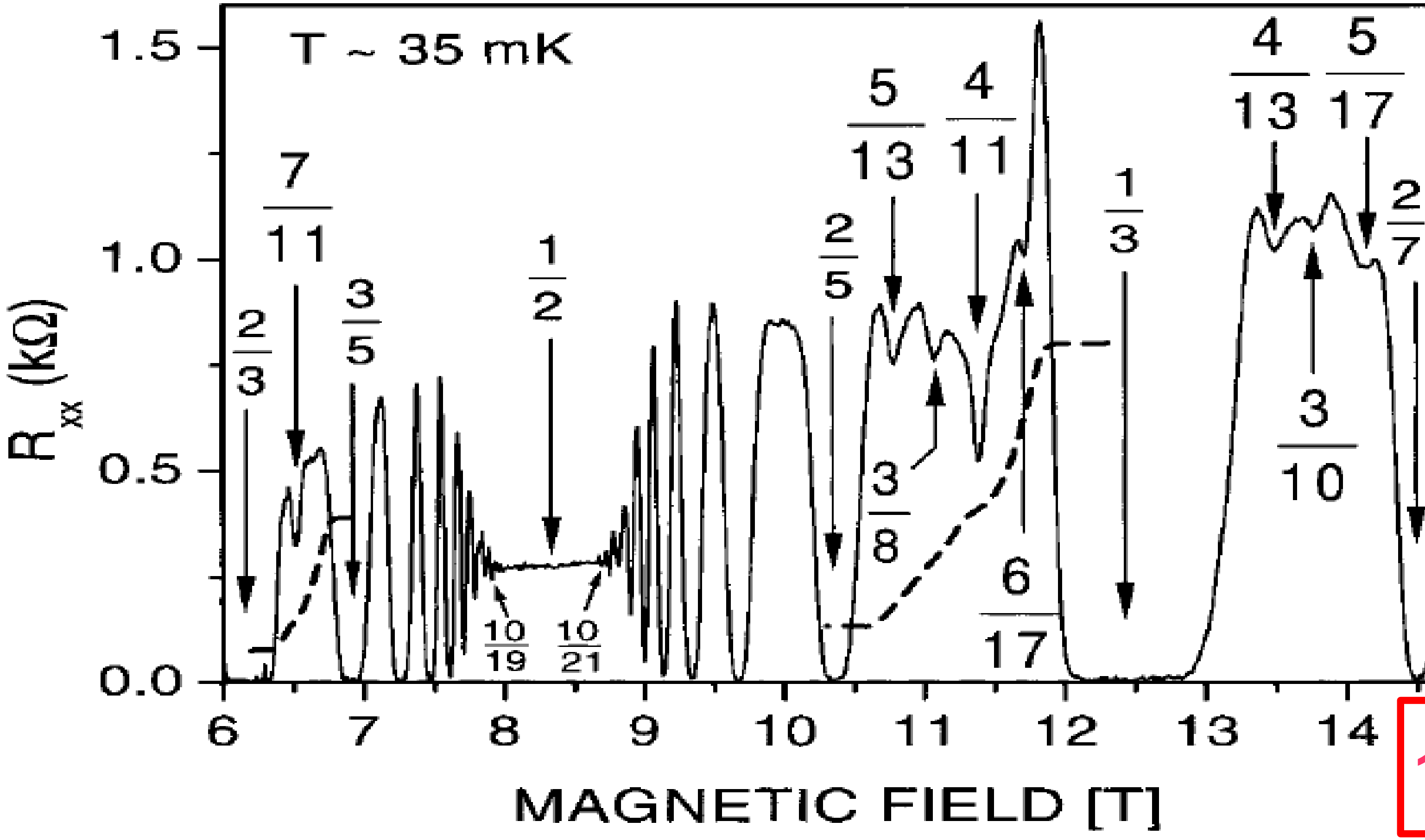
Fermions for $\nu = n + \nu*$ may be considered being at $\nu*$
but with renormalized interactions by the background of filled
n lower Landau levels

·A generic LLL state is of the form :

$$f(z_1, z_2, \ldots, z_N)\ \mathrm{e}^{-\sum_i |z_i|^2/4\ell^2}$$

For electrons we know experimentally that there are many
incompressible liquid states with unusual properties

fractional quantum Hall states in 2D electron gases

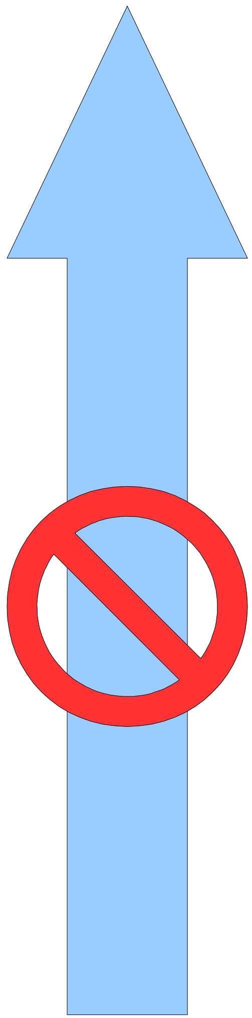


1/ν

Pan et al, PRL90, 016801 (2003)

Effective theories

·Chern-Simons Abelian theories “K-matrix”
·Chern-Simons Non-Abelian theories



In addition this is filling factor dependent !!!!

Electrons + Coulomb
N-body problem

·Microscopic wavefunctions:
·Composite fermion wavefunctions
·Clustered states based on parafermionic theories

Three arguments for electrons at ν = 1/3 :

$$\Psi^{\nu=1/3}_{Laughlin} = \prod_{i<j} (z_i - z_i)^3 \ \mathrm{e}^{-\sum_i |z_i|^2/4\ell^2}$$

· It is the exact ground state of Δδ(zi-zj) and this Hamiltonian is in
·the same phase as the Coulomb Hamiltonian (numerically)

·Can be written as a very special kind of Slater determinant

·Can be obtained from a (very simple) conformal field theory

·Note the trivial identity :

$$\prod_{i<j}(z_i - z_i)^3 = \prod_{i \neq j}(z_i - z_i) \cdot \prod_{i<j}(z_i - z_i)$$

·We recognize a van der Monde determinant !

$$\prod_{i<j}(z_i - z_i)^3 = \prod_{i \neq j}(z_i - z_i) \cdot \begin{vmatrix} 1 & \cdots & \cdots & 1 \\ z_1 & & & z_N \\ z_1^2 & & & z_N^2 \\ \vdots & & & \vdots \\ z_1^{N-1} & \cdots & \cdots & z_N^{N-1} \end{vmatrix}$$

$$\prod_{i<j}(z_i - z_i)^3 = \begin{vmatrix} \prod_{j\neq 1}(z_1 - z_j) & \cdots\ \cdots & \prod_{j\neq N}(z_N - z_j) \\ z_1 \prod_{j\neq 1}(z_1 - z_j) & & z_N \prod_{j\neq N}(z_N - z_j) \\ z_1^2 \prod_{j\neq 1}(z_1 - z_j) & & z_N^2 \prod_{j\neq N}(z_N - z_j) \\ \vdots & & \vdots \\ z_1^{N-1} \prod_{j\neq 1}(z_1 - z_j) & \cdots\ \cdots & z_N^{N-1} \prod_{j\neq N}(z_N - z_j) \end{vmatrix}$$

·A “Slater” determinant of correlated basis functions !

$$z^m \, \mathrm{e}^{-|z|^2/4\ell^2} \longrightarrow z^m \boxed{\prod_j{}' (z - z_j)} \, \mathrm{e}^{-|z|^2/4\ell^2}$$

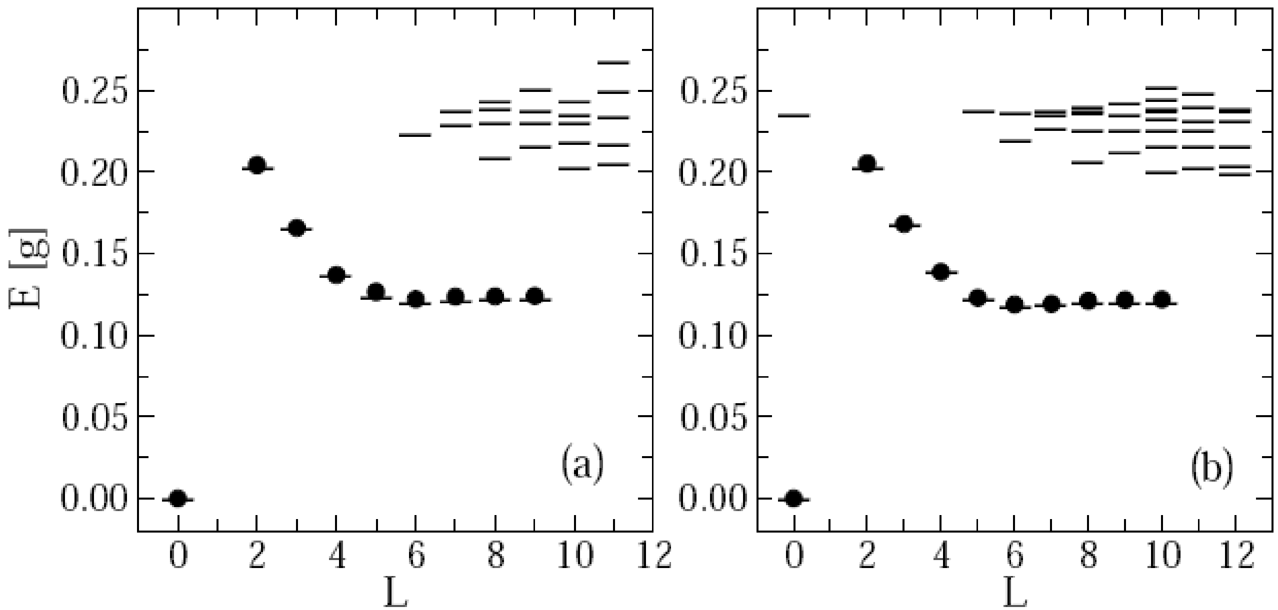
·You can play the game with higher Landau levels and project :

$$\mathcal{P}_{LLL}[\phi_{n,m}(z, z^*) \prod_j{}' (z - z_j)] \, \mathrm{e}^{-|z|^2/4\ell^2}$$

·This game is known as “composite fermions” construction
·It is highly successful for many (but not all) fractions

·The same machinery works for bosons :

$$\Psi_F(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_i) \Psi_B(z_1, z_2, \ldots, z_N)$$

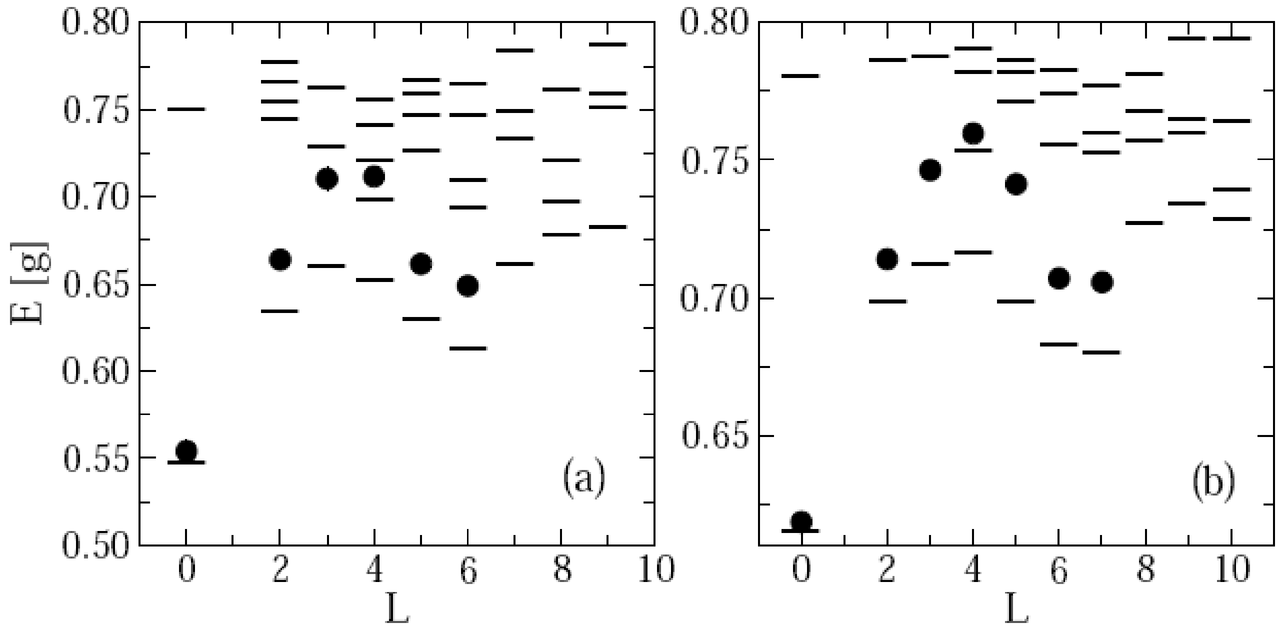


·Bosons at filling factor 1/2

·N. Regnault and TJ, PRL91,030402(2003),PRB69,134423(2004)

·CCChang,NRegnault,TJ,JKJain, PRA72,013611(2005)

·Bosons at filling factor 2/3 :



·There is an "Abelian" series of states for filling p/(p+1), p integer

·The CFT point of view:
·A naïve remark first

$$z - w = \exp(-(-\log(z - w)))$$

$$\prod_{i<j} (z_i - z_i)^3 = \langle \mathrm{e}^{i\sqrt{3}\varphi(z_1)} \ldots \mathrm{e}^{i\sqrt{3}\varphi(z_N)} \rangle$$

·Laughlin wavefunction can be expressed in terms of some correlation
·Function of a (simple!) CFT

·Given a nice CFT we can guess microscopic candidate wavefunctions !!!

·In Chern-Simons world there is a solidarity between 2+1 bulk,
·1+1 edge and 2+0 space states

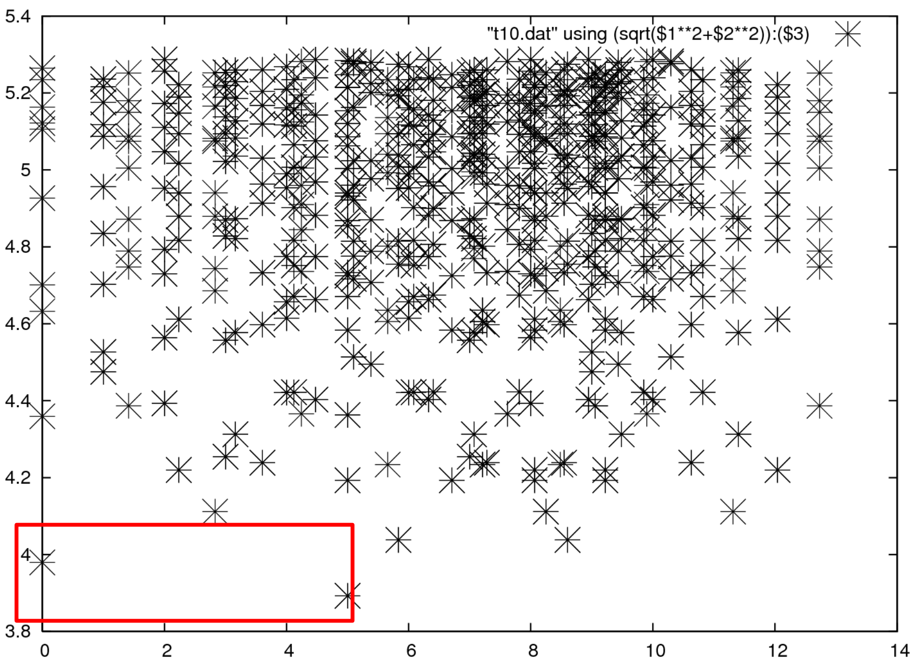
·Next to the free boson is the real (Majorana) free fermion
·This leads to the so-called Pfaffian state
·Filling factor 1 for bosons and ½ for fermions

·We use a Majorana fermion :

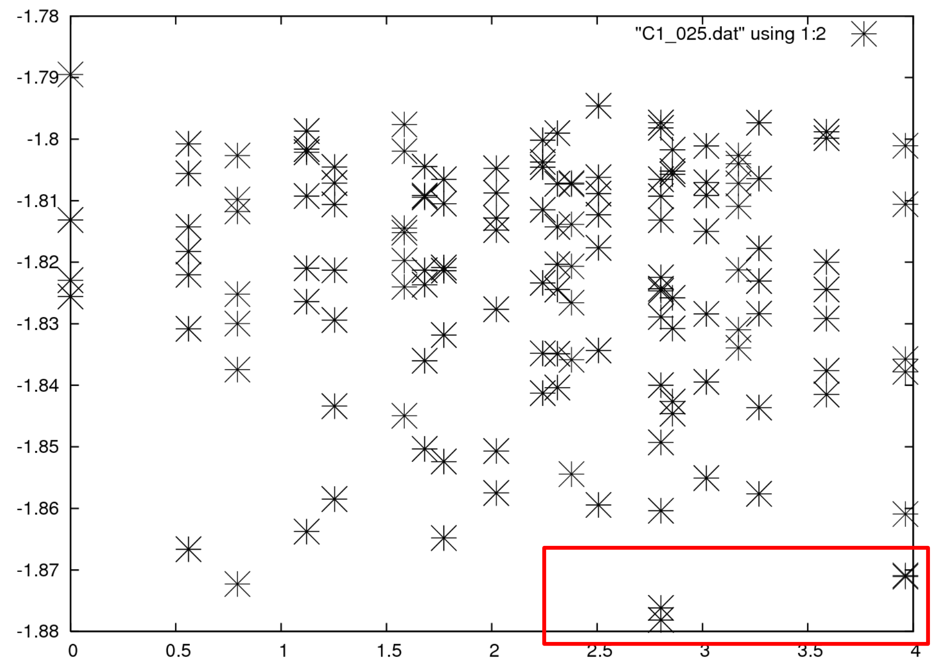
$$\tilde{\Psi}_{\mathrm{Pf}} = \langle \psi(z_1) \cdots \psi(z_N) \rangle \prod_{i<j} (z_i - z_j)^q,$$

·Evaluation leads to :

$$\tilde{\Psi}_{\mathrm{Pf}}(z_1, \ldots, z_N) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^q.$$



·10 bosons at filling 1 square unit cell



·10 fermions at filling ½ square unit cell

·One can also use the CFT of parafermions

$$\tilde{\Psi}^{(m)}_{\mathrm{para}}(z_1, \ldots, z_N) = \langle \psi_1(z_1) \cdots \psi_1(z_N) \rangle \tilde{\Psi}^{M+2/k}_{\mathrm{LJ}}$$

$$\Phi^{RR}_{\nu^*=k/2} = \mathcal{S}\left[\prod_{i_1<j_1}(z_{i_1} - z_{j_1})^2 \ldots \prod_{i_k<j_k}(z_{i_k} - z_{j_k})^2\right]$$

·Bosonic states for ν =k/2

· The k=3 is stabilized by dipolar interactions

Parafermionic clustered states have degenerate quasihole states : the building block for topological quantum computation !!!!

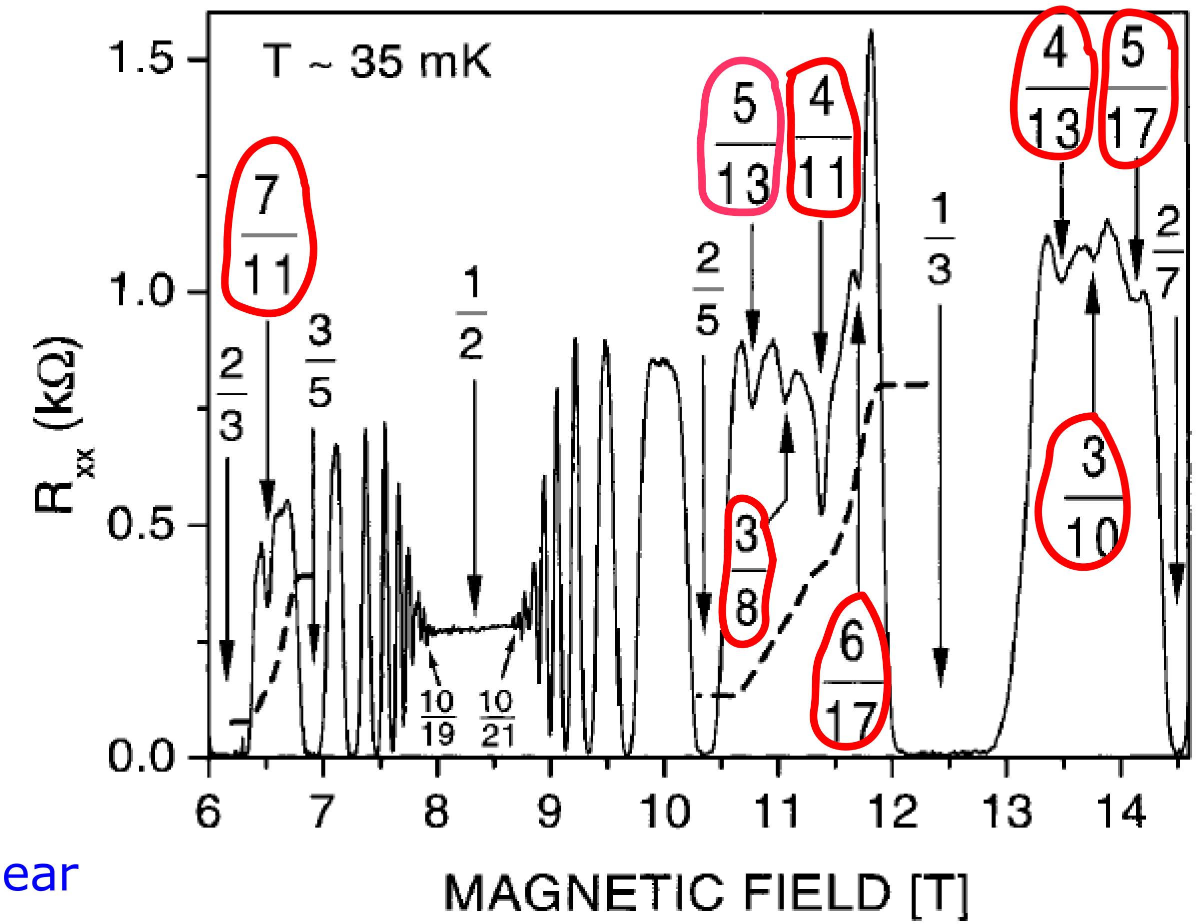
Such states may be realized for filling equal to (half)-integer

$$\mathcal{H}_k^{RR} = \sum_{i_1<\ldots<i_{k+1}} \delta^{(2)}\left(z_{i_1} - z_{i_2}\right)\ldots\delta^{(2)}\left(z_{i_k} - z_{i_{k+1}}\right)(3)$$

This problem has a zero-energy ground state which is parafermionic

Count of quasihole states is in agreement with conformal invariance

One may import the idea of clustered bosons for composite bosons made out of electrons to create new FQHE candidate states with filling k/(3k+-2)



·TJ, PRL to appear

·July 30, 2007

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PRL in 2003

- Fermions with spin have interactions in the s-wave as well
- As p-wave but zero Zeeman energy splitting
- Interesting arena for FQHE with spin degree of freedom

- Conclusion:
- So far only electrons display FQHE states
- Interactions are fixed and there is disorder

  - Needed: new arena for FQHE in ultracold atom world
  - Exploration of non-Abelian quasiparticles ?

Happy Birthday
Gora !