Wave propagation and disordered cold atomics gases

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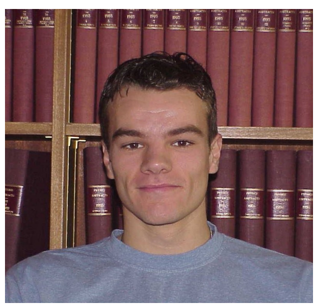
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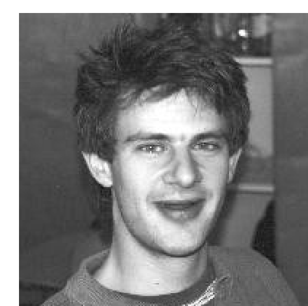
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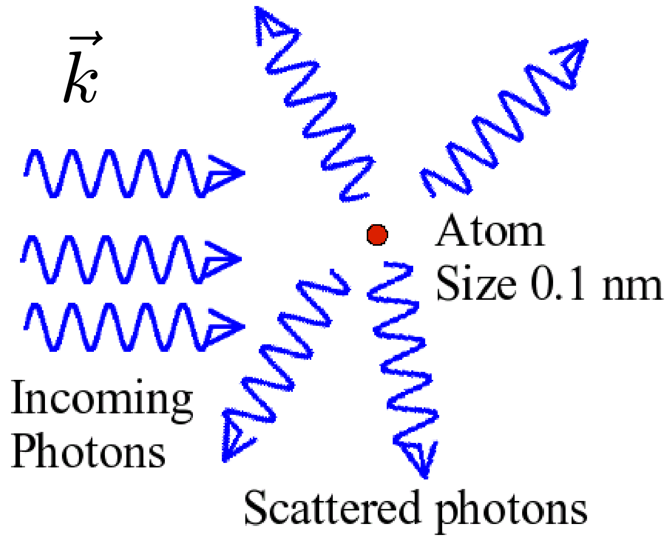
Outline

- Atom-photon interaction
- Multiple scattering, transport and diffusion
- Incoherent transport (radiation trapping)
- Coherent transport, coherent backscattering, weak localization
- Coherent backscattering of light by cold atoms
- Multiple scattering of matter waves
- Weak localization of matter waves
- Towards strong localization of matter waves

Atom-photon interaction

- Scattering amplitude $f(\theta,\phi)$
- Cross-section $\sigma(\theta,\phi) = |f(\theta,\phi)|^2$
- Total cross-section $\sigma = \iint \sigma(\theta,\phi)\, d\Omega$

  $\sigma \approx (0.1 \text{ nm})^2$ except near atomic resonances
- Recoil effect for an atom scattering a photon (typically few mm/s)



- For a Rayleigh scatterer (point dipole scatterer without internal structure, e.g. a $J=0 \rightarrow J=1$ atomic transition), the scattering amplitude near a resonance is:

$$f = \frac{3}{2k} \frac{\Gamma/2}{\delta + i\Gamma/2} \epsilon^*_{\text{out}} \cdot \epsilon_{\text{in}}$$

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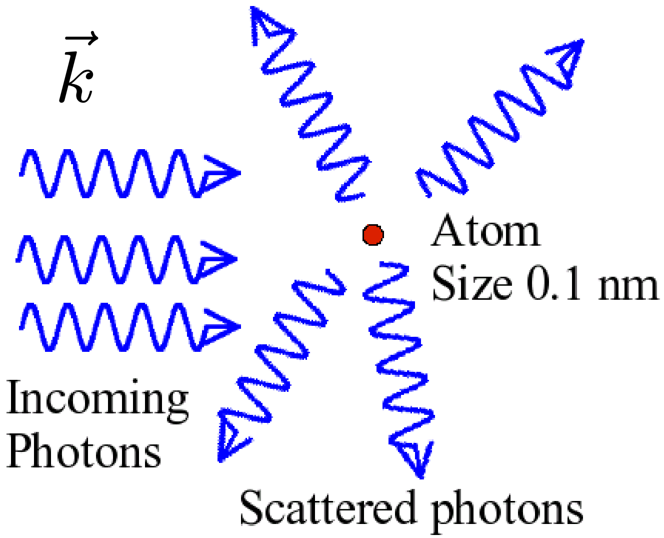

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(trivial) dependence on the incoming and outgoing polarizations

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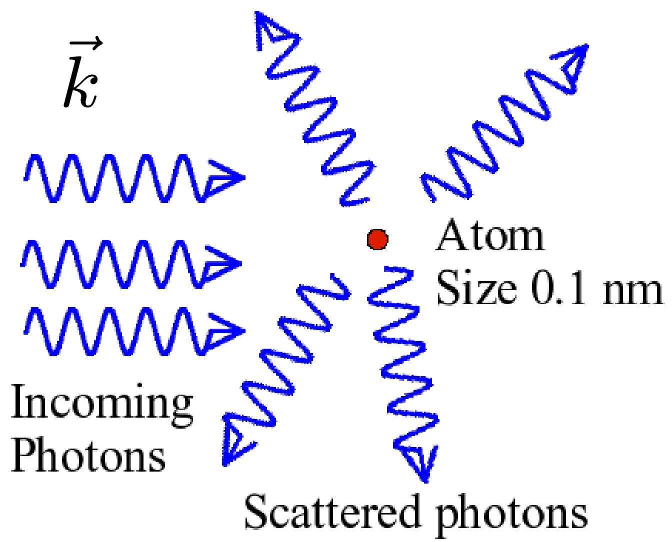

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Resonant character: $\delta = \omega_L - \omega_0$ detuning from resonance

Γ: width of the resonance (inverse of atomic lifetime)

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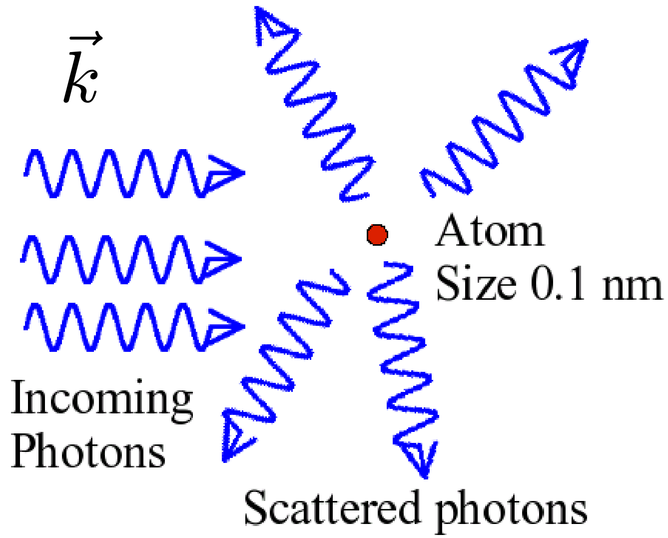

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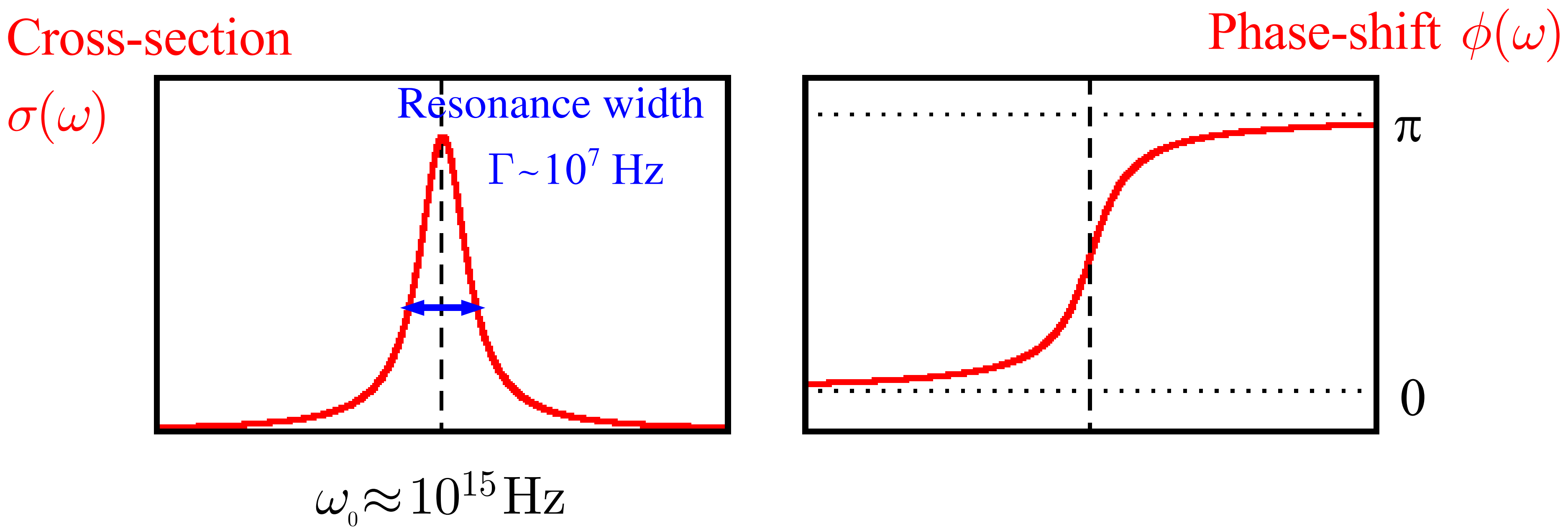
(trivial) dependence on the incoming and outgoing polarizations

Resonant cross-section of the order of λ^2 >> size of the atom

Resonant character: $\delta = \omega_{\text{L}} - \omega_0$ detuning from resonance

Γ: width of the resonance (inverse of atomic lifetime)

Atom-photon quasi-resonant interaction



- Resonant cross-section $\sim 1\,\mu\mathrm{m}^2$, 8 orders of magnitude larger than geometric cross-section.
- Wigner time delay at scattering:

$$\tau_{\text{Wigner}} = \frac{d\phi}{d\omega} = \Gamma^{-1}$$ at resonance (typically 10-100 ns)

- Very sensitive to motion of atoms (Doppler effect) $\Gamma/k \sim \mathrm{m.s^{-1}}$
- Easily saturable because of huge cross-section (few $\mathrm{mW/cm^2}$)

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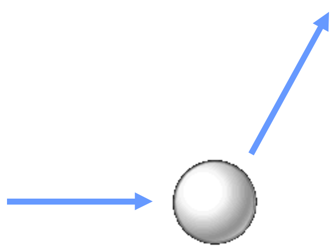
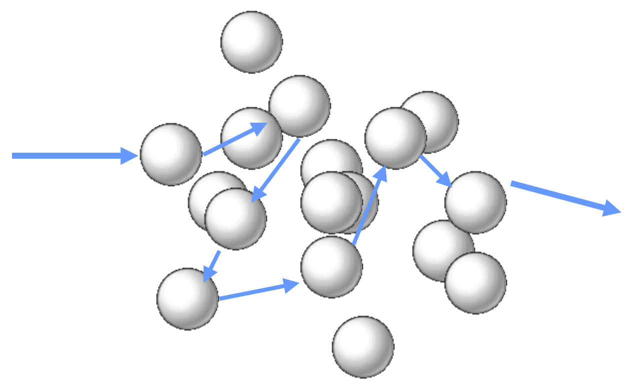
Multiple scattering and transport

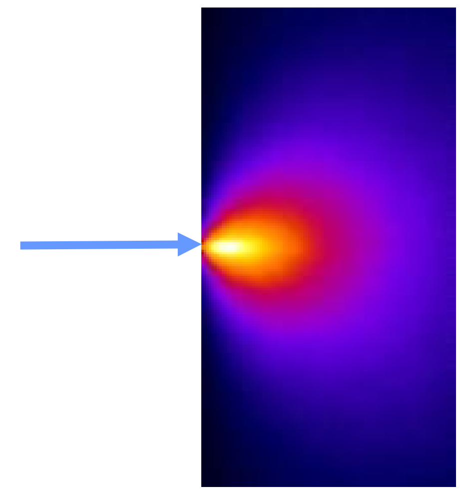
- From microscopic to macroscopic:



Microscopic scale
(scattering amplitude,
cross-section...)



Intermediate scale
Multiple scattering
Disordered medium



Macroscopic scale
Transport properties
Radiative transfer equation
Boltzmann equation
Diffusion approximation...

- Interference effects (beyond Boltzmann/RTE equation) may survive in the multiple scattering regime => mesoscopic regime

Important length scales

- Microscopic length scales:
  - size of the scatterers and of scattered particles
  - mean distance between scatterers $n^{-1/3}$
- Intermediate length scales:
  - Scattering mean free path
    n : density of scatterers
    σ : scattering cross-section
    $$\ell_S = \frac{1}{n\sigma}$$
  - Transport mean free path (Boltzmann mean free path)
    $$\ell_B = \frac{\ell_S}{1 - \langle \cos\theta \rangle}$$
    distance traveled before the direction of propagation is randomized.
  - It is directly related to the diffusion constant (v is the particle velocity), itself related to macroscopic quantities such as the conductance:
    $$D_B = \frac{v\ell_B}{3}$$
- Macroscopic length scale: size of the medium L

Usually: $n^{-1/3} \ll \ell_S, \ell_B \ll L$

Additional length scales for “wave” particles

- Wavelength (de Broglie wavelength for matter waves) λ
- Phase coherence length of the wave L_ϕ. Beyond L_ϕ, interference effects are washed out.
- Dense medium $n\lambda^3 \gg 1$
  - Effective continuous medium
  - Index of refraction...
- Dilute medium: $\lambda \ll n^{-1/3} \ll \ell_S, \ell_B \ll L$
- Interference effects may survive at the intermediate length scales, even in the presence of strong spatial disorder:

mesoscopic regime

- Visible at the macroscopic scale if $L_\phi > L$

Why atoms?

- Very well known elementary scatterer
- Large cross-section at resonance (monodisperse sample)
- Atom-atom interaction is small in dilute gases (no BEC)

Some complications

- The elementary scattering process is "very quantum"

  Recoil velocity $= \dfrac{\hbar k_{\mathrm{L}}}{M}$ = few mm/s $\quad k_{\mathrm{L}} = \dfrac{2\pi}{\lambda}$: laser wavenumber $\quad$ M : atomic mass

- Doppler effect is small, but not negligible in cold atomic gases

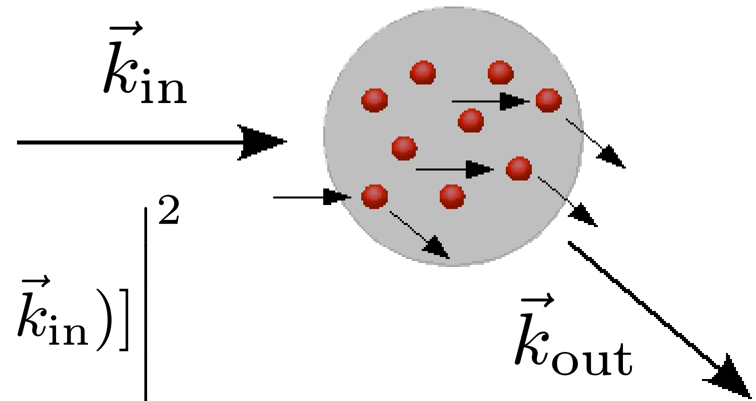
$$k_{\mathrm{L}} v_{\mathrm{atom}} \ll \Gamma \qquad \Rightarrow \qquad v_{\mathrm{atom}} \ll 1m/s$$

  Doppler shift $\quad$ Width of the atomic resonance

- Non-linear behaviour (saturation of the atomic transition)
- Internal structure (Zeeman sublevels)

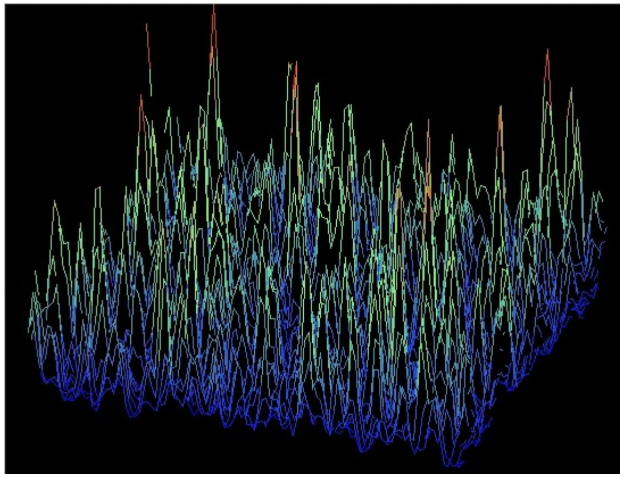
Naive view on single scattering

- The scattered intensity results from the coherent addition of scattering by each particle. For identical scattering particles:



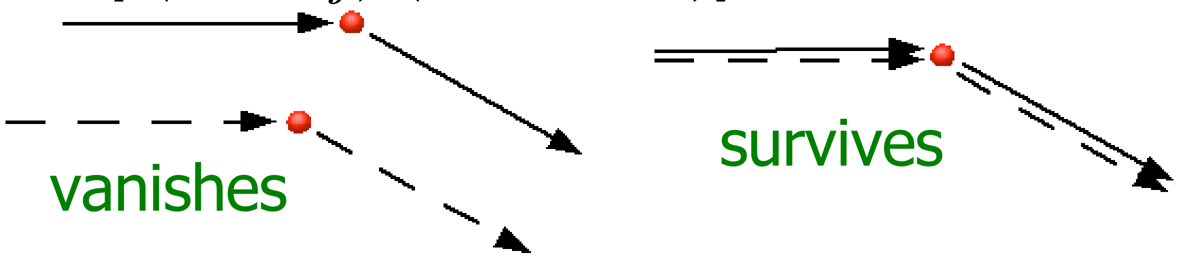
$$I(\vec{k}_{\rm in}, \vec{k}_{\rm out}) = \left| \sum_{i=1,N} f \ \exp\left[i\vec{r}_i.(\vec{k}_{\rm out} - \vec{k}_{\rm in})\right] \right|^2$$

- Ordered particles => constructive interference in some direction (Bragg diffraction).
- Disordered densely packed system ($n\lambda^3$>>1) => interference terms are washed out.
- Disordered dilute system => random interference **(SPECKLE)**



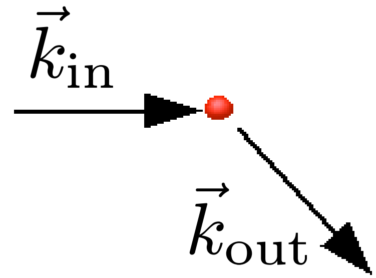
- Configuration averaging over disorder realizations: all interference terms

$\exp[i(\vec{r}_i - \vec{r}_j).(\vec{k}_{\rm out} - \vec{k}_{\rm in})]$ vanish for $i \neq j$.



Naive view on multiple scattering

- A scattered photon can be rescattered. The total scattering amplitude is a sum of terms:



single scattering

Naive view on multiple scattering

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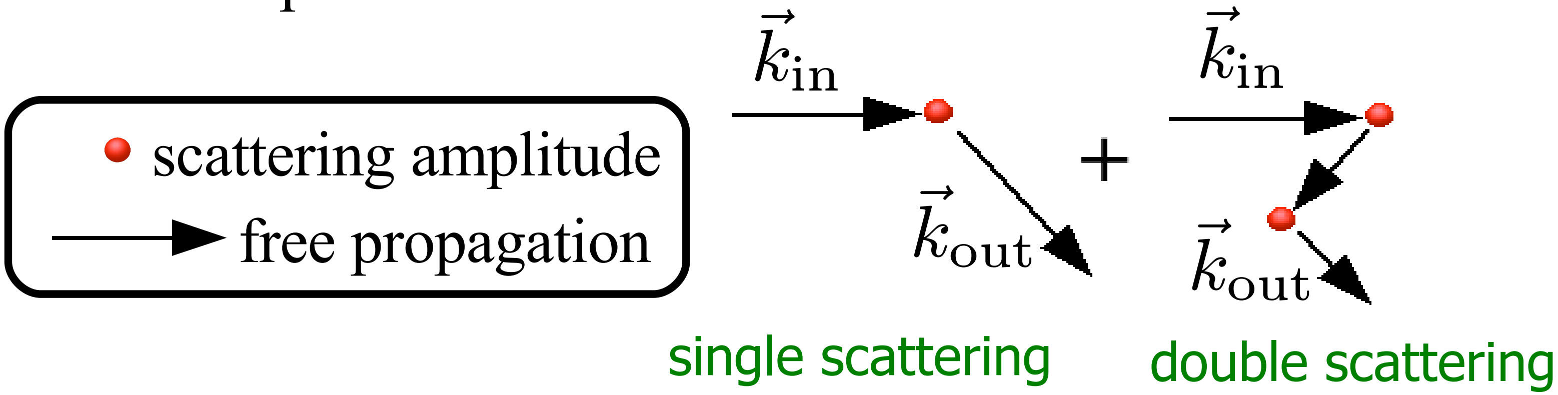

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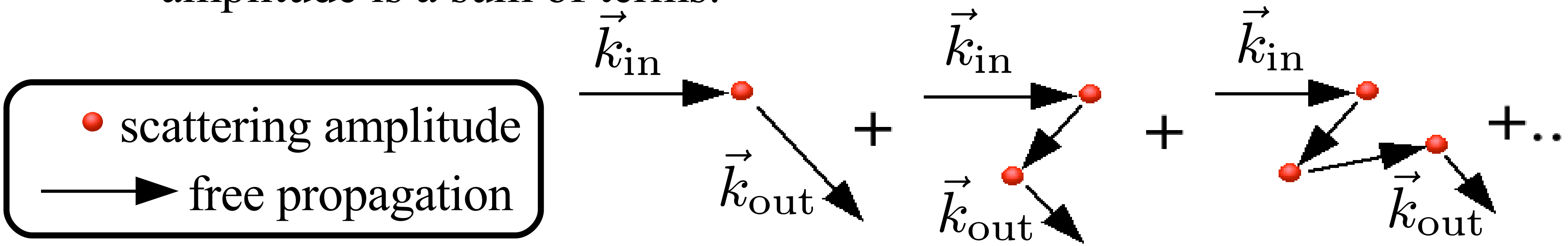

single scattering   double scattering   triple scattering

Naive view on multiple scattering

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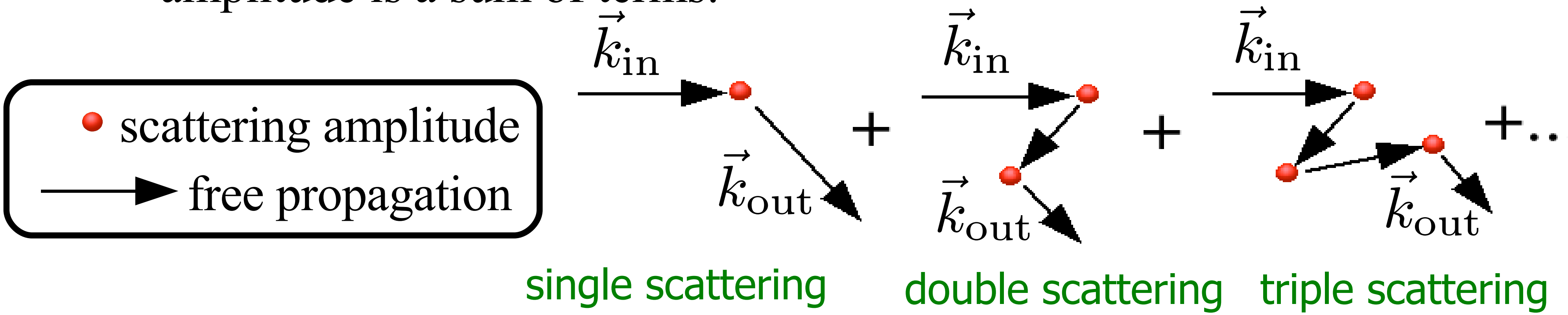


- Even in a dilute system where the lengths of the multiple scattering paths are >> λ, interference between different paths cannot be neglected (shadowing of long paths by shorter ones).
- Less naive approach: introduce an effective medium in which the photon propagates between consecutive scattering events => diagrammatic mesoscopic approach. Must take into account both index of refraction and attenuation (NO ABSORPTION).
- Configuration averaging is easy.
- Price to pay: requires various equations for the propagation of field, intensity, correlations...

Diagrammatic methods in the weak scattering regime

- Hamiltonian $H = H_0 + V(\mathbf{r})$ with V a fluctuating potential
- Compute the average Green's function $\overline{G} = \overline{1/(E-H)}$ (averaged over disorder realizations) from the unperturbed Green's function for H_0 and correlations function of the fluctuating potential

  $$\overline{G} = G_0 + G_0\overline{V}G_0 + G_0\overline{VG_0V}G_0 + \ldots$$

  Dyson equation (definition of the self-energy Σ)

  $$\overline{G} = G_0 + G_0\Sigma\overline{G}$$

- Transition operator T: $\overline{G} = G_0 + G_0\overline{T}G_0$
- The Dyson equation can be written

  $$\overline{T} = \Sigma + \Sigma G_0\overline{T}$$

- The self-energy is computed perturbatively using irreducible diagrams

  $$\Sigma = \otimes + \otimes\!-\!\!-\!\otimes\!-\!\!-\!\otimes + \ldots$$

- ImΣ represents the attenuation of a coherent mode propagating inside the medium. It is essentially the inverse of the mean free path

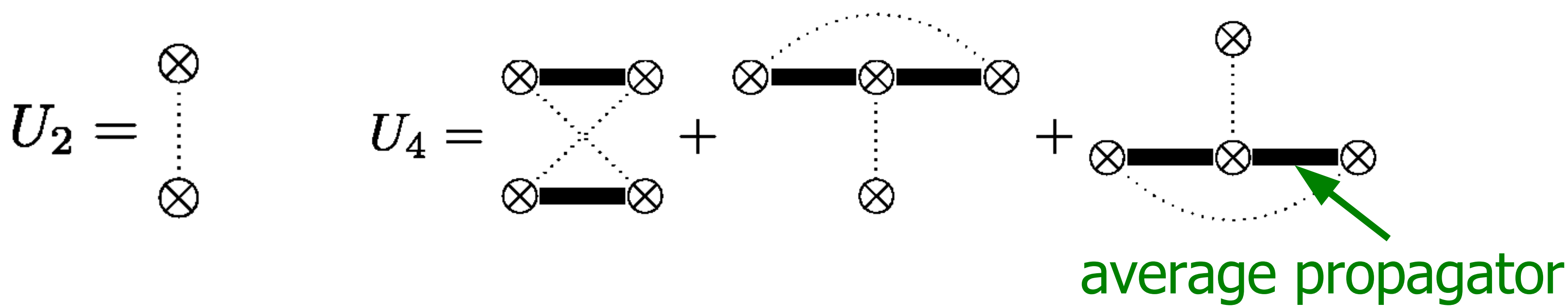
  $$\overline{G(E)} = \frac{1}{E - H_0 - \Sigma}$$

Bethe-Salpeter equation (intensity diagrams)

- Intensity requires product of field and complex conjugate, thus product of one advanced and one retarded Green's functions.
- Requires the introduction of a irreducible vertex *U*, obeying the Bethe-Salpeter equation:

$$\overline{T^\dagger \otimes T} = U + U \left[\overline{G^\dagger} \otimes \overline{G} \right] \overline{T^\dagger \otimes T}$$

- *U* can be computed perturbatively using diagrams

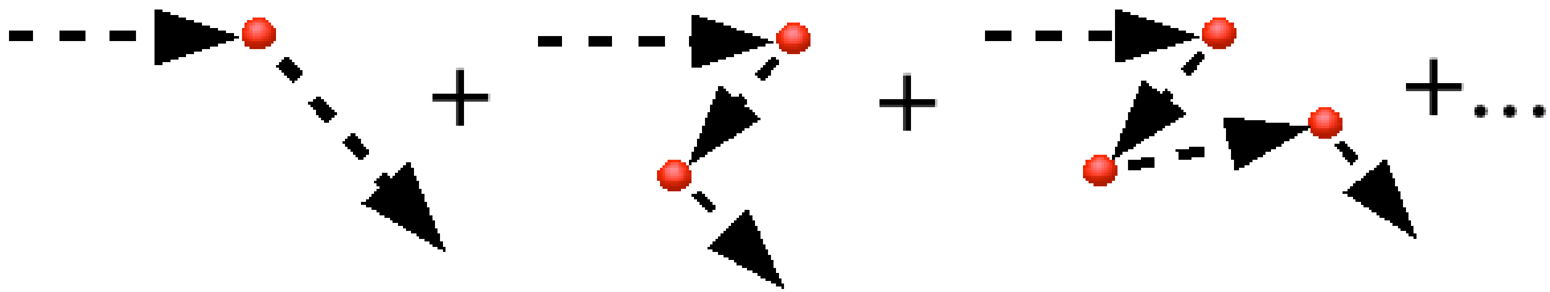


- Σ and *U* are related by the Ward identity (flux conservation, equivalent of optical theorem for single scattering) => truncation of the expansions must be done consistently on all quantities.

$$\mathrm{Im}\Sigma(\omega) = \sum_{\mathrm{k}'} U(\mathbf{k}, \mathbf{k}'; \omega)\ \mathrm{Im}\overline{G(k')}$$

Multiple scattering expansion

- Expansion in terms of the elementary scattering events (scattering amplitude) and the elementary propagation (Green's function) in the effective medium. Requires a consistent description of both processes.



scattering amplitude (cross-section σ)

propagation in the effective medium
attenuation $\exp\left(-r/\ell\right)$

Dilute medium: $\boxed{\ell = \dfrac{1}{n\sigma}}$

ℓ : mean free path
σ : cross-section
n: density of scatterers

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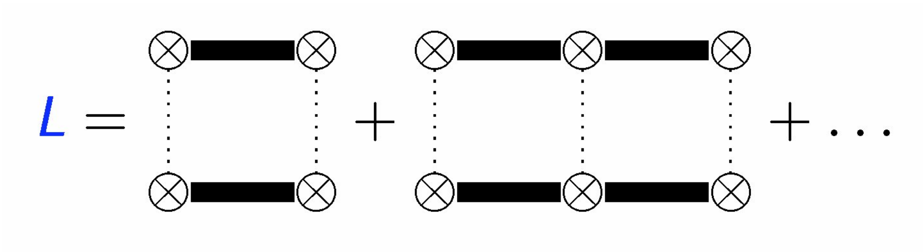
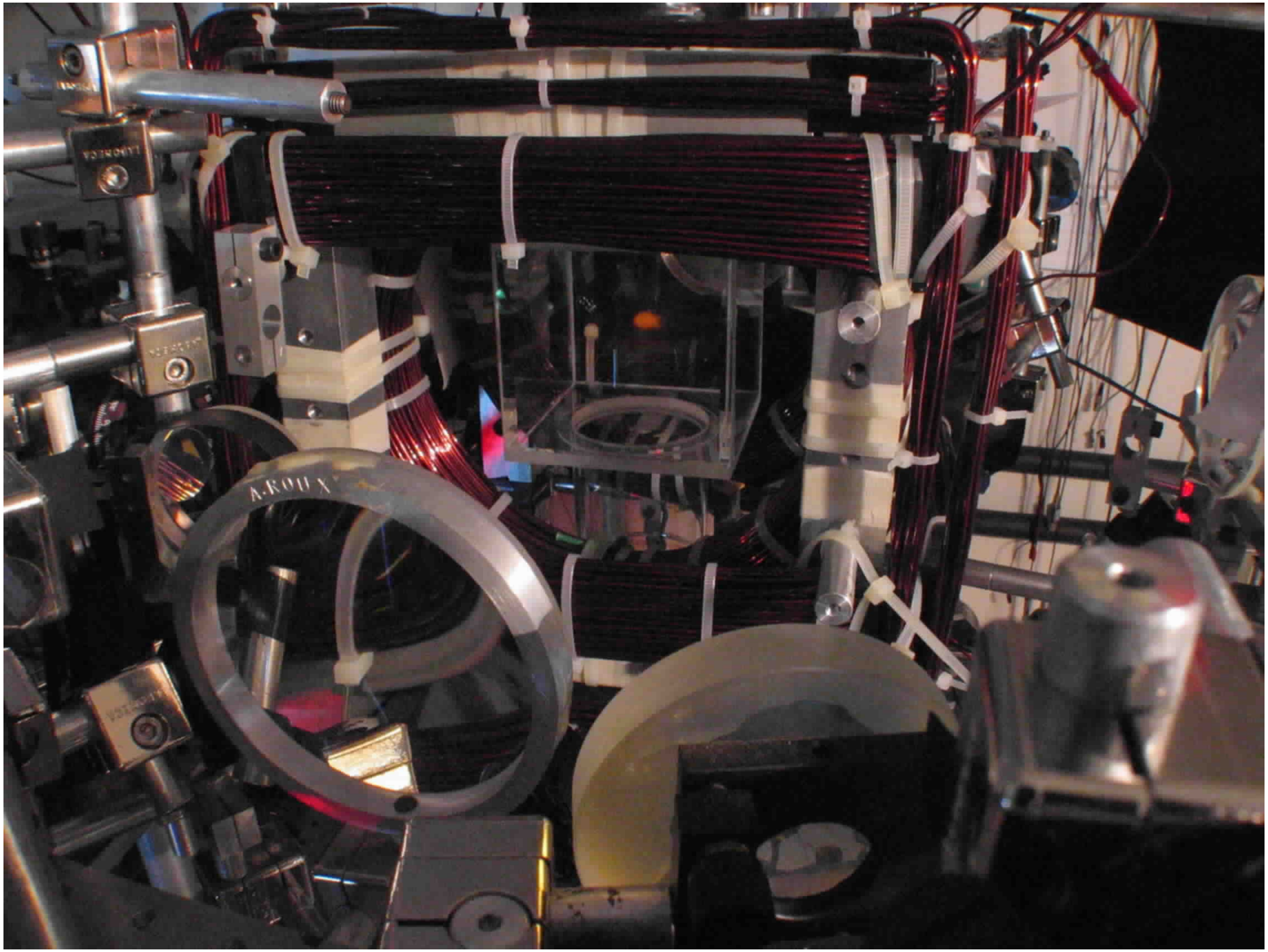
Incoherent transport

- If the mean free path ℓ is much larger than the wavelength λ of the photon, one can neglect interference between different scattering paths (washed out by configuration averaging) and keep only the "ladder" diagrams ("diffuson")



- Equivalent to Boltzmann or Radiative Transfer Equation
- At large distance (much larger than ℓ), the propagation of the intensity is diffusive => diffusion approximation.

Experimental setup

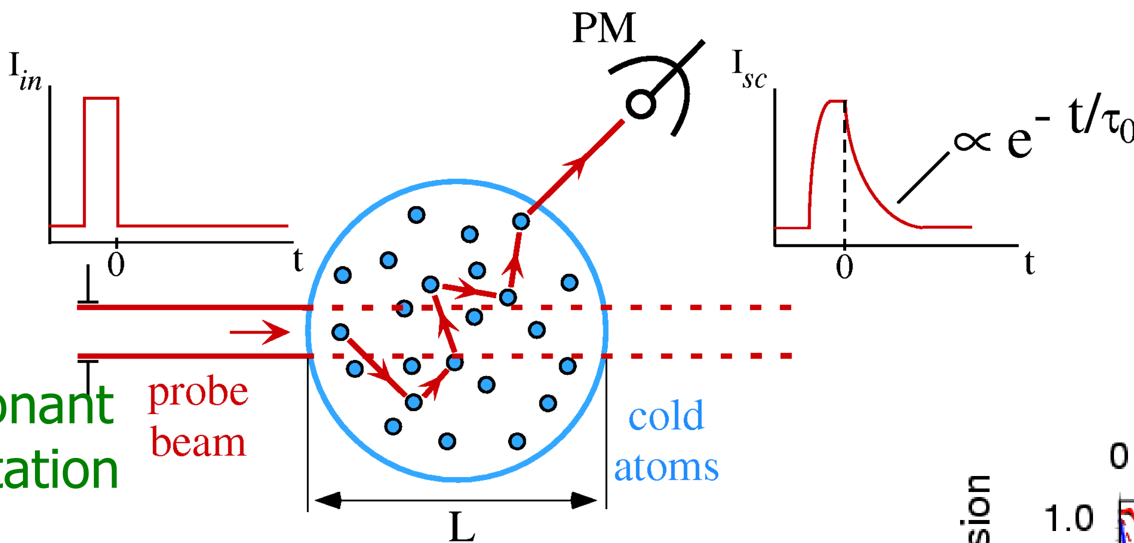


- MOT of Rubidium atoms. Large size (few mm). Optical thickness up to 40.

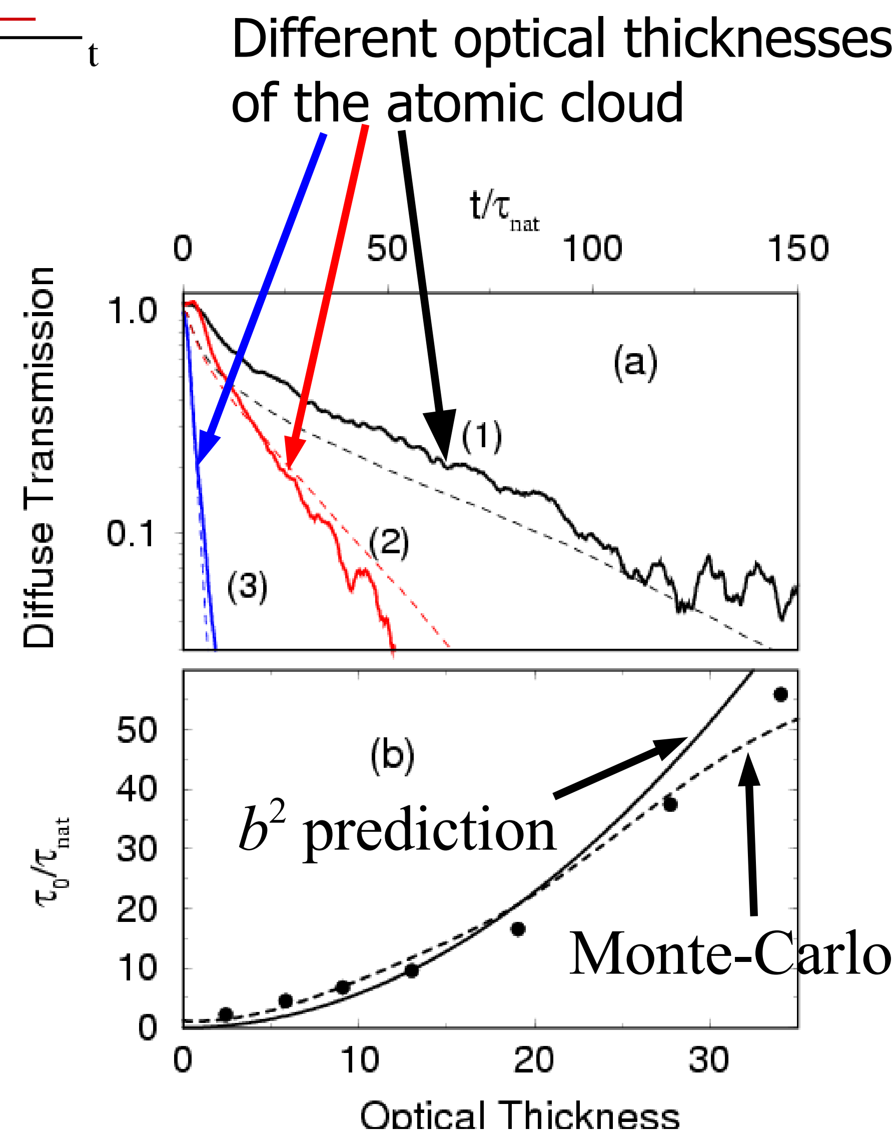
Labeyrie et al, INLN (Nice Sophia-Antipolis)

Incoherent transport (radiation trapping)

Resonant excitation



$$\tau_{\rm nat} = \Gamma^{-1} = 27\text{ns}$$



- Experimental observation: the photons are trapped in the medium during a time $\approx b^2\tau_{\rm nat} = b^2\Gamma^{-1}$
  $b \simeq L/\ell$: optical thickness
- Random walk of the photon inside the atomic medium (no interference)
- Deviation at large optical thickness, because the residual Doppler effect brings photons out of resonance.

Labeyrie et al, PRL 91, 223904 (2003)

Theoretical prediction for radiation trapping in the dilute regime ($n\lambda^3 \ll 1$)

- The multiple scattering paths followed by the photons are diffusive paths if $L \gg \ell$
- Interference effects can be neglected if $\ell \gg \lambda$
  $\Rightarrow$ Incoherent transport of radiation in the cold atomic gas
- One can calculate the mean free path and the diffusion constant from microscopic parameters:
  - Mean free path: $\ell = \dfrac{1}{n\sigma}$ typically 100 µm
  - Velocity of energy propagation:

$$v_E = \frac{\ell}{\tau}$$ τ: time delay between consecutive scattering events

$$\tau = \Gamma^{-1} + \frac{\ell}{c}$$

Scattering time delay ~ 30ns

Propagation between two scattering events ~ 10ps

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    $$\tau \approx \Gamma^{-1} \qquad v_E \approx \frac{\ell}{\Gamma} \simeq 10^4 - 10^5 \mathrm{m/s} \simeq \boxed{10^{-4} c}$$
  - Diffusion constant $D = \dfrac{v_E \ell}{3} \simeq 1 \mathrm{m}^2 \mathrm{s}^{-1}$
- Residual Doppler effect makes the photon frequency to perform a random walk. Order of magnitude (scattering order N): $\Delta\omega = k\bar{v}\sqrt{N} = k\bar{v}b$
  Monochromatic approximation valid only if $\boxed{k\bar{v}b \ll \Gamma}$

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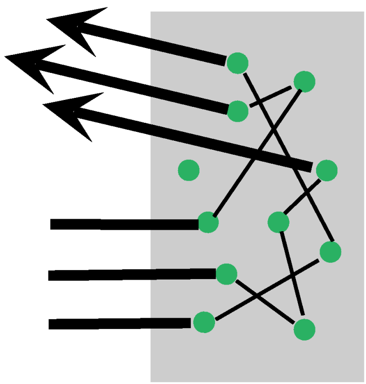
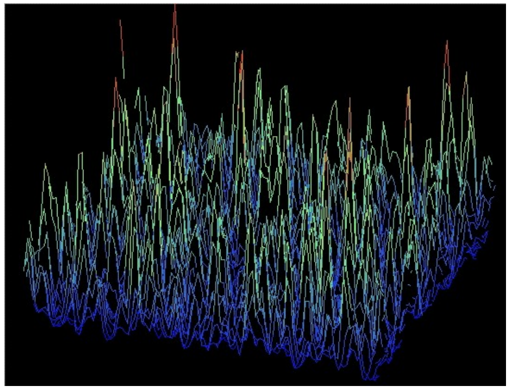
Coherent backscattering (CBS)

- In general, the interference between multiple scattering paths produces a random pattern: speckle.

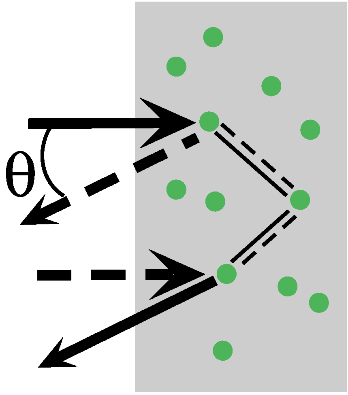


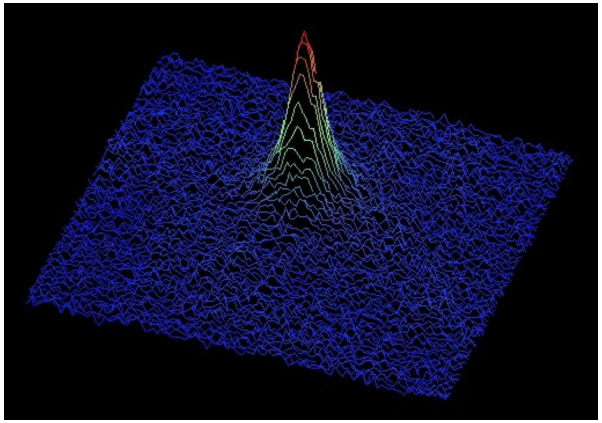


N.B.: frozen disorder polystyrene scatterers, not atoms!

sample   Experimental observation (far-field)

- When averaged over disorder (several spatial configurations), the speckle disappears, except in the backward direction => coherent back scattering (CBS).

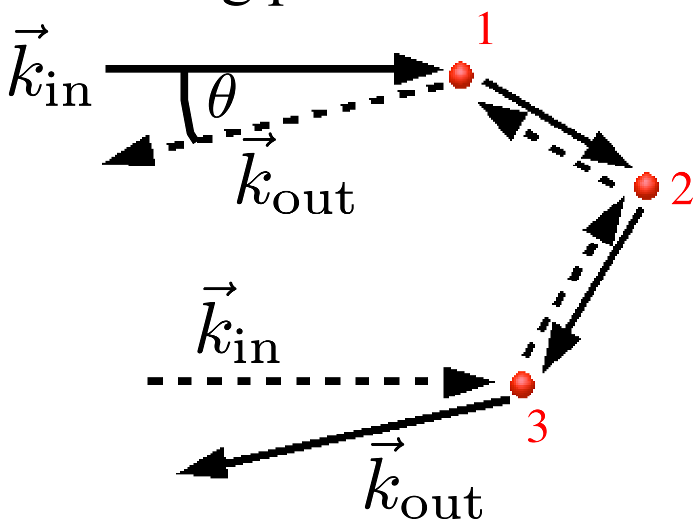




Peak around θ=0!

The physics of Coherent Back Scattering

- CBS comes from the interference between pairs of reverse multiple scattering paths:



: direct path
Scattering amplitude $f^{(123)}(\vec{k}_{\rm in}, \vec{k}_{\rm out})$

: reverse path
Scattering amplitude $f^{(321)}(\vec{k}_{\rm in}, \vec{k}_{\rm out})$

- Phase difference between the two contributions:

$$\Delta\phi = (\vec{k}_{\rm in} + \vec{k}_{\rm out}).(\vec{r}_3 - \vec{r}_1)$$

- Scattered intensity:

Diffuse (incoherent) intensity

$$I(\theta) = \left| \sum_{\text{paths } p} f_p \right|^2 = \sum_{p \neq p'} f_p \ \bar{f}_{p'} + \sum_p |f_p|^2$$

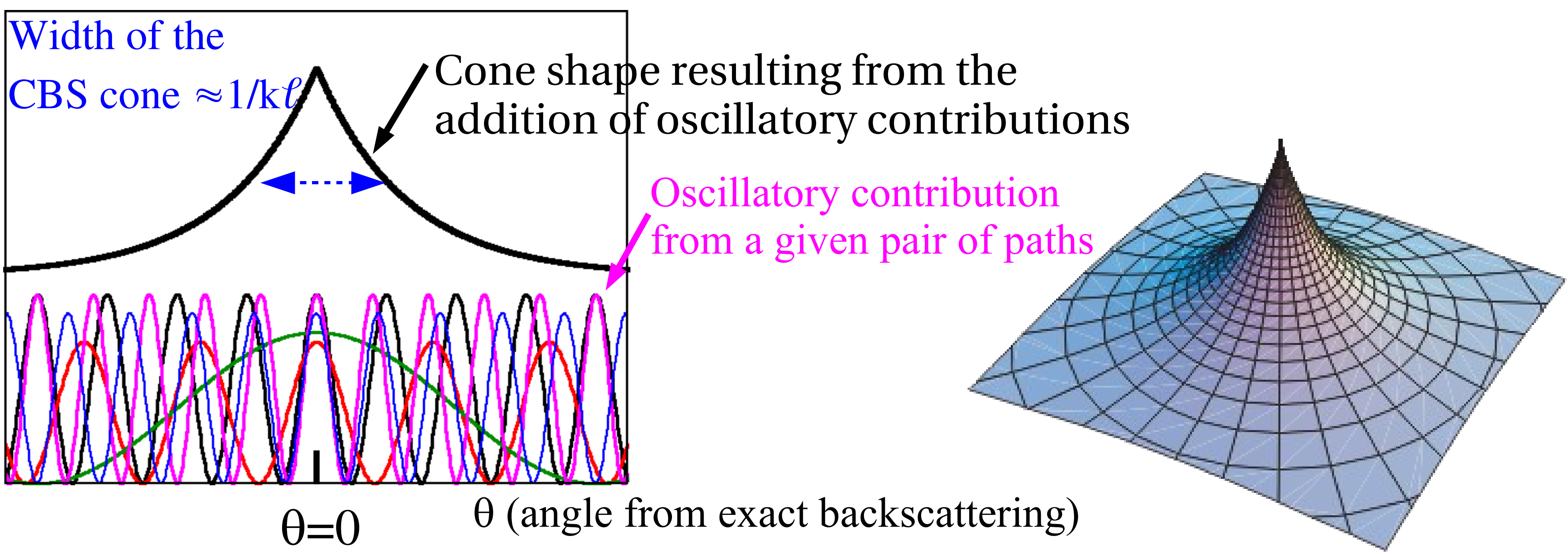
Washed out by disorder, EXCEPT if *p*'=reverse(*p*)

The physics of Coherent Back Scattering

- After configuration averaging:

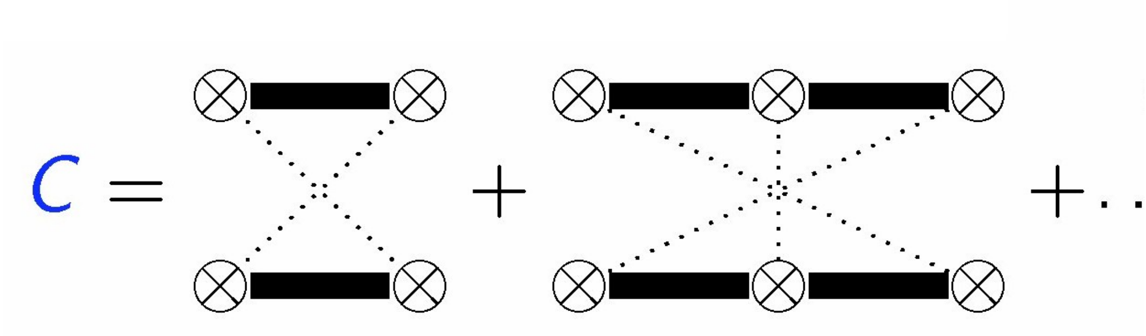
$$I(\theta) = \sum_p \left(|f_p|^2 + |f_{\bar{p}}|^2 + f_p \bar{f}_{\bar{p}} + \bar{f}_p f_{\bar{p}} \right)$$

- CBS is a 2-wave interference effect (like Young slits) ⇒ the maximum enhancement factor is 2, obtained only when the two interfering amplitudes are equal (requires good polarization channel).
- Any pair of direct/reverse paths produces a periodic modulation (interference fringes) of the scattered intensity. All these contributions are maximum (bright fringe) at back-scattering θ=0.



Crossed diagrams

- The dominant interferential contribution when $\ell >> \lambda$ comes from the “most crossed diagrams” (also known as “cooperon”):



- When reciprocity is satisfied, one can turn left-right the crossed diagrams and recover the ladder diagrams => optimal interference contrast, enhancement factor equal to 2

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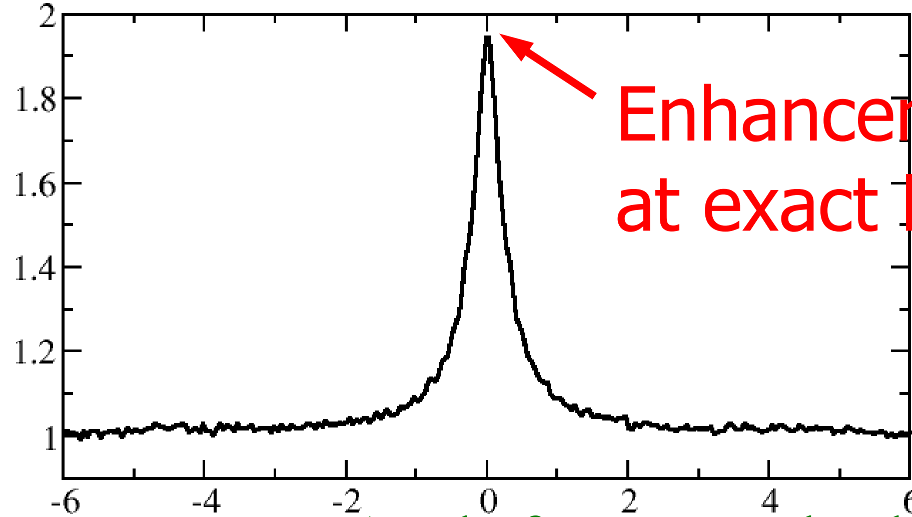
Experimental observation of CBS

- "Classical" dipole scatterers

Backscattered signal

TiO_2 particles in water



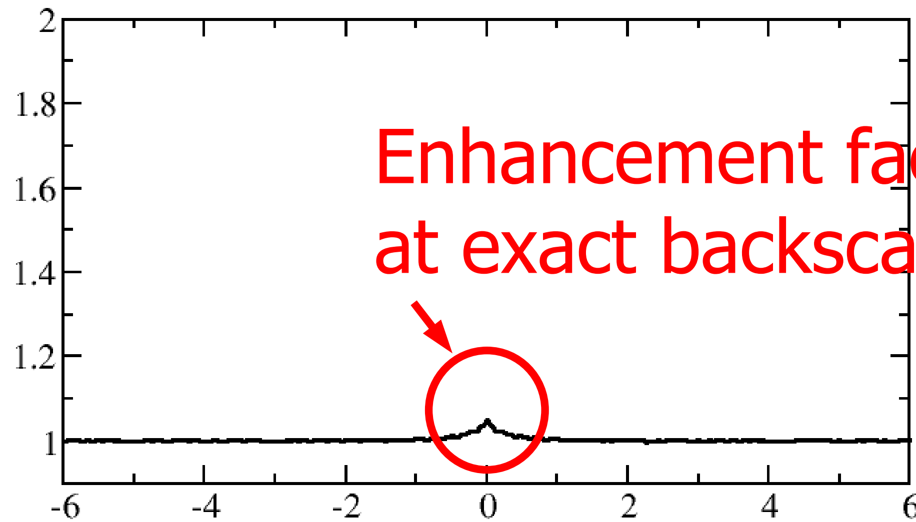
Enhancement factor at exact backscattering ~ 2

h//h polarization channel

Angle from exact backscattering (mrad)

- Dilute gas of cold Rubidium atoms

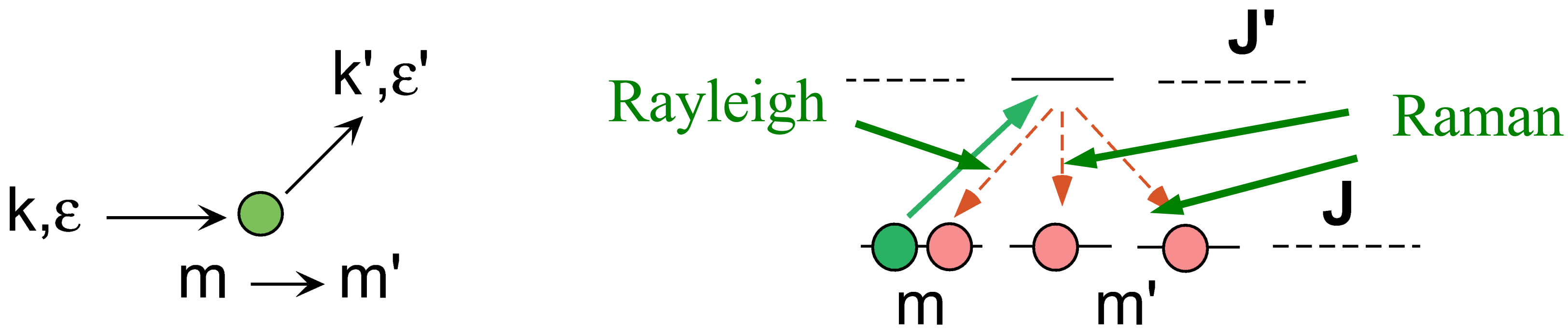
G. Labeyrie et al, Europhys. Lett. 61, 327 (2003)



Enhancement factor at exact backscattering ~ 1.04

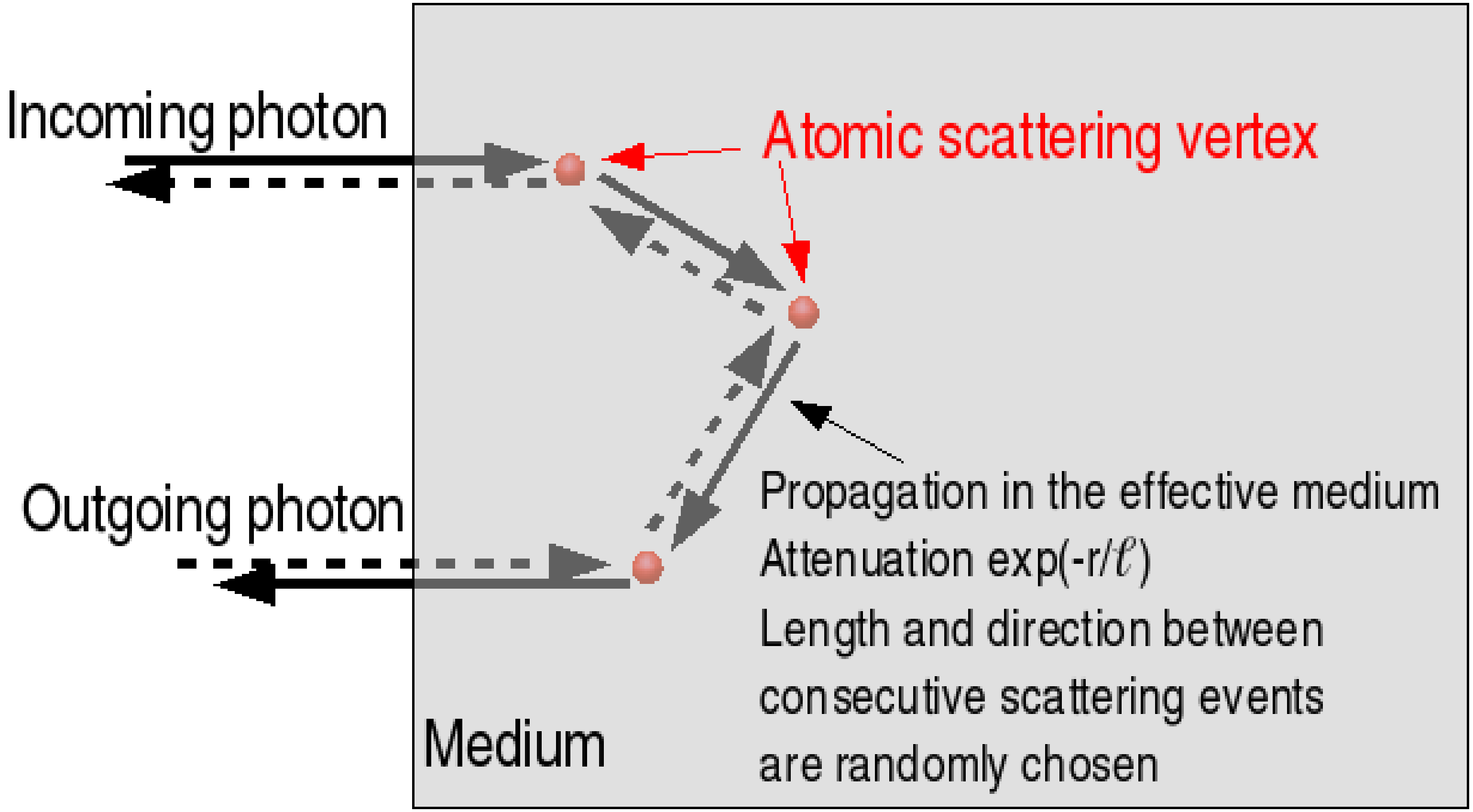
Role of the internal atomic structure

- The ground and excited states of Rubidium are degenerate. When a photon is elastically scattered, the internal atomic state (Zeeman sublevel) may remain unchanged (Rayleigh transition) or may change by 1 or 2 units (degenerate Raman transitions).



- Raman transitions do contribute to the CBS signal. The two interfering direct and reverse amplitudes are associated with the same changes of Zeeman sublevels.
- In general, the two interfering amplitudes are not equal => contrast of the two-wave interference is reduced.
- Everything can be calculated analytically, in simple geometries (semi-infinite medium). C. Mueller et al, PRA 64, 053804 (2001)
- Monte-Carlo calculation for more complicated geometries.

Schematic view of the Monte-Carlo calculation

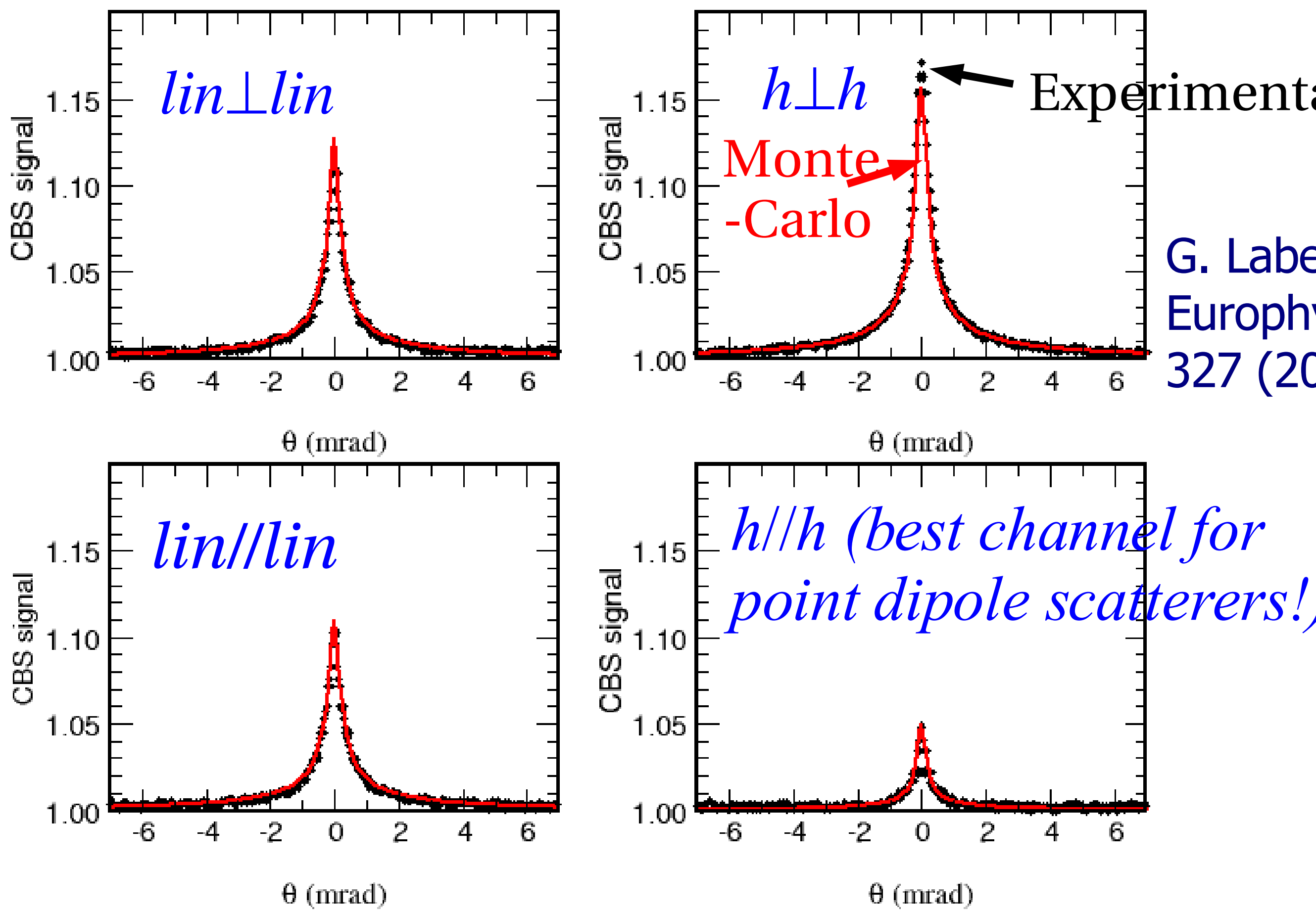


- Internal structure taken into account exactly in the atomic scattering vertex (assuming all Zeeman sublevels equally populated).
- Geometrical factors (shape of the atomic cloud, laser beam...) easily incorporated in the Monte-Carlo calculation.
- If necessary, residual Doppler and recoil effects can be incorporated.

Experimentally observed atomic CBS cones

Rubidium atom : J=3 →J=4 closed transition
Spherical atomic cloud (Gaussian density)
Optical thickness ≈26
No adjustable parameter in the Monte-Carlo calculation

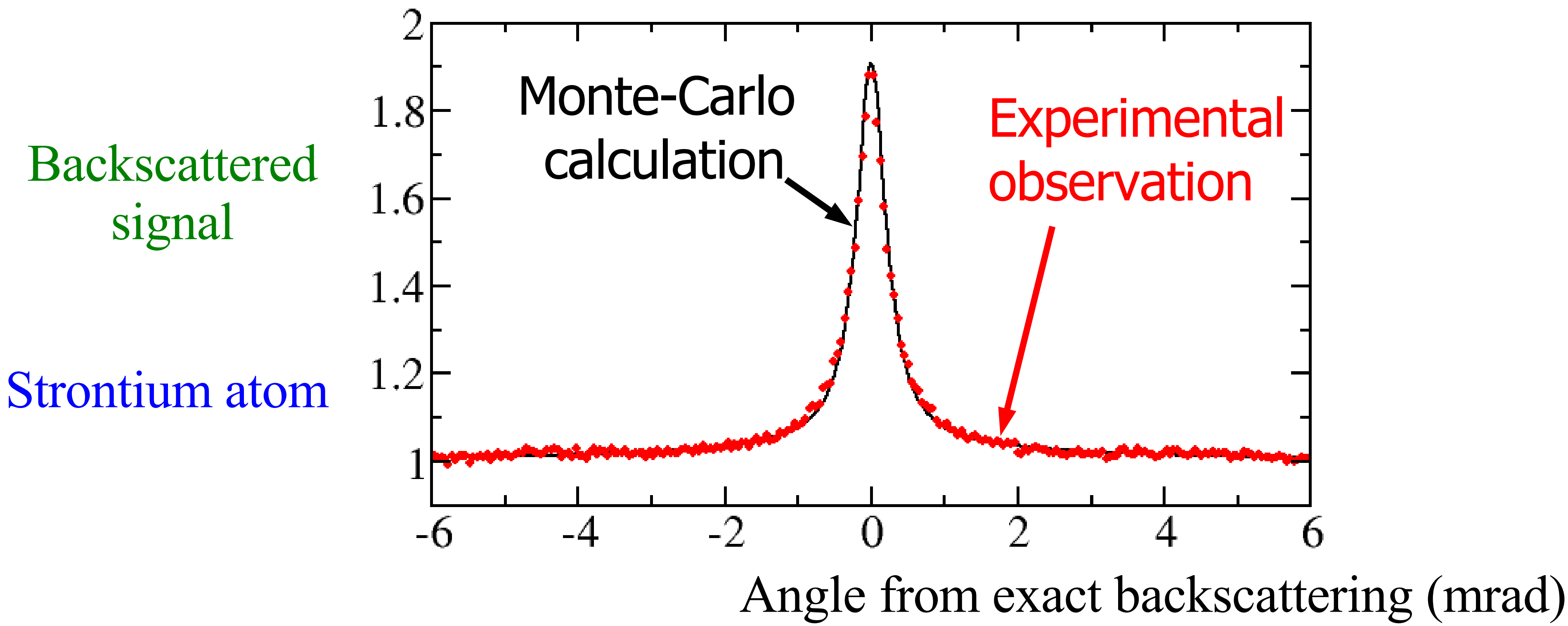
The excellent agreement shows that all important effects are taken into account and proves that the internal structure is the most important ingredient



G. Labeyrie et al, Europhys. Lett. 61, 327 (2003)

How to get rid of the internal structure?

- Simple solution: use an atom with no internal structure in the ground state, that is a $J=0 \rightarrow J=1$ atomic transition.



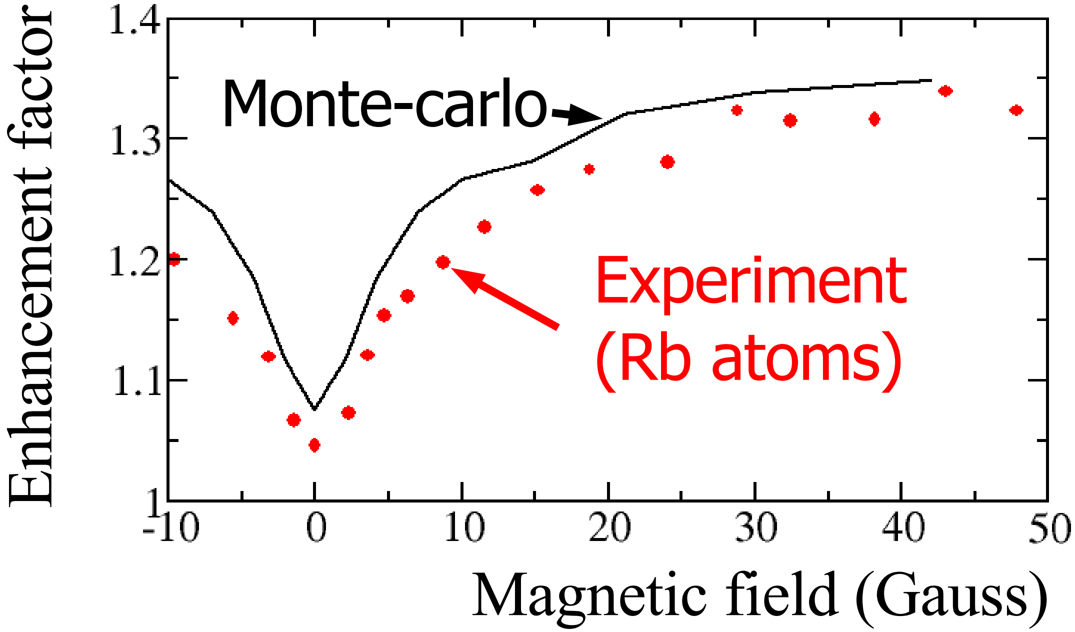
Back to an enhancement factor ~ 2!

Y. Bidel et al, PRL **88**, 203902 (2002)

How to get rid of the internal structure?

- A magnetic field splits the degenerate atomic transition (Zeeman effect). If the Zeeman effect μB is larger than the width Γ of the atomic resonance, only a single atomic transition $(J,m)\rightarrow(J',m')$ may be resonant with the incoming frequency ⇒ effective two-level atom.
- Unusual situation where breaking time-reversal symmetry **increases** interference effects!
- Completely different from previous studies on massive materials:
  - Magnetic field of the order of few Gauss instead of Teslas;
  - The scatterers themselves are affected rather than the effective medium;
  - Highly nonlinear in B.
- Very complicated situation:
  - Raman scattering changes the frequency of light;
  - Very large number of possible transitions, i.e., complicated variations of the index of refraction with frequency;
  - The medium is no longer isotropic ⇒ the polarization changes during propagation;
  - Optical pumping may take place in the medium.

Restoration of the CBS interference induced by a magnetic field

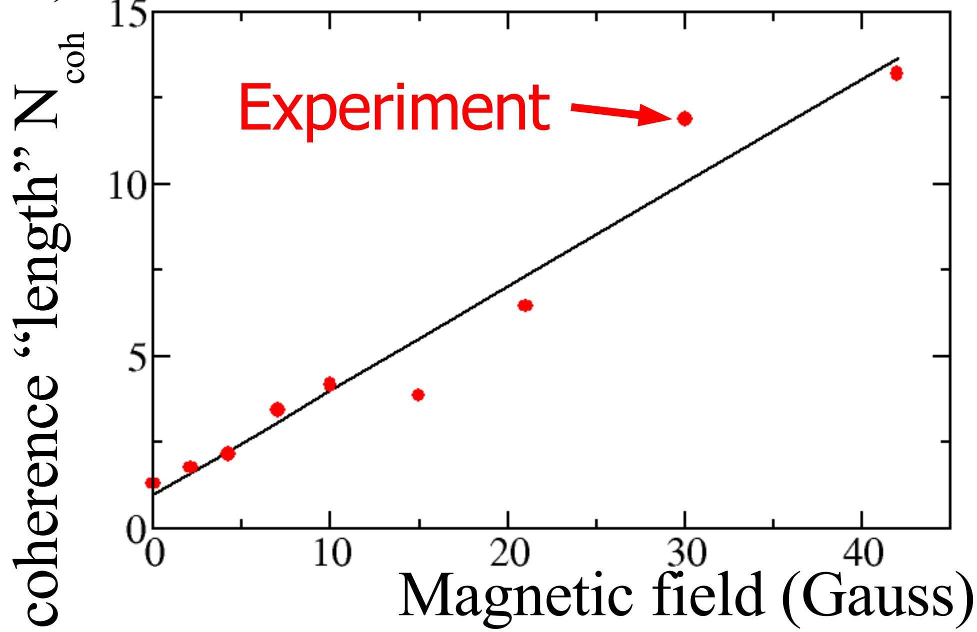


Dramatic increase of the enhancement factor from 1.05 to 1.33 for only few Gauss

O. Sigwarth et al,
Phys. Rev. Lett. (2004)

- From the Monte-Carlo calculation, extract the coherence length of the light in the cold atomic medium.
- The interference term/background ratio decreases like $\exp(-n/N_{coh})$

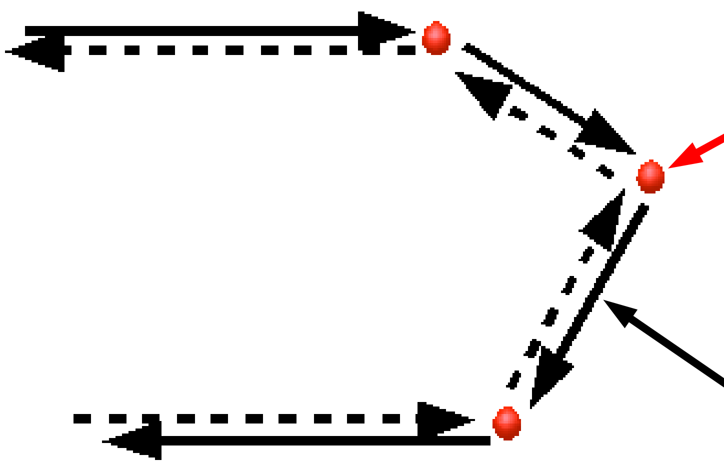
  n: number of scattering events
  N_{coh}: coherence "length".
- Study N_{coh} versus magnetic field.



Full contrast of the CBS interference is restored at large magnetic field

Effect of the residual velocity of the atoms

- Doppler effect ⇒ the scattered photon is not at the same frequency than the incoming photon.
- Recoil of the atom is small, but not completely negligible.
- On a pair of direct/reverse paths, the backscattered photons have the same frequency (≠from the incoming frequency) and thus still interfere. Intermediate frequencies (deep inside the scattering paths) are different.



Scattering cross-section and phase shifts are different along the direct and reverse paths

During the "free" propagation in the medium, the optical length and attenuation are different along the direct and reverse paths

- Imbalance between the two interfering amplitudes => interference terms and CBS are reduced.

Effect of a small residual atomic velocity

- Basic mechanism: frequency redistribution, which is diffusive at small atomic velocity $\Delta\omega^2 = Nk^2\bar{v}^2$, with N the scattering order.
- Compute interferential and non-interferential terms by averaging over the frequency diffusion.
- As long as $k\bar{v}\sqrt{N} \ll \Gamma$, frequency redistribution is smaller than Γ, the non interferential term is not affected;
- The interferential term is reduced, because the various phase shifts along the scattering path destroy phase coherence. Loss of contrast is:

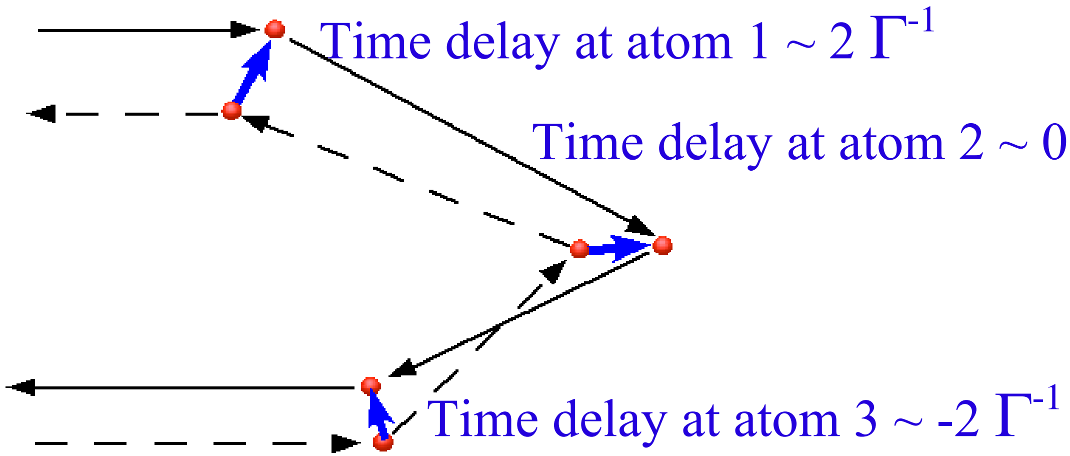
$$\exp\left(-\frac{k^2\bar{v}^2N^3}{12\Gamma^2}\right)$$

- Phase coherence length: $\boxed{L_\Phi = \ell \left(\frac{3}{2}\frac{k\bar{v}}{\Gamma}\right)^{-1/3}}$

Very severe restriction for any experiment on weak or strong localization of light in a cold atomic gas..

Destruction of CBS by the atomic motion

- Temporal point of view: atoms move between the arrival of the direct and the reverse photons.



Loss of contrast is:

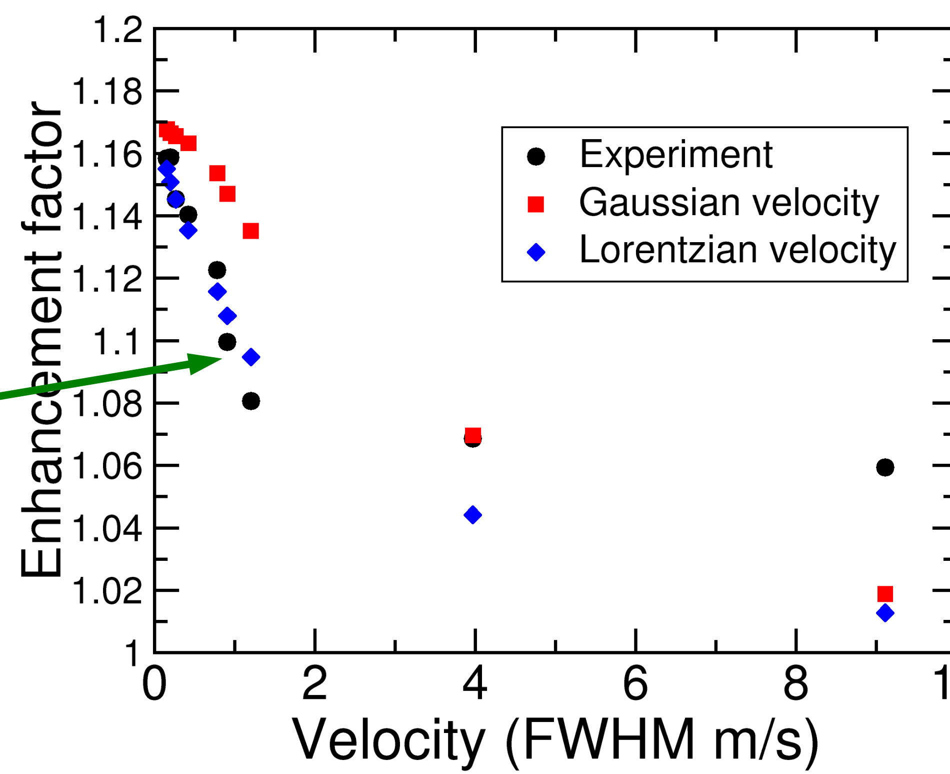
$$\exp\left(-\sum_{i=1..N}\frac{\langle(\vec{q}_i.\Delta\vec{r}_i(t_i))^2\rangle}{2}\right)$$

Labeyrie et al, PRL (2006)

- Experimental observation:
  - Loss of phase coherence is fast.

velocity = Doppler velocity/5

  - At low velocity, better agreement with Lorentzian velocity distribution.



- Atom-photon interaction
- Multiple scattering, transport and diffusion
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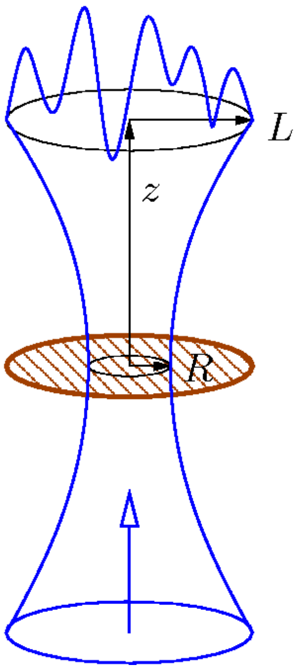
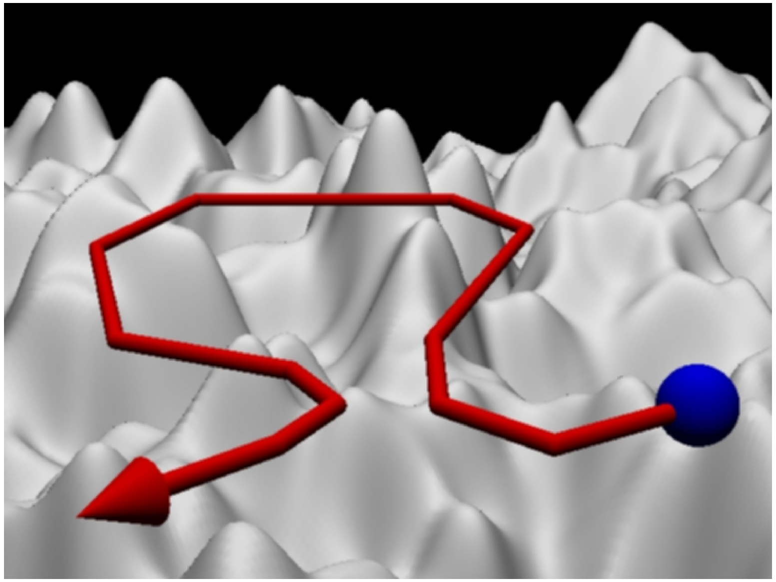
Scattering of matter waves by an optical potential

- Exchange roles of light and matter.
- Scatter matter waves by an optical potential. Example: diffraction of matter waves by a periodic optical potential.
- A disordered optical potential can be created using reflection of a far-detuned laser beam off a rough surface (speckle) or a suitable phase mask.

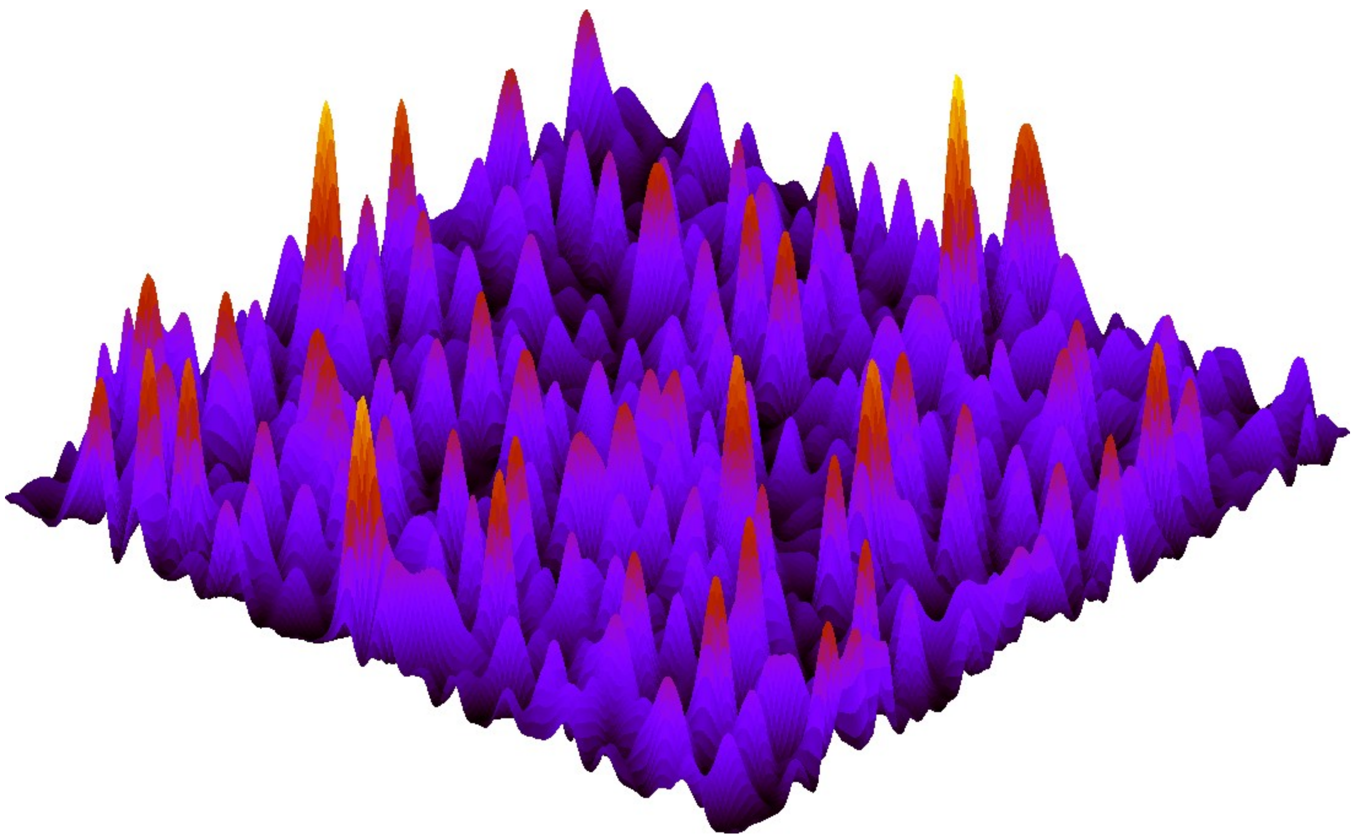




Use mesoscopic diagrammatic expansions to compute transport properties (mean free path, diffusion constant, weak localization, CBS...)

Major hypothesis: no atom-atom interaction

Typical "speckle" optical potential



Peaks can be turned into potential wells by changing the sign of the detuning δ_L : $\quad V(\mathbf{r}) \to -V(\mathbf{r})$

Properties of the optical potential

- The effective potential seen by the cold atoms is disordered, but with specific correlation functions.

Effective potential $\boxed{V(\vec{r}) = \frac{\hbar\Gamma}{8}\frac{\Gamma}{\delta_{\mathrm{L}}}\frac{I(\vec{r})}{I_{\mathrm{s}}}}$

δ_{L} : detuning

I_{s} : saturation intensity

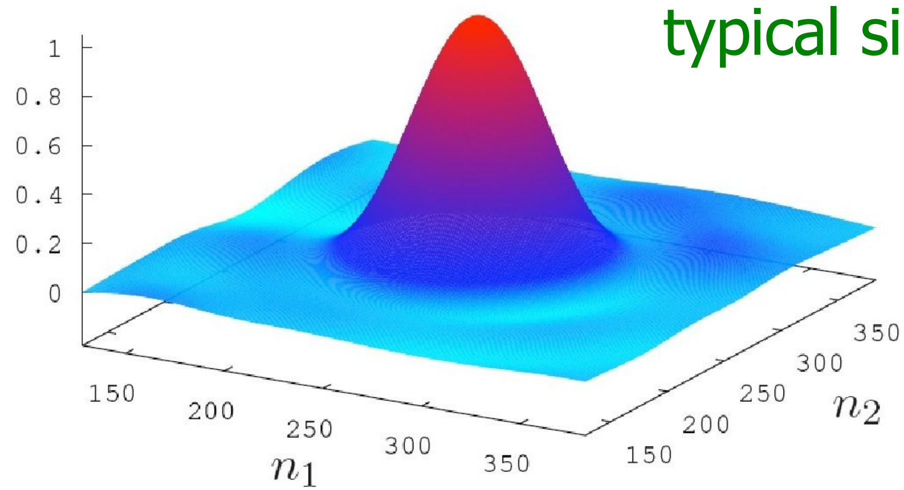
- Mean value: V_{L}; Correlation function (in 2D):

$$\langle V(\vec{r'})V(\vec{r'}+\vec{r})\rangle = V_{\mathrm{L}}^2\left(1+4\frac{J_1^2(\alpha k_{\mathrm{L}} r)}{\alpha^2 k_{\mathrm{L}}^2 r^2}\right)$$

k_{L}: laser wave vector

J_1 : Bessel function

with α the numerical aperture of the device imaging the speckle, typically α=0.1



typical size : correlation length

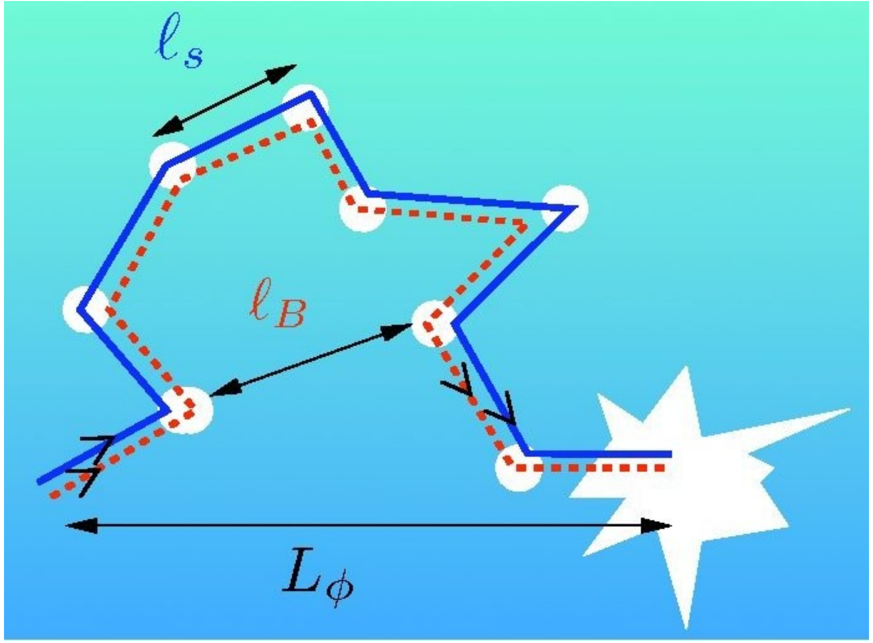
$$\zeta = \frac{1}{\alpha k_{\mathrm{L}}}$$

Beyond Hamiltonian dynamics

- Coupling with other modes of the electromagnetic field (reservoir) => spontaneous emission breaks the phase coherence of the matter wave.
- Rate is given by:

$$\Gamma_\phi \approx \frac{\Gamma}{\delta_{\mathrm{L}}} \frac{V(\mathbf{r})}{\hbar}$$

- Small far from resonance



- Coherence length:

$$L_\phi = \frac{v_{\mathrm{atom}}}{\Gamma_\phi}$$ ballistic regime

$$L_\phi = \sqrt{\frac{D_B}{\Gamma_\phi}}$$ diffusive regime

Statistical properties of the optical potential

- Important length scale: correlation length of the potential

$$\zeta = \frac{1}{\alpha k_{\mathrm{L}}}$$ typically few μm

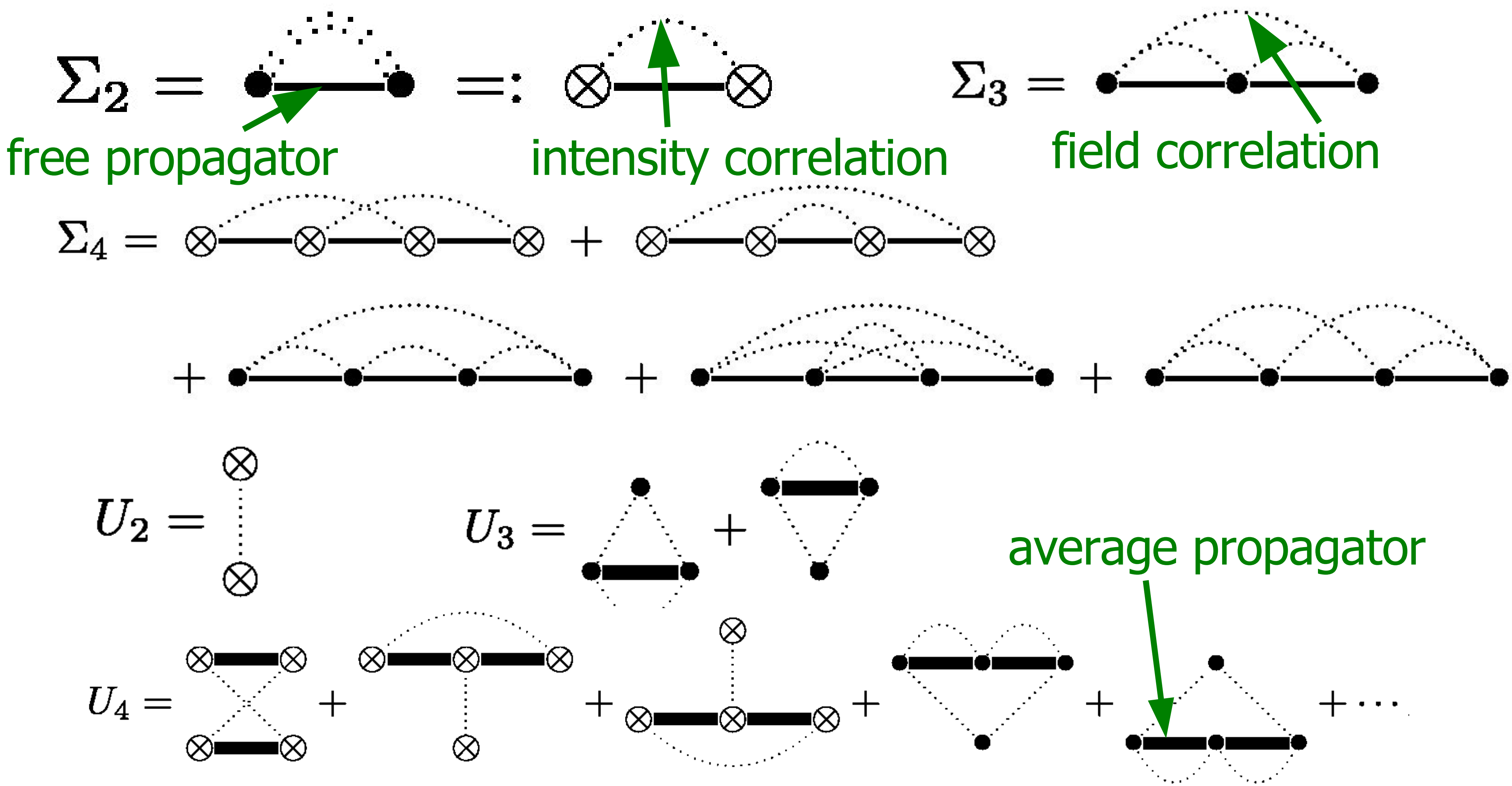
- Associated correlation energy:

$$E_\zeta = \frac{\hbar^2}{m\zeta^2} = 2\alpha^2 E_R \quad \text{with} \quad E_R = \frac{\hbar k_L^2}{m} \quad \text{the recoil energy}$$

- Typical orders of magnitude (Rubidium atom):
  - Recoil velocity : few mm/s
  - Recoil energy : few kHz (1 μK)
  - Correlation energy : few tens Hz (few nK)

Diagrammatic methods in the weak scattering regime

- Unusual features:
  - Non Gaussian fluctuations of the optical potential (∓intensity)
  - Spatial correlations of the field and the intensity
- Requires to take into account additional diagrams



Energy scales

- Correlation energy E_ζ

  Potential strength V_L

  Total energy of the atom $E = \dfrac{\hbar^2 k^2}{2m}$
- Dimensionless parameters:

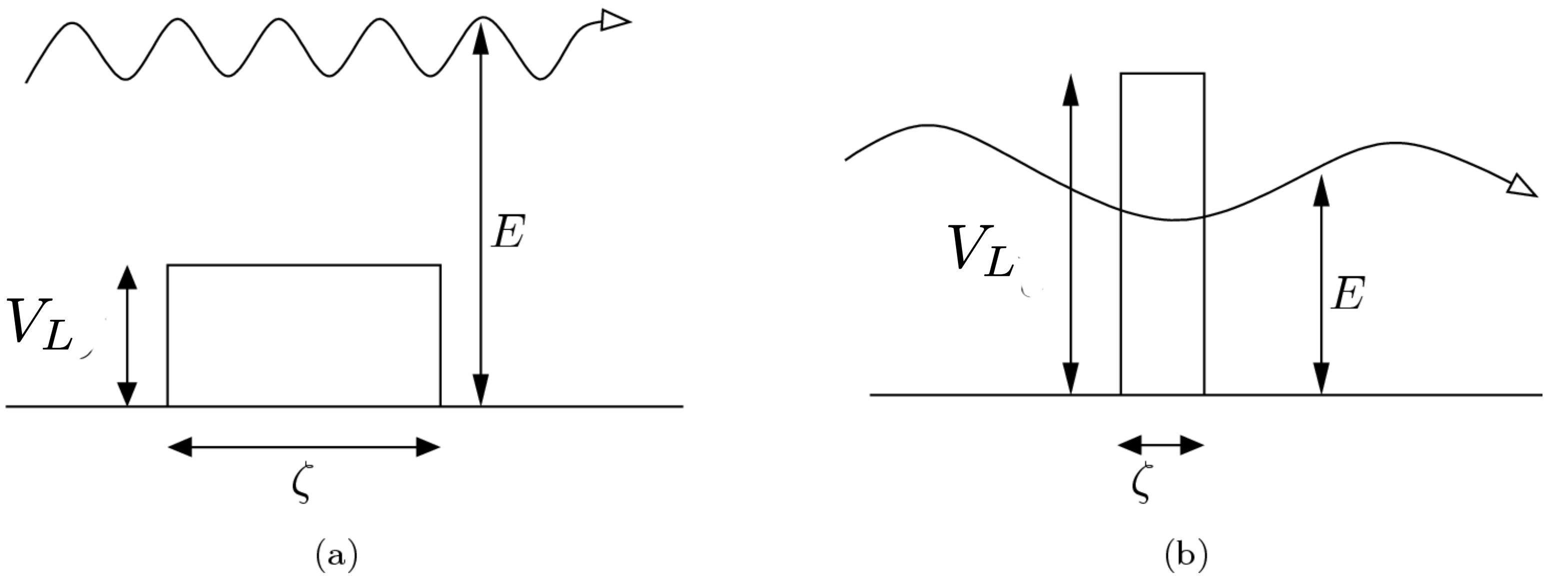
$$\boxed{\eta = \frac{V_\mathrm{L}}{E_\zeta}} \qquad \boxed{\Delta = \frac{V_\mathrm{L}^2}{E\ E_\zeta}}$$

- The weak scattering condition (~ dilute regime) for applicability of the diagrammatic expansion is:

$$\Delta \ll 1$$

Weak scattering approximation

- Two interesting limiting situations:
  - (a) Large potential fluctuations η >1. Weak scattering implies $E \gg V_L$: the atom flies well above the potential fluctuations. The de Broglie wavelength resolves all the details of the potential => essentially classical motion.
  - (b) Small potential fluctuations η <1. Weak scattering condition can be satisfied even if $E < V_L$, i.e. when atom energy lies below the average potential height. The matter wave averages out the short-range fluctuations of the potential ⇒ quantum regime.



Calculation of the mean free path

- Straightforward calculation in the weak scattering approximation (Born approximation):

$$\frac{1}{k\ell_S} = \frac{\eta^2}{k^2\zeta^2}\int \frac{d\Omega}{2\pi}\mathcal{P}(k\zeta,\theta)$$

with $\mathcal{P}(|\mathbf{k}-\mathbf{k}'|) = \zeta^2\mathcal{P}(k\zeta,\theta)$ the Fourier transform of the correlation function of the optical potential

$$\mathcal{P}(\mathbf{r}) = \left[\frac{2J_1(r/\zeta)}{r/\zeta}\right]^2$$

- Similar expression for transport mean free path:

$$\frac{1}{k\ell_B} = \frac{\eta^2}{k^2\zeta^2}\int \frac{d\Omega}{2\pi}(1-\cos\theta)\mathcal{P}(k\zeta,\theta)$$

- Boltzmann diffusion constant:

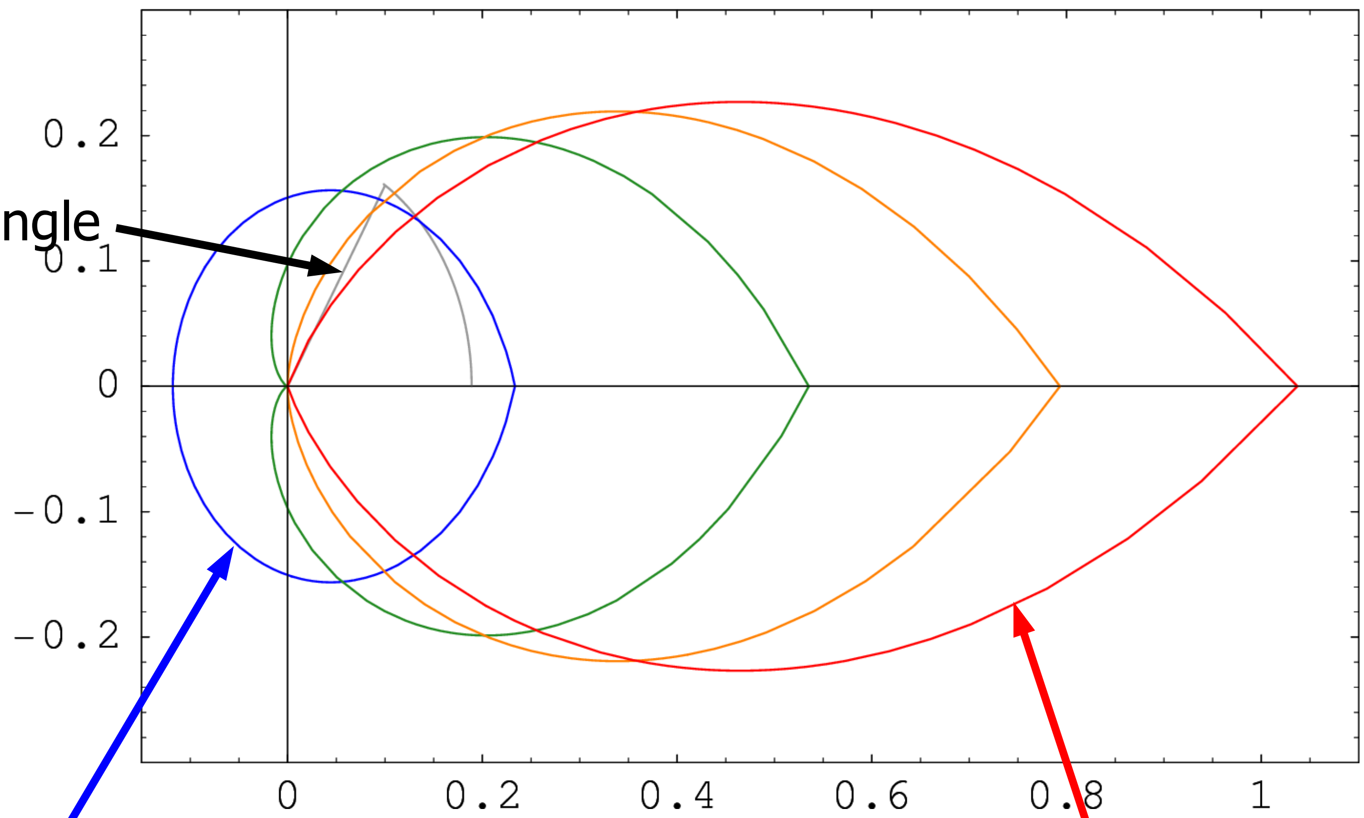
$$D_B(k) = \frac{\hbar k}{2m}\ell_B(k)$$

- Similar expressions in 3 dimensions

Scattering cross-section

- Isotropic at low energy $E \ll E_\zeta \quad k\zeta \ll 1$
- Peaked in the forward direction when energy increases

Maximum scattering angle



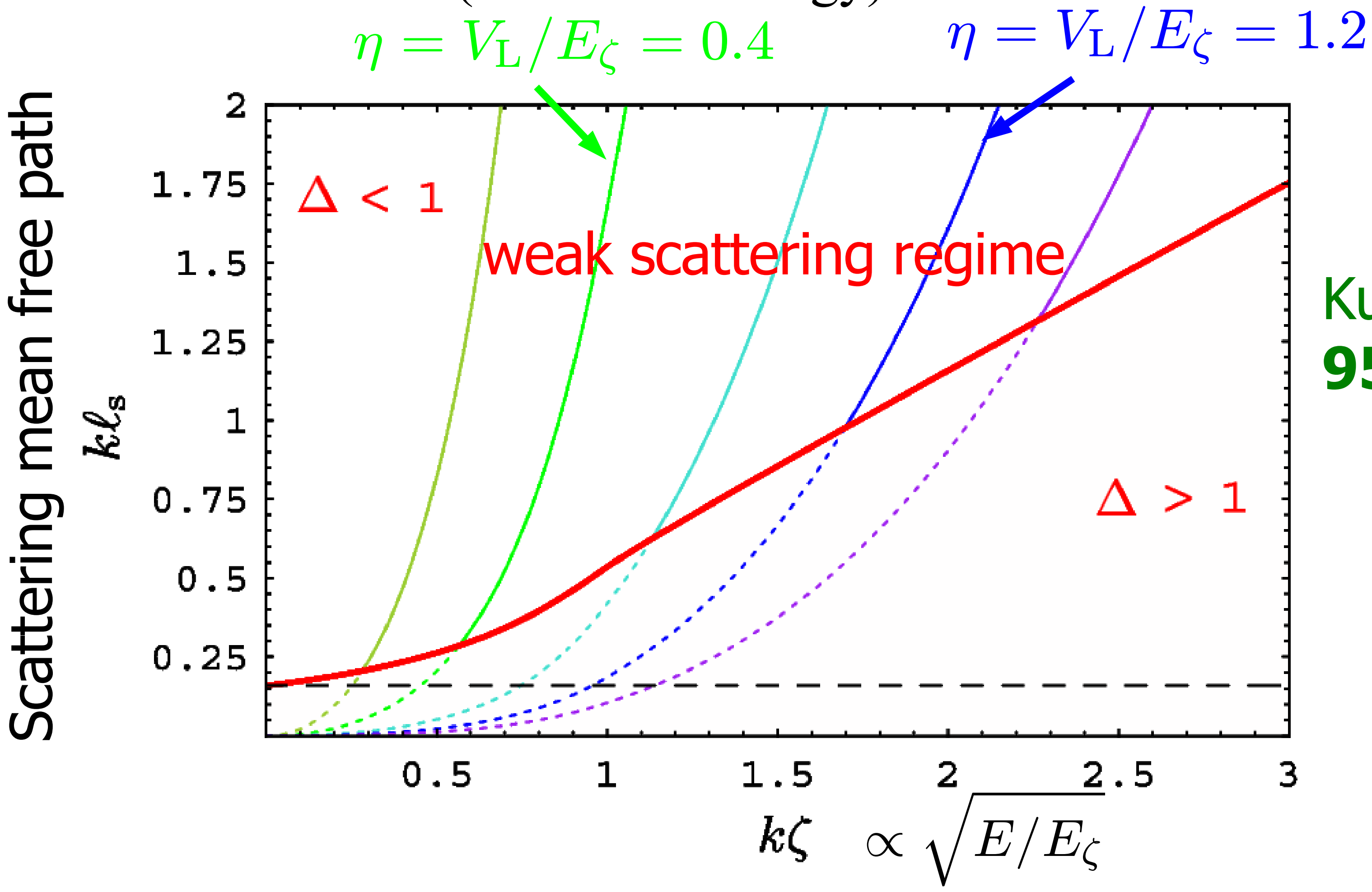
Low energy kζ=0.4

High energy kζ=1.4

Phase function (2D)

Incoherent (diffusive) transport of matter waves

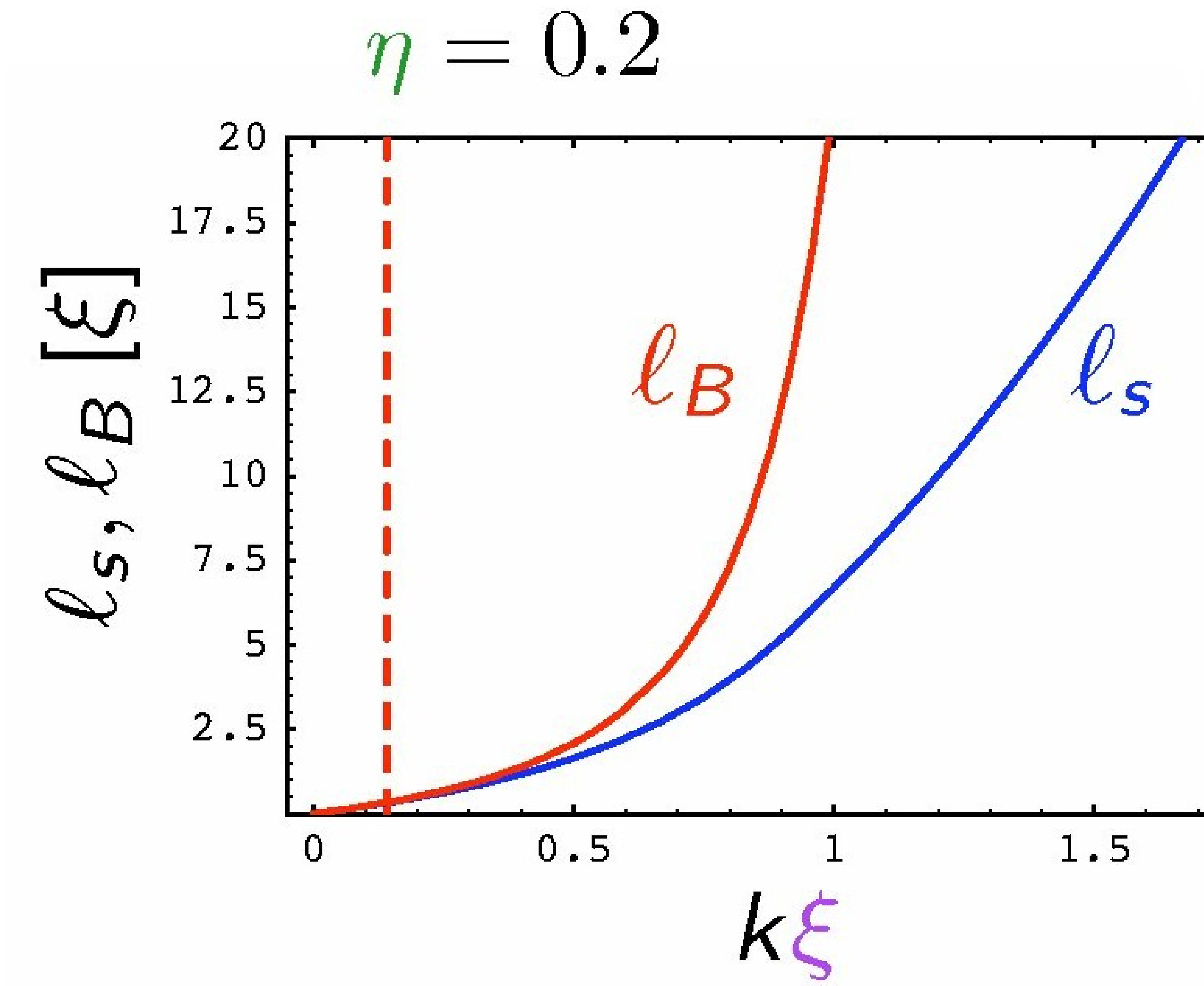
- Scattering mean free path ℓ_S
  Impossible to make a short mean free path at large energy => ultra-cold atoms (sub-recoil energy) are needed!



Kuhn et al, PRL **95**, 250403 (2005)

Incoherent (diffusive) transport of matter waves

- Transport mean free path ℓ_B



Kuhn et al, New J. Phys. 9, 161 (2007)

$$\eta = \frac{V_L}{E_\zeta} = 0.2$$

- Atom-photon interaction
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- Coherent backscattering of light by cold atoms
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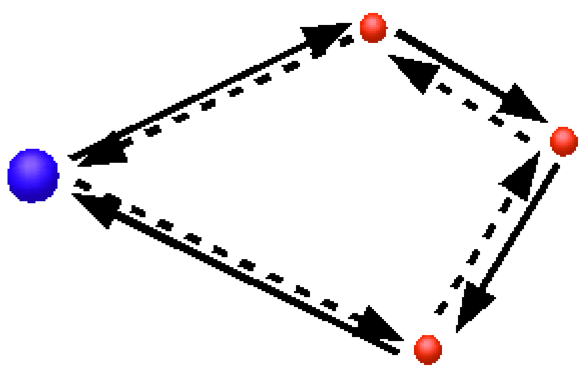
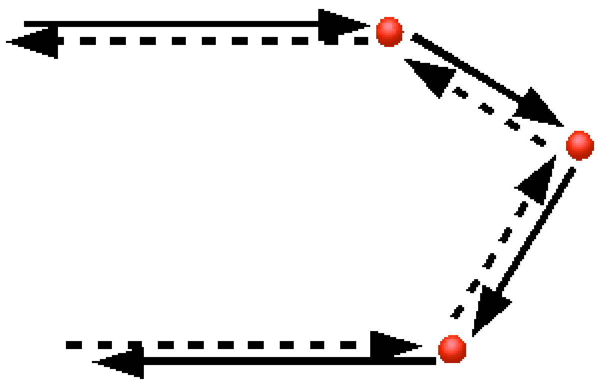
- Weak localization of matter waves
- Towards strong localization of matter waves

Weak localization

- Effect similar to coherent backscattering, but in the bulk of the medium.
- Constructive interference between pairs of time reversed scattering paths starting and ending at the same point.



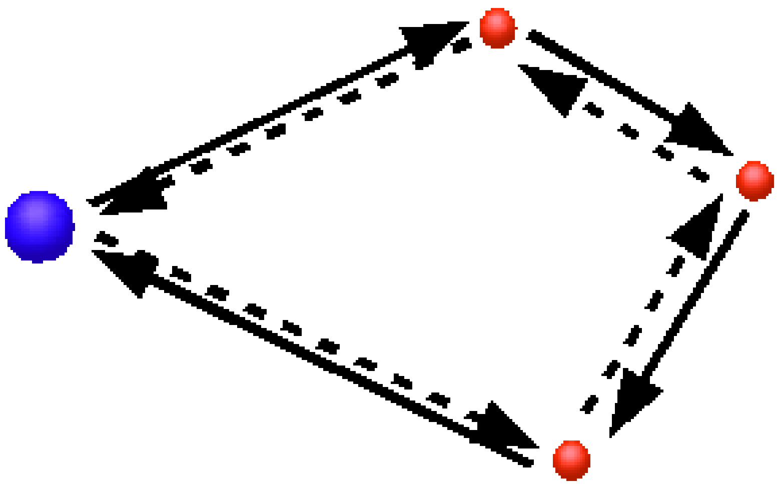
weak localization

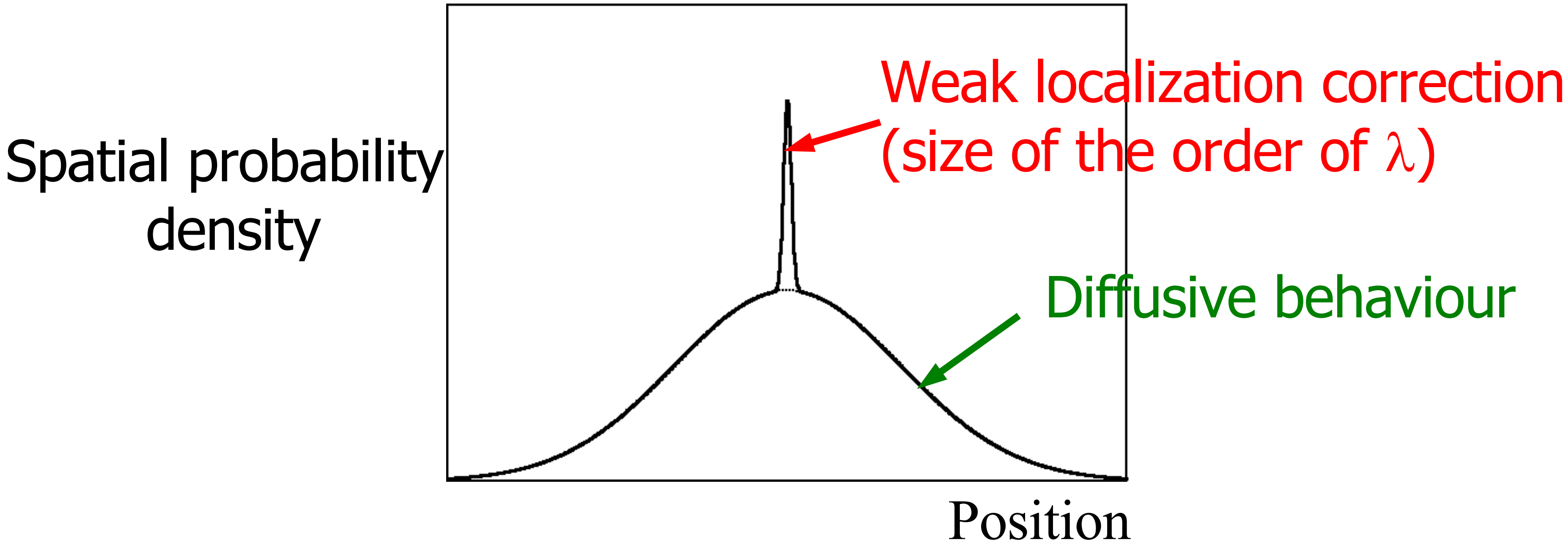


coherent backscattering

- Increased probability of return (compared to diffusive non-interferential transport) => decreased probability to leave. The diffusion constant is smaller than the Boltzmann diffusion constant.
- The relevant (small) parameter is $\dfrac{1}{k\ell_B}$
- Depends on the dimension of the system.

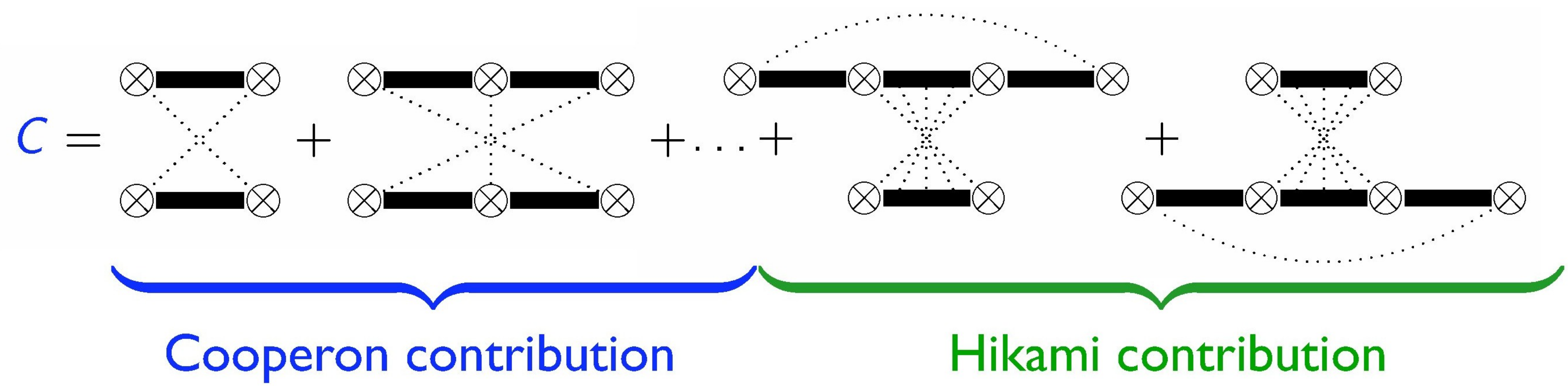
Weak localization





Weak localization

- More generally, closed loops slow down the transport => weak localization:
  - Relevant parameter $\boxed{\dfrac{\lambda}{\ell_B} \sim \dfrac{1}{k\ell_B}}$
- Associated diagrams



- In two dimensions $D = D_B\left(1 - \dfrac{2}{\pi k \ell_B}\log\dfrac{L_\star}{\ell_B}\right)$

with $\dfrac{1}{L_\star^2} = \dfrac{1}{L_\phi^2} + \dfrac{1}{L^2}$

phase coherence length

size of the medium

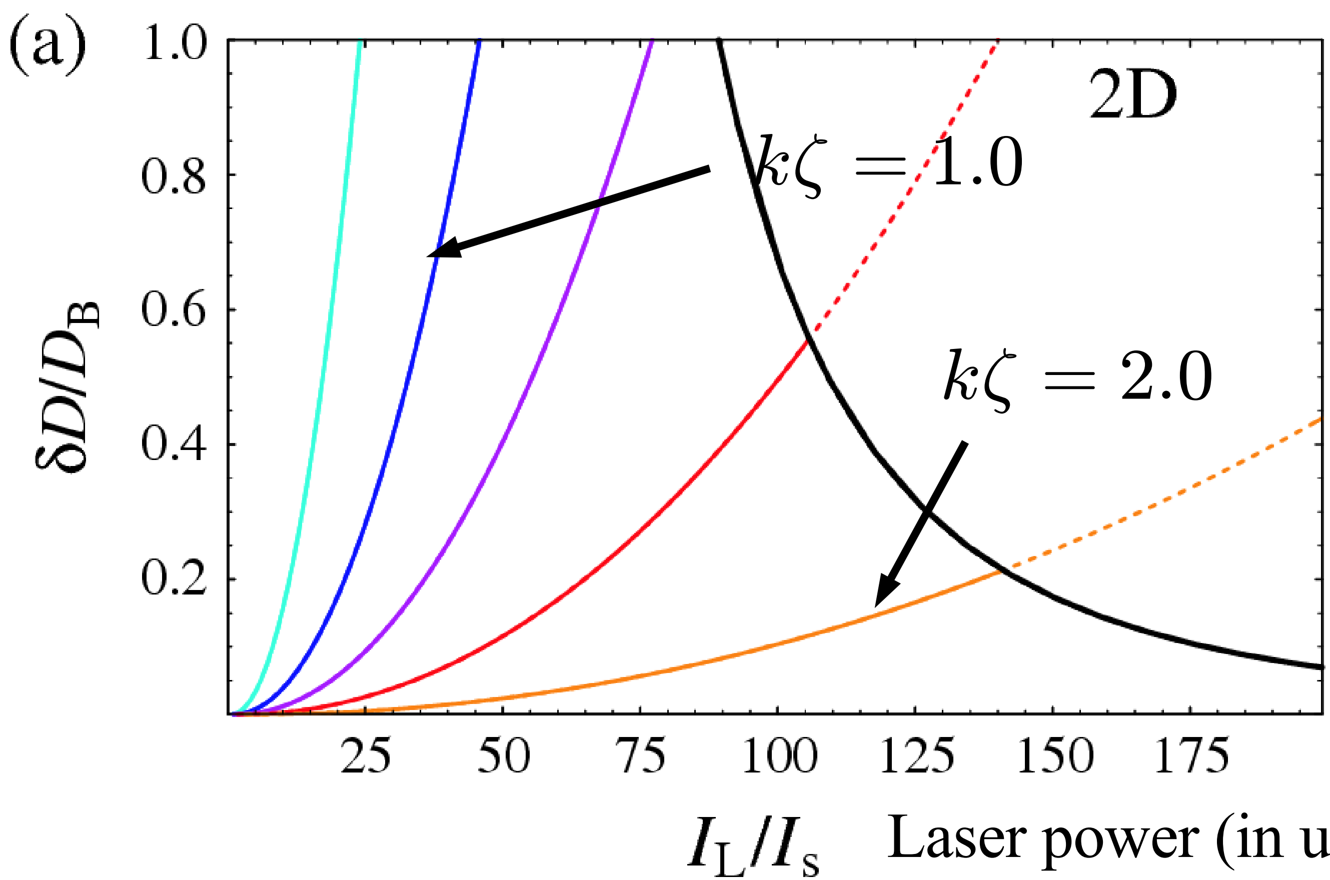
Weak localization of matter waves

- Using diagrammatic expansions, one can compute the weak localization correction to the Boltzmann (incoherent) diffusion constant

$$\frac{\delta D}{D_B} = \frac{2}{\pi}\frac{\log(L_\phi/\ell_B)}{k\ell_B}$$

ℓ_B : transport mean free path

L_ϕ: coherence length in the medium



Laser power (in units of saturation intensity)

Example for Rb atoms

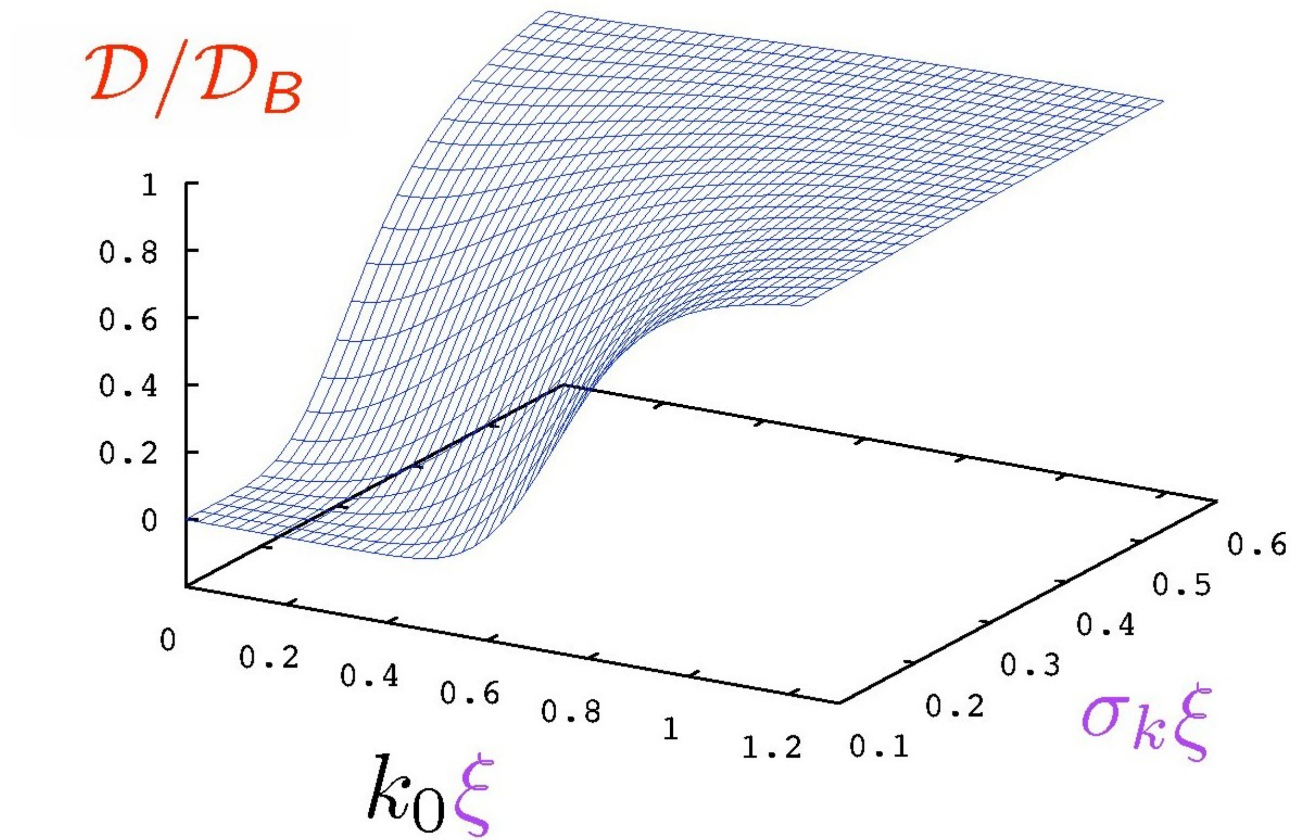
Detuning $\delta=10^6\Gamma$
Speckle size=2cm

Weak localization correction can be large!

Kuhn et al, New J. Phys. 9, 161 (2007)

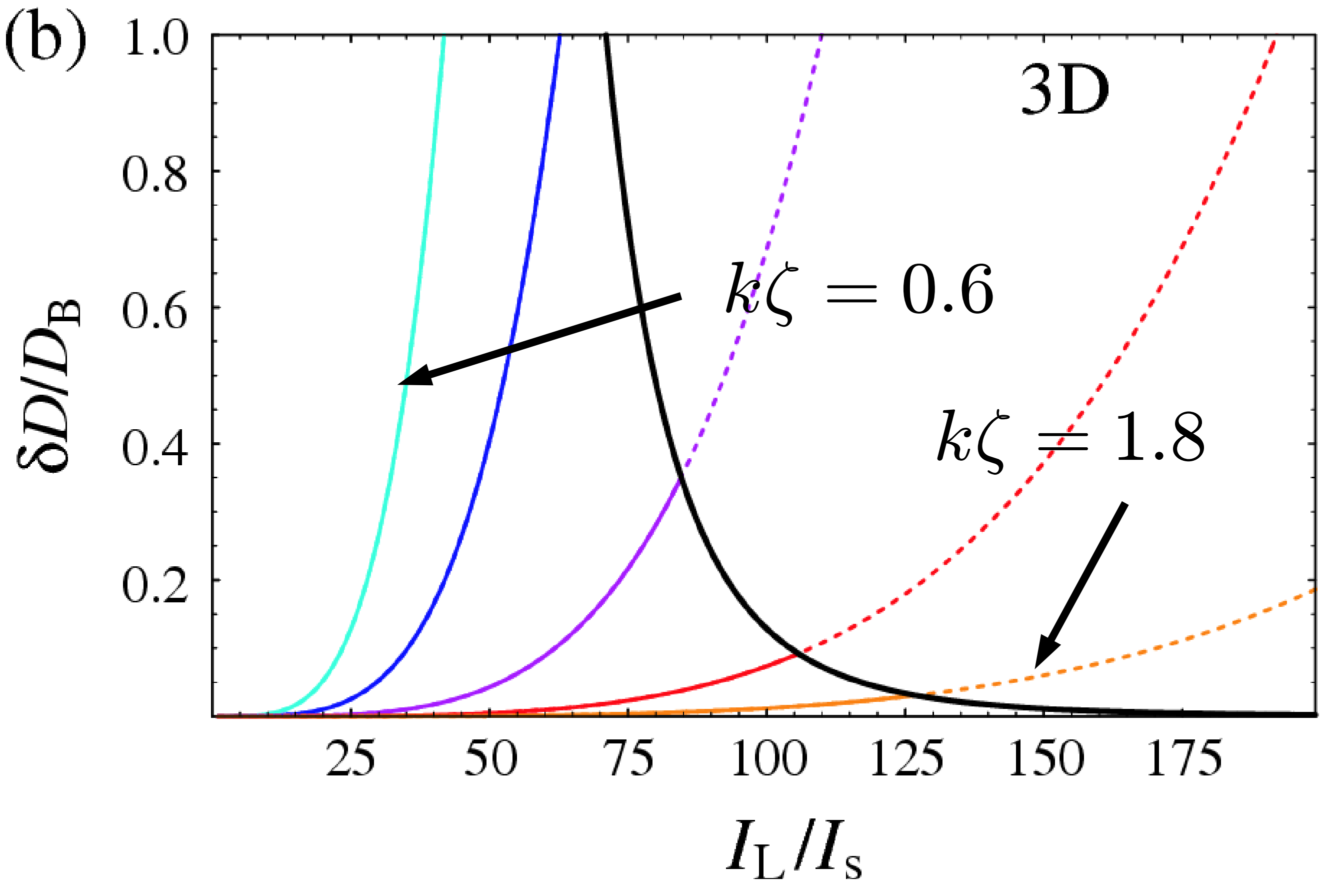
Weak localization for non-monochromatic atoms

- Assume Gaussian distribution of atomic velocity, with average momentum k_0 and variance σ_k.
- Compute spatial spreading of an initially well localized cloud.
- Visible if $k_0\zeta$ and $\sigma_k\zeta$ are slightly smaller than 1.



Weak localization of matter waves

- In 3 dimensions $$\frac{\delta D}{D_B} = \frac{3}{\pi}\frac{1}{(k\ell_B)^2}$$



Laser power (in units of saturation intensity)

Example for Rb atoms

Detuning $\delta=10^4\Gamma$
Speckle size=2cm

Weak localization correction can be large!

Kuhn et al, New J. Phys. 9, 161 (2007)

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Strong localization of matter waves

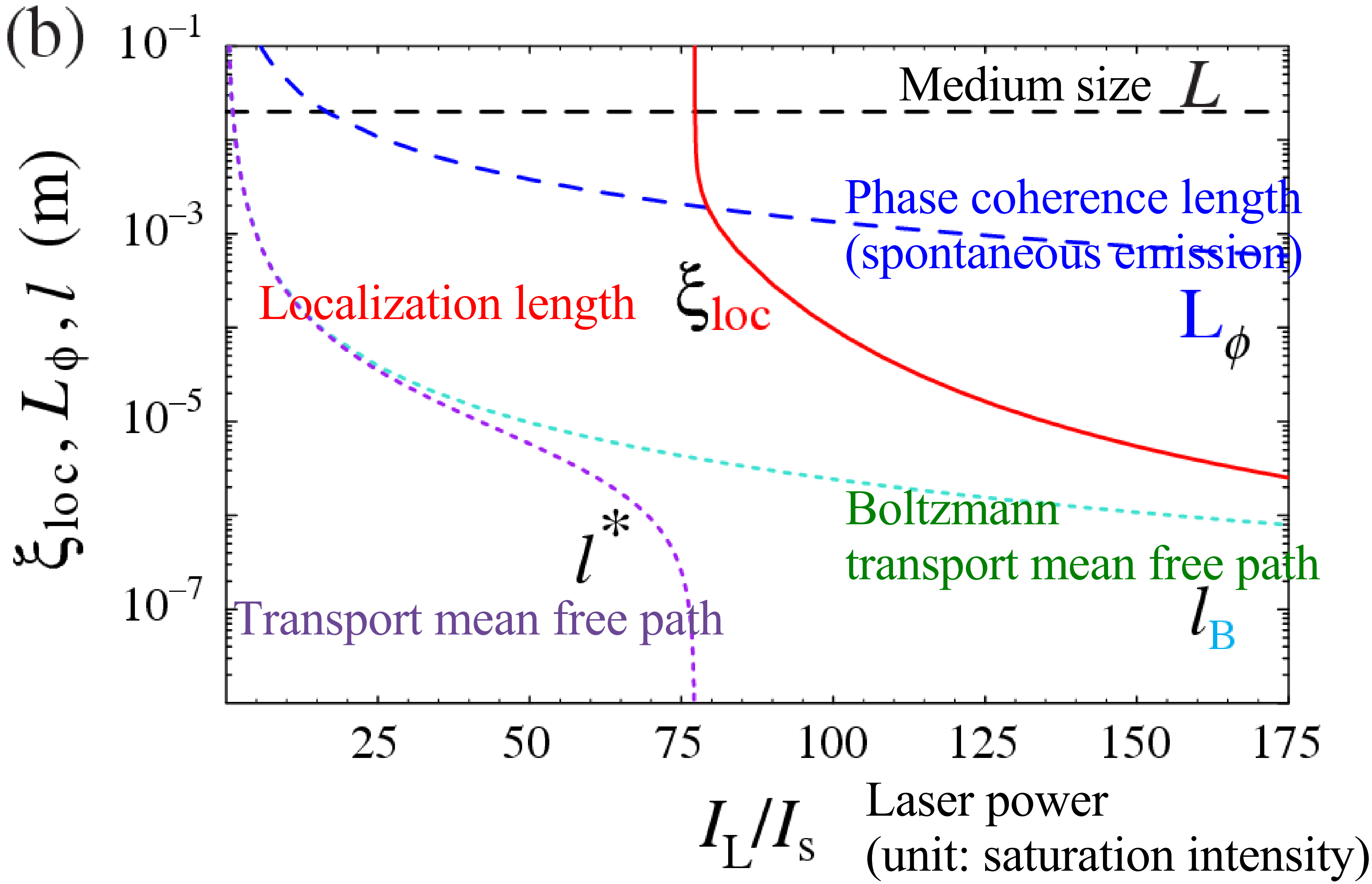
- Strong (Anderson) localization is when the diffusion constant vanishes.
  - 1D: always strong localization, for any disorder
  - 2D: critical case, exponentially long localization length
  - 3D: delocalized for weak disorder, localized for strong disorder
- Rough criterion: strong localization takes place when the weak localization correction is 100% (self-consistent theory)
- In two dimensions: $$\frac{\delta D}{D_B} = \frac{2}{\pi} \frac{\log(L_\star/\ell_B)}{k\ell_B}$$

  Onset of strong localization $$\boxed{L_\star = \ell_B \exp\left(\frac{\pi}{2} k\ell_B\right)}$$
- In the localized regime, the localization length ξ_{loc} is given by

$$\frac{1}{L_\star^2} = \frac{1}{L_\phi^2} + \frac{1}{L^2} + \frac{1}{\xi_{\mathrm{loc}}^2} \qquad \text{where } L_\star = \ell_B \exp\left(\frac{\pi}{2} k\ell_B\right)$$

Example of strong localization in 2D

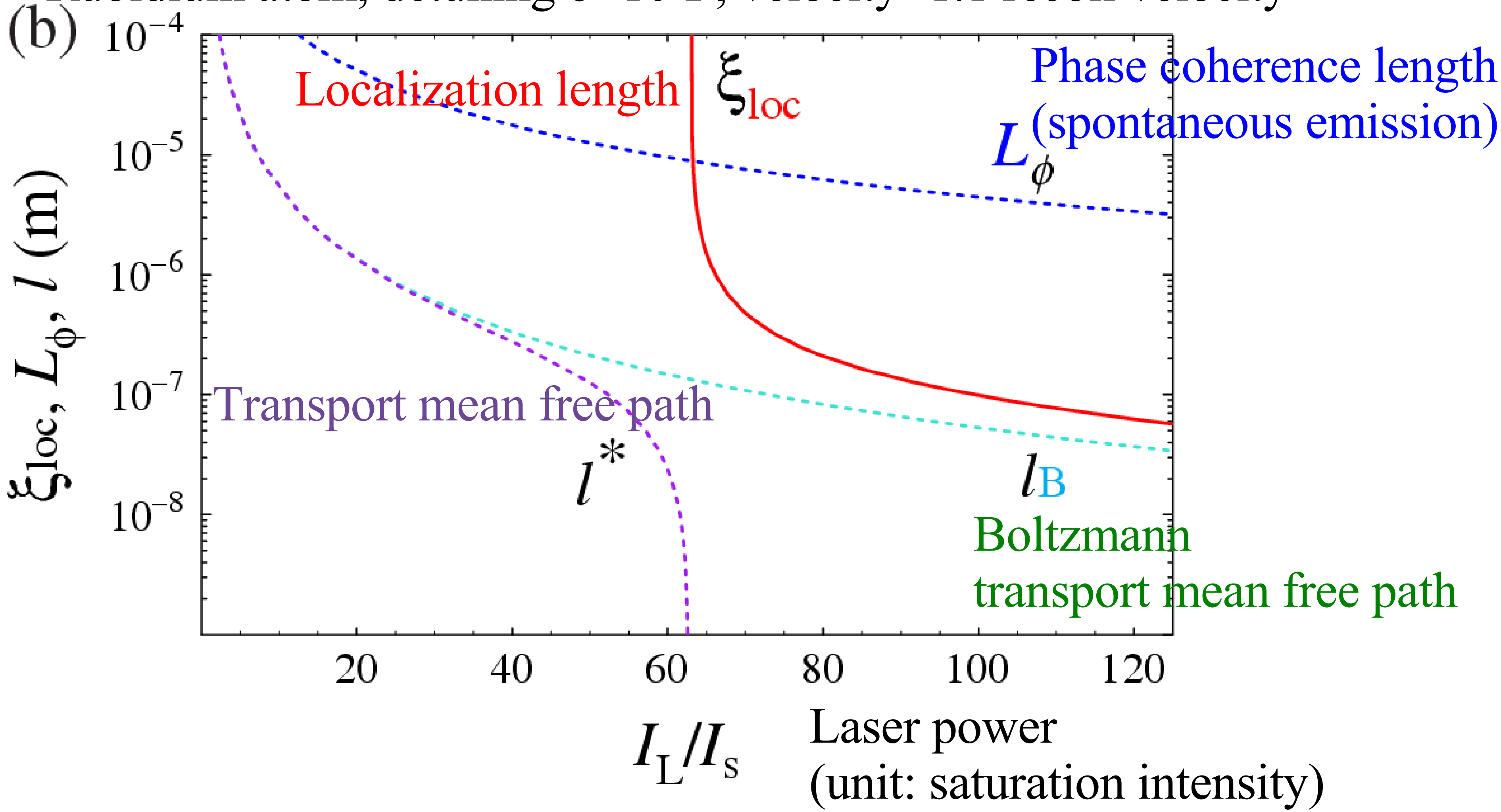
- Rubidium atom, detuning $\delta=10^6\Gamma$, velocity=recoil velocity/8.



Kuhn et al, PRL, 95, 250403 (2005) and New J. Phys. 9, 161 (2007)

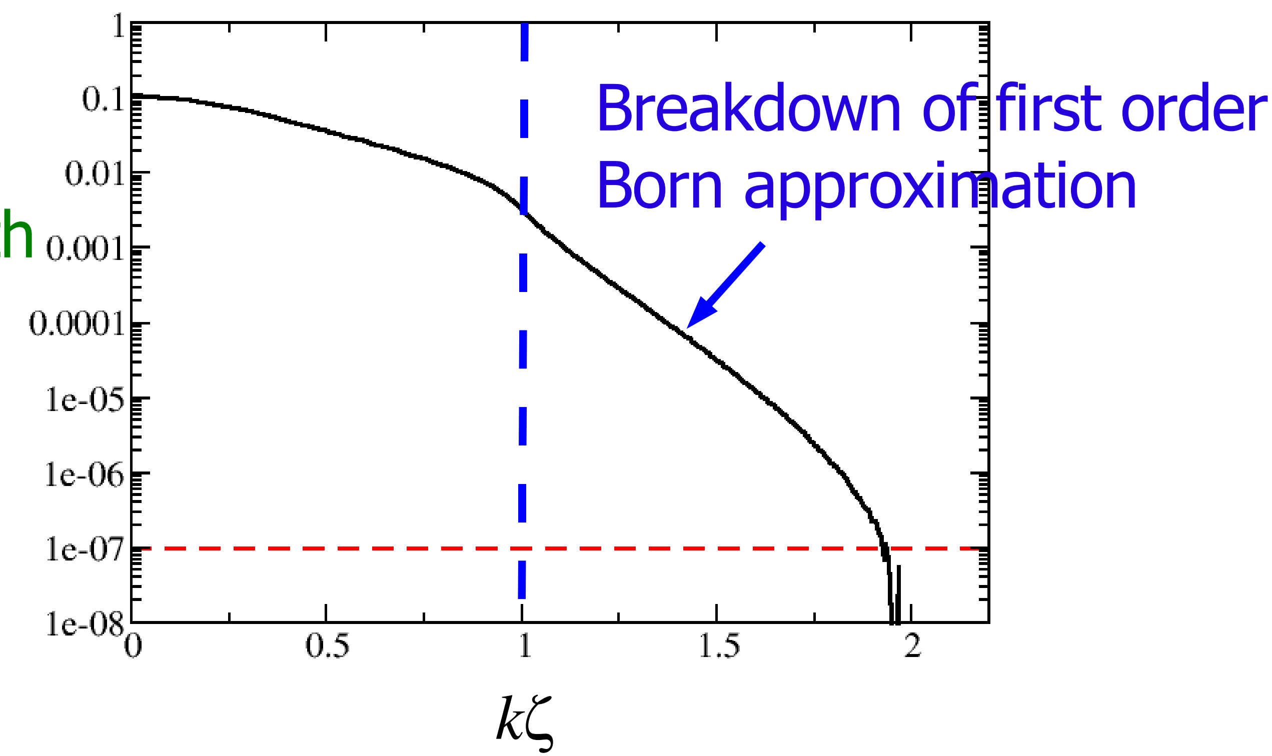
Strong localization in 3D

- Onset of strong localization (infinite system, perfect phase coherence) $\boxed{k\ell_B = \sqrt{\frac{3}{\pi}}}$ Ioffe-Regel criterion
- Rubidium atom, detuning $\delta=10^4\Gamma$, velocity=1.1 recoil velocity



Localization in one dimension

- Strong transverse confinement of cold atoms (see Ph. Bouyer's talk).
- Generic behaviour: exponential localization at any energy, for arbitrary small and correlated disorder.
- Because the Fourier transform of the potential has compact support, the localization length diverges at lowest order of the Born approximation if $k\zeta>1$ (see L. Sanchez-Palencia's talk).
- Using transfer matrix method, it is easy to numerically compute the localization length.

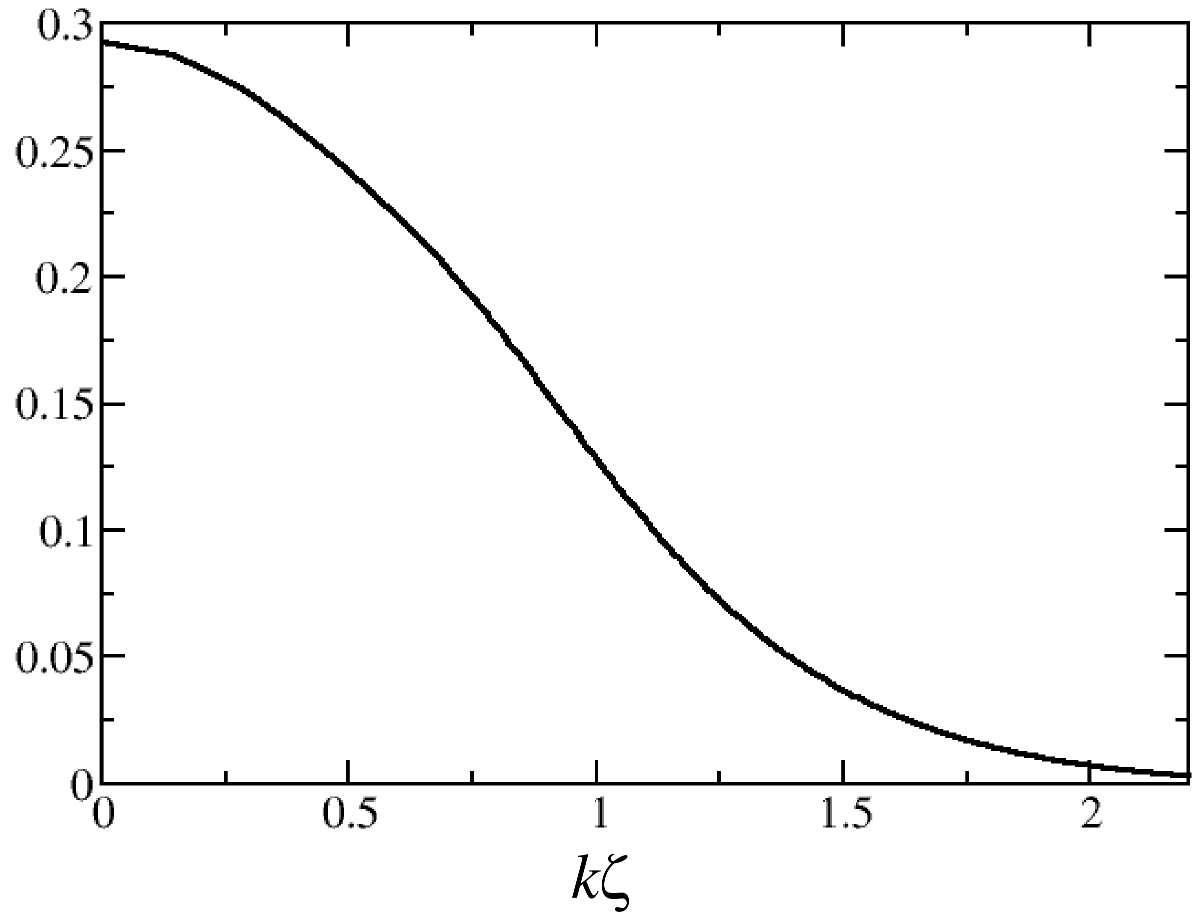


Localization in one dimension

- At larger height of the optical potential η, nothing spectacular happens at $k\zeta=1$

1/localization length

$\eta=0.5$



B.Grémaud, private communication

Experimental results?

- Several experiments performed with quasi-1D ultra-cold atomic gases in a disordered optical potential:
  - Orsay (Bouyer et al)
    - D. Clément et al, PRL 95, 170409 (2005)
    - D. Clément et al, NewJournal of Physics 8, 165 (2006)
    - L. Sanchez-Palancia et al, cond-mat/0612670 and 0610389
  - Florence (Inguscio's group)
    - J.E. Lye et al, PRL 95, 070401 (2005)
    - L. Fallani et al, cond-mat/0603655
  - Hannover (Ertmer's group)
    - T. Schulte et al, PRL 95, 170411 (2005)
- The three experiments are in the regime where atom-atom interaction is dominant.
- A BEC (Bose-Einstein condensate) in the weak interaction regime, in the mean-field approach can be described by a single wavefunction obeying a non-linear Gross-Pitaevskii equation:

$$\left(\frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) + \underbrace{g|\psi(\mathbf{r})|^2}_{\text{Interaction term}}\right)\psi(\mathbf{r}) = \mu\psi(\mathbf{r})$$

μ: chemical potential

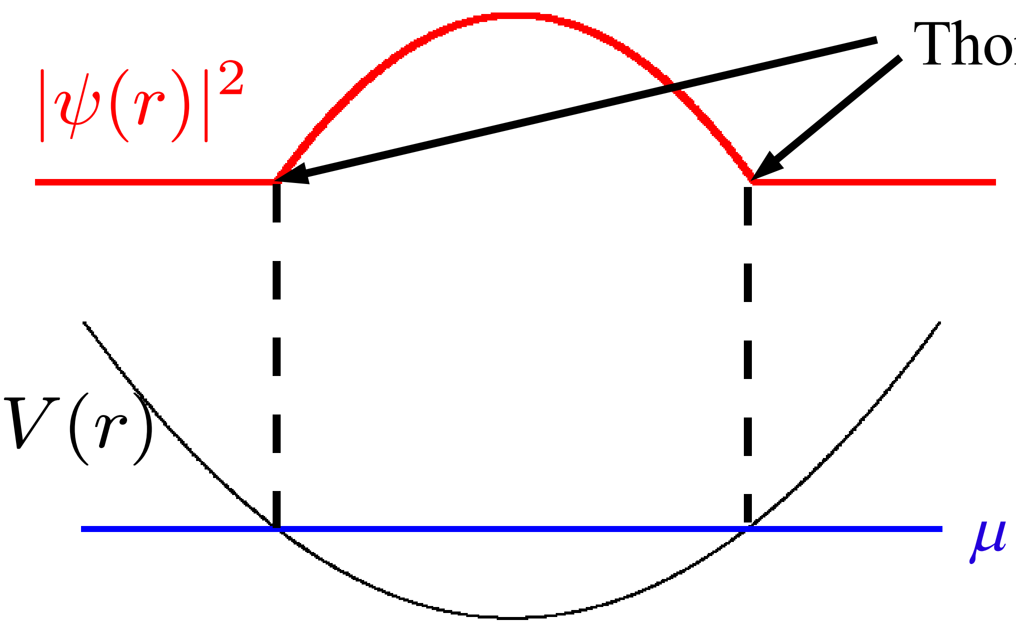
Thomas-Fermi regime

- For large atom-atom interaction, the kinetic energy is negligible

$$\left(\cancel{\frac{\mathbf{p}^2}{2m}} + V(\mathbf{r}) + g|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}) = \mu\psi(\mathbf{r})$$

$$|\psi(\mathbf{r})|^2 = \frac{\mu - V(\mathbf{r})}{g}$$ for $\mu > V(\mathbf{r})$, 0 otherwise

No tunneling, no de Broglie wavelength, no quantum effect!



In the presence of disorder, the condensate accumulates at the potential minima and the interaction term smoothes out short range potential fluctuations => “trivial localization”

Summary and perspectives

- Cold atoms can be used for studies of transport and localization in complex disordered systems. Advantages: good control, convenient time and length scales...
- Light propagation in a cold atomic gas:
  - Radiation trapping (no interference). Slowing down of photons because of highly resonant interaction.
  - Interference effects may survive in disordered systems: coherent backscattering, (weak localization)...
  - Various phenomena limit the phase coherence: internal atomic structure, residual atomic velocity. Observation of strong localization is not straightforward.
- Matter wave in disordered optical potential:
  - Independent atoms: (almost) everything can be calculated. Observation of weak localization would require ultra-cold atoms. Strong localization should not be much more difficult.
  - With atom-atom interaction: more difficult problem. Work in progress when interaction is a perturbation.

Atomic dynamics

- For simplicity, consider a two-level atom: $|g\rangle$ and $|e\rangle$
- Interaction with a monochromatic (NOT single mode) laser field, $\mathbf{E}(\mathbf{r})\cos\omega_{\mathrm{L}} t$ close to $|g\rangle \rightarrow |e\rangle$ resonance.
- Hamitonian is (**D** is the dipole operator):

$$H = \frac{\mathbf{p}^2}{2m} + \hbar\omega_0|e\rangle\langle e| - \mathbf{D}.\mathbf{E}(\mathbf{r},t) + H_R - \mathbf{D}.\mathbf{E}_R(\mathbf{r})$$

Atomic kinetic energy

Excited state contribution

Interaction with laser field

Interaction with other modes of the electromagnetic field (reservoir)

- Far from resonance $\delta_L = \omega_L - \omega_0 \gg \Gamma$ the coupling with the reservoir can be neglected.
- Rotating wave approximation and write

$$|\psi\rangle = \psi_g|g\rangle + \psi_e \exp(-i\omega_{\mathrm{L}}t)|e\rangle$$

Atomic dynamics

- Then

$$i\hbar\partial_t\psi_g = -\frac{\hbar^2}{2m}\nabla^2\psi_g + \frac{\hbar\Omega^*(\mathbf{r})}{2}\psi_e$$

$$i\hbar\partial_t\psi_e = -\frac{\hbar^2}{2m}\nabla^2\psi_e + \frac{\hbar\Omega(\mathbf{r})}{2}\psi_g - \hbar\delta_L\psi_e$$

with $\hbar\Omega(\mathbf{r}) = -\mathbf{d}.\mathbf{E}(\mathbf{r})$ Rabi frequency $\mathbf{d} = \langle e|\mathbf{D}|g\rangle$

- Low intensity (no saturation) $\psi_e(\mathbf{r}) \approx \psi_g(\mathbf{r})\ \Omega(\mathbf{r})/2\delta_L$
  - $\psi_g(\mathbf{r})$ obeys an effective Schrödinger equation with:

$$H_g = \frac{\mathbf{p}^2}{2m} + \frac{\hbar|\Omega(\mathbf{r})|^2}{4\delta_L}$$

  - Optical potential:

$$\boxed{V(\mathbf{r}) = \frac{\hbar|\Omega(\mathbf{r})|^2}{4\delta_L} = \frac{\hbar\Gamma}{8}\frac{\Gamma}{\delta_L}\frac{I(\mathbf{r})}{I_s}}$$

$I(\mathbf{r})$: laser intensity

I_s : saturation intensity