Disordered Quantum Systems

Boris Altshuler
Physics Department, Columbia University and NEC Laboratories America

Collaboration: Igor Aleiner , Columbia University

Part 1: Introduction

Part 2: BCS + disorder

INSTITUT HENRI POINCARE

Centre Emile Borel

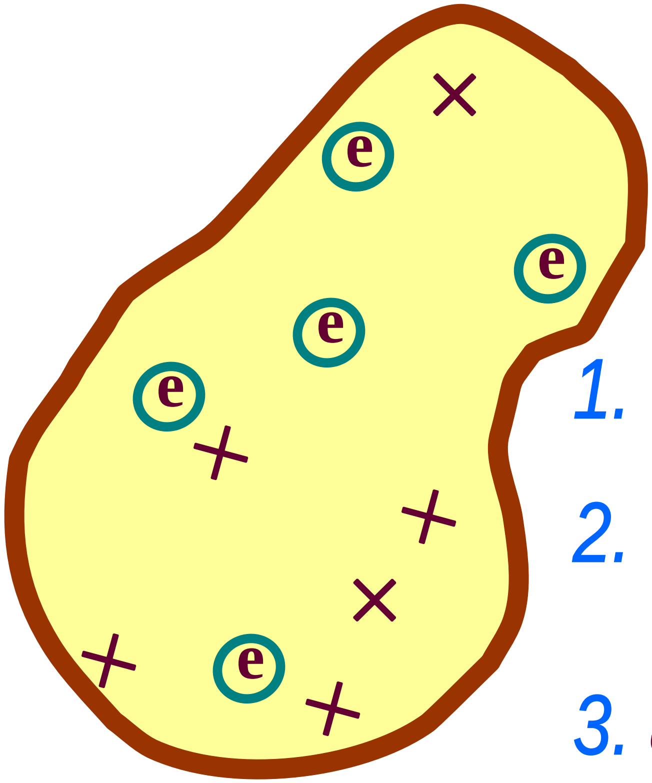
Gaz quantiques

23 avril - 20 juillet 2007

0. Previous talk:

Disorder + Interactions in a Fermi Liquid

Zero-dimensional Fermi Liquid



Quantum Dot

1. *Disorder (×impurities)*
2. *Complex geometry*

} *chaotic one-particle motion*

3. ***e-e*** *interactions*

Realizations:

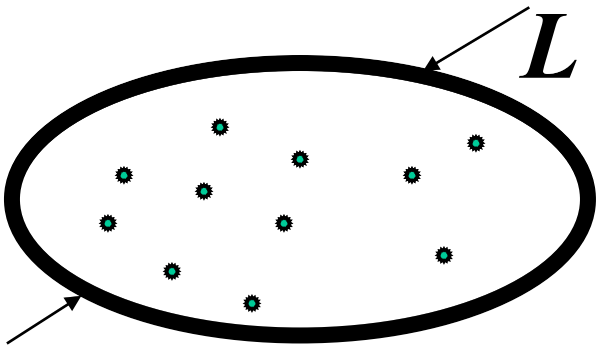
- Metallic clusters
- Gate determined confinement in 2D gases (e.g. ***GaAs/AlGaAs***)
- Carbon nanotubes
-
-

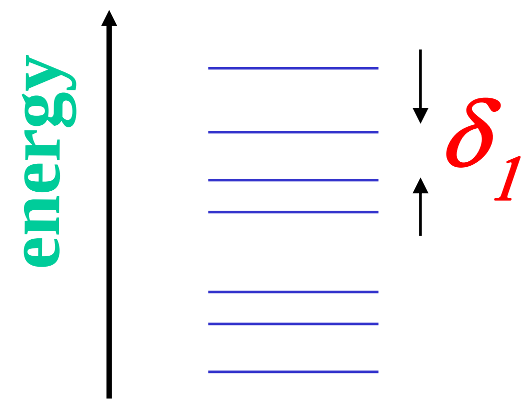
One-particle problem (*Thouless, 1972*)

Energy scales

1. Mean level spacing

$$\delta_1 = 1/\nu \times L^d$$





L *is the system size;*

d *is the number of dimensions*

2. Thouless energy

$$E_T = hD/L^2$$

D *is the diffusion const*

E_T *has a meaning of the inverse diffusion time of the traveling through the system or the escape rate (for open systems)*

$$g = E_T / \delta_1$$

dimensionless Thouless conductance

$$g = Gh/e^2$$

Thouless Conductance and One-particle Quantum Mechanics

0 **1** ***g***

Localized states	Extended states
Insulator	*Metal*
Poisson spectral statistics	**Wigner-Dyson spectral statistics**

$N \times N$ Random Matrices

Quantum Dots with dimensionless conductance g

$N \to \infty$

$g \to \infty$

The same statistics of the random spectra and one-particle wave functions (eigenvectors)

Zero Dimensional Fermi Liquid

Finite System → Thouless energy E_T

$$\varepsilon \ll E_T \xrightarrow{def} 0D$$

At the same time, we want the typical energies, ε, to exceed the mean level spacing, δ_1:

$$\delta_1 \ll \varepsilon \ll E_T$$

$$g \equiv \frac{E_T}{\delta_1} \gg 1$$

Matrix Elements

$$\hat{H}_{\text{int}} = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \sigma,\sigma'}} M_{\alpha\beta\gamma\delta} a^{+}_{\alpha,\sigma} a^{+}_{\beta,\sigma'} a_{\gamma,\sigma} a_{\delta,\sigma'}$$

Matrix Elements $M_{\alpha\beta\gamma\delta}$

Diagonal - $\alpha,\beta,\gamma,\delta$ are equal *pairwise*

$\alpha=\gamma$ and $\beta=\delta$ or $\alpha=\delta$ and $\beta=\gamma$ or $\alpha=\beta$ and $\gamma=\delta$

Offdiagonal - *otherwise*

It turns out that

in the limit $g \to \infty$

- *Diagonal matrix elements are much bigger than the offdiagonal ones*

$$M_{\text{diagonal}} >> M_{\text{offdiagonal}}$$

- *Diagonal matrix elements in a particular sample do not fluctuate - selfaveraging*

Toy model:

*Short range **e-e** interactions*

$$U(\vec{r}) = \frac{\lambda}{\nu}\delta(\vec{r})$$

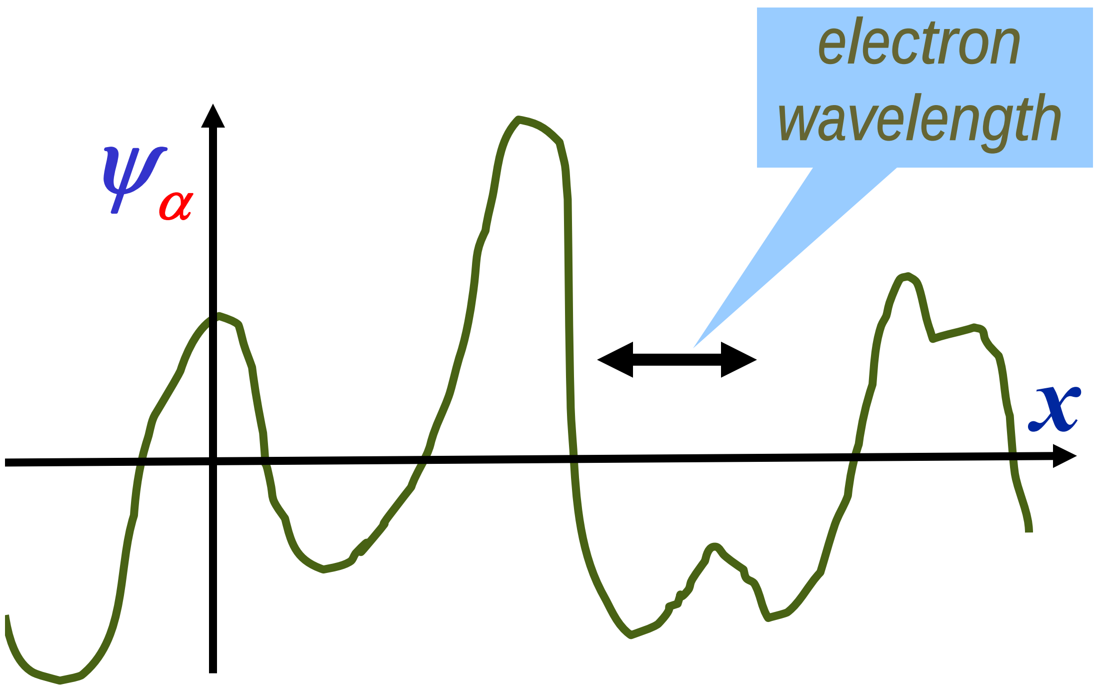
λ *is dimensionless coupling constant*

ν *is the electron density of states*

$$M_{\alpha\beta\gamma\delta} = \frac{\lambda}{\nu}\int d\vec{r}\,\psi^*_\alpha(\vec{r})\psi^*_\beta(\vec{r})\psi_\gamma(\vec{r})\psi_\delta(\vec{r})$$

$\psi_\alpha(\vec{r})$

one-particle eigenfunctions



$\Psi_\alpha(x)$ *is a random function that rapidly oscillates*

$$|\psi_\alpha(x)|^2 \geq 0$$

$$\psi_\alpha(x)^2 \geq 0$$

*as long as **T**-invariance is preserved*

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha}$$

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c \hat{n}^2 + J\hat{S}^2 + \lambda_{BCS} \left(\sum_{\alpha} a^{+}_{\alpha\uparrow} a^{+}_{\alpha\downarrow} \right) \left(\sum_{\alpha} a_{\alpha\uparrow} a_{\alpha\downarrow} \right)$$

Three coupling constants

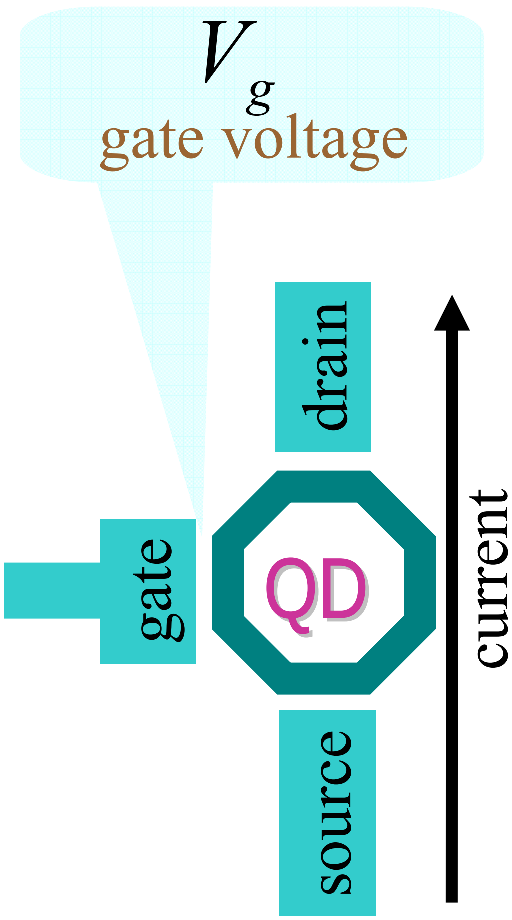
E_c determines the charging energy

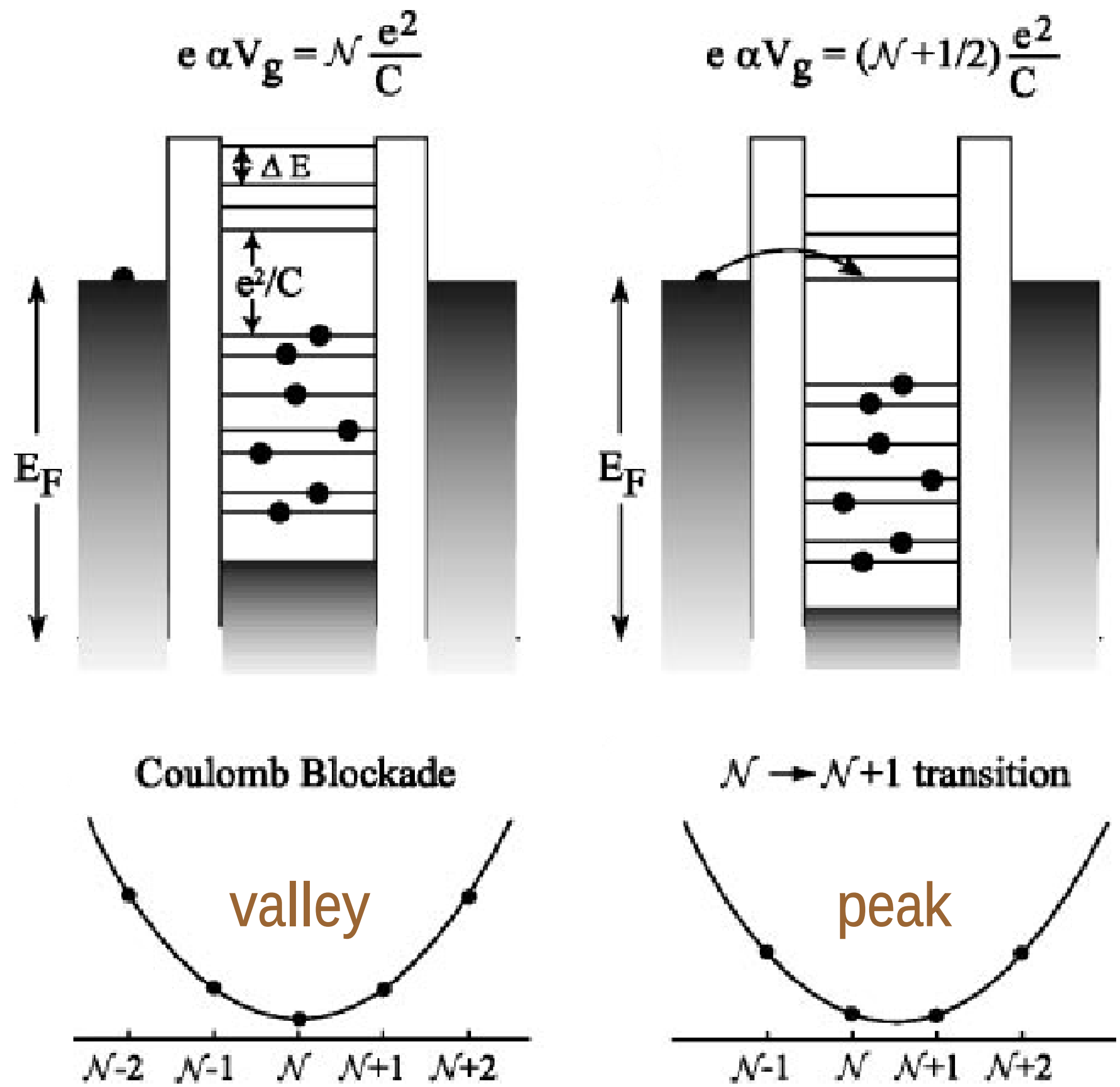
J describes the spin exchange interaction

λ_{BCS} determines effect of BCS-like pairing

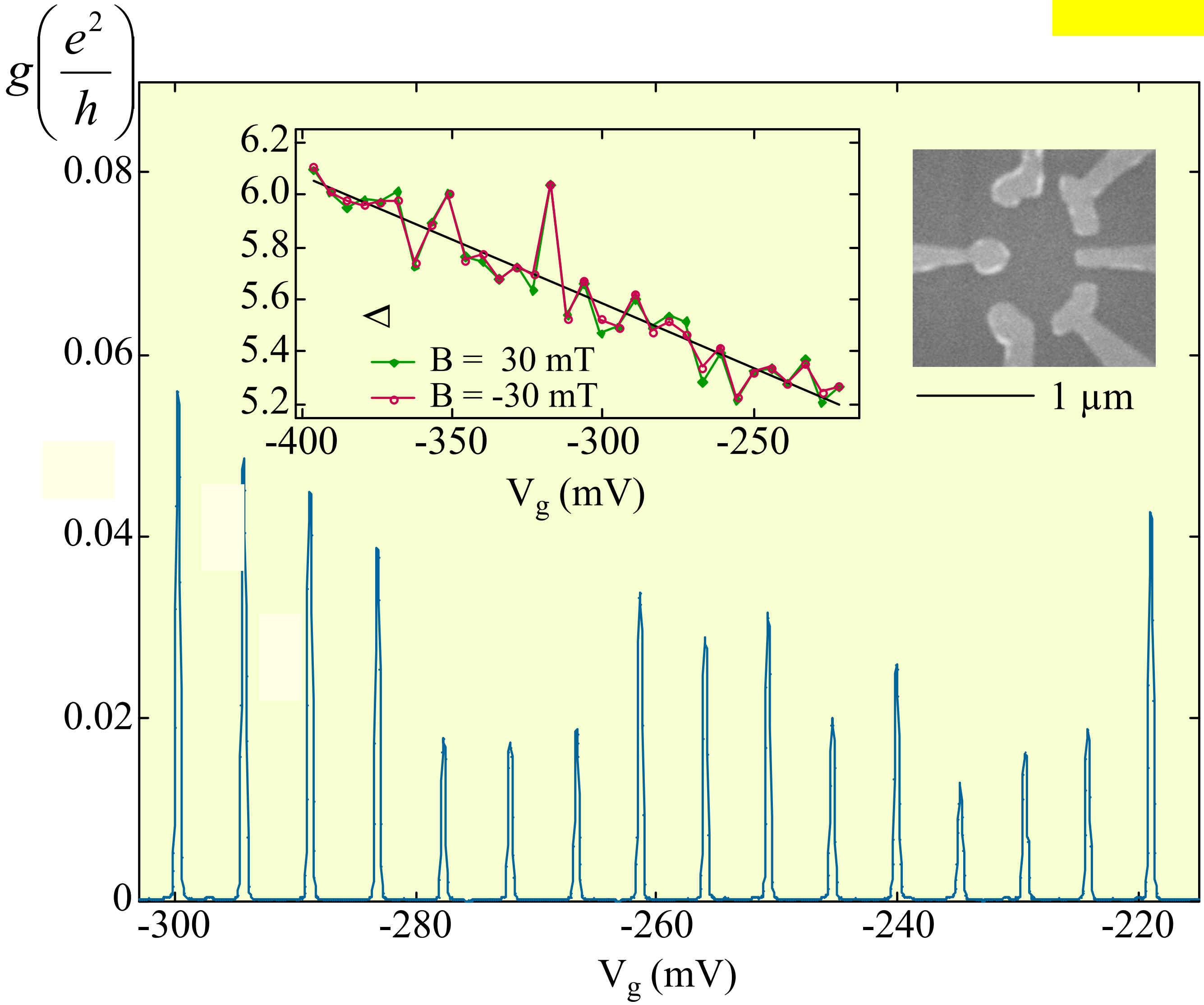
Selfaveraging!

Example 1: Coulomb Blockade





$$E_c = \frac{e^2}{2C}$$



Coulomb Blockade Peak Spacing
Patel, et al. PRL 80 4522 (1998)
(Marcus Lab)

CONCLUSIONS

One-particle chaos + moderate interaction of the electrons $\mapsto$ to a rather simple Hamiltonian of the system, which can be called Zero-dimensional Fermi liquid.

The main parameter that justifies this description is the Thouless conductance, which is supposed to be large

Excitations are characterized by their one-particle energy, charge and spin, but not by their momentum.

These excitations have the lifetime, which is proportional to the Thouless conductance, i.e., is long.

This approach allows to describe Coulomb blockade (renormalization of the compressibility), as well as the substantial renormalization of the magnetic susceptibility and effects of superconducting pairing

BCS + Disorder

I. Superconductor – Insulator transition in two dimensions

Anderson theorem and beyond

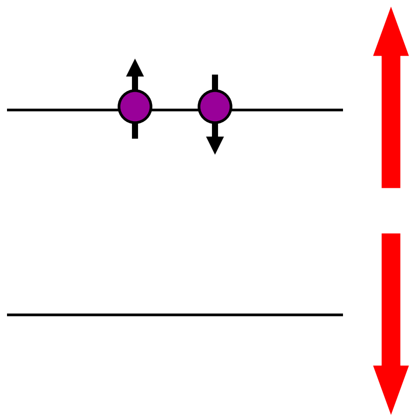
$$\hat{H}_{BCS} = \sum_{\alpha,\sigma=\uparrow,\downarrow} \varepsilon_\alpha a^+_{\alpha\sigma} a_{\alpha\sigma} + \lambda_{BCS} \sum_{\alpha,\beta} a^+_{\alpha\uparrow} a^+_{\alpha\downarrow} a_{\beta\uparrow} a_{\beta\downarrow}$$

Anderson spin chain

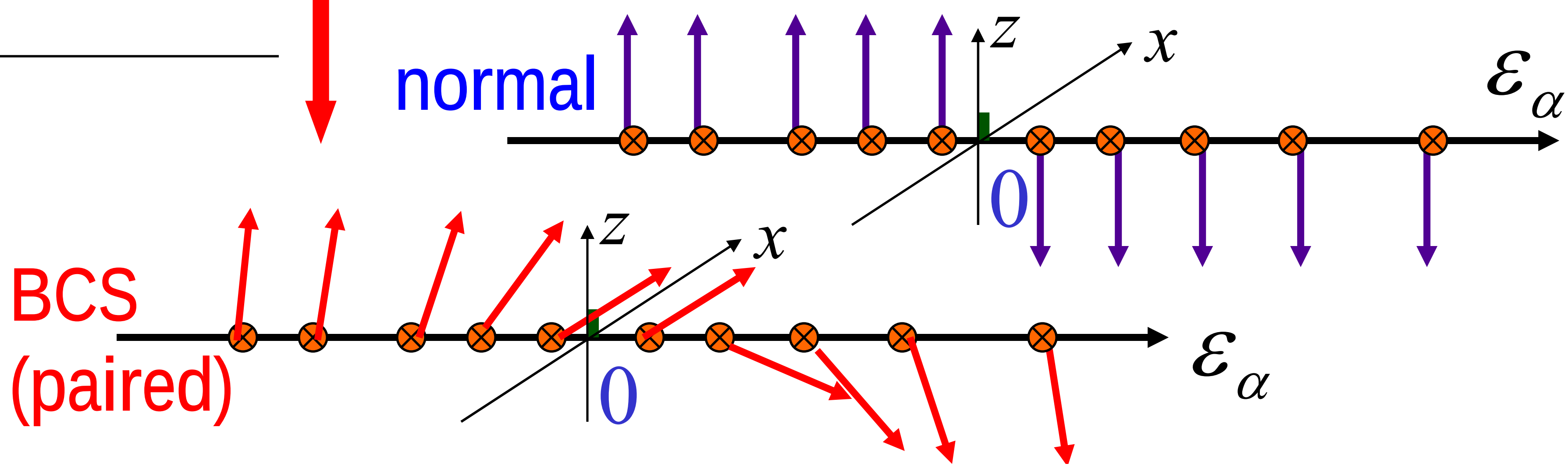
P.W. Anderson:
Phys. Rev. 112,1800, 1958

$$\hat{K}^z_\alpha = \frac{1}{2}\left(\sum_{\sigma=\uparrow,\downarrow} \varepsilon_\alpha a^+_{\alpha\sigma} a_{\alpha\sigma} - 1\right) \qquad \hat{K}^+_\alpha = a^+_{\alpha\uparrow} a^+_{\alpha\downarrow} \qquad \hat{K}^-_\alpha = a_{\alpha\uparrow} a_{\alpha\downarrow}$$

SU_2 algebra
spin 1/2



$$\hat{H}_{BCS} = \sum_{\alpha,\sigma=\uparrow,\downarrow} \varepsilon_\alpha \hat{K}^z_\alpha + \lambda_{BCS} \sum_{\alpha,\beta} \hat{K}^+_\alpha \hat{K}^-_\beta$$



ANDERSON THEOREM

Neither superconductor order parameter Δ nor transition temperature T_c depend on ***disorder****, i.e. on* g

Provided that

Δ is homogenous in space

This is just the universal limit !!

i.e. the limit $g \to \infty$

ANDERSON THEOREM

***Neither** superconductor order parameter Δ **nor** transition temperature T_c **depend** on **disorder**, i.e. on g*

Provided that

Δ is homogenous in space

This is just the *universal* limit !!

i.e. the limit $g \to \infty$

Corrections

at large, but finite g

***Ovchinnikov* 1973, *Maekawa & Fukuyama* 1982**

$$\frac{\delta T_c}{T_c} \propto \frac{-1}{g}\left(\log\frac{\hbar}{T_c\tau}\right)^3 \qquad T_c\tau < \hbar$$

!

Large, but finite g

1. Offdiagonal matrix elements are random and small:

a) ***zero average***

$$\left\langle M_{offdiagonal} \right\rangle = 0$$

b) ***fluctuations***

$$\left\langle \left(M_{offdiagonal} \right)^2 \right\rangle = 0\left(g^{-2} \right)$$

2. Diagonal matrix elements - corrections $O(g^{-1})$

a) ***average***

$$\left\langle M_{diagonal} \right\rangle - \lambda\delta_1 = 0\left(\frac{1}{g} \right)$$

b) ***fluctuations***

$$\left\langle \left(M_{diagonal} \right)^2 \right\rangle - \left\langle M_{diagonal} \right\rangle^2 = 0\left(\frac{1}{g^2} \right)$$

3. In two dimensional system (pancake) of the size L

$$\left\langle M_{diagonal} \right\rangle = \lambda\delta_1 \left[1 + \frac{\#}{g} \ln\left(\frac{L}{l} \right) \right]$$

l is mean free path

$$\frac{\delta T_c}{T_c} \propto \frac{-1}{g}\left(\log\frac{\hbar}{T_c\tau}\right)^3 \qquad T_c\tau < \hbar$$

Interpretation :

$$T_c = \theta_D \exp\left(-\frac{1}{\lambda_{BCS}}\right)$$

SC temperature

Debye temperature

Dimensionless BCS coupling constant

$$\lambda_{BCS} = \left[\ln\left(T_c/\theta_D\right)\right]^{-1}$$

Pure BCS interaction (no Coulomb repulsion):

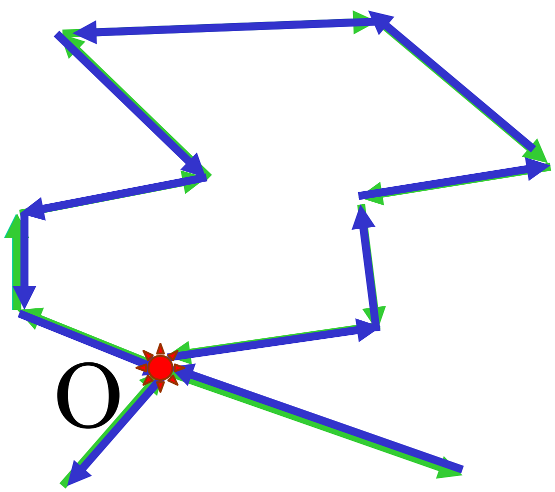
$$\lambda_{eff} = \lambda_{BCS}\left[1 + \frac{\#}{g}\ln\left(\frac{1}{T_c\tau}\right)\right]$$

Interpretation continued: "weak localization" logarithm

$$\lambda_{eff} = \lambda_{BCS}\left[1 + \frac{\#}{g}\ln\left(\frac{1}{T_c\tau}\right)\right]$$

Q: Why logarithm?

A: Return probability



$$\frac{d\log g}{d\log L} = \beta(g)$$

$$\beta(g)\xrightarrow[g\to\infty]{}(d-2)-\frac{\#}{g}$$

$$g(L) = g_0 - \#\ln\left(\frac{L}{l}\right)$$

$$\lambda_{BCS} = \left[\ln\left(T_c / \theta_D \right) \right]^{-1}$$

$$\lambda_{eff} = \lambda_{BCS} \left[1 + \frac{\#}{g} \ln\left(\frac{1}{T_c \tau} \right) \right]$$

•In the universal ($g=\infty$) limit the effective coupling constant equals to the bare one - **Anderson theorem**

•If there is only BCS attraction, then **disorder increases** T_c and Δ by optimizing spatial dependence of Δ. **!**

T_c **and** Δ reach maxima at the point of Anderson localization

Problem in conventional superconductors:
Coulomb Interaction

Interpretation continued: Coulomb Interaction

Anderson theorem - the gap is homogenous in space.

Without Coulomb interaction adjustment of the gap to the random potential strengthens superconductivity.

Homogenous gap in the presence of disorder violates electroneutrality; Coulomb interaction tries to restore it and thus suppresses superconductivity

$$\lambda_{eff} = \lambda_{BCS} - \frac{\#}{g}\ln\left(\frac{\hbar}{T_c\tau}\right)$$

Perturbation theory:

$$\frac{\delta T_c}{T_c} = \delta\left(\frac{-1}{\lambda}\right) = \frac{\delta\lambda}{\lambda_{BCS}^2} \propto -\frac{1}{g}\ln\left(\frac{\hbar}{T_c\tau}\right)\left[\ln\left(\frac{\theta_D}{T_c}\right)\right]^2$$

$$\lambda_{eff} = \lambda_{BCS} - \frac{\#}{g} \ln\left(\frac{\hbar}{T_c \tau} \right)$$

$$\lambda_{BCS} << 1$$

Effective interaction constant vanishes, when g is still $>> 1$

$$T_c = \frac{\hbar}{\tau} \left[\frac{\sqrt{g} - \ln\left(\hbar / \tau T_{c0} \right)}{\sqrt{g} + \ln\left(\hbar / \tau T_{c0} \right)} \right]^{\frac{\sqrt{g}}{2}}$$

Finkelshtein (1987)
renormalization group

Aleiner (unpublished)
BCS-like mean field

CONCLUSION:

T_c and Δ both vanish at

$$g = g^*$$

$$g^* = \left(\ln \frac{\hbar}{T_c \tau} \right)^{-2} = \lambda_{BCS}^{-2} >> 1$$

QUANTUM PHASE TRANSITION
Theory of Dirty Bosons

Fisher, Grinstein and Girvin 1990
Wen and Zee 1990
Fisher 1990

Only phase fluctuations of the order parameter are important near the superconductor - insulator transition

CONCLUSIONS

1. **Exactly at the transition point and at $T \rightarrow 0$ conductance tends to a universal value g_{qc}**

$$g_{qc} = \frac{4e^2}{h} \approx 6K\Omega \quad ??$$

2. **Close to the transition point magnetic field and temperature dependencies demonstrate universal scaling**

Granular Superconducting Films

E_J – Josephson energy

E_c – charging energy

$E_J > E_c$ superconductor

$E_J < E_c$ insulator

?

Problem: E_c is renormalized

Important parameter:

tunneling conductance between grains

$$g_t$$

Ambegoakar & *Baratoff* (1963)

$$E_J = g_t \Delta$$

Small g_t : vortex energy ~ E_J

quasiparticle energy ~ $\Delta >> E_J$

It is easier to create vortices than quasiparticles

dirty boson model might be relevant

Large g_t ?

Conductance near the transition

Ambegaokar, Eckern & Schon (1982, 1984)

Fazio & Schon (1990)

Albert Schmid (1983)

Chakravarty, Kivelson, Zimani & Halperin(1987)

When tunneling conductance g_t exceeds unity,

charging energy gets renormalized - SCREENING !

$$\widetilde{E}_c(\omega) = \frac{E_c}{1 + \# g_t \dfrac{E_c}{\omega}}$$

$$E_c^{eff} = \widetilde{E}_c(\Delta) = \frac{E_c}{1 + \# g_t \dfrac{E_c}{\omega}} \xrightarrow{g_t >> 1} \frac{\Delta}{g_t}$$

$$E_c^{eff} \approx \frac{\Delta}{g_t}$$

Ambegaokar, Eckern & Schon

$$E_J \approx g_t \Delta$$

Ambegaokar & Baratof

Therefore

$$E_c^{eff} \approx E_J \quad \Longleftrightarrow \quad g_t \approx 1$$

Q: How to match this result with Finkelshtein's formula for homogenous films ?

Charging energy of a grain or a Josephson junction

$$\tilde{E}_c = \frac{E_c}{1+\frac{E_c}{\delta_1}\frac{1/\tau_{esc}}{-i\omega+1/\tau_{esc}}}\frac{1}{\tau_{esc}} = g_t\delta_1$$

This is an analog of the RPA result in the continuous case

$$U_{eff}(q,\omega) = \frac{U_0(q)}{1+U_0(q)\nu\frac{Dq^2}{-i\omega+Dq^2}}$$

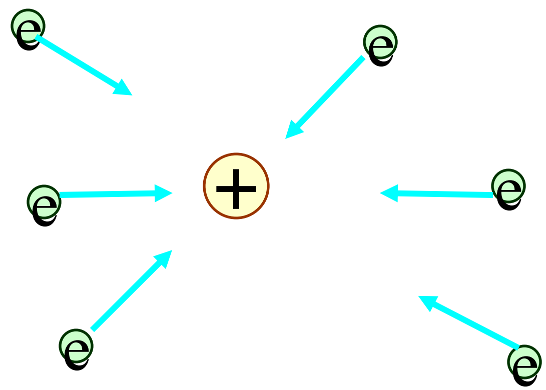
Indeed $E_c/\delta_1 \Leftrightarrow U\nu$ and $1/\tau_{esc} \Leftrightarrow Dq^2$

If $E_c >> \delta_1$ and $\omega\tau_{esc} << 1$, then $E_c^{eff} = \delta_1$

At $\omega\tau_{esc} > 1$ we obtain

$$\tilde{E}_c = \frac{E_c}{1+\frac{E_c}{-i\omega\tau_{esc}\delta_1}} = \frac{E_c}{1+g_t\frac{E_c}{-i\omega}}$$

$\xrightarrow{\Delta\rightarrow -i\omega}$ ***Ambegaokar, Eckern & Schon*** result



dynamical screening

$$U_{eff}(q,\omega)=\frac{U_0(q)}{1+U_0(q)\nu\dfrac{Dq^2}{-i\omega+Dq^2}}$$

? DISSIPATION

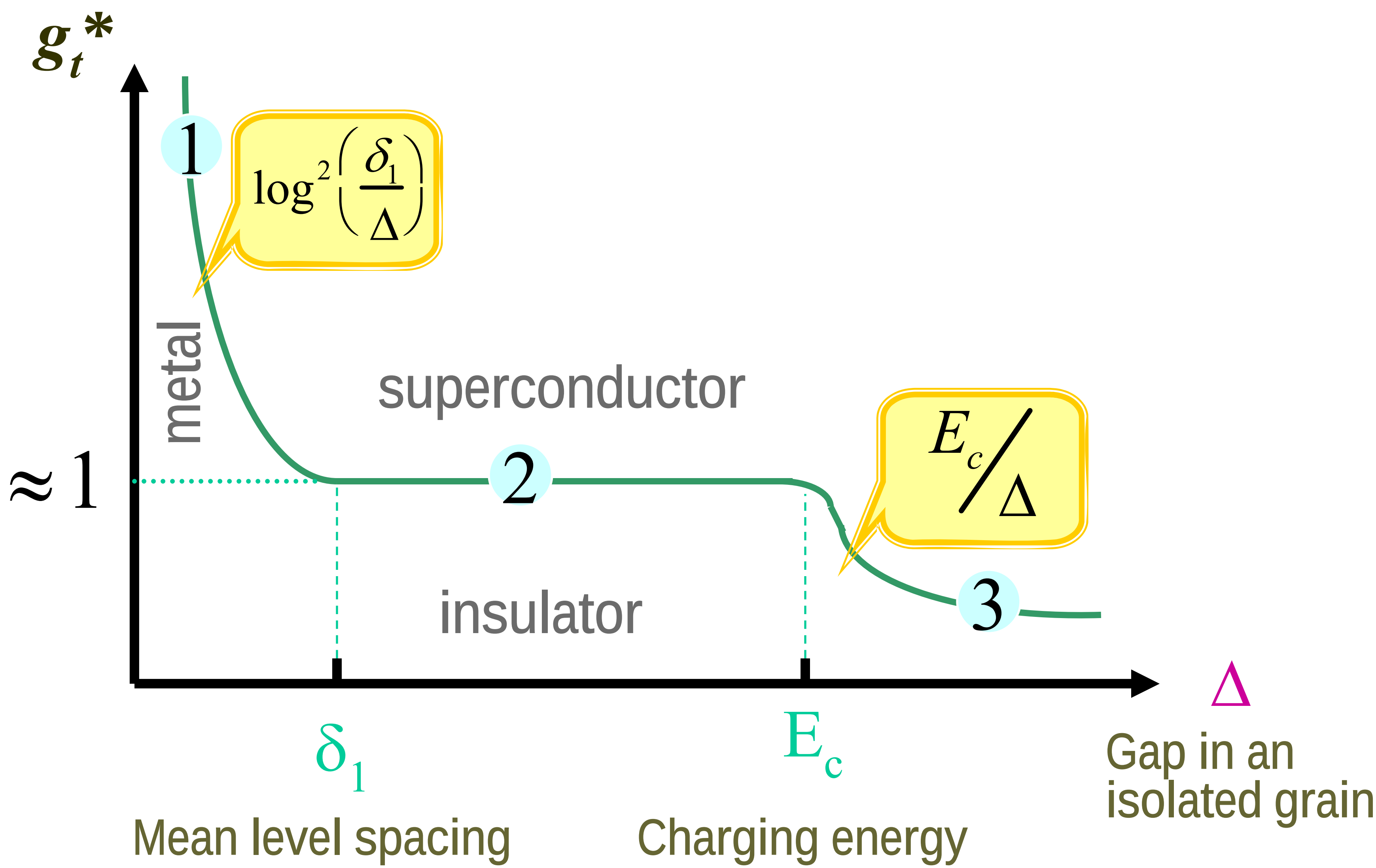
or

DYNAMICAL SCREENING ?

$$\Delta_{eff} = g\delta_1 \left[\frac{\sqrt{g} - \ln\left(g\delta_1 / \Delta \right)}{\sqrt{g} + \ln\left(g\delta_1 / \Delta \right)} \right]^{\frac{\sqrt{g}}{2}}$$

$$E_c^{\,eff} = \frac{E_c}{1 + \# \dfrac{E_c g_t}{\Delta + g_t \delta_1}} \xrightarrow{g_t >> 1} \begin{cases} \dfrac{\Delta}{g_t} & \dfrac{\Delta}{\delta_1} >> g_t >> \dfrac{E_c}{\Delta} \\[2ex] E_c & g_t << \dfrac{E_c}{\Delta} \end{cases}$$

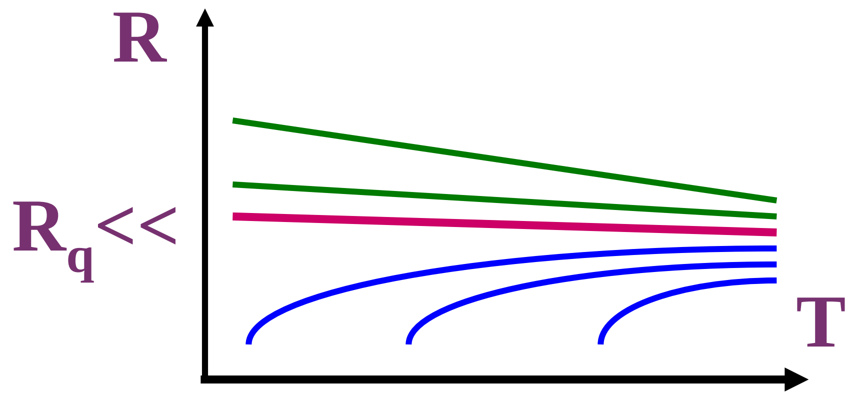
THREE REGIMES



1. superconductor - metal transition

$\Delta < \delta_1;$

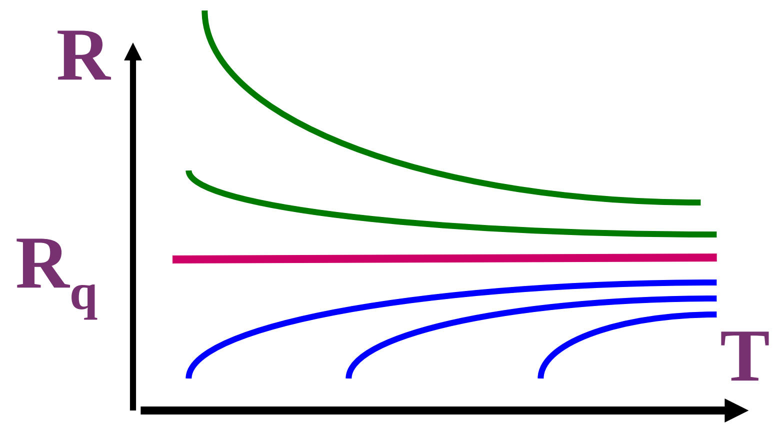
$g_t^* >> 1$



2. superconductor - insulator transition

$\delta_1 < \Delta < E_c$

$g_t^* \approx 1$

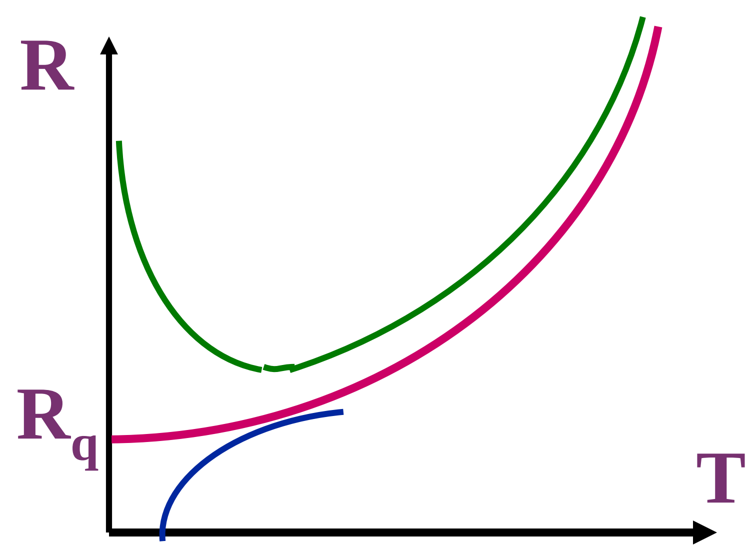


3. superconductor -insulator transition

$E_c << \Delta$

$E_c \approx E_J; \; E_J = g_T \Delta$

$g_t^* = E_c / \Delta << 1$



Superconducting-Insulating Transition in Two-Dimensional *a*-MoGe Thin Films

Ali Yazdani* and Aharon Kapitulnik
Department of Applied Physics, Stanford University, Stanford, California 94305
(Received 29 November 1994)

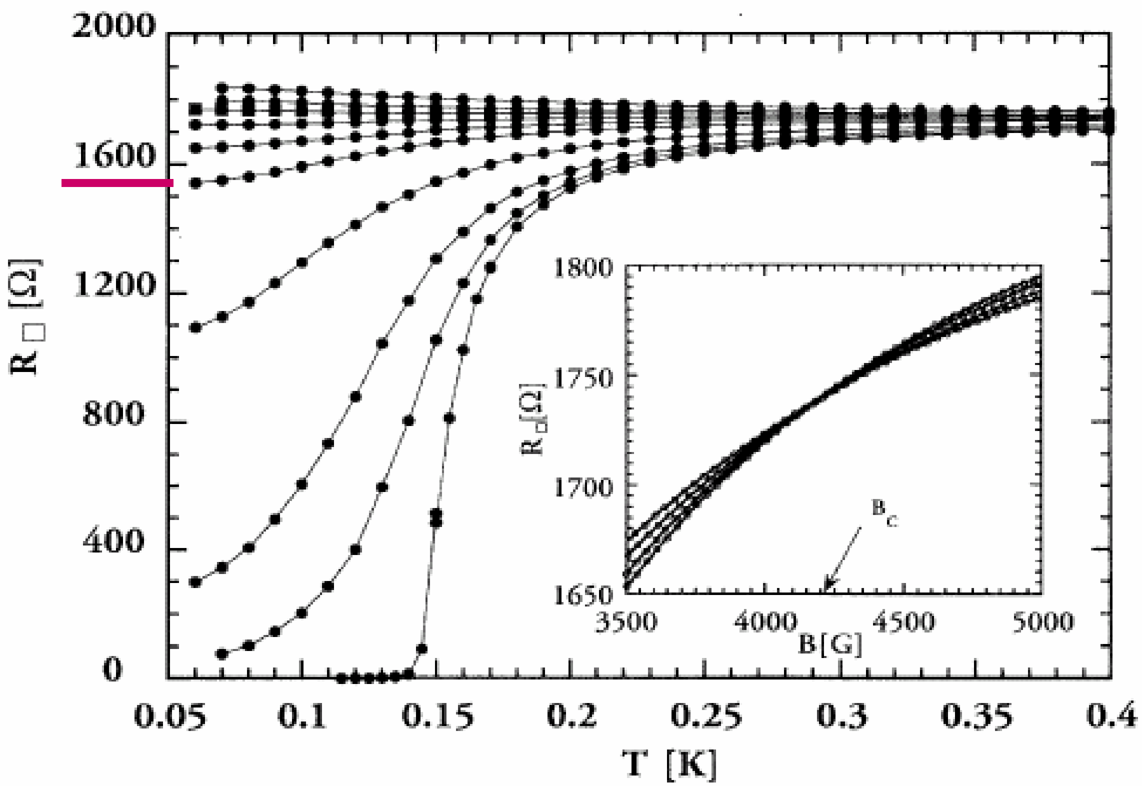


FIG. 1. Zero bias resistance of sample 2 plotted versus temperature at $B = 0, 0.5, 1.0, 2.0, 3.0, 4.0, 4.4, 4.5, 5.5, 6$ kG. In the inset, $R_{\square}(B, T, E = 0)$ for the same sample measured versus field, at $T = 80, 90, 100, 110$ mK.

Scaling of the Insulator-to-Superconductor Transition in Ultrathin Amorphous Bi Films

Y. Liu, K. A. McGreer,[(a)] B. Nease, D. B. Haviland,[(b)] G. Martinez, J. W. Halley, and A. M. Goldman
Center for the Science and Application of Superconductivity and School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 15 May 1991)

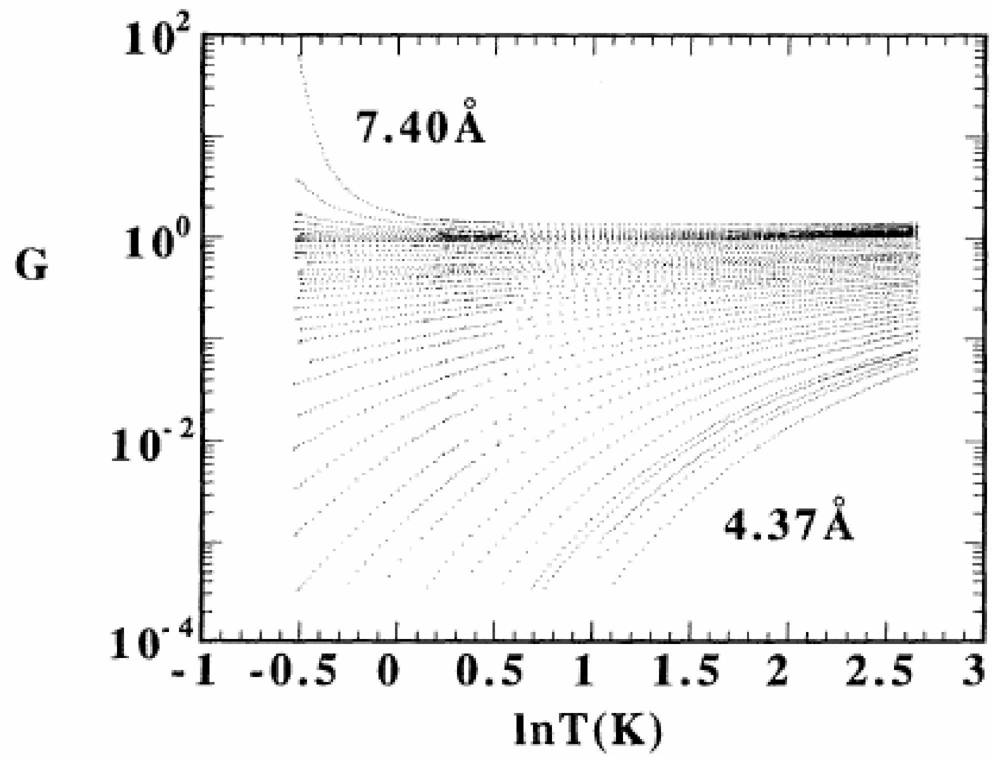


FIG. 1. Logarithm of the conductance G, in units of $4e^2/h$, vs $\ln(T)$ for a number of different Bi films. The thickness of the first (thinnest) and last (thickest) films are indicated.

PHYSICAL REVIEW B VOLUME 54, NUMBER 14 1 OCTOBER 1996-II

Quantum phase transitions in two dimensions: Experiments in Josephson-junction arrays

H. S. J. van der Zant, W. J. Elion, L. J. Geerligs, and J. E. Mooij
Department of Applied Physics and Delft Institute of Microelectronics and Submicron-technology (DIMES), Delft University of Technology Lorentzweg 1, 2628 CJ Delft, The Netherlands
(Received 21 May 1996)

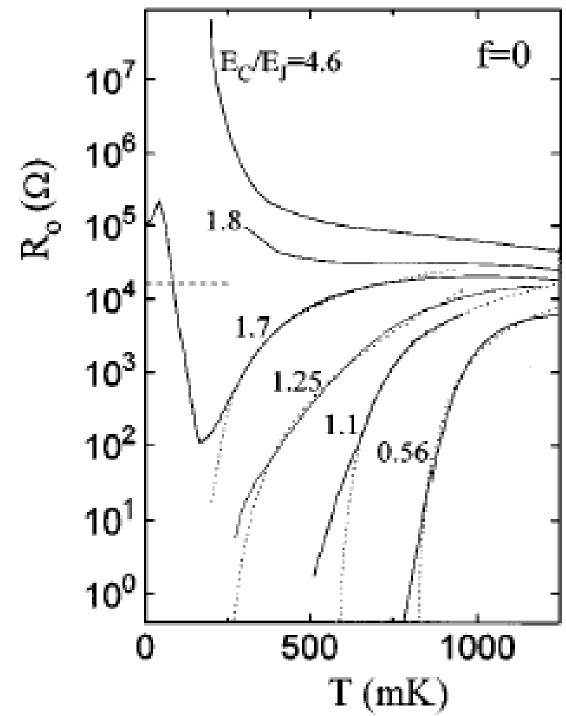


FIG. 5. The zero-field linear resistance per junction measured as a function of temperature for six different arrays. Dotted lines are fits to the vortex-KTB square-root cusp formula. The dashed horizontal line shows the zero-temperature universal resistance ($8R_q/\pi=16.4$ kΩ) of the S–I transition at $f=0$.

I. Theory:

1. *Homogenous films - Finkel'shtein's theory is relevant;* **$g^* >> 1$**

2. *Granular films - three regimes*
   If $\delta_1 < \Delta < E_c$, *then the critical conductance is of the order of the quantum conductance.*

3. ***Universalities ???***

II. Experiment

If all of the three energy scales E_J ; Δ and E_c *are of the same order, then the transition is nearby*

BCS + Disorder

II. Superconductor above paramagnetic limit

Clogston – Chandrasekhar transition

Tunnelinig conductance: finite bias anomalies

VOLUME 77, NUMBER 6 PHYSICAL REVIEW LETTERS 5 AUGUST 1996

Zeeman Splitting of the Coulomb Anomaly: A Tunneling Study in Two Dimensions

Wenhao Wu,* J. Williams, and P. W. Adams
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803
(Received 13 March 1996)

We report measurements of the tunneling density of states (DOS) of ultrathin Al films in which superconductivity is quenched by a parallel magnetic field, $H_\parallel > 48$ kG. The normal state DOS not only displays the usual logarithmic zero bias anomaly (ZBA) but also a new anomaly, superimposed on the ZBA, that appears at bias voltages $V \sim 2\mu_B H_\parallel/e$, the electron Zeeman splitting. Measurements in tilted fields reveal that the Zeeman feature collapses with increasing perpendicular field component, suggesting that it is associated with the Cooper electron-electron interaction channel. [S0031-9007(96)00791-0]

VOLUME 79, NUMBER 21 PHYSICAL REVIEW LETTERS 24 NOVEMBER 1997

Tunneling Anomaly in a Superconductor above the Paramagnetic Limit

I. L. Aleiner[1] and B. L. Altshuler[1,2]
[1]*NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540*
[2]*Physics Department, Princeton University, Princeton, New Jersey 08544*
(Received 23 April 1997)

We study the tunneling density of states (DOS) in superconducting systems driven by Zeeman splitting E_Z into the paramagnetic phase. We show that, even though the BCS gap disappears, superconducting fluctuations cause a strong DOS singularity in the vicinity of energies $-E^*$ for electrons polarized along the magnetic field and E^* for the opposite polarization. The position of the singularity $E^* = \frac{1}{2}(E_Z + \sqrt{E_Z^2 - \Delta^2})$ (where Δ is the BCS gap at $E_Z = 0$) is universal. We find analytically the shape of the DOS for different dimensionalities of the system. For ultrasmall grains the singularity has the form of the hard gap, while in higher dimensions it appears as a significant though finite dip.

PHYSICAL REVIEW B VOLUME 58, NUMBER 9 1 SEPTEMBER 1998-I

Theory of tunneling anomalies in superconductors above the paramagnetic limit

Hae-Young Kee
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855

I. L. Aleiner
NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540
and Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook, New York 11794

B. L. Altshuler
NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540
and Physics Department, Princeton University, Princeton, New Jersey 08544
(Received 17 February 1998)

We study the tunneling density of states (DOS) in superconducting systems driven by a Zeeman splitting E_Z into the paramagnetic phase. We show that, even though the BCS gap disappears, superconducting fluctuations cause a strong DOS singularity in the vicinity of energies $-E^*$ for electrons polarized along the magnetic field and E^* for the opposite polarization. The position of this singularity $E^*=\frac{1}{2}(E_Z+\sqrt{E_Z^2-\Delta^2})$ (where Δ is BCS gap at $E_Z=0$) is universal. We found analytically the shape of the DOS for different dimensionalities of the system. For ultrasmall grains the singularity has the shape of a hard gap, while in higher dimensions it appears as a significant though finite dip. Spin-orbit scattering, and an orbital magnetic field suppress the singularity. Our results are qualitatively consistent with recent experiments in superconducting films.

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \sum_{\alpha} \varepsilon_\alpha n_\alpha$$

$$\hat{H}_{int} = eV\hat{n} + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\hat{K}^+\hat{K}.$$

I. *Excitations are similar to the excitations in a disordered Fermi-gas.*

II. *Small decay rate*

III. *Substantial renormalizations*

Isn't it a Fermi liquid ?

Fermi liquid behavior follows from the fact that different wave functions are almost uncorrelated

Is there a spin-charge separation?

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\sum_{\alpha\beta}\hat{K}_\alpha^{\ +}\hat{K}_\beta .$$

$$\hat{K}_\alpha^+ = a_{\alpha,\uparrow}^+ a_{\alpha,\downarrow}^+$$

Commute with the one-particle part of the Hamiltonian

The interaction part of the hamiltonian does not mix single occupied orbitals with empty or double occupied ones – Blocking effect .

(V.G.Soloviev, 1961)

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

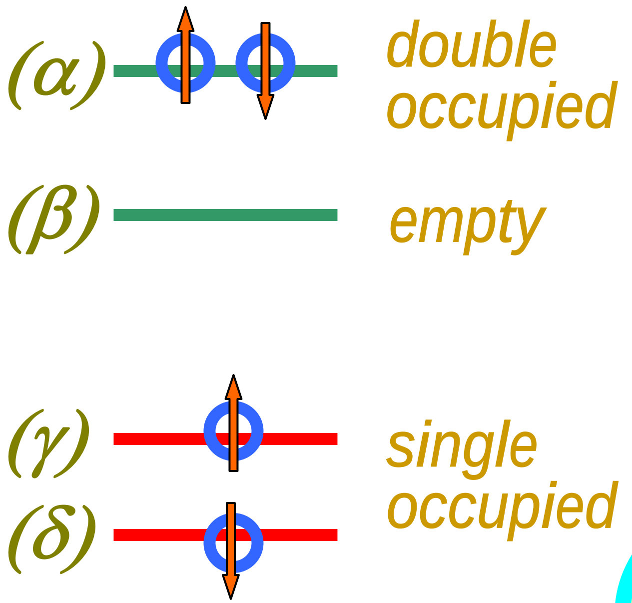
$$\hat{H}_0 = \sum \varepsilon_\alpha n_\alpha$$

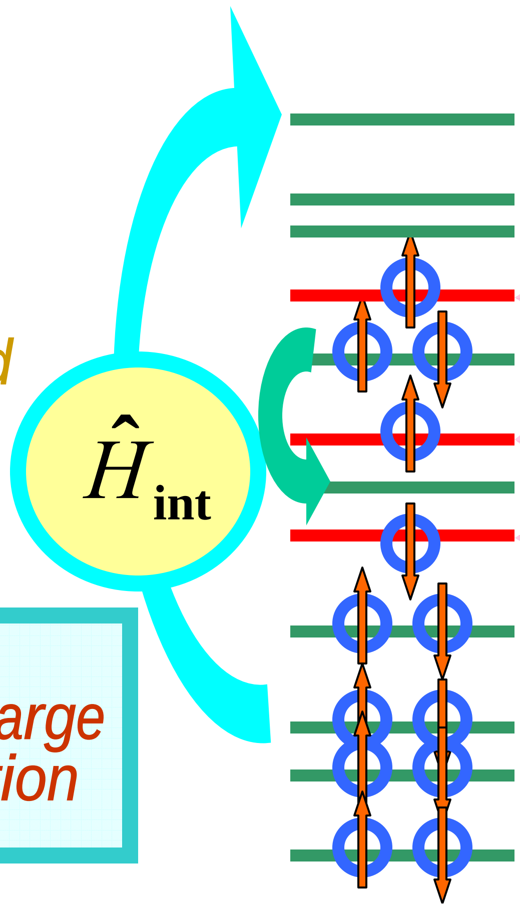
$$\hat{H}_{\text{int}} = eV\hat{n} + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\sum a^+_{\alpha\uparrow}a^+_{\alpha\downarrow}a_{\beta\uparrow}a_{\beta\downarrow}$$

$\hat{H}_0$ commutes with $\hat{n}$, $\hat{n}^2$ and $\hat{S}^2$, but does not commute with $\sum a^+_{\alpha\uparrow}a^+_{\alpha\downarrow}a_{\beta\uparrow}a_{\beta\downarrow}$.

$a^+_{\alpha\uparrow}a^+_{\alpha\downarrow}a_{\beta\uparrow}a_{\beta\downarrow}$ mixes (α) and (β)

at the same time $a^+_{\alpha\uparrow}a^+_{\alpha\downarrow}a_{\beta\uparrow}a_{\beta\downarrow}$ (γ,δ) $= 0$





This single-occupied states are not effected by the interaction.

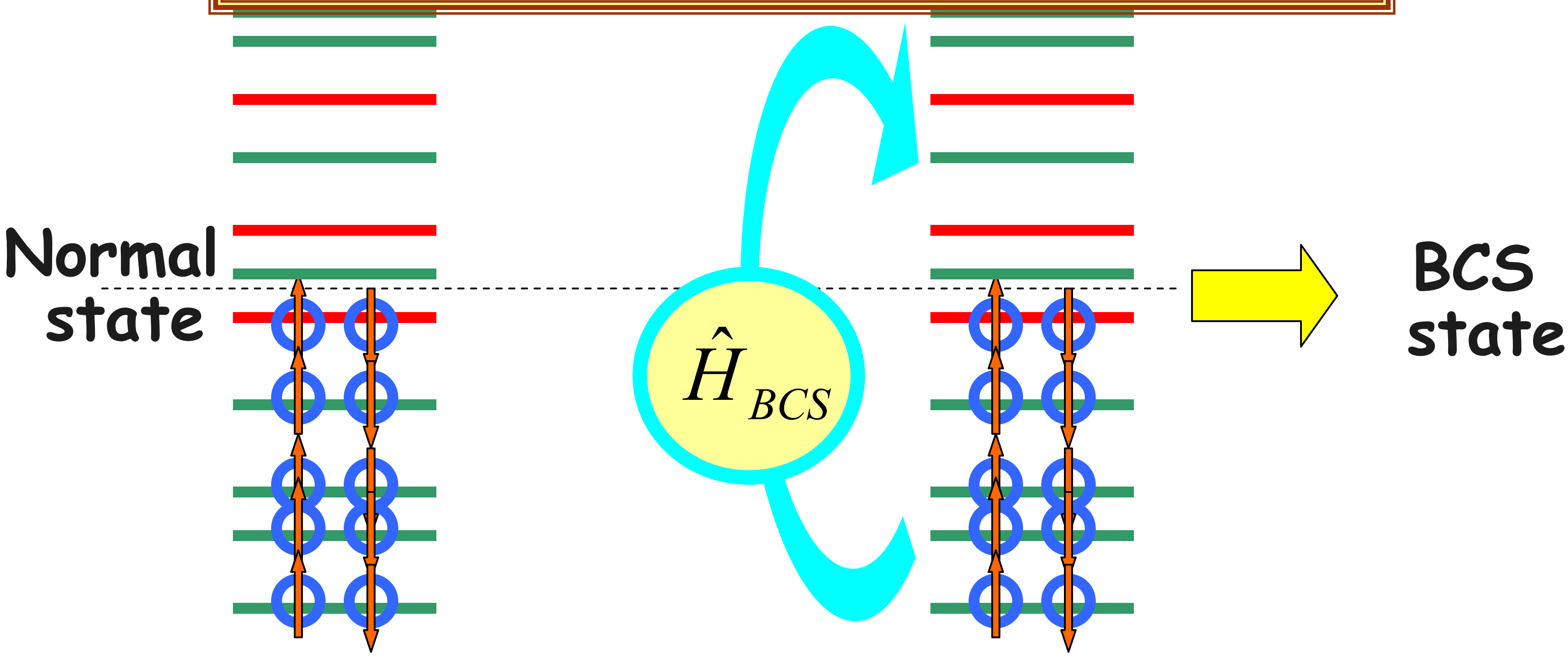
They are blocked

The Hilbert space is separated into two independent Hilbert subspaces

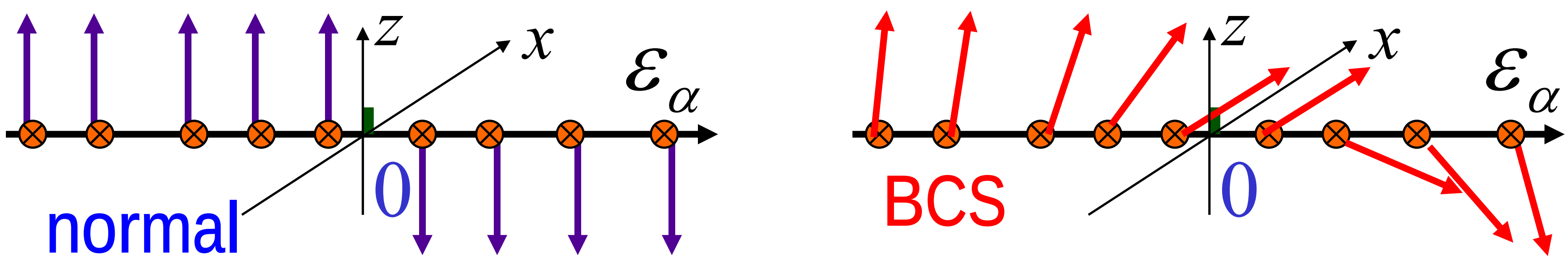
Charges and spins ??

Blocking effect $\overset{?}{=}$ Spin -charge separation

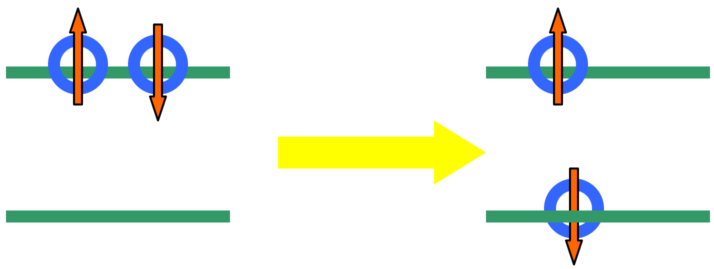
$$\hat{H}_{BCS} = \sum \varepsilon_{\alpha}\left(a^{+}_{\alpha\uparrow}a_{\alpha\uparrow} + a^{+}_{\alpha\downarrow}a_{\alpha\downarrow}\right) + \lambda_{BCS}\sum a^{+}_{\alpha\uparrow}a^{+}_{\alpha\downarrow}a_{\beta\uparrow}a_{\beta\downarrow}$$

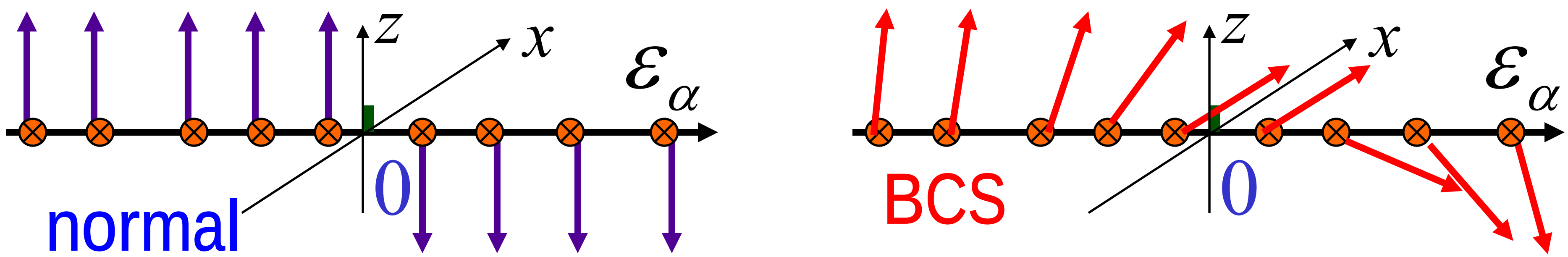


In terms of the isospins:



Blocking:

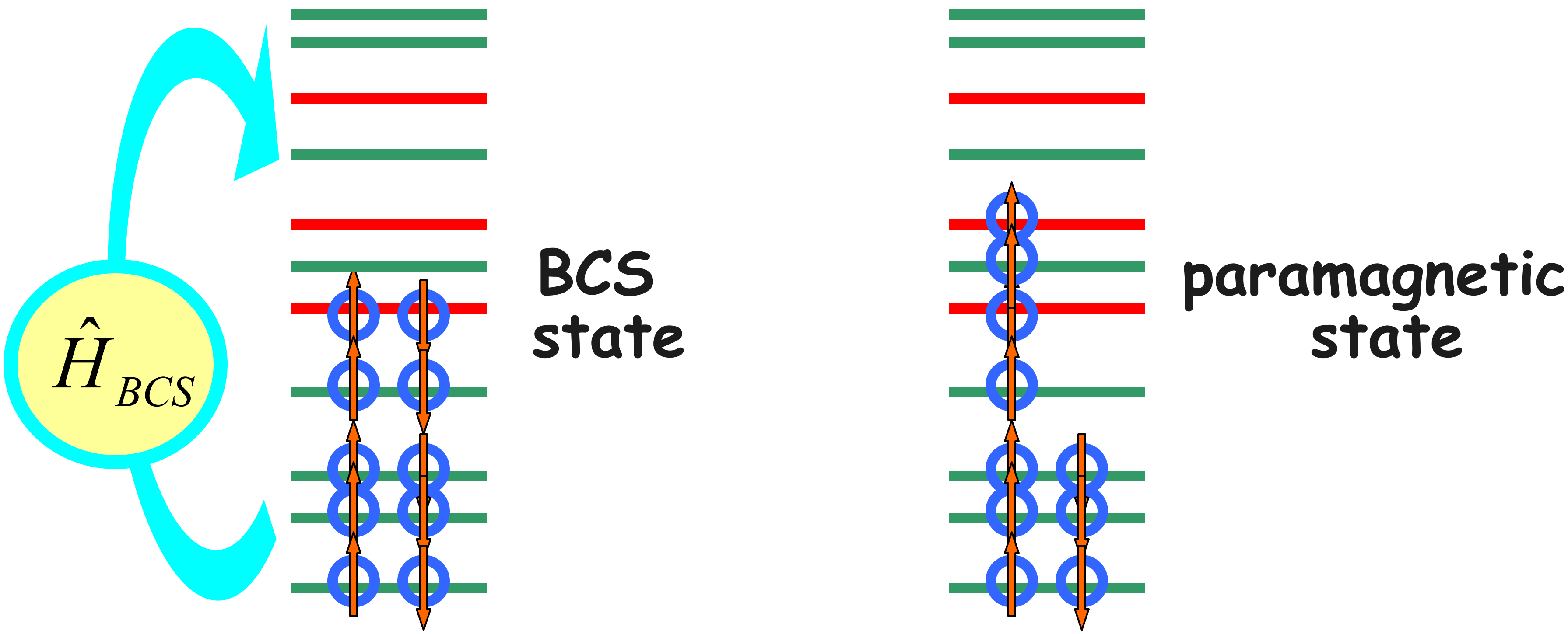




Therefore:

Blocking reduces the BCS gap

+ Zeeman splitting E_Z (small size or || magn. field):



First pair breaking: Energy loss 2Δ

Energy gain E_Z

But as a result the gap becomes smaller and it is easier to break a new pair ⇒ First order phase transition (Clogston – Chandrasekhar)

First order phase transition (Clogston – Chandrasekhar)

Coexistance of the two phases: $\Delta < E_Z < 2\Delta$

"True" transition: $E_Z = \sqrt{2}\Delta$

Tunneling Density of States and its anomalies

Tunneling Phenomena in Solids

Lectures presented at the 1967 NATO Advanced Study Institute at Risö, Denmark, 1967

Edited by

ELIAS BURSTEIN
Department of Physics
University of Pennsylvania
Philadelphia, Pennsylvania

and

STIG LUNDQVIST
Institute of Theoretical Physics
Chalmers Tekniska Högskola
Göteborg, Sweden

Chapter 3

Metal–Insulator–Metal Tunneling

I. Giaever
General Electric Research and Development Center
Schenectady, New York

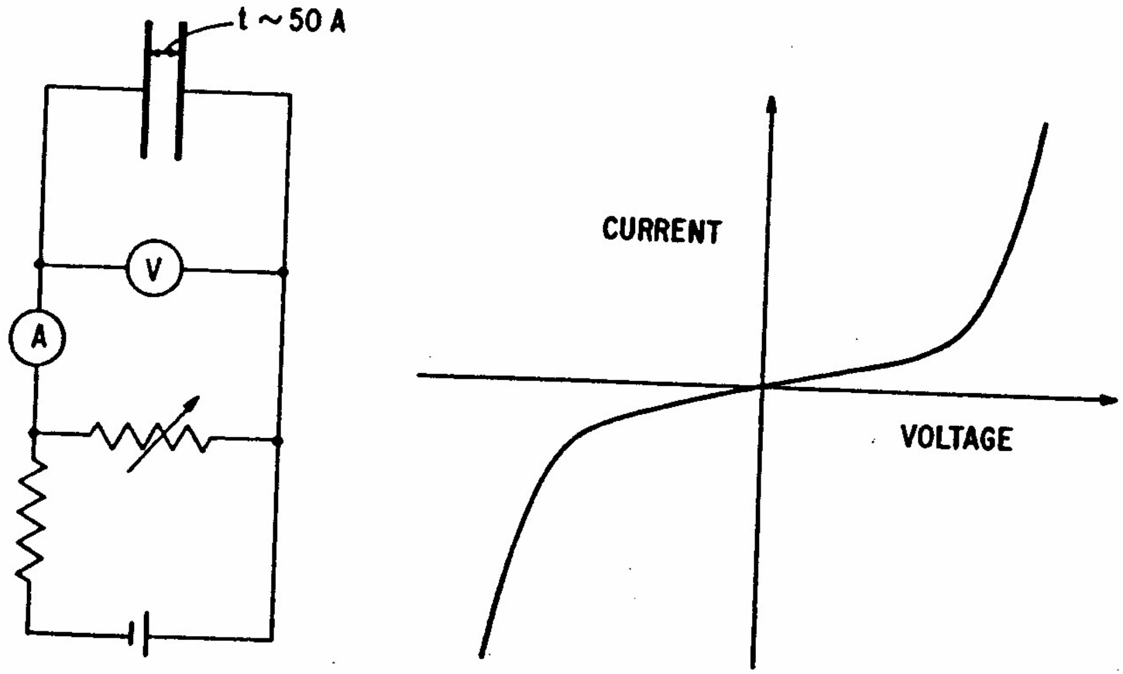
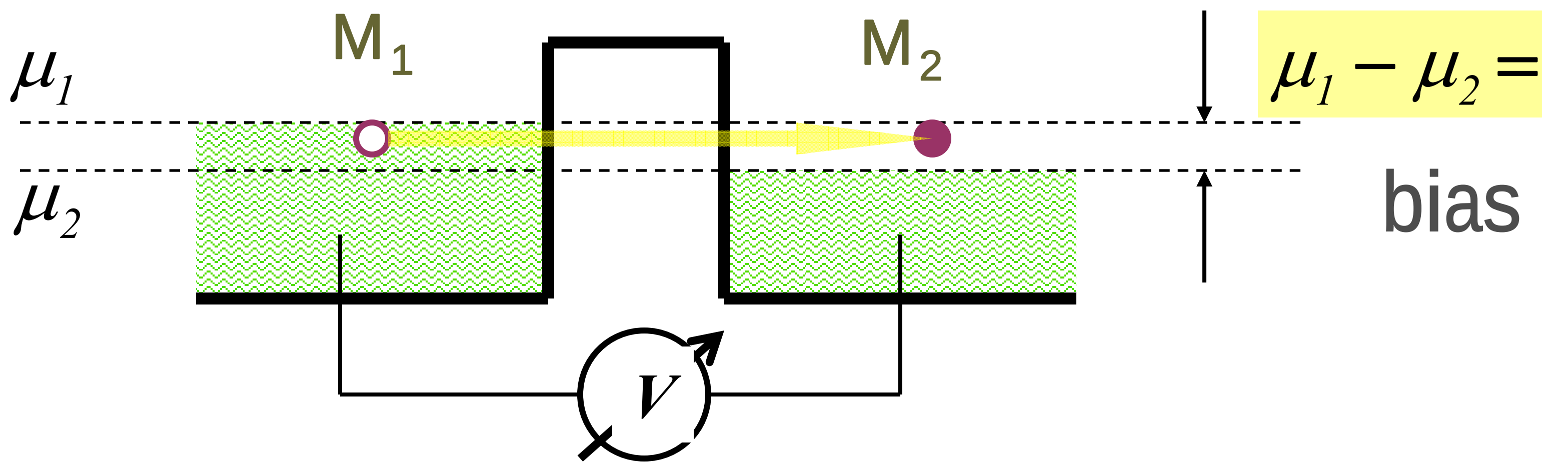


Fig. 1. Schematic drawing of a tunneling experiment. If the capacitor plates are spaced about 50 Å apart or less, a tunnel current will be easily observable. The current–voltage characteristic will be nearly symmetric about zero, linear at low voltages (below ~ 0.1 V), and nonlinear at higher voltages.

Tunneling Density of States



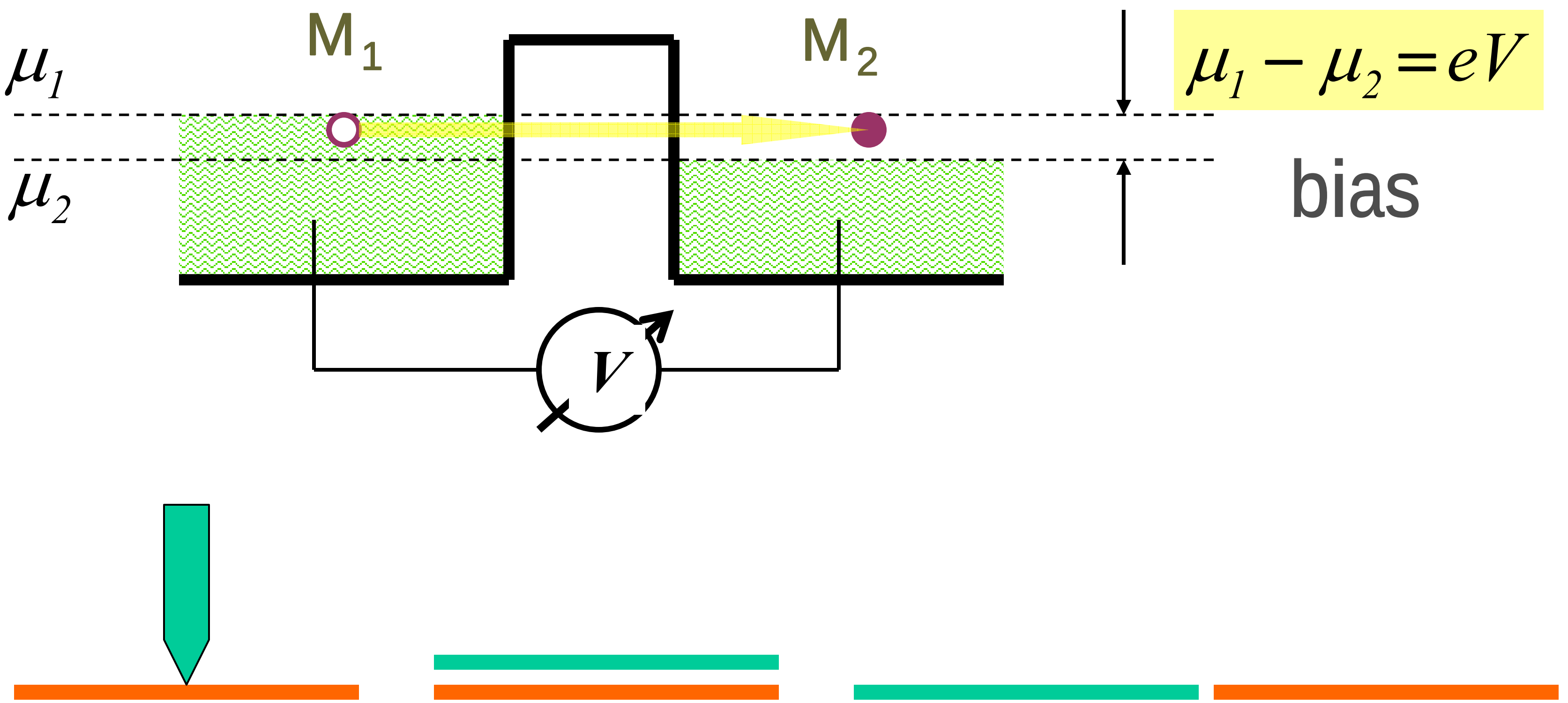
$\mu_1 - \mu_2 = eV$

$$G(V) \equiv \frac{d\,I(V)}{dV} \propto \nu_1(\mu)\nu_2(\mu) \approx const$$

tunneling probability

Depends on the bias only on the scale of the Fermi energy ?

Tunneling Density of States



A charge is created at $t=0$

VOLUME 4, NUMBER 9 PHYSICAL REVIEW LETTERS MAY 1, 1960

DIRECT OBSERVATION OF POLARONS AND PHONONS DURING TUNNELING IN GROUP 3-5 SEMICONDUCTOR JUNCTIONS

R. N. Hall, J. H. Racette, and H. Ehrenreich
General Electric Research Laboratory, Schenectady, New York

(Received April 6, 1960)

First observation of the Zero Bias Anomaly

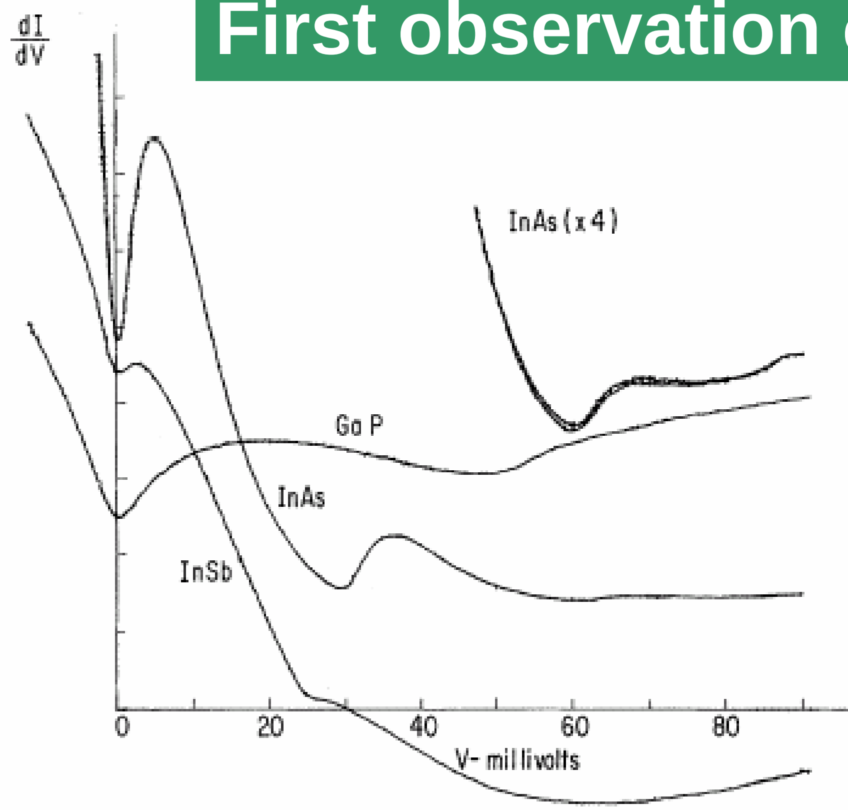


FIG. 1. Conductance (dI/dV) in arbitrary units vs voltage for several group 3-5 junctions.

Zero Bias Anomaly (ZBA)

Tunneling conductance, G_t,
is determined by the product of the tunneling probability, W,
and the densities of states in the electrodes, $\nu_t(\varepsilon = eV)$.

Originally ZBA was attributed to W:
- Paramagnetic impurities inside the barrier (Appelbaum-Andersdon theory) for the maximum of G_t.
- Phonon assisted tunneling for the minimum.

Now it is accepted that in most of cases

ZBA is a hallmark of the interactions between the electrons.

In other words, it is better to speak in terms of anomalies in the tunneling DoS.

In the presence of the disorder ZBA appears already at the level of the Hartree – Fock approximation i.e. in the first order in the perturbation theory in the interaction.

Correction to the DoS in the disordered case:

BA & A.G. Aronov, **Solid St. Comm. 30, 115 (1980).**
BA, A.G. Aronov, & P.A. Lee, **PRL, 44, 1288 (1980).**

Effect appears already in the first order in the perturbation theory
⇒ its sign is not determined

Correction to the DoS in the disordered case:

BA & A.G. Aronov, Solid St. Comm. 30, 115 (1980).
BA, A.G. Aronov, & P.A. Lee, PRL, 44, 1288 (1980).

$$\delta\nu(\varepsilon) = \frac{\lambda_d}{\varepsilon\left(\hbar D / \varepsilon\right)^{d/2}} \propto \begin{cases} -\sqrt{\varepsilon} & d=3 \\ \log \varepsilon & d=2 \\ \dfrac{1}{\sqrt{\varepsilon}} & d=1 \end{cases}$$

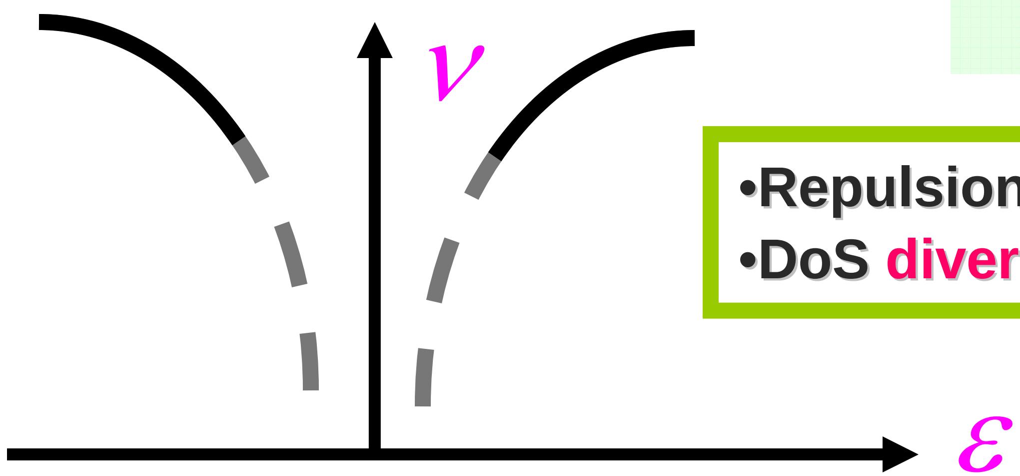
Effect appears already in the first order in the perturbation theory ⇒ its sign is not determined

ε electron energy counted from the Fermi level

D diffusion constant of the electrons

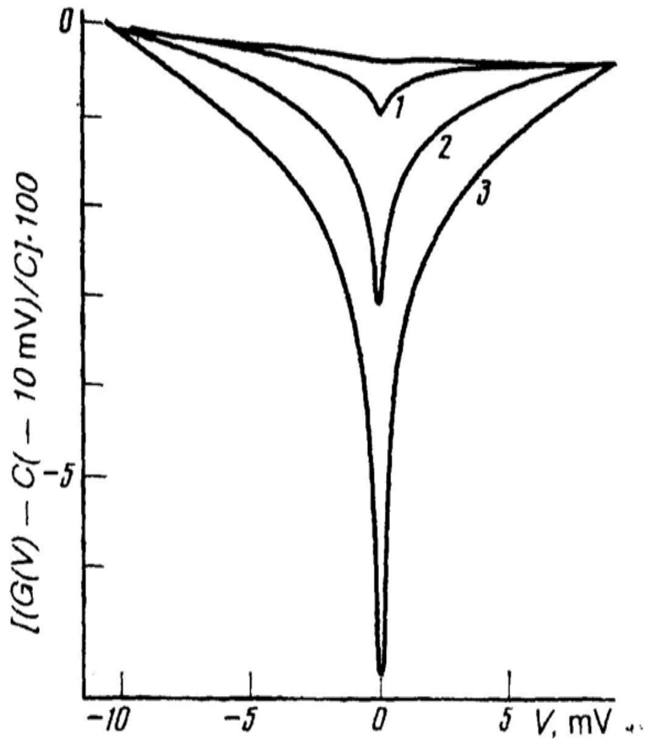
d # of the dimensions

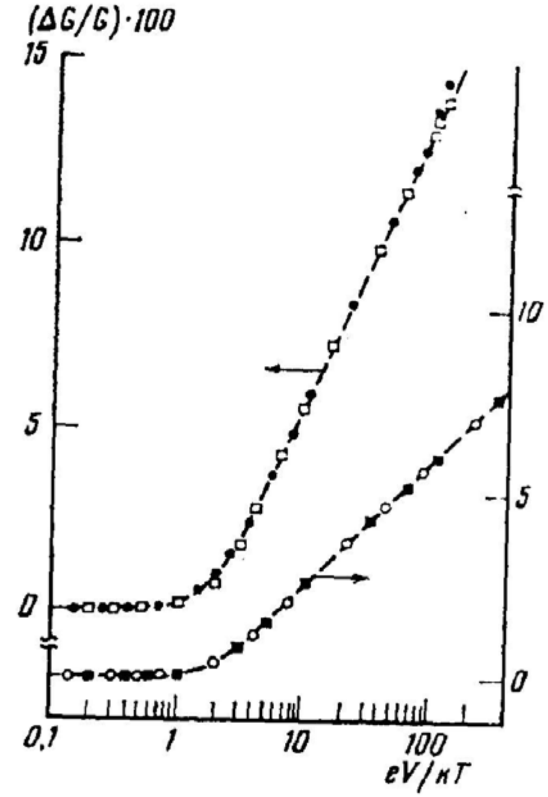
λ effective coupling constant;
$\lambda > 0$ -repulsion



- **Repulsion - minimum in the DoS;**
- **DoS diverges at low dimensions**

Zero Bias Tunneling Anomaly





The conductivity of the tunnel junctions ***Al-I-Al*** **(T=0.4K, B=3.5T)** for 2D films with different **$R_{\square}$: 1 – 40 Ω, 2 – 100 Ω, 3 - 300 Ω.** Right panel: comparison with the theoretical prediction for the interaction-induced ZBA.

Gershenson et al, *Sov. Phys. JETP* 63, 1287 (1986)

Tunneling Density of States (DoS) $\nu(\varepsilon)$

Role of the Friedel Oscillations

K.A. Matveev, D.Yue, and L.I. Glazman Phys. Rev. Lett., v.71, p.3351 (1993)
A.M. Rudin, I.L. Aleiner, and L.I. Glazman; Phys. Rev. v.B71, #15 (1997)



$$\nu(\varepsilon) = \frac{1}{\text{volume}} \sum_{\alpha} \delta(\varepsilon - \varepsilon_{\alpha}) = -\frac{2}{\pi} \int \operatorname{Im} G^{R}(\vec{r}, \vec{r}) \frac{d\vec{r}}{\text{volume}}$$

Tunneling Density of States (DoS) $\nu(\varepsilon)$

Role of the Friedel Oscillations

K.A. Matveev, D.Yue, and L.I. Glazman Phys. Rev. Lett., v.71, p.3351 (1993)
A.M. Rudin, I.L. Aleiner, and L.I. Glazman; Phys. Rev. v.B71, #15 (1997)

DoS at a given point $\vec{R}$ in space is determined by the quantum mechanical amplitude to come back to this point

Tunneling Density of States (DoS) $\nu(\varepsilon)$

DoS at a given point $\vec{R}$ in space is determined by the quantum mechanical amplitude to come back to this point

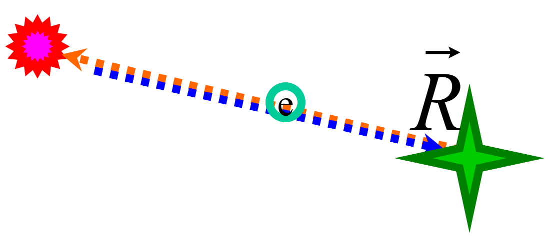
0) No disorder
No interactions between the electrons

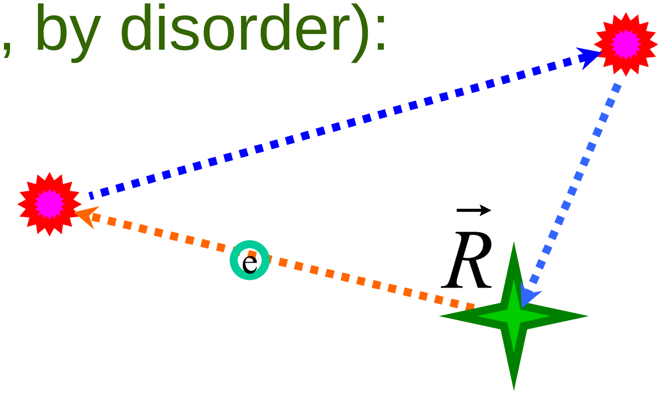
→ Non of the classical trajectories returns to the original point → DoS is a smooth function of the energy

(Energy ε is counted from the Fermi level)

$$\nu(\varepsilon) \propto \left(\varepsilon + \varepsilon_F\right)^{-1+d/2} \approx const$$

1) Such classical trajectories appear as soon as translation invariance is violated (e.g., by disorder):

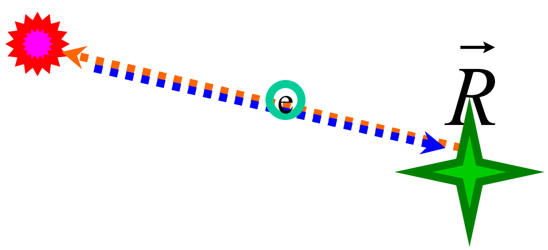




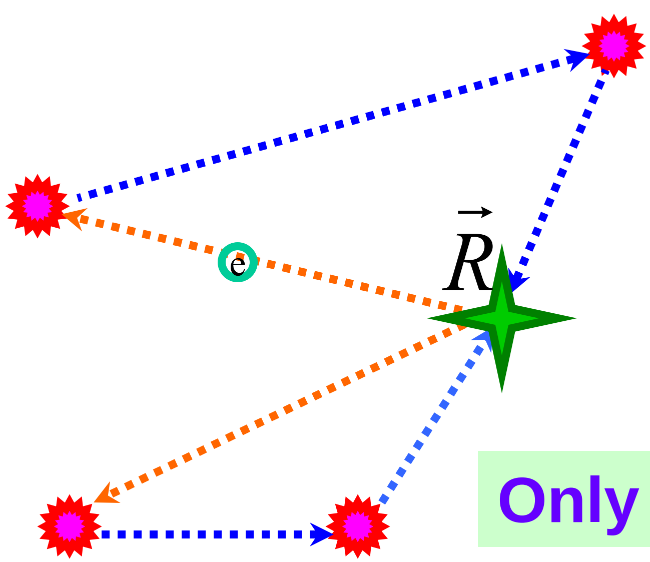
Tunneling Density of States (DoS) $\nu(\varepsilon)$

DoS at a given point $\vec{R}$ in space is determined by the quantum mechanical amplitude to come back to this point

1) **Such classical trajectories appear as soon as translation invariance is violated (e.g., by disorder):**



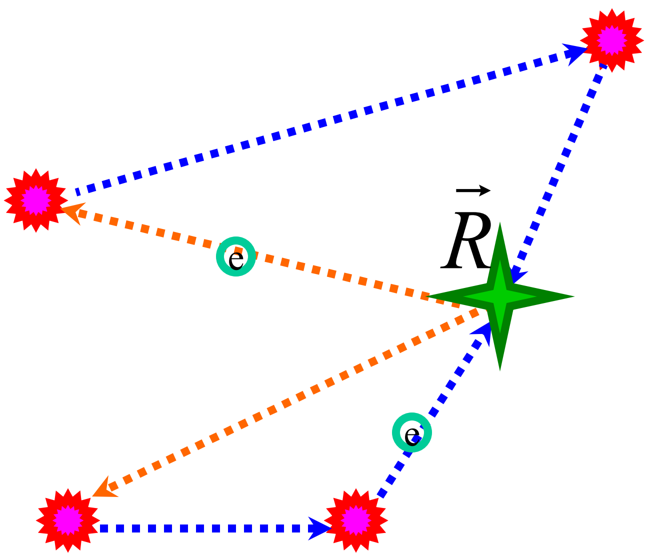
The return amplitude contains the phase factor. The phase $\varphi = 2k_F R$ is large (if the distance between the original point and the impurity exceeds the Fermi wavelength). The correction to the DoS vanishes when averaged over the sample volume



Different trajectories are characterized by different phase factors

$$\left\langle e^{i\varphi} \right\rangle_{disorder} = 0$$

Only mesoscopic fluctuations



Different trajectories have different phase factors

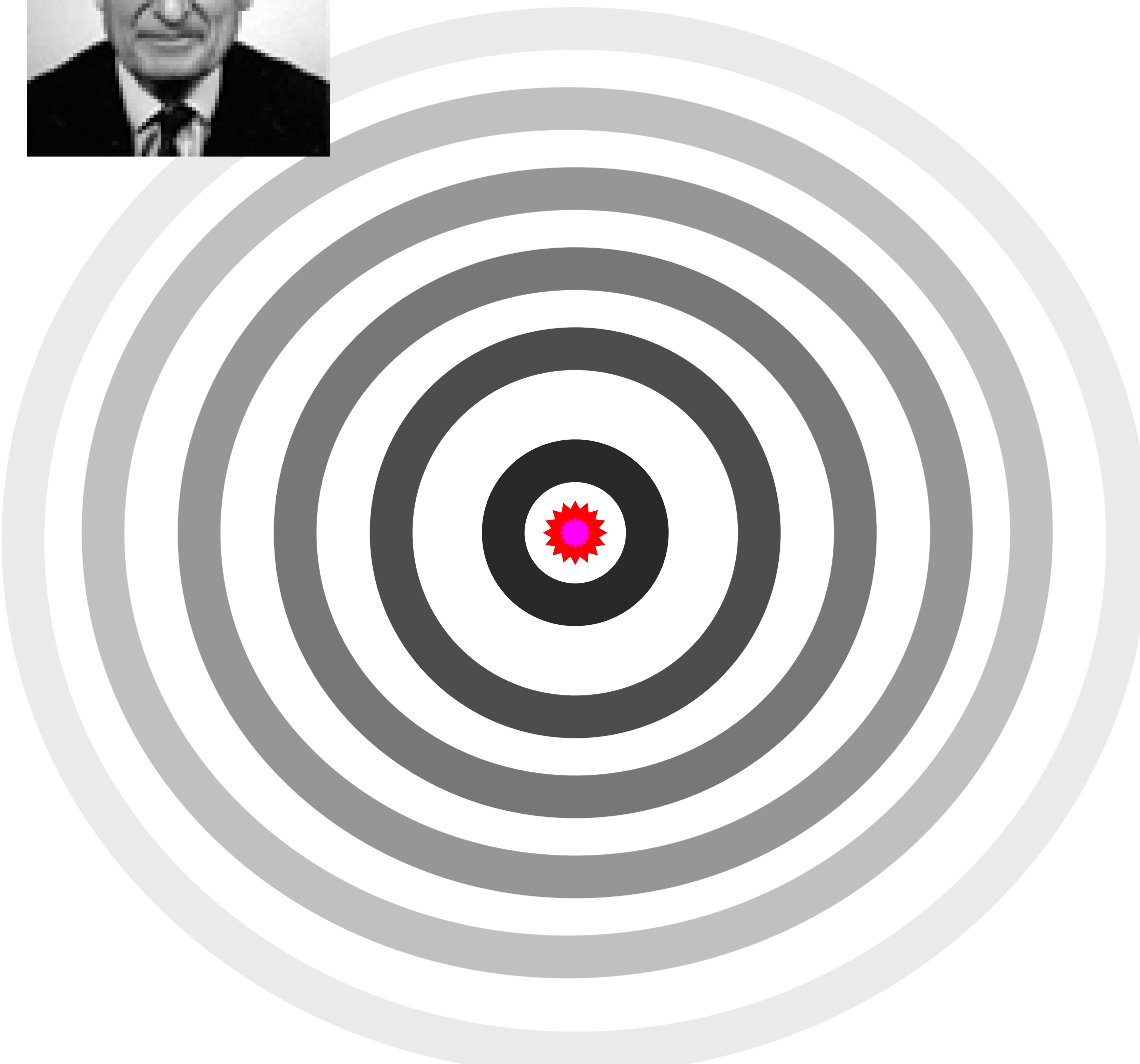
$$\left\langle e^{i\varphi} \right\rangle_{disorder} = 0$$

Without electron-electron interactions (averaged) DoS is not effected by the disorder.

Only mesoscopic fluctuations

Friedel Oscillations

$$\delta\rho(\vec{r}) \propto \frac{\sin(2k_F r)}{r^d}$$



Electron density oscillates as a function of the distance from an impurity.

The period of these oscillations is determined by the Fermi wave length.

The amplitude of the oscillations decays only algebraically.

These oscillations are not screened

Single impurity (ballistic) case
Compensation of Phases

An electron right after the tunneling finds itself at a point R. It moves, then

(i) gets scattered off an impurity at a point O,

(ii) gets scattered off the Friedel oscillation **created by the same impurity** (interaction !!!) , and

(iii) returns to the point R .

Phase factor at small angle θ :

$$\sin(2k_F r)e^{ik\left|\vec{R}-\vec{r}\right|}e^{ikr}e^{ikR} \approx$$

$$e^{2i(k-k_F)r} = \exp\left(2i\,\varepsilon r / v_F\right)$$

No oscillations in the limit

$$\varepsilon \to 0;\ r_\varepsilon \to \infty$$

ZBA !

An electron right after the tunneling finds itself at a point R. It moves, then

- **(i)** gets scattered off an impurity at a point O,
- **(ii)** gets scattered off the Friedel oscillation created by the same impurity, and
- **(iii)** returns to the point R .

No oscillations in the limit $\varepsilon \to 0$

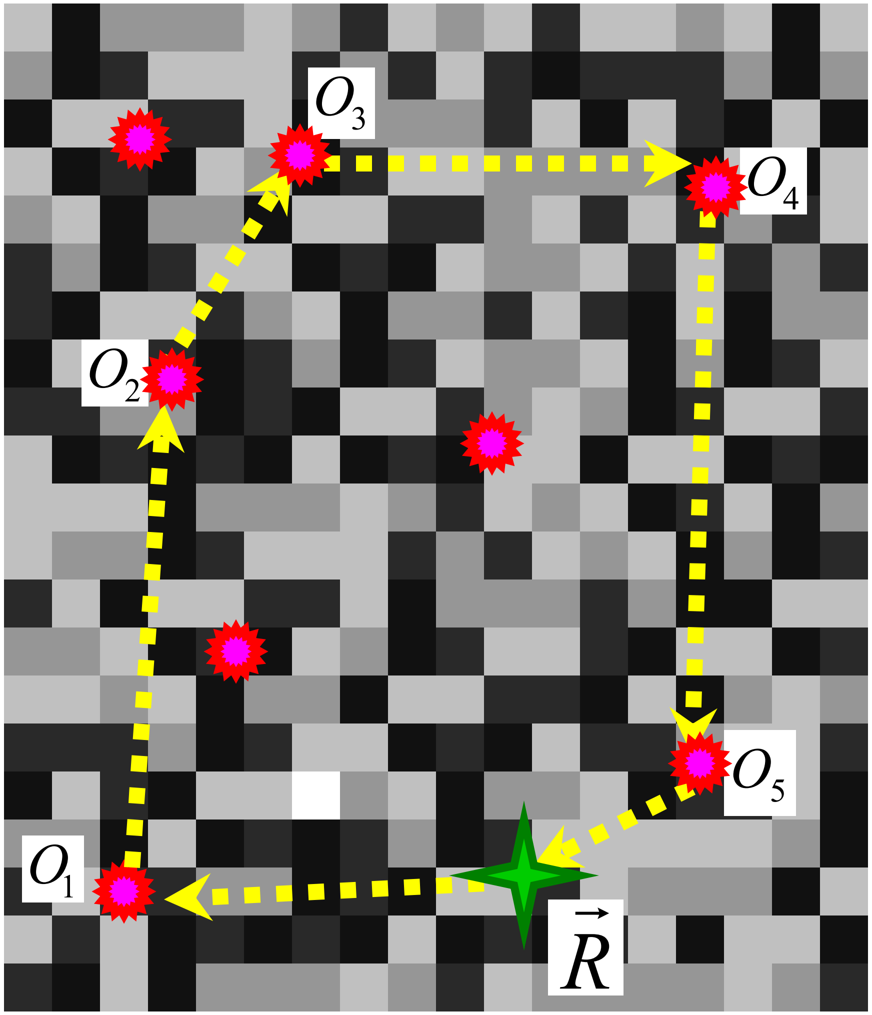
Phase fluctuates only when $r > r_\varepsilon$ where

$$r_\varepsilon \approx {v_F}/{\varepsilon} \to \infty$$

→ ZBA **!**

Important: this effect exists already in the first order of the perturbation theory in the interaction between the electrons (between the probe electron and the Friedel oscillation), i.e., in the Hartree-Fock approximation. As a result the DoS correction as well as ZBA can have arbitrary sign.

Multiple impurity scattering - diffusive case. Compensation of Phases



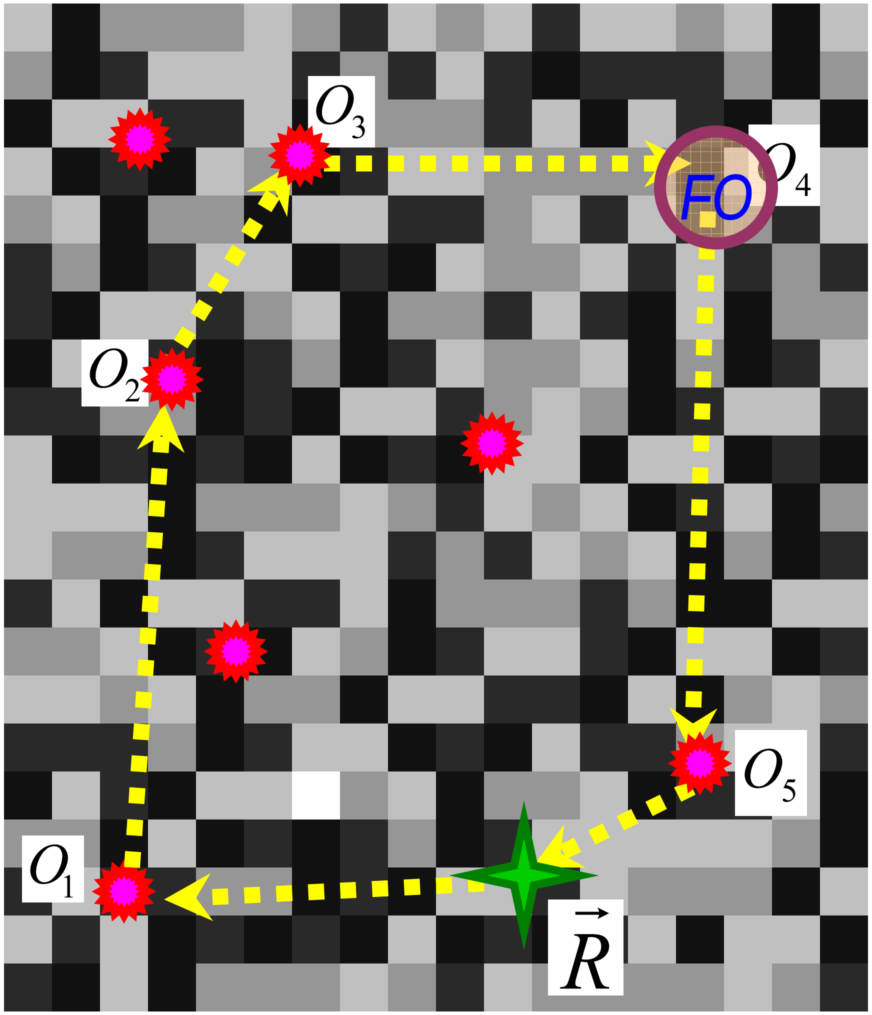
"Messy" Friedel oscillations - combination of the Friedel oscillations from different scatterers

$$\delta\rho(\vec{r}) \propto \sum_{paths\ \alpha} A_\alpha \sin\left(k_F L_\alpha\right)$$

$$\alpha = \left\{O_1, O_2, O_3, \ldots, O_n,\right\}$$ a path

L_α total length of this path

Multiple impurity scattering - diffusive case.
Compensation of Phases



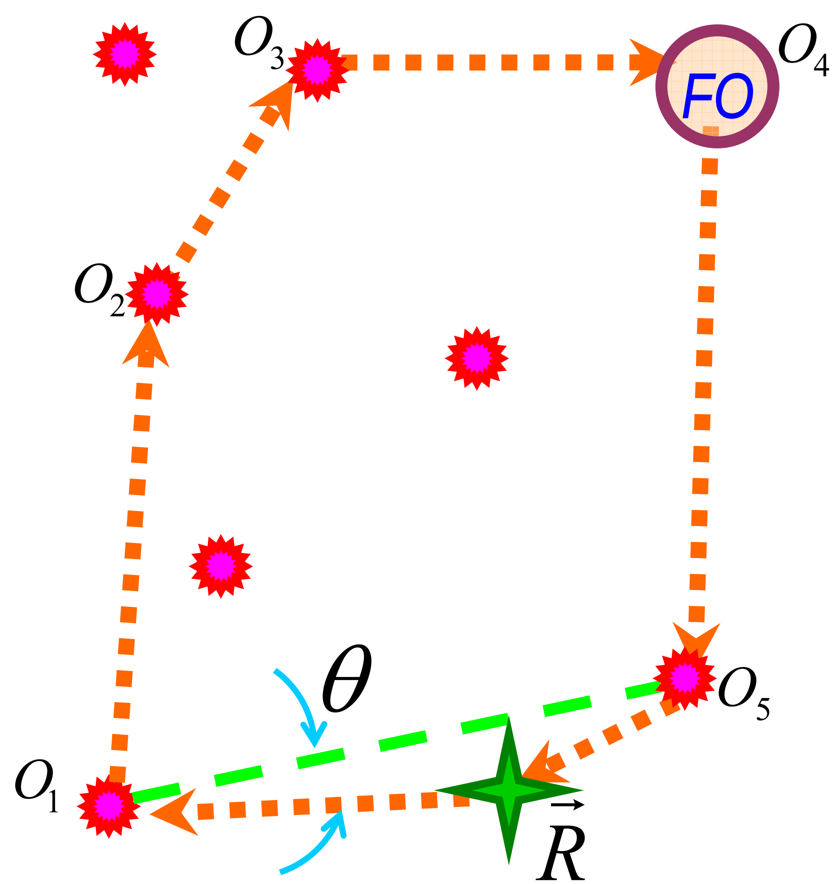
"Messy" Friedel oscillations - combination of the Friedel oscillations from different scatterers

$$\delta\rho(\vec{r}) \propto \sum_{paths\ \alpha} A_\alpha \sin\left(k_F L_\alpha\right)$$

$$\alpha = \left\{O_1, O_2, O_3, \ldots, O_n,\right\}$$ **a path**

L_α **total length of this path**

Multiple impurity scattering - diffusive case
Compensation of Phases



"Messy" Friedel oscillations - combination of the Friedel oscillations from different scatterers

$$\delta\rho(\vec{r}) \propto \sum_{paths\ \alpha} A_\alpha \sin\left(k_F L_\alpha\right)$$

$$\alpha = \{O_1, O_2, O_3, \ldots, O_n,\}$$ a path

L_α total length of this path

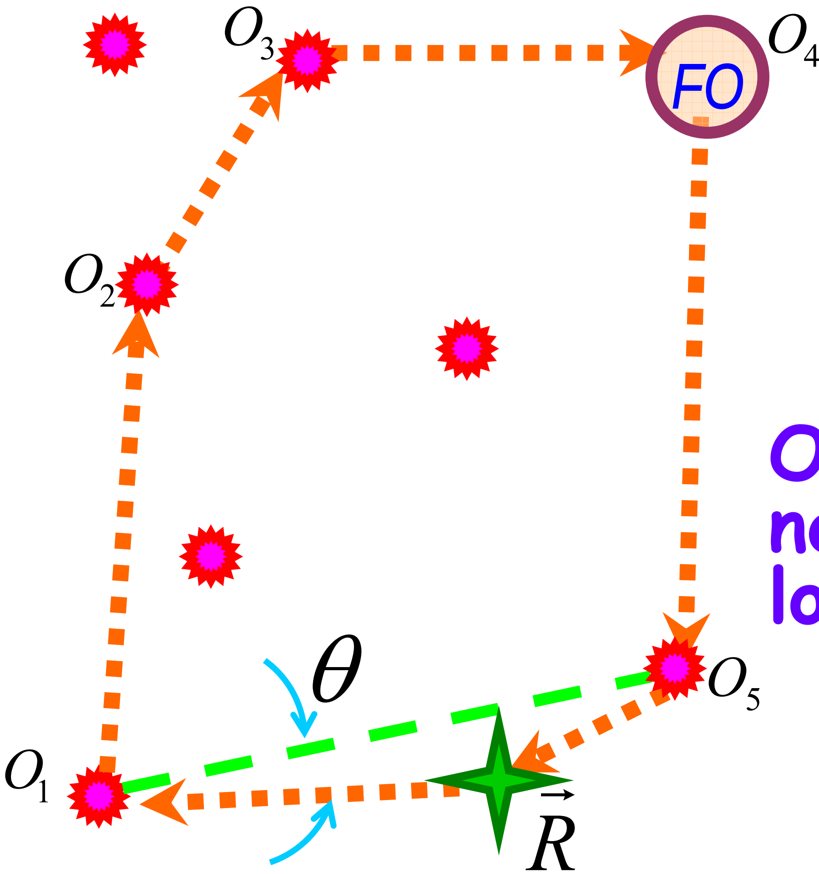
phase factor at small angle θ:

$$\sin\left(k_F L_\alpha\right) e^{ikL_\alpha} \approx \exp\left(\frac{i\varepsilon L_\alpha}{v_F}\right)$$

Again, oscillations are not important as long as

$$L_\alpha < r_\varepsilon \approx \frac{v_F}{\varepsilon} \to \infty$$

Multiple impurity scattering - diffusive case
Compensation of Phases



phase factor at small angle θ:

$$\sin(k_F L_\alpha)e^{ikL_\alpha} \approx \exp\left(\frac{i\varepsilon L_\alpha}{v_F}\right)$$

Oscillations are not important as long as

$$L_\alpha < r_\varepsilon \approx \frac{v_F}{\varepsilon} \to \infty$$

Magnitude of the correction to the DoS is determined by the return probability

If the interaction is not weak, the relative corrections to the DoS are the same as the weak localization corrections to the conductivity

Zeeman Splitting of the Coulomb Anomaly: A Tunneling Study in Two Dimensions

Wenhao Wu,* J. Williams, and P. W. Adams
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803
(Received 13 March 1996)

We report measurements of the tunneling density of states (DOS) of ultrathin Al films in which superconductivity is quenched by a parallel magnetic field, $H_{\parallel} > 48$ kG. The normal state DOS not only displays the usual logarithmic zero bias anomaly (ZBA) but also a new anomaly, superimposed on the ZBA, that appears at bias voltages $V \sim 2\mu_B H_{\parallel}/e$, the electron Zeeman splitting. Measurements in tilted fields reveal that the Zeeman feature collapses with increasing perpendicular field component, suggesting that it is associated with the Cooper electron-electron interaction channel. [S0031-9007(96)00791-0]

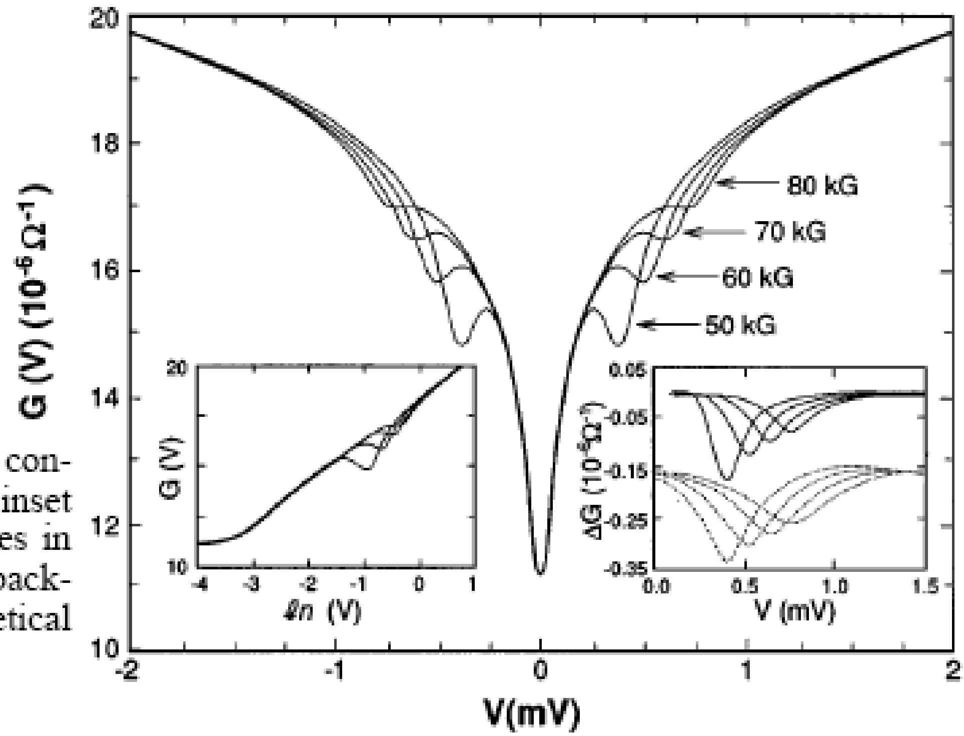
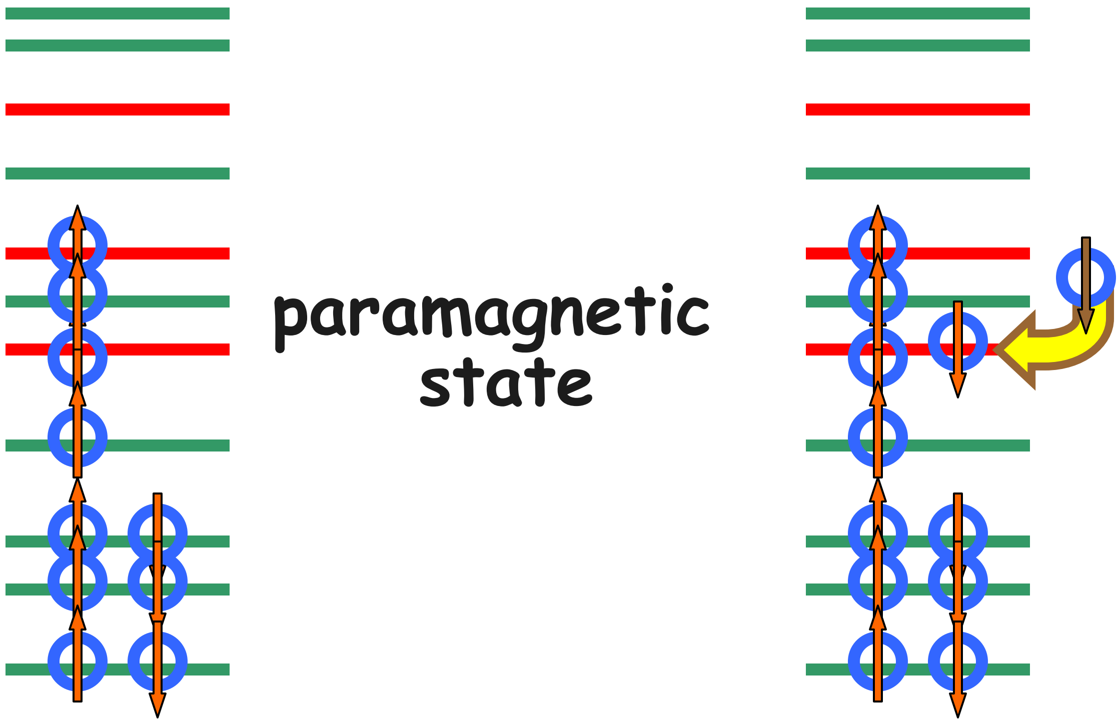


FIG. 2. The main figure is the normal state tunneling conductance of the film in Fig. 1 at several $H_{\parallel}$. The left inset shows the $\ln(V)$ dependence of the ZBA. The solid curves in the right inset show the Zeeman anomaly with the $\ln(V)$ background subtracted off. The dashed curves are the theoretical DOS of Ref. [3] and have been shifted down for clarity.

Tunneling into paramagnetic state

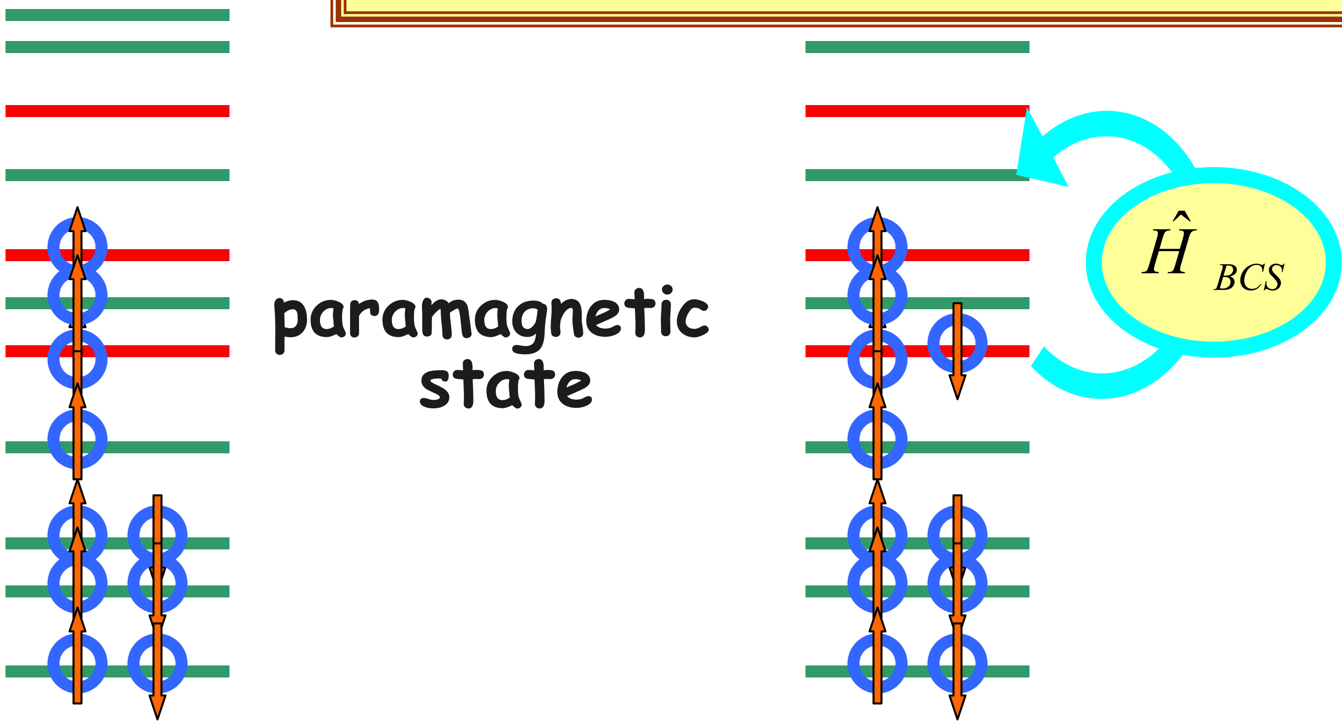


Tunneling of a particle with the "opposite spin" unblocks the orbital state!

BCS Hamiltonian mixes this state with other unblocked (unoccupied) states.

Tunneling into paramagnetic state

$$\hat{H}_{BCS} = \sum \varepsilon_\alpha \left(a^+_{\alpha\uparrow} a_{\alpha\uparrow} + a^+_{\alpha\downarrow} a_{\alpha\downarrow} \right) + \lambda_{BCS} \sum a^+_{\alpha\uparrow} a^+_{\alpha\downarrow} a_{\beta\uparrow} a_{\beta\downarrow}$$

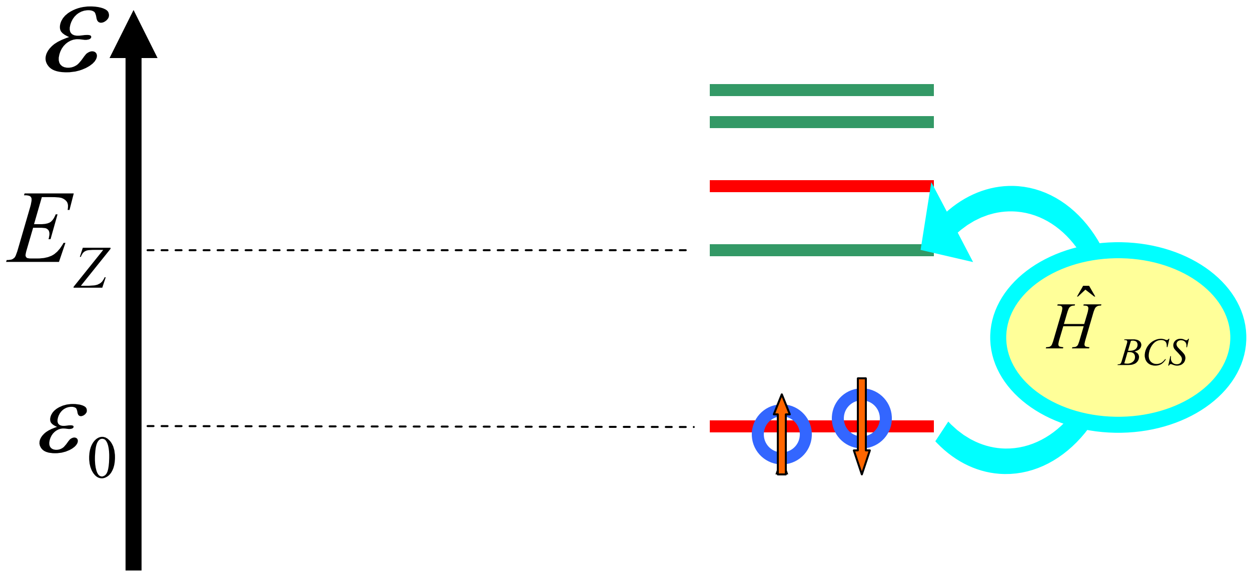


Tunneling of a particle with the "opposite spin" unblocks the orbital state!

BCS Hamiltonian mixes this state with other unblocked (unoccupied) states.

Qualitative picture

$$\hat{H}_{BCS} = \sum \varepsilon_\alpha \left(a^+_{\alpha\uparrow} a_{\alpha\uparrow} + a^+_{\alpha\downarrow} a_{\alpha\downarrow}\right) + \lambda_{BCS} \sum a^+_{\alpha\uparrow} a^+_{\alpha\downarrow} a_{\beta\uparrow} a_{\beta\downarrow}$$

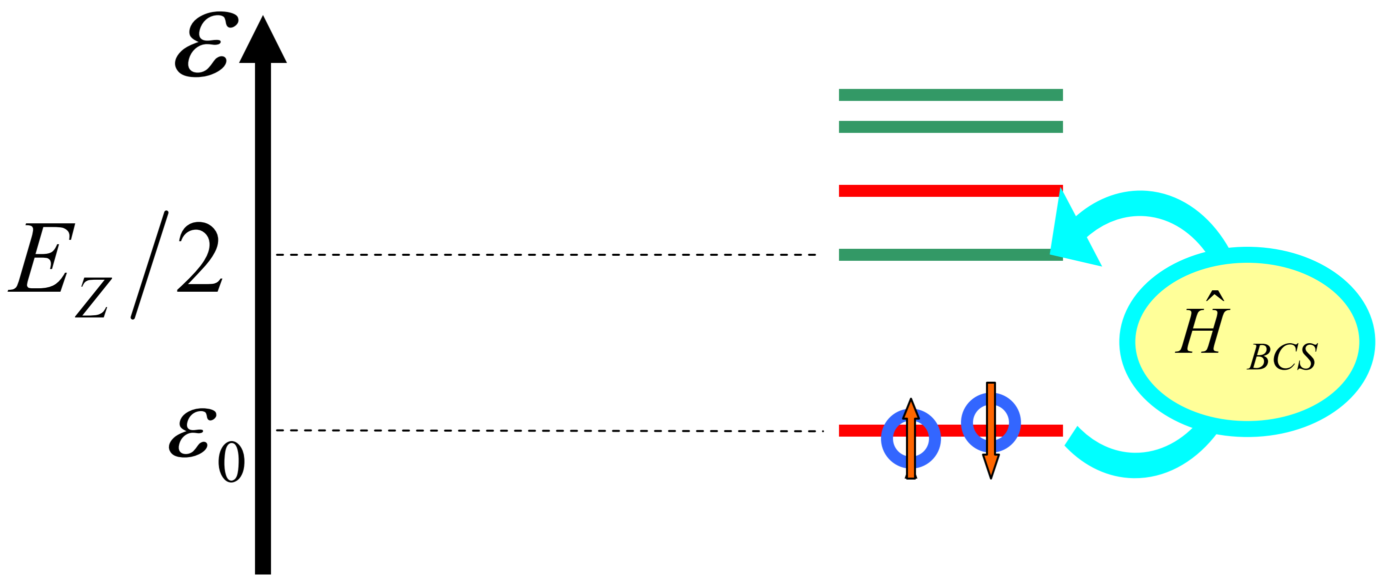


Tunneling of a particle with the "opposite spin" unblocks the orbital state!

BCS Hamiltonian mixes this state with other unblocked (unoccupied) states.

Qualitative picture

$$\hat{H}_{BCS} = \sum \varepsilon_\alpha \left(a^+_{\alpha\uparrow} a_{\alpha\uparrow} + a^+_{\alpha\downarrow} a_{\alpha\downarrow} \right) + \lambda_{BCS} \sum a^+_{\alpha\uparrow} a^+_{\alpha\downarrow} a_{\beta\uparrow} a_{\beta\downarrow}$$



Schrodinger eqn

(all energies are measured in units of the mean level spacing)

$$E^{(2)} \psi_\alpha = 2\varepsilon_\alpha \psi_\alpha + \lambda_{BCS} \sum_{\varepsilon_\alpha > E_Z/2} \frac{1}{2\varepsilon_\alpha - E^{(2)}}$$

$$\frac{2}{2\varepsilon_0 - E^{(2)}} + \ln\left(\frac{\Delta_b}{E_Z - E^{(2)}} \right) = 0 \qquad \Delta_b = \theta_D e^{2/\lambda_{BCS}}$$

One electron excitation energy $E^{(1)} \equiv E^{(2)} - E_\uparrow$

$$E^{(1)} = E_Z - \frac{\Delta_b}{2} \pm \sqrt{\left(\frac{E_Z - \Delta_b}{2} - \varepsilon_0 \right)^2 + 2\Delta_b}$$

$$E^{(1)} = E_Z - \frac{\Delta_b}{2} \pm \sqrt{\left(\frac{E_Z - \Delta_b}{2} - \varepsilon_0\right)^2 + 2\Delta_b}$$

⇒ DoS gap at $E_b^* \equiv E_Z - \frac{\Delta_b}{2}$

More accurate calculation – other electrons are taken into account

$$E^* = \frac{E_Z}{2} + \sqrt{\left(\frac{E_Z}{2}\right)^2 - \left(\frac{\Delta}{2}\right)^2}$$