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SECONDARY INSTABILITY OF FREE SHEAR FLOWS

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Three-dimensional instabilities of saturated two-dimensional vortical states of planar free shear flows are shown to exist for spanwise scales much smaller than those at which classical linear modes are unstable. These modes grow on convective time scales, extract their energy from the mean flow, and persist to moderately low Reynolds numbers. Their growth rates are comparable to the most rapidly growing inviscid instability and two-dimensional subharmonic (pairing) modes. In contrast to two-dimensional modes, the three-dimensional modes do not appear to saturate in quasi-steady states; they seem to lead directly to chaos.

We present here some new results on secondary instabilities of viscous mixing layers and planar jets; details of the results given here are discussed by Brachet & Orszag (1983). The undisturbed velocity profiles are assumed to be **u = tanh (z/a)** for a planar mixing layer and **u = sech (z/a)** for a planar jet. Such free shear flows differ from wall bounded flows in the sense that they have inflectional points and hence are inviscidly unstable.

The central role of secondary three-dimensional instabilities in the transition to turbulence of wall bounded flows, such as plane Poiseuille flow, has recently been established (Herbert 1983, Orszag 1983, Orszag & Patera 1980, 1983). With inflectional free shear flows, one may expect that the transition process is basically simpler due to the possible dominance of the essentially two-dimensional instabilities of the primary shear flow as well as the subharmonic instabilities leading to vortex pairing. Indeed, a wide variety of experimental results for free shear flows demonstrates the central role played by two-dimensional processes (Miksad 1972, Bernal et al 1979, Browand & Troutt 1980). On the other hand, Pierrehumbert & Widnall (1982) have shown that secondary three-dimensional 'translative' instabilities are also potent in mixing layers.

We have investigated the two- and three-dimensional stability of the two-dimensional, viscous, vortical flows that result from the primary and pairing instabilities of free shear flows. We use methods similar to those used by Orszag & Patera in studying wall bounded flows. The problem is studied by solving the Navier-Stokes equations as an initial-value problem using pseudospectral methods based on expanding the flow field in Fourier series in x and y and in mapped Chebyshev polynomial expansions in z. The initial conditions are constructed as a superposition of Orr-Sommerfeld eigenmodes of the basic flow, which are also computed numerically by spectral techniques. The results of purely two-dimensional studies show the important role of the primary viscous instability and the pairing modes, but there is no evidence of 'chaos' in the finite-amplitude developing flow; the two dimensional flows appear to saturate into ordered flow states (see Patnaik et al 1976). These saturated two-dimensional states are unstable to infinitesimal three-dimensional perturbations; it is these 'secondary' instabilities that is the main focus of our work.

Our numerical studies demonstrate the following properties of the secondary three-dimensional instabilities. First, they are strong for spanwise scales at which the basic flow is linearly stable. At Reynolds number R, they persist to scales of order $\mathbf{a}/\mathbf{R}^{1/2}$ while the primary instability disappears if the scale of the mode is smaller than order **a** (see Figure 1). Second, the secondary instability is effective even for moderately low Reynolds numbers. Third, its growth rate scales with the convective (not the dissipative) parameters of the flow. Fourth, the instability seems to lead directly to chaotic flows; at finite-amplitudes, there is no evidence that the secondary instability saturates into ordered, laminar flow states. Finally, the energetics of the instability is similar to that of the secondary instability in wall-bounded shear flows; the growing three-dimensional modes extract their energy from the basic flow with the two-dimensional disturbance acting only as a catalyst for this process. When the spatial scale of the disturbance is too small, the instability is turned off because viscous dissipation becomes larger than the transfer from the basic flow into the disturbance, the latter process being relatively independent of spatial scale.

One main conclusion from our work is that transitional processes may, in fact, be more complex for free-shear flows than for non-inflectional flows (typical of wall-bounded situations). With non-inflectional basic flows, the primary linear instabilities grow on viscous time scales so they cannot compete with three-dimensional secondary instabilities which grow on convective time scales. However, for free shear flows, inviscid two-dimensional instabilities exist that grow on convective time scales and, hence, may compete effectively with the three-dimensional secondary instabilities. With this competition, the evolution of free shear flows in transitional regimes may well depend significantly on the past history of the flow and on details of the environment in which the flow occurs.

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REFERENCES

Bernal, L.P., Breidenthal, R.E., Brown, G.L., Konrad, J.H. & Roshko, A. 1979 On the development of three dimensional small scales

in turbulent mixing layers. In Proc. 2nd Int. Symp. on Turbulent Shear Flows, Imperial College, London.
Brachet M.E. & Orszag, S.A. 1983 Secondary Instability of Free Shear Flows. Submitted to J. Fluid Mech.
Browand, F.K., & Troutt, T. R. 1980 A note on spanwise structure in the two-dimensional mixing layer. J. Fluid Mech. 97, 771.
Herbert, T. 1983 Modes of secondary instability in plane Poiseuille flow. This Volume.
Miksad, R.W. 1972 Experiments on the non-linear stages of free-shear-layer transition. J. Fluid Mech. 56, 695.
Orszag, S. A. 1983 Instabilities and turbulence. This Volume.
Orszag, S. A. & Patera, A. T. 1980 Subcritical transition to turbulence in plane channel flows. Phys. Rev. Lett. 45, 989.
Orszag, S. A. & Patera, A. T. 1983 Secondary instability of wall-bounded shear flows. J. Fluid Mech. 128, 347.
Patnaik, P. C., Sherman, F. S. & Corcos, G. M. 1976 A numerical simulation of Kelvin-Helmholtz waves of finite amplitude. J. Fluid Mech. 73, 215.
Pierrehumbert, R. T. & Widnall, S. E. 1982 The two- and three-dimensional instabilities of a spatially periodic shear layer. J. Fluid Mech. 114, 59.

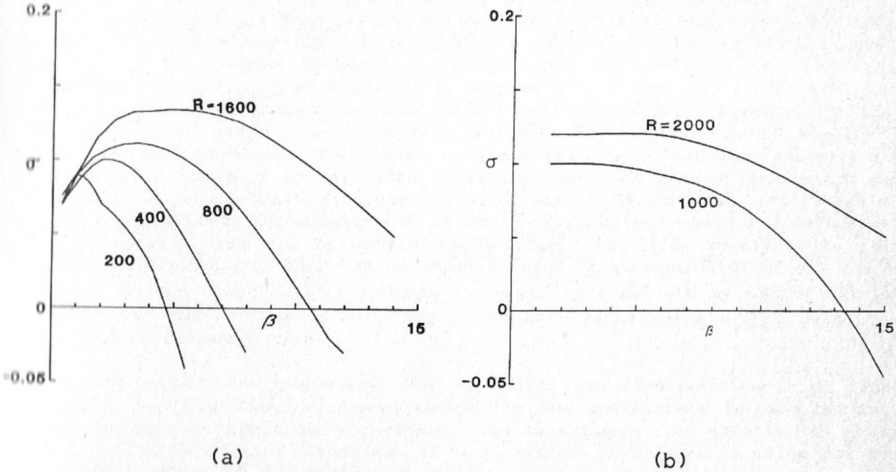

Figure 1. (a) A plot of the computed growth rate of the three-dimensional secondary instability for a tanh-profile mixing layer plotted as a function of spanwise wavenumber β for various Reynolds numbers R. The streamwise wavenumber is $\alpha = 0.4$ with $a = 1$ in the mean velocity profile. Note that the instability persists up to spanwise wavenumbers of order $1/3R^{\frac{1}{2}}$. (b) Same as (a), except that the plot is made for the sech-profile jet with $a = 1$ and streamwise wavenumber $\alpha = 1.5$.