# Problem Set for Exercise Session No.6

Course: Mathematical Aspects of Symmetries in Physics,
ICFP Master Program (for M1)

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## 1 Differential

(1) Let us consider differentiable manifolds $M_1, M_2, M_3$ of class $C^\infty$ and $C^\infty$ maps $f : M_1 \to M_2$ and $g : M_2 \to M_3$. We also denote a point on $M_1$ by $p$. Prove the following properties of the differentials (below $d(...)$ means the differential of a map $(...)$):

1. $d(g \circ f)_p = dg_{f(p)} \circ df_p$.
2. $d(id_{M_1})_p = id_{T_pM_1}$. Here $id_{M_1}$ and $id_{T_pM_1}$ are identity maps from $M_1$ to $M_1$ and $T_pM_1$ to $T_pM_1$, respectively.
3. When $f$ is a diffeomorphism, then $df_p$ is an isomorphism and $(df_p)^{-1} = d(f^{-1})_{f(p)}$.

(2) Answer the following questions:

1. Let us consider a map $\psi : \mathbb{R}P^2 \to \mathbb{R}$ defined by
$$\psi([x : y : z]) = \frac{z^2}{x^2 + y^2 + z^2} \, .$$
By using the coordinates defined in Problem Set No.5, confirm that $\psi$ is $C^\infty$. Find points $p \in \mathbb{R}P^2$ at which the differential $d\psi_p : T_p\mathbb{R}P^2 \to T_{\psi(p)}\mathbb{R}$ vanishes.
2. Let us consider a map $\pi : S^2 \to \mathbb{R}P^2$ defined by $\pi(x, y, z) = [x : y : z]$ for $(x, y, z) \in S^2$. Show that, at any point $p \in S^2$, the differential $d\pi_p : T_pS^2 \to T_{\pi(p)}\mathbb{R}P^2$ is an isomorphic map.