A new quantum glass phase: the superglass

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1 Motivations
   • Supersolidity of He$^4$

2 The glass transition of classical liquids
   • Phenomenology
   • Mean field spin glass models for the glass transition

3 The quantum glass transition
   • Quantum p-spin and QREM
   • Helium 4: Monte Carlo results

4 A model for the superglass phase
   • Mapping on classical diffusive dynamics
   • The phase diagram
   • Quantum slow dynamics
   • Condensate fluctuations
   • Superfluid properties
   • Perspectives

5 Lattice models
   • Disordered Bose-Hubbard model: the Bose glass
   • Quantum Biroli-Mézard model: a superglass?
   • Solution of Bose-Hubbard models on the Bethe lattice
Motivations

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5. The phase diagram
6. Quantum slow dynamics
7. Condensate fluctuations
8. Superfluid properties
9. Perspectives

Lattice models

5. Disordered Bose-Hubbard model: the Bose glass
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7. Solution of Bose-Hubbard models on the Bethe lattice

Conclusions
Motivations: supersolidity of He\textsuperscript{4}

Non-classical rotational inertia observed in solid He\textsuperscript{4} (Kim and Chan)

Possible interpretation: supersolidity

- Supersolidity excluded in perfect He\textsuperscript{4} crystals (Boninsegni, Ceperley et al.)
- Supersolidity strongly enhanced by fast quenches (Rittner and Reppy)
- History dependent response and some evidence for aging (Davis et al.)
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Conclusions
Phenomenology

Classical particle system (e.g. Lennard-Jones like potential)
No external disorder

Huge increase of the viscosity (or density relaxation time) in a small range of temperature

Second order phase transition: jump in compressibility
First six decades of dynamic slowing down is well described by Mode-Coupling Theory (MCT)

- MCT predicts power-law divergence, $\tau \sim (T - T_c)^{-\gamma}$, with too large $T_c$
- The divergence is "activated" $\tau \sim \exp(A/(T - T_0))$ instead
- Activation is neglected in MCT (mean field theory)

Two steps relaxation:
1. Intra-cage vibrational motion ($\tau_\beta$)
2. Structural relaxation ($\tau_\alpha$)
Mean field spin glass models

A mean field model for the glass transition: the *p-spin model*:

\[ H = \sum_{i<j<k} J_{ijk} S_i S_j S_k \]

\( S_i \) Ising spins
\( J_{ijk} \) independent Gaussian random variables with zero average

- Liquid phase: dynamics is described by MCT-like equations
- "Activated" liquid phase: \( e^{N\Sigma} \) states are populated
- Glass phase: "condensation", finite number of ground states

In a suitable limit (infinite number of spin in each interaction) reduces to the **Random Energy Model (REM)**: \( 2^N \) levels \( E_i \), i.i.d. Gaussian variables
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Quantum p-spin and QREM

Quantum p-spin in a transverse field: (Goldschmidt; Cugliandolo et al.; Jorg et al.)

$$H = \sum_{i<j<k} J_{ijk} S_i^z S_j^z S_k^z - \Gamma \sum_i S_i^x$$

For infinite-body interaction: quantum REM, full spectrum
First order quantum phase transition (paramagnet → glass) at $T = 0$

At $T = 0$, slow dynamics in the glass but not in the paramagnet; no slowing down observed on approaching $\Gamma_c$ from above.
Quantum Monte Carlo simulation of He\textsuperscript{4} at high pressure $P > 32$ bar
Quench from the liquid phase down in the solid phase (Boninsegni et al.)

Density-density correlations similar to the liquid (large Lindemann ratio)

ODLRO observed in the one-particle density matrix $\rightarrow$ BEC, superfluidity
At $P = 32$ bar, $n_0 = 0.5\%$ and $\rho_s/\rho = 0.6$
Helium 4: Monte Carlo results

Amorphous condensate wavefunction: $n(r - r') \sim n_0 \phi(r)\phi(r')$

Plot of $\phi(x, y, z)$ on slices at fixed $z$

Many open problems

What is the nature of the transition?
Is it accompanied by slow dynamics in the liquid phase?
Where does superfluidity come from?
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Mapping on classical diffusive dynamics

- General mapping: Quantum Hamiltonian $\leftrightarrow$ Fokker-Planck operator
- Diffusive dynamics (Brownian motion, Langevin equation):
  \[
  \gamma_i \frac{dx_i}{dt} = -\frac{\partial}{\partial x_i} U_N(x_1, \ldots, x_N) + \eta_i(t), \quad i = 1, \ldots, N,
  \]
- Evolution of probability $P(x_i; t)$: Fokker-Planck eq. $\partial_t P = -H_{FP} P$
- Equilibrium distribution $P_G = \exp(-\beta U_N)/Z$, $H_{FP} P_G = 0$
  All other eigenvectors $H_{FP} P_E = E P_E$ such that $E > 0$
- Associated quantum (Hermitian) Hamiltonian: $H = P_G^{-1/2} H_{FP} P_G^{1/2}$
- Ground state $\Psi_G(x_i) = \sqrt{P_G(x_i)}$ is a Jastrow wavefunction
  Full spectrum of $H$ equal to spectrum of $H_{FP} \Rightarrow$ access to real time quantum dynamics

Remarks:
- $H$ has special properties! No inverse mapping in general...
- Jastrow wavefunctions are good variational ground states for He$^4$
The phase diagram

We choose $U_N(x_i) = \sum_{i<j} V_{HS}(x_i - x_j)$ (classical Hard Spheres)
Quantum potential: sticky Hard Sphere + sticky three-body interactions
Glass transition on increasing density

Solid phases are ”classical”: small Lindemann ratio
Finite $n_0$ but very small in both crystal and glass phases
Slow dynamics approaching the glass phase

Density-density correlation function:
- \( F_{cl}(q, t) = \langle \rho_q(t) \rho_{-q}(0) \rangle = \int_0^\infty \frac{d\omega}{2\pi} \rho_q(\omega) e^{-\omega t} \)
- \( F_Q(q, t) = \langle 0 | \{ \rho_q(it), \rho_q(0) \} | 0 \rangle = \int_0^\infty \frac{d\omega}{2\pi} \rho_q(\omega) \cos(\omega t) \)

Separation of time scales: \( \rho_q(\omega) = \rho_\beta(\omega \tau_\beta) + \rho_\alpha(\omega \tau_\alpha) \) with \( \tau_\beta \ll \tau_\alpha \)
For \( \tau_\beta \ll t \ll \tau_\alpha \):
- the contribution of \( \rho_\beta(\omega \tau_\beta) \) decays to zero
- the contribution of \( \rho_\alpha(\omega \tau_\alpha) \) is the same since \( e^{-\omega t} \sim \cos(\omega t) \sim 1 \)
hence \( F_{cl}(q, t) \sim F_Q(q, t) \sim \int_0^\infty \frac{d\omega}{2\pi} \rho_\alpha(\omega \tau_\alpha) \Rightarrow \text{same plateau!} \)
Condensate fluctuation in the glass

In the glass state $\tau_\alpha = \infty \rightarrow$; liquid freezes in many possible states
Amorphous density profile $\rho_\alpha(r)$ and condensate profile $\phi_\alpha(r)$

$$g_\phi(r - r') \propto \sum_\alpha p_\alpha \phi_\alpha(r) \phi_\alpha(r')$$
correlation function of condensate fluctuations
Superfluid properties

Superfluidity requires a linear spectrum ("phonons"): \( v_c \leq \min_k [\epsilon(k)/k] \)

In our model \( e(\rho) \equiv 0 \Rightarrow \) sound velocity \( c = \frac{d}{d\rho} \rho^2 \frac{de}{d\rho} = 0 \Rightarrow v_c = 0 \)

(follows from a special symmetry that allows to map \( H \) into a Fokker-Planck operator)

Introduce a perturbation \( \delta v(r) \); then \( \delta e(\rho) = \frac{\rho}{2} \int dr \ g(r) \ \delta v(r) \)

- sound velocity \( c \neq 0 \Rightarrow \rho_s \neq 0 \)
- first order transition at \( \rho_K \)
  
  [very weak jump in \( e'(\rho) = P/\rho^2 \)]
Perspectives

Weak points in the theory:

- "Classical"-like solids, small Lindemann ratio and superfluid fraction
- "Ad hoc" inclusion of phonons
- New quantum phase transition: first order with slow dynamics. How general?
- Quantitative computation for He\(^4\), cold atoms…
  \[\rho_K\text{ for He}\(^4\) is 10 times larger than the one of Boninsegni et al.\]
- What happens at finite temperature?

Possible strategies:

- Better variational wavefunctions: Shadow and Jastrow with three body interactions; should give larger Lindemann ratio and \(\rho_s\)
- Quantum Mode Coupling Theory (Reichmann and Miyazaki)
- Replica computation at finite temperature
- Leggett bound: relation between \(\rho(r)\) and \(\rho_s\), apply to superglass
  It seems that disorder does not help superfluidity
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Disordered Bose-Hubbard model: the Bose glass

\[ H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i (\mu + \varepsilon_i)n_i \]

\[ \varepsilon_i \in [-\Delta, \Delta] \text{ quenched external disorder} \]

- Mott insulator: one particle/site
  Strong localization \( \Rightarrow \) no BEC, \( \rho_s = 0 \)
  Zero compressibility

- Bose glass: additional defects
  Anderson localization
  Finite compressibility

No frustration, no RSB
No slow dynamics
Quantum Biroli-Mézard model: a superglass?

\[ H = -J \sum_{<i,j>} (a_i^\dagger a_j + a_j^\dagger a_i) + \sum_{<i_1,\ldots,i_k>} V(n_{i_1}, \ldots, n_{i_k}) - \sum_i \mu n_i \]

Classical model \((J = 0)\): glass transition similarly to Hard Spheres
Self-generated disorder, RSB, slow dynamics

Add quantum fluctuations \((J \neq 0)\)
A quantum glass transition? Slow dynamics? Aging?
Nature of the transition (first or second order)?

Strategy: solve the model on the Bethe lattice
Solution of Bose-Hubbard models on the Bethe lattice

- Solution of functional recurrence equations for the local action
- Gives back DMFT for $Z \rightarrow \infty$
- Successfully tested on the ordered Bose-Hubbard

Work in progress... (with G. Semerjian and M. Tarzia)
Conclusions

Our results:

- A semi-realistic model for interacting Bosons displays a superglass phase
- First order quantum glass transition with real time slow dynamics
- Variational calculation for more realistic potentials
- Possibility of exact solution for Bethe lattice models

Related works:

- Quantum Mode Coupling Theory (Reichmann, Miyazaki)
- B-DMFT (Vollhardt, Hofstetter, et al.)
- Monte Carlo simulations (Boninsegni, Prokof’ev, Svistunov, et al.)