Random field spin models beyond one loop: a mechanism for decreasing the lower critical dimension

Pierre Le Doussal and Kay Jörg Wiese

CNRS-Laboratoire de Physique Théorique de l’Ecole Normale Supérieure, 24 rue Lhomond, 75005 Paris, France.

(Dated: October 28, 2005)

The functional RG for the random field and random anisotropy $O(N)$ sigma-models is studied to two loop. The ferromagnetic/disordered (F/D) transition fixed point is found to next order in $d = 4 + \epsilon$ for $N \gg N_c$ ($N_c = 2.8347408$ for random field, $N_c = 9.44121$ for random anisotropy). For $N < N_c$, the lower critical dimension $d = d_{lc}$ plunges below $d_{uc} = 4$: we find two fixed points, one describing the quasi-ordered phase, the other is novel and describes the F/D transition. $d_{lc}$ can be obtained in an $(N_c-N)$-expansion. The theory is also analyzed at large $N$ and a glassy regime is found.

It is important for numerous experiments to understand how the spontaneous ordering in a pure system is changed by quenched substrate impurities. One class of systems are modeled by elastic objects in random potentials (so-called random manifolds, RM). Another class are $O(N)$ classical spin models with ferromagnetic (Ferro) couplings in presence of random fields (RF) or anisotropies (RA). The latter describe amorphous magnets [1]. Examples of RF are liquid crystals in porous media [2], He-3 in aerogels [3], nematic elastomers [4], and ferroelectrics [5]. The XY random field case $N = 2$ is common to both classes and describes periodic RM such as charge density waves, Wigner crystals and vortex lattices [6]. Larkin showed [7] that the well-understood pure fixed points (FP) of both classes are perturbatively unstable to weak disorder for $d < d_{uc}$ ($d_{uc} = 4$ in the generic case). For a continuous symmetry (i.e. the RF Heisenberg model) it was proven [8] that order is destroyed below $d = 4$. This does not settle the difficult question of the lower critical dimension $d_{lc}$ as a weak-disorder phase can survive below $d_{uc}$, associated to a non-trivial FP, as predicted in $d = 3$ for the Bragg-glass phase with quasi long-range order (QLRO) [9]. For the random field Ising model $N = 1$ (RFIM) it was argued [10], then proven [11] that the ferromagnetic phase survives in $d = 3$. Developing a field theory to predict $d_{lc}$, and the exponents of the weak-disorder phase and the Ferro/Disordered (F/D) transition, has been a long-standing challenge. Both extensive numerics and experiments have not yet produced an unambiguous picture. Among the debated issues are the critical region of the 3D RFIM [12] and the possibility of a QLRO phase in amorphous magnets [2,13,14].

A peculiar property shared by both classes is that observables are identical to all orders to the corresponding ones in a $d = 2$ thermal model [15]. This dimensional reduction (DR) naively predicts $d_{lc} = 4$ for the weak-disorder phase in a RF with a continuum symmetry [16] and no Ferro order for the $d = 3$ RFIM, which is proven wrong [11]. It also predicts $d_{lc} = 6$ for the F/D transition FP. While there is agreement that multiple local minima are responsible for DR failure, constructing the field theory beyond DR is a formidable challenge. Recent attempts include a reexamination of the $\phi^4$ theory (i.e. soft spins) for the F/D transition near $d = 6$ [17].

Previous large-$N$ approaches failed to find a non-trivial FP, but a self-consistent resummation including the $1/N$ corrections hinted at exponents different from DR (without succeeding in computing them) from a solution breaking replica symmetry [18].

As for the pure $O(N)$ model, an alternative to the soft-spin version (near $d = 6$) is the sigma model near the lower critical dimension (here presumed to be $d = 4$). In 1985 D.S. Fisher [19] noticed that an infinite set of operators become relevant near $d = 4$ in the RF $O(N)$ model. These were encoded in a single function $R(\phi)$ for which Functional RG equations (FRG) were derived to one loop, but no new FP was found. For a RM problem [20] it was found that a cusp develops in the function $R(\phi)$ (the disorder correlator), a crucial feature which allows to obtain non-trivial exponents and evade DR. A fixed point for the RF model was later found [9] in $d = 4 - \epsilon$ for $N = 2$. It was noticed only very recently [21] that the 1-loop FRG equations of Ref. [19] possess fixed points in $d = 4 + \epsilon$ for $N \geq 3$, providing a description of the long-sought critical exponents of the F/D transition.

In spite of these advances, many questions remain. Constructing FRG beyond one loop (and checking its internal consistency) is highly non-trivial. Progress was made for RM [22, 23], and one hopes for extension to RF. Some questions necessitate a 2-loop treatment, e.g. for the depinning transition, as shown in [24]. In RF and RA models the 1-loop analysis predicted some repulsive FP in $d = 4 + \epsilon$ (for larger values of $N$), and some attractive ones [9,25] in $d = 4 - \epsilon$. The overall picture thus suggests a lowering of the critical dimension, but how it occurs remains unclear. Finally the situation at large $N$ is also puzzling. Recently, via a truncation of exact RG [24] it was claimed that DR is recovered for $N$ large.

Our aim in this Letter is twofold. We reexamine the overall scenario for the fixed points and phases of the $O(N)$ model using FRG. This requires the FRG to two loop. Here we present selected results, details are presented elsewhere [27]. We find a novel mechanism for how the lower critical dimension is decreased below $d = 4$ for $N < N_c$ at some critical value $N_c$. We obtain a description of the bifurcation which occurs at $N_c$, and below $N_c$ we find two perturbative FPs. Thanks to 2-loop terms $d_{lc}$ can be computed in an expansion.
in $N_{c} - N$, and the Ferro/Para FP below $d = 4$ is found. A study at large $N$ indicates that some glassy behavior survives there.

Let us consider $O(N)$ classical spins $\vec{n}(x)$ of unit norm $\vec{n}^{2} = 1$. To describe disorder-averaged correlations one introduces replicas $\vec{n}_{a}(x)$, $a = 1, \ldots, k$, the limit $k = 0$ being implicit everywhere. The starting model is a non-linear sigma model, of partition function $Z = \int D[\vec{n}] e^{-\beta S[\vec{n}]}$, action:

$$\mathcal{S}[\pi] = \int d^{d}x \left[ \frac{1}{2T_{0}} \sum_{\alpha} \left( \nabla^{2} \pi_{\alpha} \right)^{2} - \frac{1}{2T_{0}} \sum_{\alpha} M_{0} \sigma_{\alpha} \right. $$
$$+ \left. \frac{1}{2T_{0}} \sum_{\alpha} \sum_{b} \mathcal{R}_{0} \left( \vec{n}_{\alpha} \vec{n}_{b} \right) \right] , \tag{1}$$

where $\vec{n}_{a} = (\sigma_{a}, \vec{\pi}_{a})$ with $\sigma_{a}(x) = \sqrt{1 - \vec{\pi}_{a}(x)^{2}}$. A small uniform external field $\sim M_{0}(1, \vec{0})$ acts as an infrared cutoff. Fluctuations around its direction are parameterized by $(N-1)$ $\pi$-modes. The ferromagnetic exchange produces the 1-replica part, while the random field yields the 2-replica term $\mathcal{R}_{0}(z) = z$ for a bare Gaussian RF. RA corresponds to $\mathcal{R}_{0}(z) = z^{2}$. As shown in [19] one must include a full function $\mathcal{R}_{0}(z)$, as it is generated under RG. It is marginal in $d = 4$.

To obtain physics at large scales, one computes perturbatively the effective action $\Gamma[\vec{n}(x)]$. It expands in gradients near a uniform background configuration $n_{0}^{0}$, and split-ted in $1-, 2-$ and higher-replica terms. From rotational invariance it is natural to look for $\Gamma$ in the form $\Gamma[\vec{n}]$ with $\vec{n}_{a} \rightarrow \vec{n}_{a}^{R} = (\sigma_{a}^{R}, \vec{\pi}_{a}^{R})$, $\sigma_{a} \rightarrow \sigma_{R}^{a} = \sqrt{1 - (\vec{\pi}_{a}^{R})^{2}}$, $\pi_{a} \rightarrow \pi_{R}^{a} = Z^{-1/2} \pi_{a}$, $T_{0} \rightarrow T_{R} = T_{0}/Z_{R}$, $M_{0} \rightarrow M_{R} = M_{0} \sqrt{Z_{R}}/Z_{T}$, $m = \sqrt{M_{R}}$ the renormalized mass of the $\vec{n}_{a}$ modes, and $\mathcal{R}_{0}(\vec{n}_{a} \vec{n}_{b}) \rightarrow m^{2} R^{2}(\vec{n}_{a}^{R} \vec{n}_{b}^{R})$. Higher vertices generated under RG are irrelevant by power-counting, hence discarded. Renormalization of $T$ contributes to the flow of $R$, and one sets $T = 0$ at the end.

One computes $Z, Z_{R}$ and $R_{R}$ perturbatively in $R_{0}$ and extracts $\beta$ and $\gamma$ functions $\beta[\mathcal{R}](z) = -m \partial_{m} \mathcal{R}(z), \gamma = -m \partial_{m} \ln Z$ and $\Gamma_{\gamma} = -m \partial_{m} \ln Z_{T}$, derivatives taken at fixed $\mathcal{R}_{0}, T_{0}, M_{0}$. Although calculation of the $Z$-factors is simplified due to DR, anomalous contributions appear from the non-analyticity of $\mathcal{R}(z)$. To compute $\mathcal{R}(z)$, one chooses a pair of uniform background fields $(n_{a}^{0}, n_{b}^{0})$ for each $(a, b)$. We use a basis for the fluctuating fields (to be integrated over) such that $\vec{n}_{a} = (\sigma_{a}, \eta_{a}), \vec{n}_{b} = (\sigma_{b}, \eta_{b}, \vec{\rho}_{b})$, where $\eta_{a}$ lies in the plane common to $(\vec{n}_{a}^{0}, \vec{n}_{b}^{0})$, and $\vec{\rho}_{b}$ along the perpendicular $N-2$ directions; both have diagonal propagators. Denoting $\vec{n}_{a}^{0} \vec{n}_{b}^{0} = \cos \phi_{ab}$, one has $\vec{n}_{a} \vec{n}_{b} = \cos \phi_{ab}\sigma_{a}\sigma_{b} + \eta_{a}\eta_{b} + \sin \phi_{ab}(\sigma_{a}\eta_{b} - \eta_{a}\sigma_{b}) + \vec{\rho}_{a} \vec{\rho}_{b}$. One gets factors of $\gamma_{a} = 0$ from the contraction of $\vec{\rho}$. Our calculation to 2 loops results in the flow-equation for the function $\mathcal{R}(\phi) = \mathcal{R}(z = \cos \phi)$, and $\epsilon = 4 - d$:

$$\partial_{t} \mathcal{R}(\phi) = \epsilon \mathcal{R}(\phi) + \frac{1}{2} \mathcal{R}^{\prime}(\phi)^{2} - \mathcal{R}^{\prime\prime}(0) \mathcal{R}^{\prime}(\phi) + (N - 2) \left[ \frac{1}{2} \frac{\mathcal{R}^{\prime\prime}(\phi)^{2}}{\sin^{2}\phi} - \cot \phi \mathcal{R}^{\prime\prime}(\phi) \mathcal{R}^{\prime}(\phi) \right] $$
$$+ \frac{1}{2} \left( \mathcal{R}^{\prime\prime\prime}(\phi) - \mathcal{R}^{\prime\prime}(0) \right) \mathcal{R}^{\prime}(\phi)^{2} + (N - 2) \left[ \frac{\cot \phi}{\sin^{4}\phi} \mathcal{R}^{\prime\prime\prime}(\phi) - \frac{5 + \cos 2\phi}{4 \sin^{4}\phi} \mathcal{R}^{\prime}(\phi)^{3} \mathcal{R}^{\prime\prime}(\phi) + \frac{1}{2 \sin^{2}\phi} \mathcal{R}^{\prime\prime\prime}(\phi) \mathcal{R}^{\prime\prime}(\phi)^{3} \right] $$
$$- \frac{1}{4 \sin^{4}\phi} \mathcal{R}^{\prime\prime\prime}(0) \left( 2(2 + \cos 2\phi) \mathcal{R}^{\prime}(\phi)^{2} - 6 \sin 2\phi \mathcal{R}^{\prime}(\phi) \mathcal{R}^{\prime\prime}(\phi) + (5 + \cos 2\phi) \sin^{2}\phi \mathcal{R}^{\prime\prime\prime}(\phi) \mathcal{R}^{\prime\prime}(\phi) \right) ^{2} \mathcal{R}^{\prime\prime}(\phi) - \frac{N + 2}{8} \mathcal{R}^{\prime\prime\prime}(0)^{2} \mathcal{R}^{\prime\prime}(\phi) - \frac{N - 2}{4} \cot \phi \mathcal{R}^{\prime\prime\prime}(0)^{2} \mathcal{R}^{\prime\prime}(\phi) + (N - 2) \left[ \frac{\mathcal{R}^{\prime\prime}(0) - \mathcal{R}^{\prime\prime}(0)^{2} + \gamma_{a} \mathcal{R}^{\prime\prime\prime}(0)^{2}}{2} \mathcal{R}^{\prime\prime}(\phi) \right] \tag{2}$$

with $\partial_{t} = -m \partial_{m}$, and the last factor proportional to $R(\phi)$ is $-2\gamma_{T}$ and takes into account the renormalization of temperature. Thanks to the anomalous terms, arising from a non-analytic $R(\phi)$, this $\beta$-function preserves a (at most) linear cusp (i.e. finite $\mathcal{R}^{\prime\prime\prime}(0)$), and reproduces for $N = 2$ the previous $2$-loop results for the periodic RM [22]. For $N > 2$, anomalous contributions are determined following [28]. $\gamma$ is found as

$$\gamma = (N - 1) \mathcal{R}^{\prime\prime}(0) + \frac{3N - 8}{8} \mathcal{R}^{\prime\prime\prime}(0)^{2} , \tag{3}$$

either via a calculation of $\langle \sigma_{a} \rangle$ [30], or of the mass corrections, a result consistent with the $\beta$ function [23] [31]. The determination of $\gamma_{T}$ is more delicate [32], and we have allowed for an anomalous contribution $\gamma_{a}$, whose effect is minor, and discussed below. The correlation exponents (standard definition [21]) are obtained as $\eta = \epsilon - \gamma$, $\eta = \gamma_{T} - \gamma$ at the FP. [2] has the form:

$$\partial_{t} R = \epsilon R + B(R, R) + C(R, R, R) + O(R^{4}) \tag{4}$$

We now discuss its solution, first in the RF case, and setting $\gamma_{a} = 0$. The 1-loop flow-equation (setting $C = 0$) admits, in dimensions larger than 4, a fixed point $R_{F}^{*}\partial D$ with a single repulsive direction, argued by Feldman to describe the F/D zero temperature transition. This is true only for $N > N_{c}$. For $N < N_{c}$ this fixed point disappears and instead an attractor

$\begin{array}{lll}
\text{FIG. 1: (Color online) Phase diagram. D = disordered, F = ferromagnetic, QLRO = quasi long-range order.}
\end{array}$
fixed point $R^*_\text{QGRO}$ appears which describes the Bragg glass for $N = 2$. We have determined $N_c = 2.8347408$ and the solution $R_c(u)$ which satisfies $B(R_c, R_c)|_{N = N_c} = 0$. It is formally the solution at $\epsilon = 0$. Since the FRG flow vanishes to one loop along the direction of $R_c$, examination of the 2-loop terms is needed to understand what happens at $N = N_c$. In particular the F/D transition should still exist for $N < N_c$, though it cannot be found at one loop. It is not even clear a priori whether it remains perturbative.

The scenario found is perturbative, accessible within a double expansion in $\sqrt{\epsilon}$ and $N - N_c$. To this aim, we write the leading terms in $N - N_c$ and $\epsilon$ of (4), namely

$$\partial_t R = \epsilon R + B_c(R, R) + C_c(R, R, R) + (N - N_c) B_N(R, R),$$

$$B_c(\ldots) = B(\ldots)|_{N = N_c}, \quad C_c(\ldots) = C(\ldots)|_{N = N_c}$$

(5)

One looks for a fixed-point solution of the form $R(u) = g R_c(u) + g^2 \delta R(u)$, with $g > 0$, $R''(0) = -1$, and its flow. This analysis is done numerically and leads to the flow shown schematically on Fig. 1. The RG-flow projected onto the direction of $g$ is equivalent to

$$\partial_t g = \epsilon g + 1.092(N - N_c)g^2 + 2.352g^3.$$  

(6)

As a solution of the functional flow near $N_c$, its simplicity is surprising. Setting $g = (N - N_c)f$, there are three FP:

$$\frac{\epsilon}{(N - N_c)^2} - 1.092f + 2.352f^2 = 0, \quad f = 0.$$  

(7)

For $N > N_c$ the physical branch is $f < 0$. As seen in Fig. 1, for $d > 4$ there is a ferro phase (i.e. $f = 0$ is attractive) and an unstable FP describing the F/D transition, given by the negative branch of (7). At $N = N_c$ one sees from (6) that the F/D fixed point is still perturbative, but in a $\sqrt{\epsilon}$ expansion for $g$ (and for the critical exponents). For $N < N_c$, the physical side is $f > 0$ and there are two branches on Fig. 2 corresponding to two non-trivial fixed points. One is the infrared attractive FP for weak disorder which describes the Quasi-Ordered ferromagnetic phase; the second one is unstable and describes the transition to the disordered phase with a flow to strong coupling. These two fixed points exist only for $\epsilon < \epsilon_c$ and annihilate at $\epsilon_c$. The lower critical dimension of the RF-model for $N < N_c$ is lowered from $d = 4$ to

$$d^\text{RF}_{\epsilon_c} = 4 - \epsilon_c \approx 4 - 0.1268(N - N_c)^2 + O((N - N_c)^3).$$  

(8)

Note that the mechanism is different from the more conventional criterion $d - 4 + \eta(d) = 0$ at $d = d_{\epsilon_c}$.

The same analysis for the random anisotropy class yields $N_c = 9.44721$. The equivalent of (6) becomes $\partial_t g = \epsilon g + 0.549(N - N_c)g^2 + 47.69g^3$, leading to $d^\text{RA}_{\epsilon_c} \approx 4 - 0.00158(N - N_c)^2$. Although it yields $d_{\epsilon_c}(N = 3) \approx 3.93$ and no QLRO phase in $d = 3$, naive extrapolation should be taken with caution given the high value of $N_c$. Numerical values for $d_{\epsilon_c}$ are changed for a $\gamma_0 \neq 0$, but the scenario is robust for $\gamma_0 < \gamma_c$.

We now discuss the FRG flow-equations for $N$ large. From a truncated exact RG Tarjus and Tissier (TT) [26] found: that the linear cusps of the F/D phase transition for $d > 4$ vanishes for $N > N^*(d)$, i.e. $R''(0^+) = 0$; and that the non-analyticity becomes weaker as $N$ increases (as $|\phi^*|/n \sim N$). Analytical study of the derivatives of (6) confirms the existence of this peculiar FP to two loop and predicts $N^*(d, 2p)$ beyond which the set of $\{ R^{(2k)}(0) \}$ for $k \leq p$ admits a stable FP, with $R^{(2k-1)}(0^+)$ for $k \leq p$ and $R^{(2k-1)}(0^+)$ for $k > p$. We find:

$$N^*(d) = N^*(d, 4) = 18 + 49\epsilon/5 + \ldots$$

(9)

which yields a slope roughly twice the one of Fig. 1 of [26]. This remarkable FP raises some puzzles. Although weaker than a cusp its non-analyticity should imply some (weaker) metastability in the system. It is thus unclear whether DR is fully restored: to prove it one should rule out feedback from anomalous higher-loop terms in exponents or the $\beta$-function.

Finally, one also wonders about its basin of attraction. As shown in Fig. 1, the FRG flow for $R''''(0)$ is still to large values if its bare value is large enough, indicating some tendency to glasy behaviour.

To explore these effects we now study the F/D phase transition at large $N$ and $d > 4$. We obtain, both at large $N$ and fixed $d$ (extending Ref. [23]), and to one loop, the flow equation for the rescaled $\tilde{R}(z = \cos \phi) = NR(\phi)/|\epsilon|$:

$$\partial_t \tilde{R} = -\tilde{R} + 2\tilde{R}^2 - \tilde{R}^2 \tilde{R} z + \frac{1}{2} \tilde{R}^2 = 0.$$  

(10)

We denote $y(z) = \tilde{R}(z)$, $y_0 = \tilde{R}(1) = -NR''(0)/|\epsilon|$ and $r_4 = NR''''(0)/|\epsilon|$. There are two analytic FPs $\tilde{R}(z) = z - 1/2$ and $\tilde{R}(z) = z^2/2$, corresponding both to $y_0 = 1$ and to $r_4 = 1$ and $r_4 = 4$ respectively. This agrees with the flow of the derivatives for analytic $R(\phi)$: $\partial_t y_0 = y_0(y_0 - 1)$, and at $y_0 = 1$: $\partial_t r_4 = \frac{1}{2}(r_4 - 1)(r_4 - 4)$. The first FP is the large-$N$ limit of the TT fixed point, the second is repulsive and divides the region where $r_4 \to \infty$ (non-analytic $R(\phi)$) in a finite RG time $l_c$ (Larkin scale). For $y_0 > 1$, we find a family of NA fixed points with a linear cusp, parameterized by an
integer $n \geq 2$, s.t. $y_0 = n/(n-1)$, $z = y - (y_0 - 1)(y/y_0)^n$. The solutions with $n$ (i.e., $z(y)$) odd correspond to random anisotropy ($R'(\phi) = R'(\phi + \pi)$). The $n = 2$ RF fixed point is $R(\phi) = 2\cos(\phi) + \frac{8\sqrt{2}}{3}\sin^3(\phi/2) - \frac{4}{3}$. To elucidate their role, we obtained the exact solution for the flow both below $l_\gamma$, i.e., $z = \frac{y_0}{y_0} + (y_0 - 1)\Phi(\frac{y_0}{y_0}) (\Phi(x)$ parameterizes the bare disorder, $\Phi(1) = 0$), and above $l_\gamma$, with an anomalous flow for $y_0$. Matching at $l_\gamma$ yields the critical manifold for RF disorder, defined from the conditions that $\Phi'(w) = \Phi(w)/w = 1$ has a root 0 $\leq w \leq 1$. It is different from the naive DR condition $y_0 = 1$, valid for small $r_2$. The $n = 2$ FP corresponds to bare disorder such that the root $w = 0$. Hence it is multicritical [33].

Generic initial conditions within the critical F/P manifold flow back to the TT FP i.e. the linear cusp decreases to zero [35]. This however occurs only at an infinite scale, hence we expect a long crossover within a glassy region, characterized by a cusp, and metastability on finite scales [36]. The large-$N$ limit here is subtle. Taking $N \to \infty$ at fixed volume on a bare model with $R_0(z) = z$ yields only the analytic FP, equivalent to a replica-symmetric saddle point. Higher monomials $z^l$ are generated in perturbation theory, at higher order in 1/$N$. Thus, for $N$ large but fixed and infinite size, one must first coarse grain to generate a non-trivial function $R_0(z)$, before taking the limit of $N \to \infty$.

In conclusion we obtained the 2-loop FRG functions for the random field and anisotropy $\sigma$-models. We found a new fixed point and a scenario for the decrease of the lower critical dimension. This rules out the scenario left open at one loop that the bifurcation close to $d = 4$ simply occurs within the (quasi-) ordered phase.

---

[27] P. Le Doussal and K.J. Wiese, to be published.
[30] For $N = 2$, one can also use $\sin = \psi$ and a RM calculation with $\langle \sigma_n \rangle \approx e^{\frac{\psi}{2} N^2}$, since the field correlator is gaussian up to $O(\epsilon^2)$.
[31] Reexpressing [31] in $\hat{R}$, [32] is the rescaling term $\gamma z \hat{R}(z)$.
[32] 1-loop corrections to correlations at non-zero momentum are anisotropic $\sim \mu_{ij}(v)$ (see (13) in [28]) in presence of a background $\tilde{h}_i^a$ there, $v_i$ there. This yields formally $\gamma' = 1$, $\gamma'' = (N - 4)/(8(N - 2))$.
[33] The coefficient in [8] and [9] becomes 2.35(1 - $\gamma_a/\gamma_0$) with $\gamma_a \approx 2.04$, and similarly for RA 17.6(1 - $\gamma_a/\gamma_0$). The scenario reverses (fig. 2) is flipped w.r.t. the f-axis). The FP, for $\gamma < \gamma_c$ the scenario above $d = 4$. This scenario would imply a F/D fixed point inaccessible to FRG, contradicting [21]. It is unlikely, since [32] suggests $\frac{1}{2} \leq \gamma_c \leq \frac{1}{4}$.
[34] We thank G. Tarjus and M. Tissier for pointing out this important fact.
[35] This is true only on one side of the multicritical FP. The other side, if accessible from physically realizable bare disorder, would correspond to a strong disorder regime.