Multi-photon entanglement for revealing anyonic features and beyond

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QIAP

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Symposium
Topological quantum computing
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MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK GARCHING

MPQ

Outline

Multi-qubit entanglement

applications & classifications

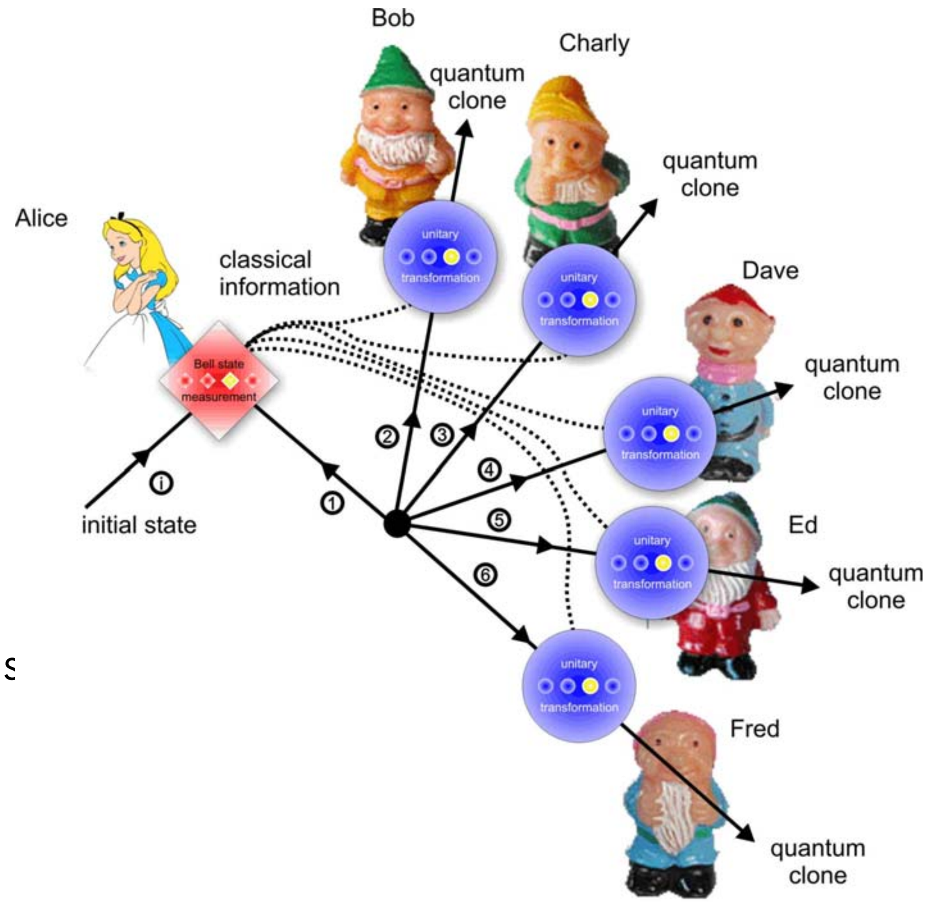
Anyonic features with multi-photon entanglement

toric code
implementation of a minimal system

Beyond: how to entangle more photons

UV enhancement cavity and Dicke state

Multi-qubit entanglement

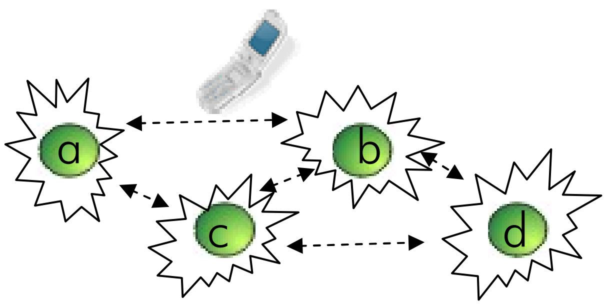


Resource for quantum information:

teleportation, telecloning,
one-way quantum computing,
decoherence-free communication,
anyonic features, …

Classification of entangled states:

via SLOCC (stochastic local operations
and classical communication)



Classification via SLOCC

two qubits — one class

- $|\psi^+\rangle \;=\; \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

Bennett et al. PRA 53 *(1996)*

three qubits — two classes

- $|GHZ_3\rangle \;=\; \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
- $|W_3\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$

Dür et al. PRA 62 *(2000)*

four qubits — richer structure, e.g. subsets of continuous SLOCC inequivalent states (e.g. $|GHZ_4\rangle$, $|D_4^{(2)}\rangle$, ...)

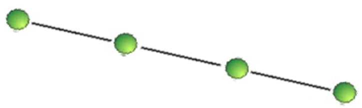
Verstraete et al. PRA 65 *(2002)*
Lamata et al. PRA 75 *(2007)*

(...)

Phenomenological classification

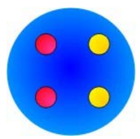
Classification according to state specific properties, e.g.

Graph states: N-vertices (corresponding to N-qubits) & edges (Ising-type next-neighbour interaction), e.g. cluster states, GHZ states [anyonic features]



Hein et al. PRA ***69*** *(2004)*

Dicke states: eigenstates of $\begin{pmatrix} J^2 \\ J_z \end{pmatrix}$ with eigenvalues $\begin{pmatrix} j(j+1) \\ m \end{pmatrix}$



J … total spin
J_z … spin in z-direction,

Dicke Phys. Rev. ***93*** *(1954)*

symmetric Dicke states [more photons]:

$$|D_N^{(m)}\rangle = \begin{pmatrix} N \\ m \end{pmatrix}^{-1/2} \sum_i \Pi_i(|1^{\otimes m} 0^{\otimes (N-m)}\rangle)$$

e.g. $|D_6^{(3)}\rangle = 1/\sqrt{20} \sum_i \Pi_i(|000111\rangle)$

(…)

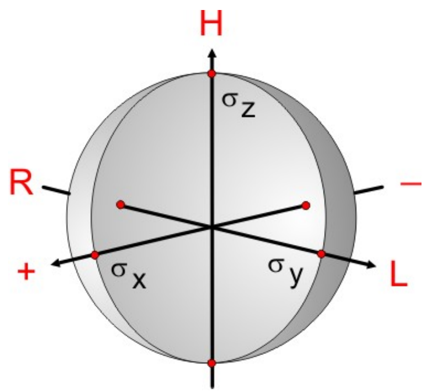
Entangled state observation

We use ...

Photons: ideally suited for communication tasks

- negligible decoherence, easy to transmit
- simple polarization encoding of qubits

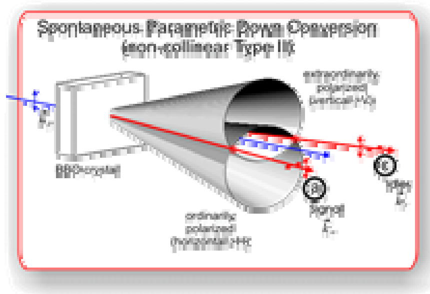
$$|0\rangle \rightarrow |H\rangle \quad |1\rangle \rightarrow |V\rangle$$



but

- low interaction between photons

Entangled state observation:



[entanglement from] photon source
SPDC (spontaneous parametric down conversion)
+ linear optical network
(beam splitters, wave plates, phase shifters)
+ conditional detection

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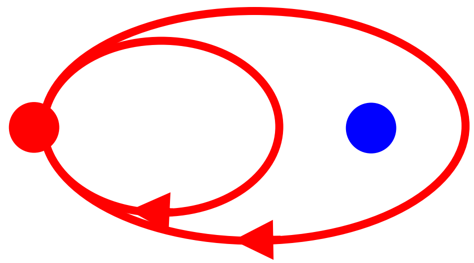
What are anyons?

Performing two exchanges of two particles
= rotating one particle around the other

3 spatial dimensions:

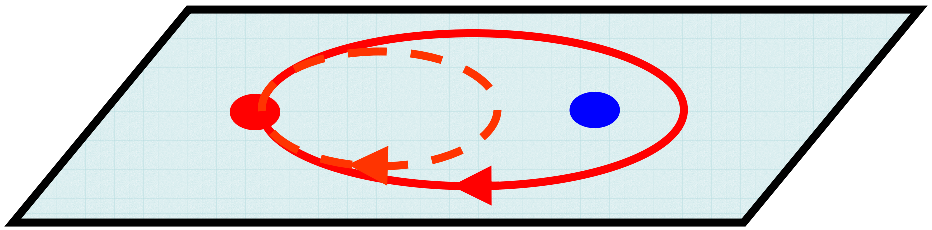


Bosons $|\Psi\rangle \rightarrow |\Psi\rangle$

Fermions $|\Psi\rangle \rightarrow e^{i2\pi}|\Psi\rangle$

e.g.
Pauli, Phys. Rev. ***58*** *(1940)*

2 spatial dimensions (e.g. quantum Hall effect):



Anyons $|\Psi\rangle \rightarrow e^{i2\phi}|\Psi\rangle$

Wilczek, Phys. Rev. Lett. ***48 & 49*** *(1982)*

Where can anyons be found?

Toric code: topological 2d system realising anyons as local excitations

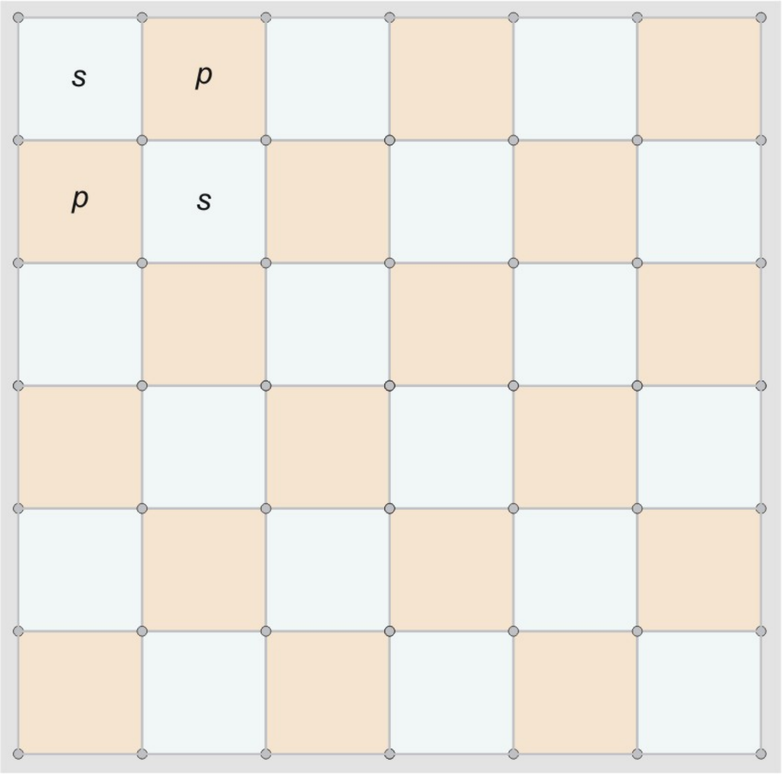
Kitaev, Ann. Phys. ***303*** *(2003)*

- square lattice made up from **s/p** plaquettes
- qubits at vertices
- interaction Hamiltonian

$$\mathcal{H} = \; -\sum_p \sigma^z_{p1}\sigma^z_{p2}\sigma^z_{p3}\sigma^z_{p4} \\ \qquad -\sum_s \sigma^x_{s1}\sigma^x_{s2}\sigma^x_{s3}\sigma^x_{s4}$$

- gound state

$$|\xi\rangle = \prod_s \frac{1}{\sqrt{2}}\left(\mathbb{1} + \sigma^x_{s1}\sigma^x_{s2}\sigma^x_{s3}\sigma^x_{s4}\right)|00\ldots0\rangle$$



Excitations in the Toric code

Toric code: three types of local excitations

e type

$$\boxed{|e\rangle = \sigma^z|\xi\rangle}$$

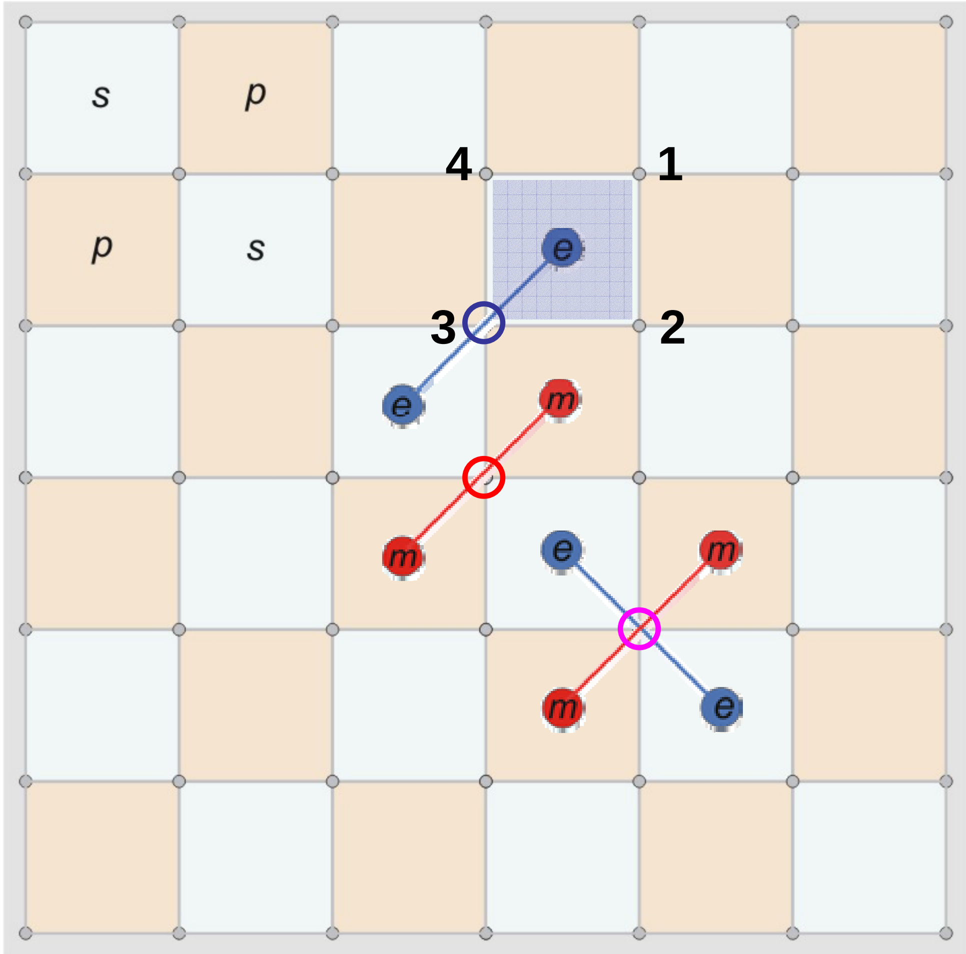
m type

$$\boxed{|m\rangle = \sigma^x|\xi\rangle}$$

ε type

$$\boxed{|\epsilon\rangle = \sigma^z\sigma^x|\xi\rangle = i\,\sigma^y|\xi\rangle}$$

local excitation, e.g. **e** type
eigenvalue of plaquette
operator $\sigma^x_{s1}\sigma^x_{s2}\sigma^x_{s3}\sigma^x_{s4}$
becomes -1



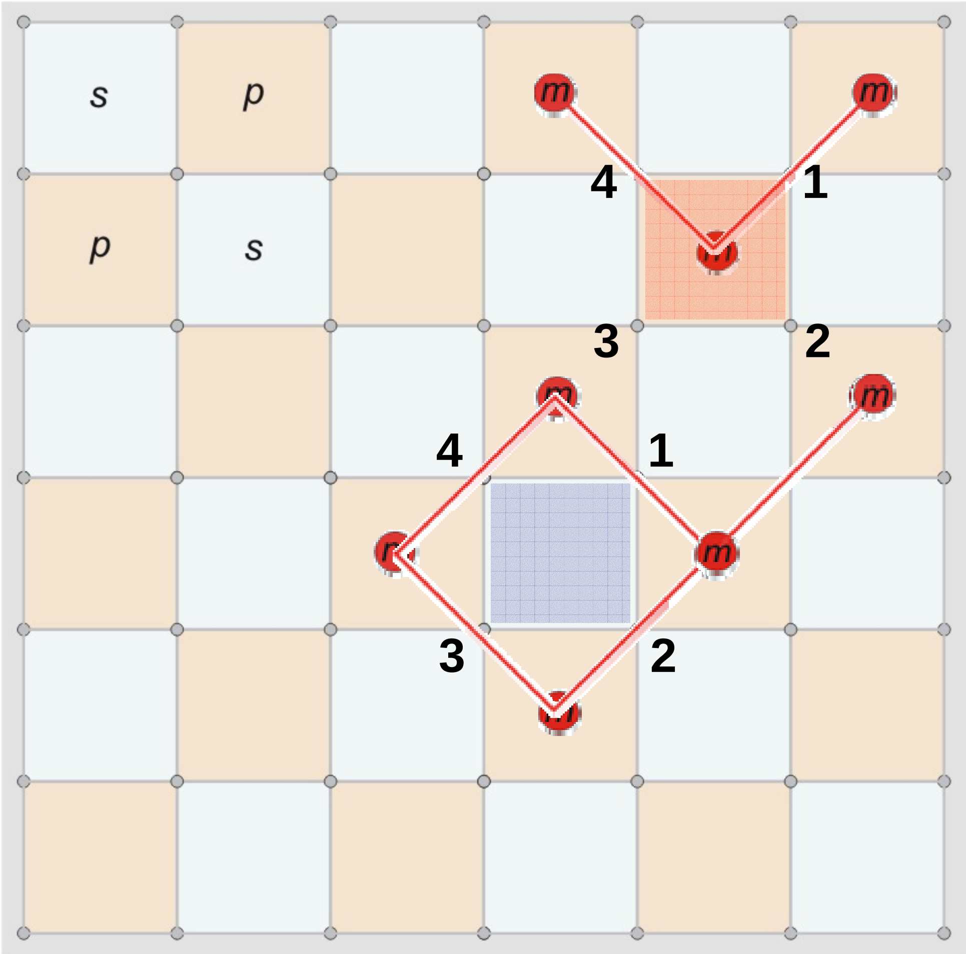
Excitations in the Toric code

Toric code: moving and annihilating excitations

$\sigma_1^x|\xi\rangle = |m\rangle$
plaquette operator
$\sigma_{p1}^z\sigma_{p2}^z\sigma_{p3}^z\sigma_{p4}^z$ -1

$\sigma_4^x\sigma_1^x|\xi\rangle$
plaquette operator
$\sigma_{p1}^z\sigma_{p2}^z\sigma_{p3}^z\sigma_{p4}^z$ +1

contractible loop
(i.e. empty plaquette)
$\sigma_1^x\sigma_4^x\sigma_3^x\sigma_2^x|\xi\rangle = |\xi\rangle$

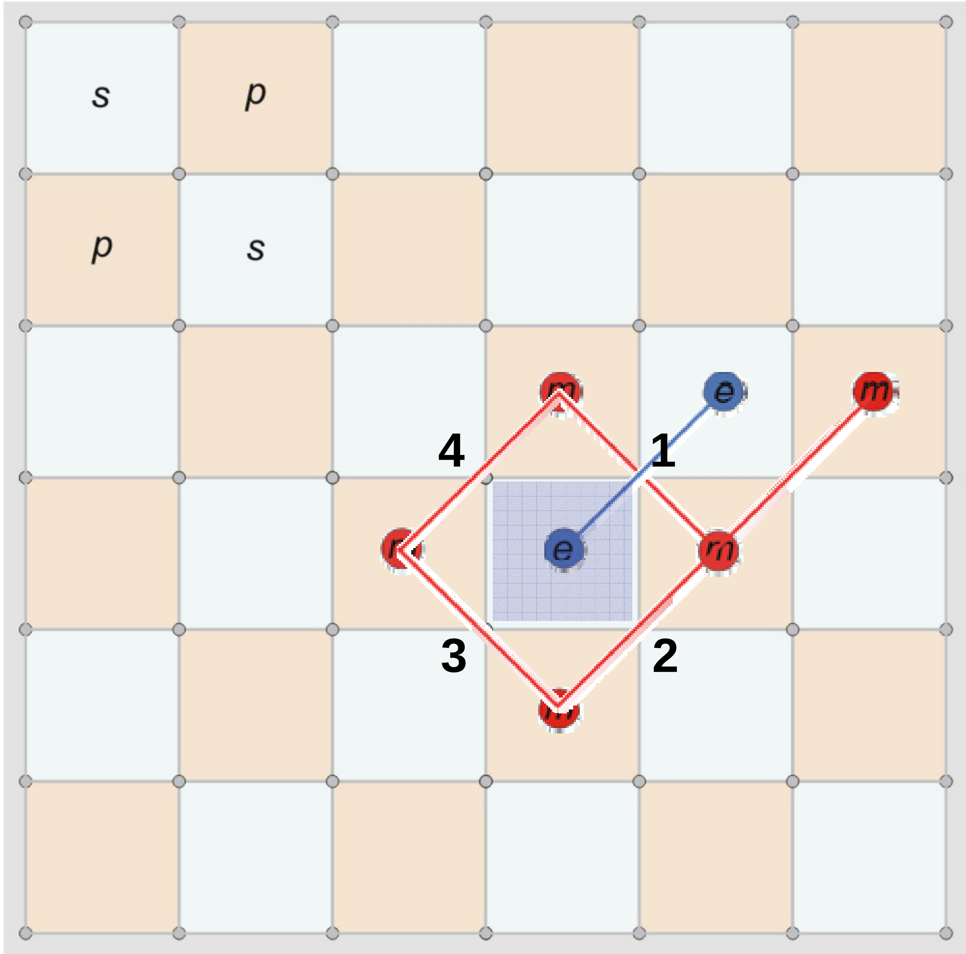


Excitations in the Toric code

Toric code: non-contractible loop (i.e. populated plaquette)

$$|\Psi_{\text{ini}}\rangle = \sigma_1^z|\xi\rangle = |e\rangle$$

$$\sigma_1^x\sigma_4^x\sigma_3^x\sigma_2^x|\Psi_{\text{ini}}\rangle$$
$$= -\sigma_1^z(\sigma_1^x\sigma_4^x\sigma_3^x\sigma_2^x|\xi\rangle)$$
$$= -|\Psi_{\text{ini}}\rangle$$



Excitations in the Toric code

Cycling *m* around *e* yields a phase of π

m and *e* behave anyonic with respect to each other

(for bosons & fermions no phase is expected)

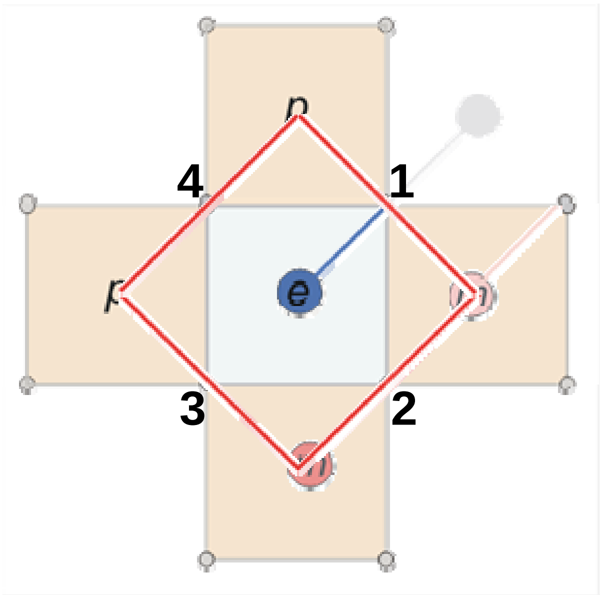
Implementation of small system

System: lattice of a single **s** plaquette (bounded by 4 **p** plaquettes)

- ground state

$$|\xi\rangle = \frac{1}{\sqrt{2}}(|0_1 0_2 0_3 0_4\rangle + |1_1 1_2 1_3 1_4\rangle)$$

4 qubits in a GHZ state and
local Pauli rotations on qubits
simulating anyonic features



Pachos et al., quant-ph 0710.0895 (2007)

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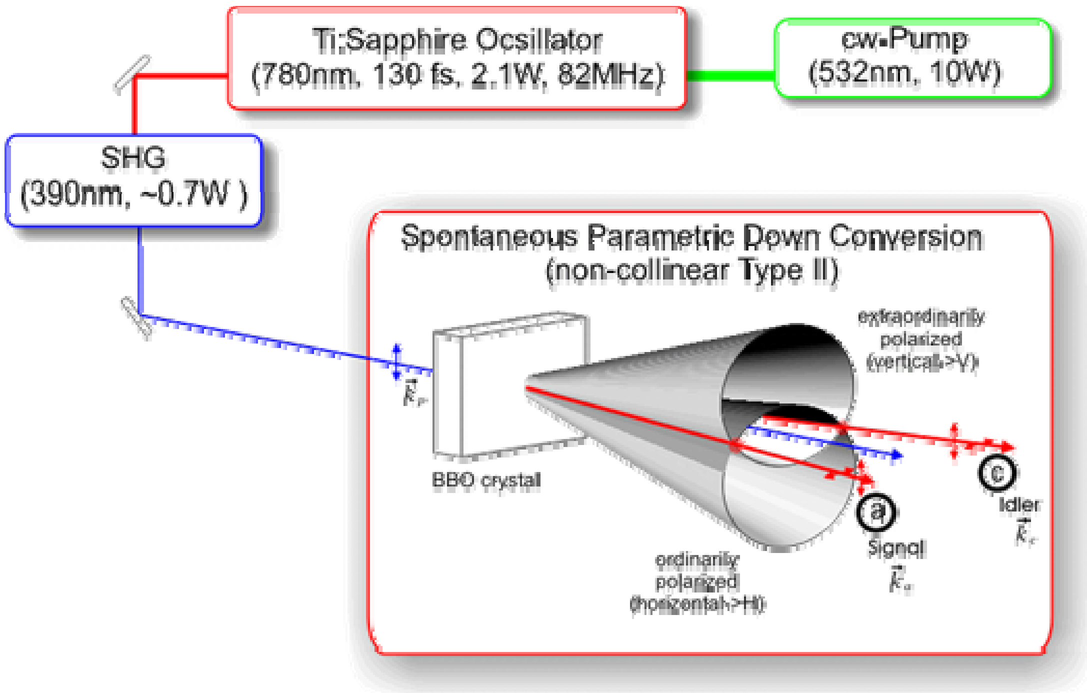

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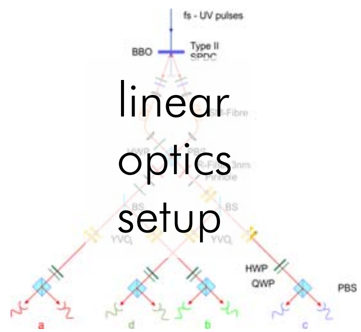


linear optics setup

2 pump photons convert:
$$
\begin{aligned}
|\psi_4\rangle_{ac} &\propto \left(a_H^\dagger c_V^\dagger + a_V^\dagger c_H^\dagger\right)^2 |\mathrm{vac}\rangle \\
&= \left(a_H^{\dagger\,2} c_V^{\dagger\,2} + a_V^{\dagger\,2} c_H^{\dagger\,2} + 2 a_H^\dagger a_V^\dagger c_H^\dagger c_V^\dagger\right) |\mathrm{vac}\rangle
\end{aligned}
$$

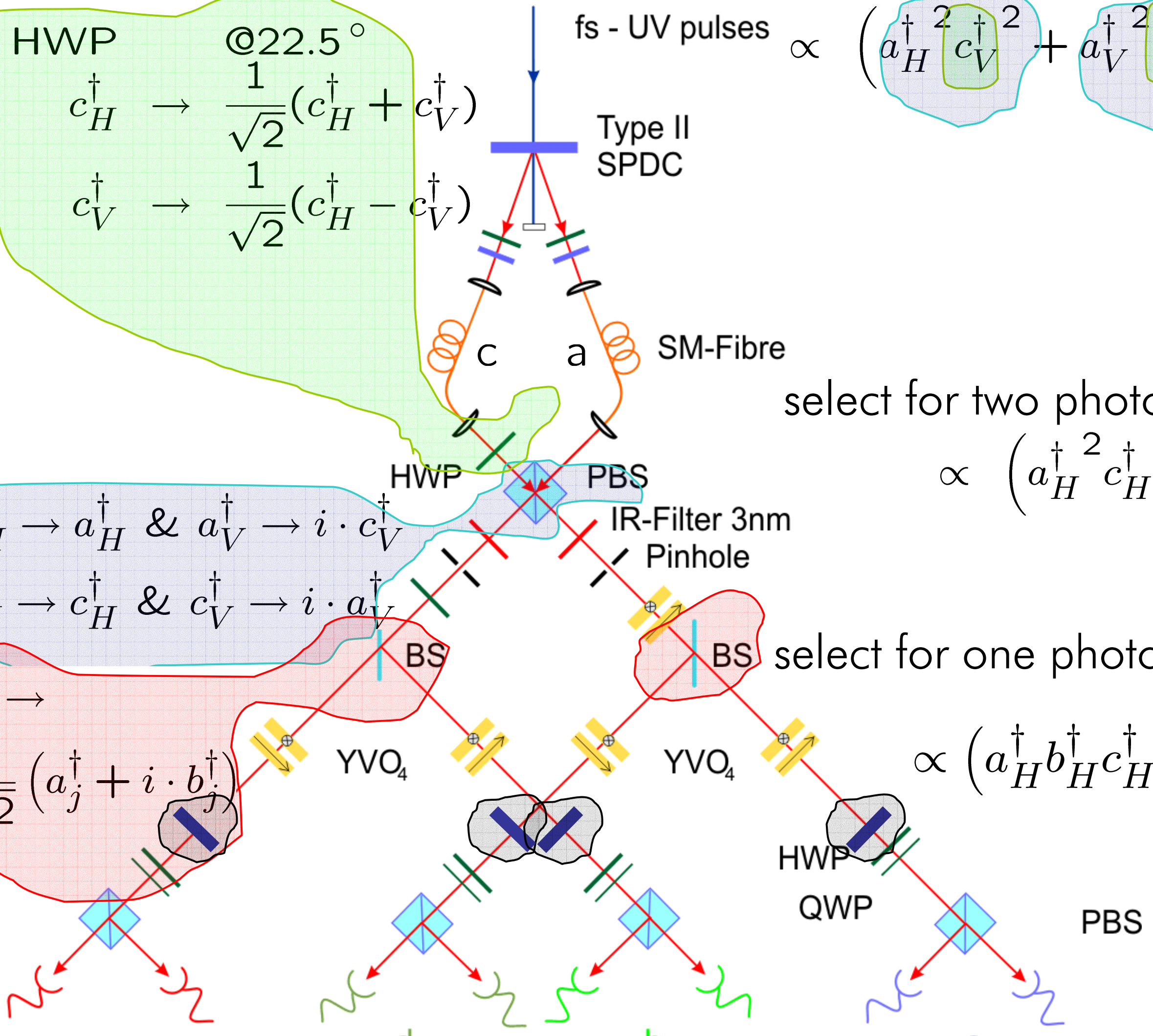
Ground state

HWP @22.5°

$$c_H^\dagger \rightarrow \frac{1}{\sqrt{2}}(c_H^\dagger + c_V^\dagger)$$

$$c_V^\dagger \rightarrow \frac{1}{\sqrt{2}}(c_H^\dagger - c_V^\dagger)$$

$$\propto \left(a_H^{\dagger\,2} c_V^{\dagger\,2} + a_V^{\dagger\,2} c_H^{\dagger\,2} + 2 a_H^\dagger a_V^\dagger c_H^\dagger c_V^\dagger\right)|\text{vac}\rangle$$



select for two photons in each mode {a,c}

$$\propto \left(a_H^{\dagger\,2} c_H^{\dagger\,2} + a_V^{\dagger\,2} c_V^{\dagger\,2}\right)|\text{vac}\rangle$$

$$a_H^\dagger \rightarrow a_H^\dagger \;\&\; a_V^\dagger \rightarrow i \cdot c_V^\dagger$$

$$c_H^\dagger \rightarrow c_H^\dagger \;\&\; c_V^\dagger \rightarrow i \cdot a_V^\dagger$$

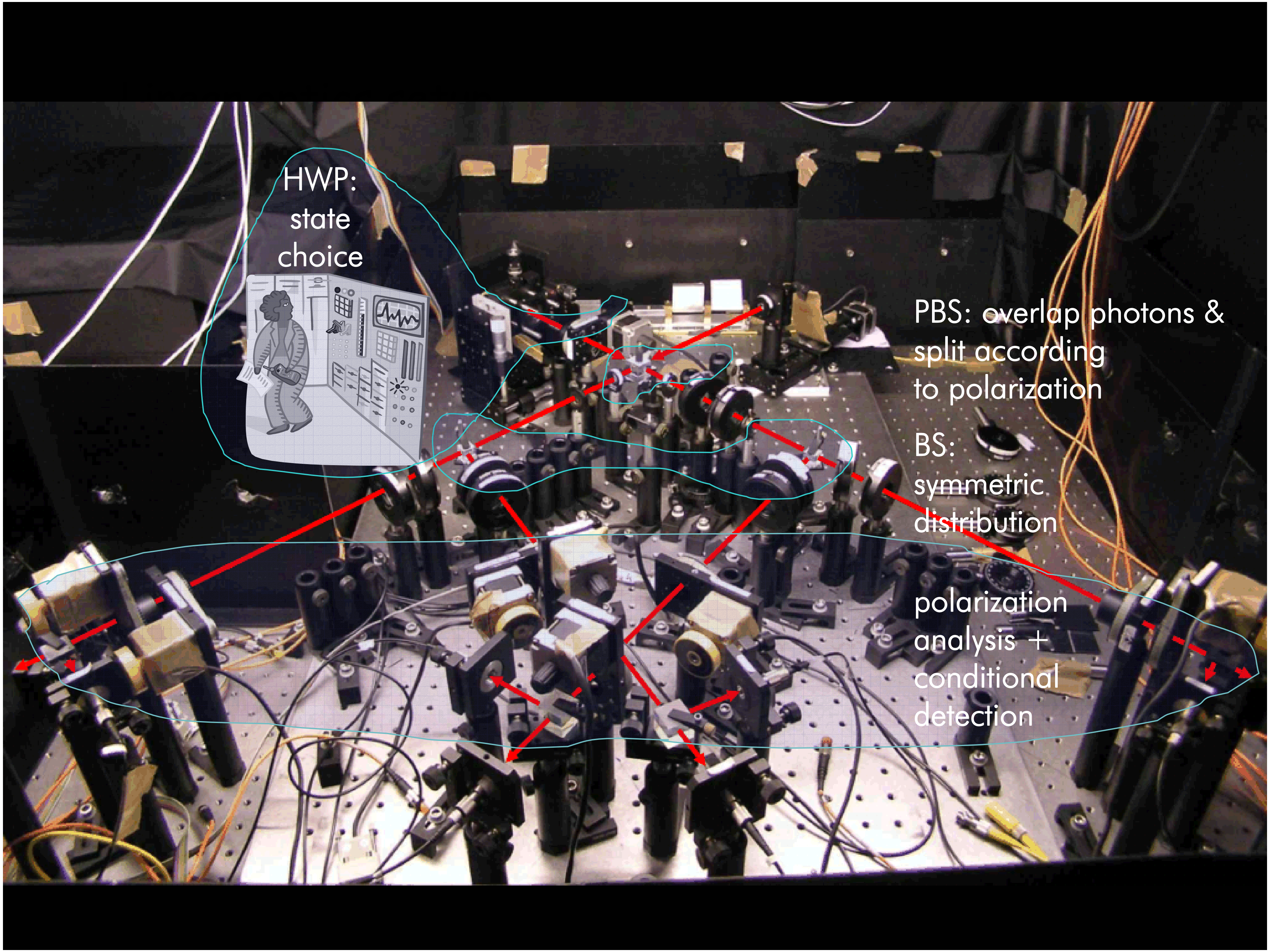
$$a_j^\dagger \rightarrow \frac{1}{\sqrt{2}}\left(a_j^\dagger + i \cdot b_j^\dagger\right)$$

select for one photon in each mode {a,b,c,d}

$$\propto \left(a_H^\dagger b_H^\dagger c_H^\dagger d_H^\dagger + a_V^\dagger b_V^\dagger c_V^\dagger d_V^\dagger\right)|\text{vac}\rangle$$

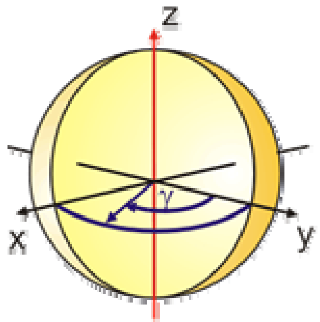
local rotations
for anyonic
manipulations

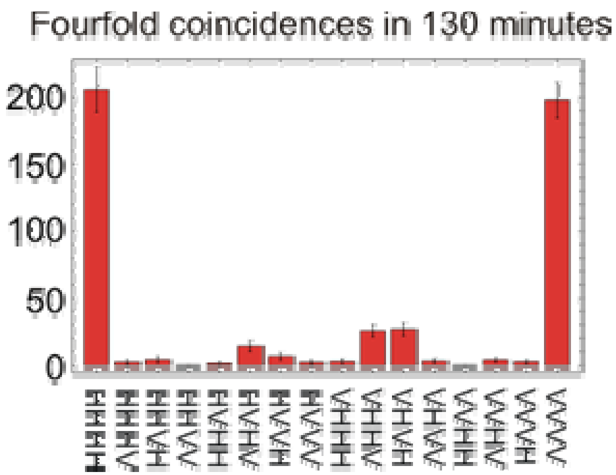
$$\boxed{|\xi\rangle = 1/\sqrt{2}(|\text{HHHH}\rangle + |\text{VVVV}\rangle)}$$


HWP:
state
choice
PBS: overlap photons &
split according
to polarization
BS:
symmetric
distribution
polarization
analysis +
conditional
detection

Ground state $|\xi\rangle = 1/\sqrt{2}(|\mathrm{HHHH}\rangle + |\mathrm{VVVV}\rangle)$

Measurement in computational basis (H/V)



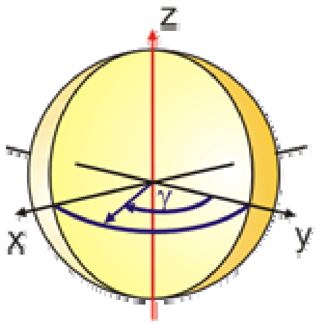


$$P_{\mathrm{HHHH}} = 0.41 \pm 0.03$$

$$P_{\mathrm{VVVV}} = 0.39 \pm 0.03$$

Ground state $|\xi\rangle = 1/\sqrt{2}(|\text{HHHH}\rangle + |\text{VVVV}\rangle)$

Coherent superposition: correlation function

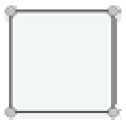


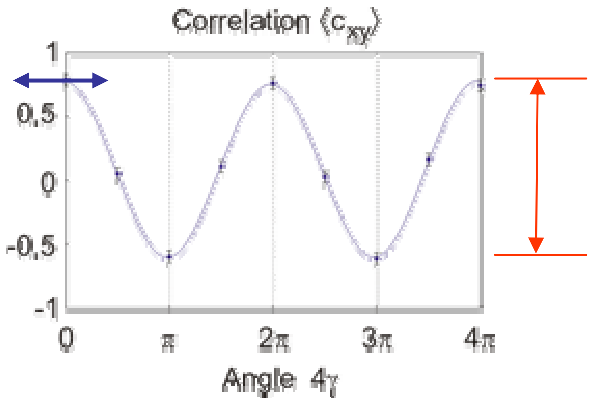
$$c_{xy}(\gamma) = \sigma_1(\gamma) \otimes \sigma_2(\gamma) \otimes \sigma_3(\gamma) \otimes \sigma_4(\gamma)$$

$$\sigma_i(\gamma) = (\cos\gamma)\sigma_i^y + (\sin\gamma)\sigma_i^x$$

$$|GHZ\rangle = 1/\sqrt{2}(|\text{HHHH}\rangle + e^{i\phi}|\text{VVVV}\rangle)$$

$$\langle c_{xy}(\gamma)\rangle_{\text{GHZ}} = \boxed{\mathcal{V}}\cos(4\gamma + \boxed{\phi})$$





$$\boxed{\mathcal{V} = 68.3 \pm 1.1\,\%}$$

$$\boxed{(0.02 \pm 0.01)\cdot\pi}$$

Fidelity
(if >0.5 then genuine four-qubit entangled)

$$F = \tfrac{1}{2}(\mathcal{V} + P_{\text{HHHH}} + P_{\text{VVVV}})$$

$$\boxed{F = 74.5 \pm 2.2\,\%}$$

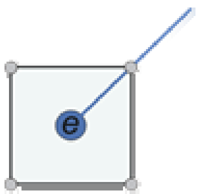
Toth and Gühne, Phys. Rev. Lett. ***94*** *(2005)*

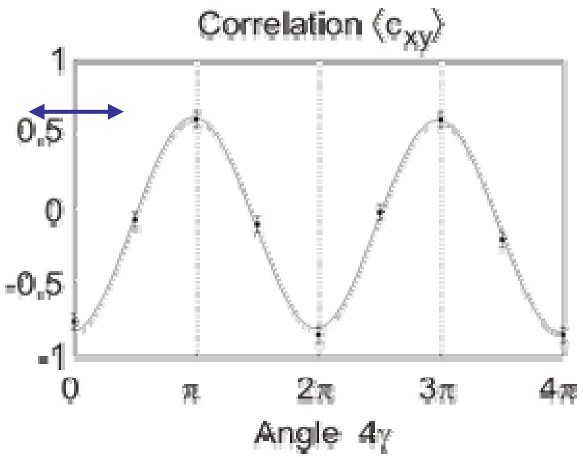
Sackett et al., Nature ***404*** *(2000)*

Excited state

Creating excitation **e**

$$|e\rangle = \sigma_1^z|\xi\rangle = 1/\sqrt{2}(|HHHH\rangle - |VVVV\rangle)$$

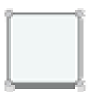


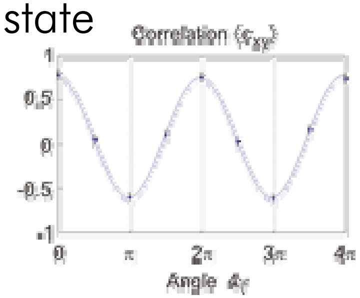


$$F = 74.9 \pm 2.8\,\%$$

$$\boxed{(1.02 \pm 0.01)\cdot\pi}$$

Comparison:
ground state

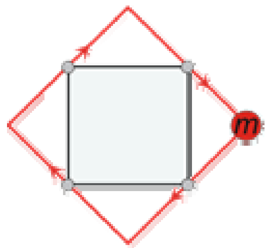


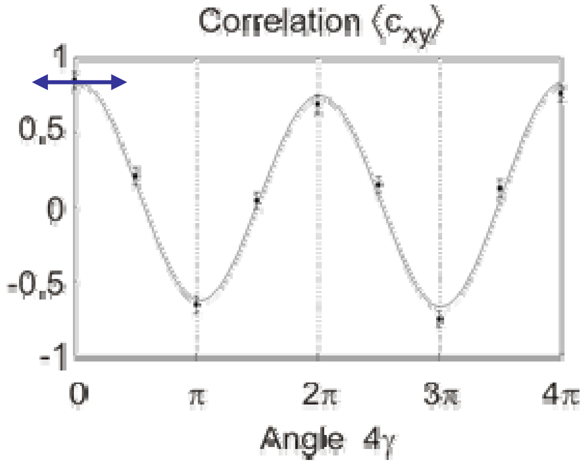


Loops

Move *m* around empty plaquette

$$\sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x|\xi\rangle = |\xi\rangle = 1/\sqrt{2}(|\mathrm{HHHH}\rangle + |\mathrm{VVVV}\rangle)$$

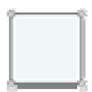


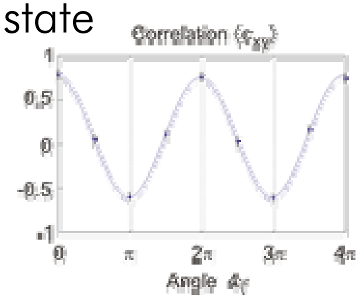


$$\boxed{F = 76.6 \pm 3.4\,\%}$$

$$\boxed{(-0.02 \pm 0.02)\cdot\pi}$$

Comparison:
ground state

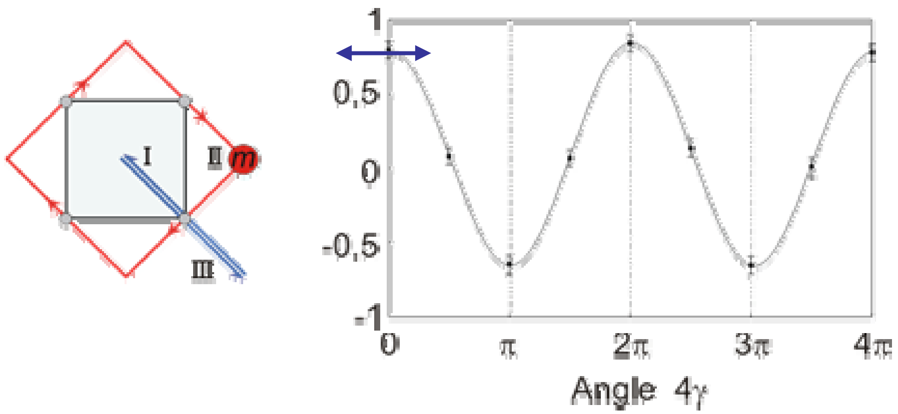




Loops

i) Generate **e**, ii) move *m* around and iii) annihilate **e**

$$\sigma_2^z(\sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x)\sigma_2^z|\xi\rangle = -|\xi\rangle = -1/\sqrt{2}(|HHHH\rangle + |VVVV\rangle)$$

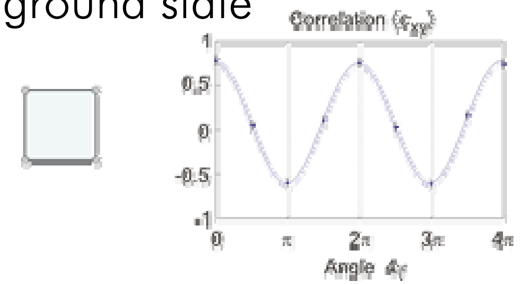


$F = 76.8 \pm 2.8\,\%$

$\boxed{(-0.01 \pm 0.01)\cdot\pi}$

„Invisible“ global phase

Comparison:
ground state



Interference procedure

i) Superposition of creating excitation **e** or not

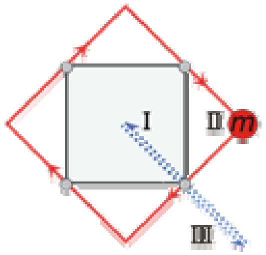
$$(\sqrt{\sigma_4^z})^{-1}|\xi\rangle = \mathrm{e}^{-i\pi/4}(|\xi\rangle + i|e\rangle)/\sqrt{2}$$

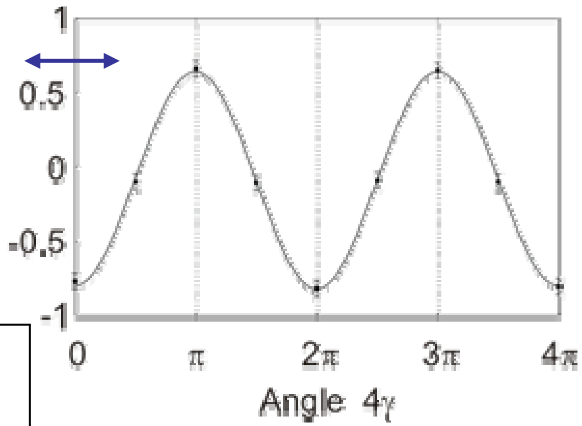
ii) Moving *m* around

$$(\sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x)(\sqrt{\sigma_4^z})^{-1}|\xi\rangle = \mathrm{e}^{-i\pi/4}(|\xi\rangle - i|e\rangle)/\sqrt{2}$$

iii) Inverse of i)

$$\sqrt{\sigma_4^z}(\sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x)(\sqrt{\sigma_4^z})^{-1}|\xi\rangle = -i|e\rangle$$
$$= -i/\sqrt{2}(|\mathrm{HHHH}\rangle - |\mathrm{VVVV}\rangle)$$



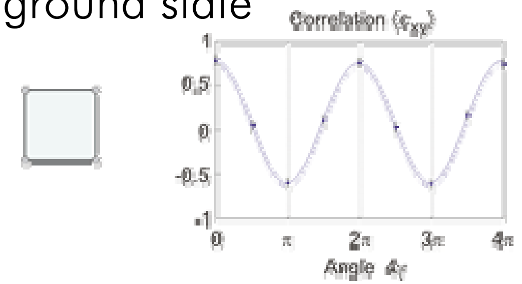


$F = 75.8 \pm 2.5\,\%$

$(1.00 \pm 0.01) \cdot \pi$

„Invisible“ global phase into relative phase

Comparison:
ground state



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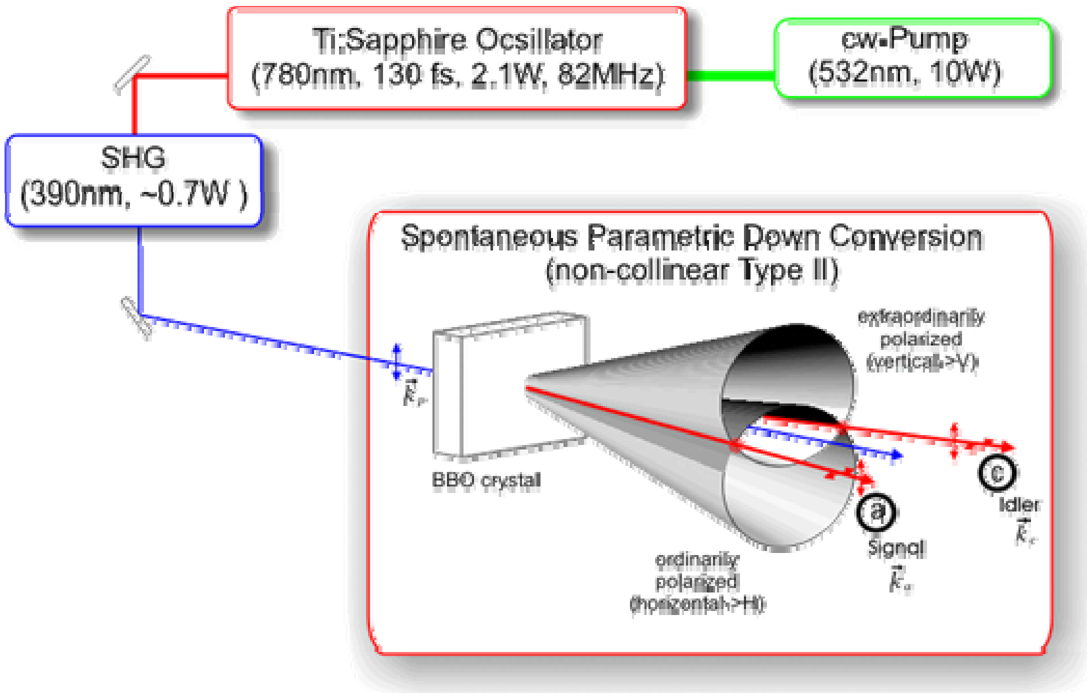

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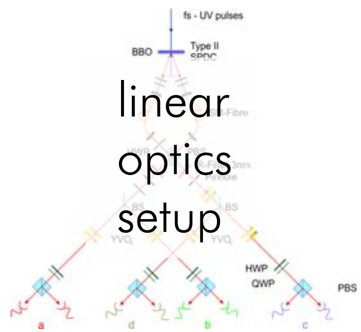


linear
optics
setup

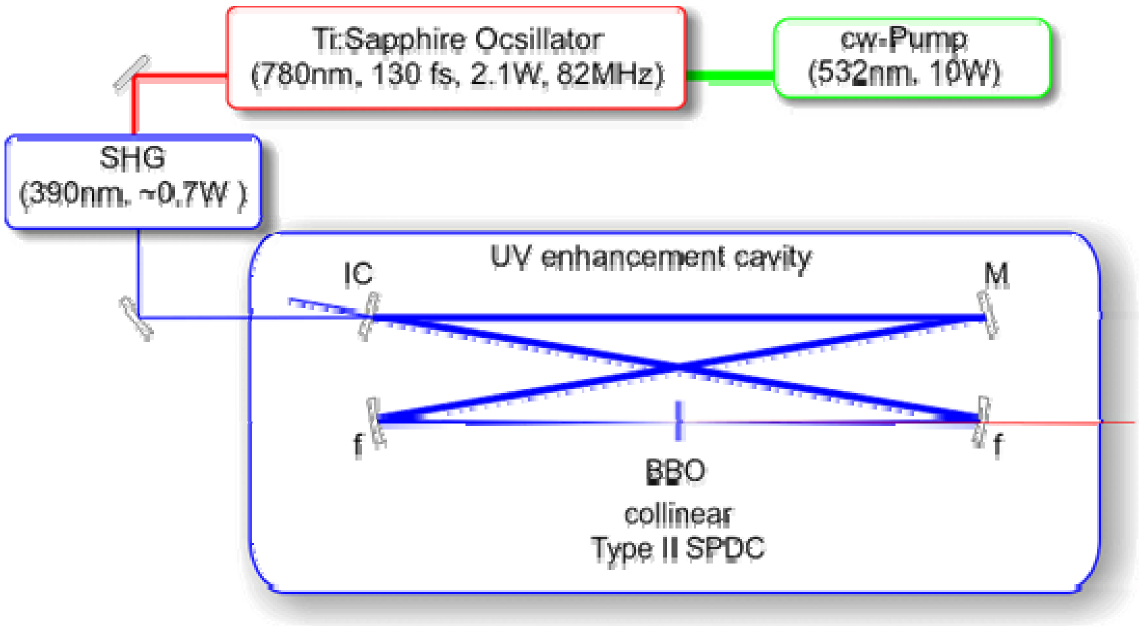
4-photon count rates: per minute

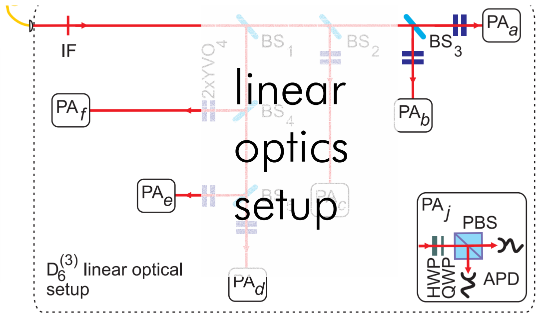
6-photon count rates: per day

-> too long measurement times

More photons

UV enhancement cavity: up to 7.5W UV power

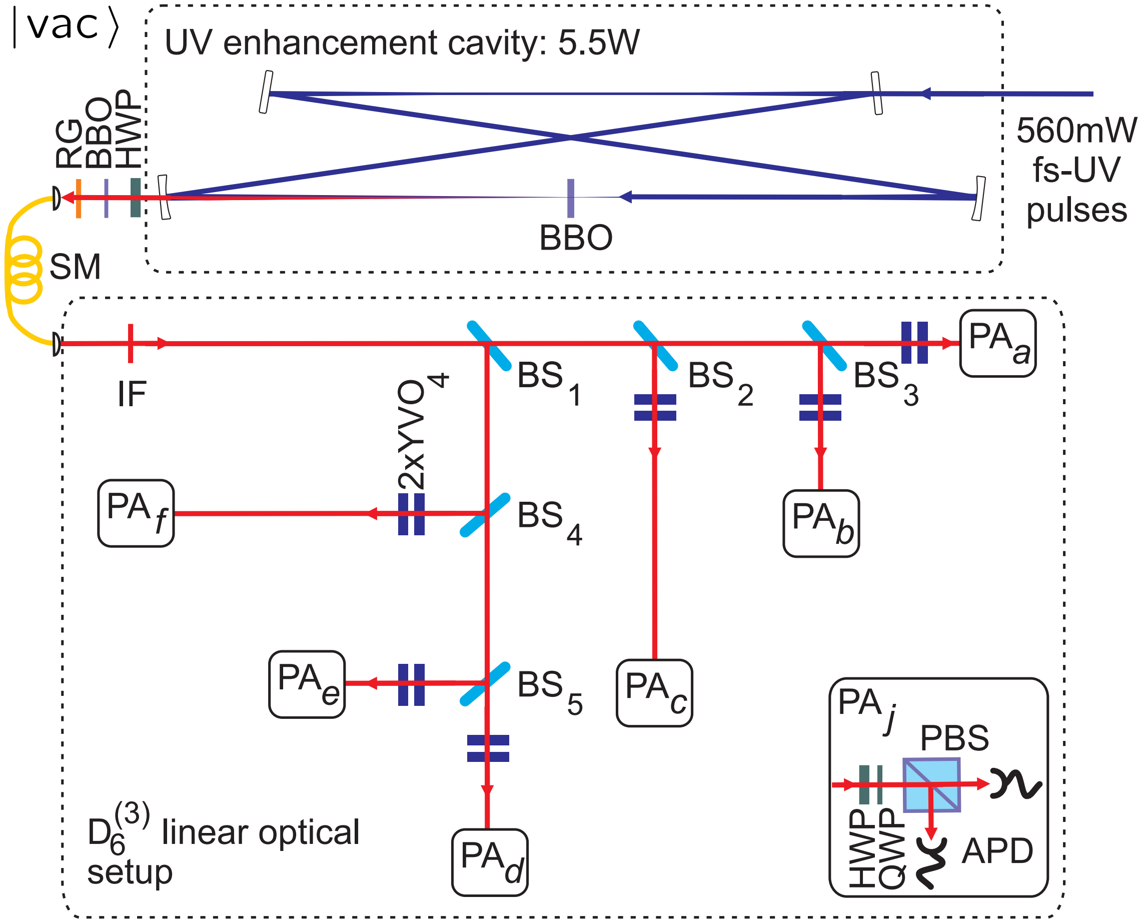




6 photon count rates: per minute !!!

Dicke state

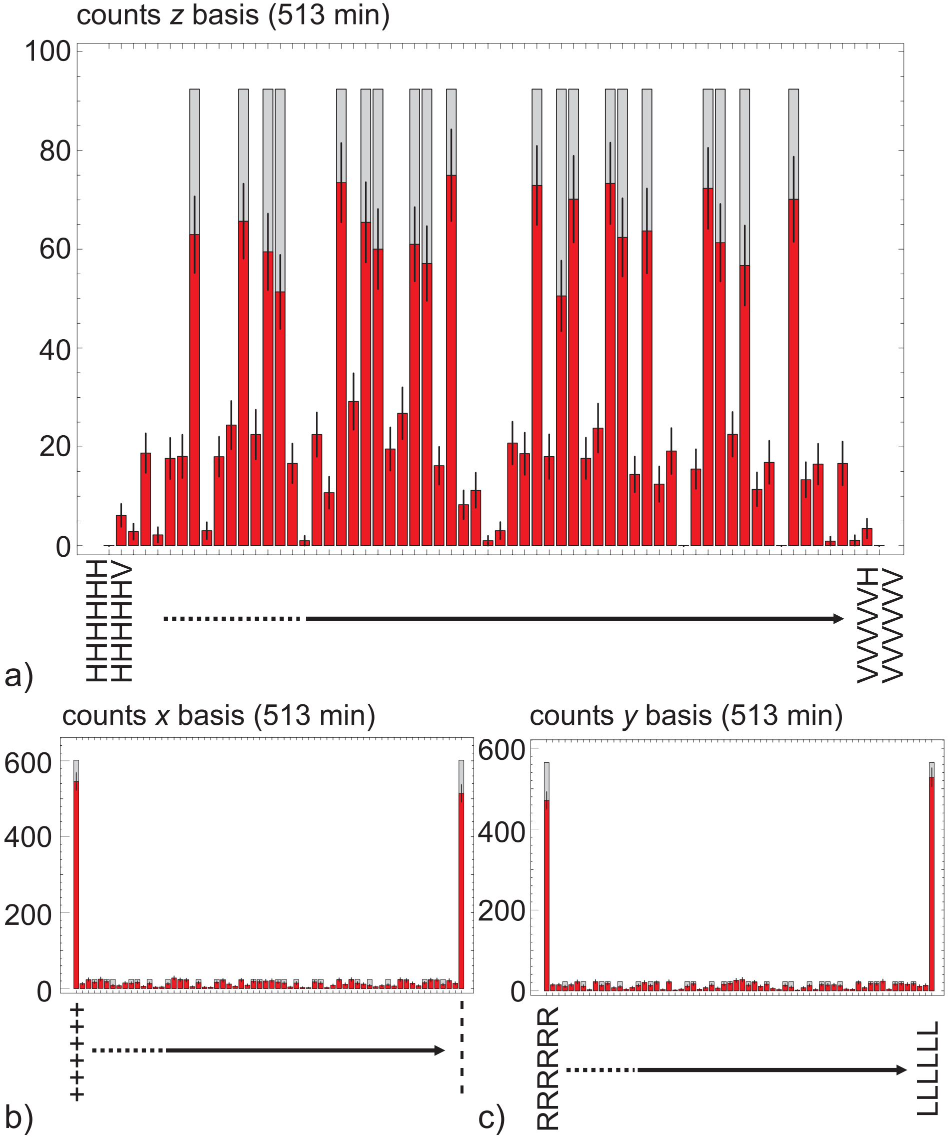
$$|\psi_6\rangle_a \propto \left(a_H^\dagger a_V^\dagger\right)^3 |\mathrm{vac}\rangle$$



$$|D_6^{(3)}\rangle = 1/\sqrt{20}\sum_i \Pi_i(|HHHVVV\rangle)$$

Wieczorek et al., quant-ph 0903.2213 (2009)

Results $|D_6^{(3)}\rangle = 1/\sqrt{20}\sum_i \Pi_i(|HHHVVV\rangle)$



Fidelity:

$$F = \langle D_6^{(3)} |\, \rho \,| D_6^{(3)} \rangle$$
$$= 0.65 \pm 0.02$$

Dicke states 6 -> 5 -> 4

Dicke state as resource for other entangled states:

$$|D_6^{(3)}\rangle = 1/\sqrt{20}\sum_i \Pi_i(|HHHVVV\rangle)$$

$$P(\theta,\phi) := |\theta,\phi\rangle\langle\theta,\phi|$$

$$|\theta,\phi\rangle = \cos\theta\,|H\rangle + e^{i\phi}\sin\theta\,|V\rangle$$

$$|\Delta_5\rangle = \cos\theta\,|D_5^{(3)}\rangle + e^{-i\phi}\sin\theta\,|D_5^{(2)}\rangle$$

2 SLOCC classes:

(i) $\theta = \{0, \pi/2\} \rightarrow \{|D_5^{(3)}\rangle, |D_5^{(2)}\rangle\}$

(ii) $\theta \in (0, \pi/2) \rightarrow \cos\theta\,|D_5^{(3)}\rangle + e^{-i\phi}\sin\theta\,|D_5^{(2)}\rangle$

Wieczorek et al., Phys. Rev. A ***79*** *(2009)*

Wieczorek et al., quant-ph 0903.2213 (2009)

Dicke states 6 -> 5 -> 4

$$|\Delta_5\rangle = \cos\theta\,|D_5^{(3)}\rangle + e^{-i\phi}\sin\theta\,|D_5^{(2)}\rangle$$

$$|D_5^{(2)}\rangle$$

projection onto

$\langle H|$ $\quad$ $\langle V|$

$$|D_4^{(2)}\rangle \qquad |D_4^{(1)}\rangle \equiv |W_4\rangle$$

$$1/\sqrt{2}(|D_5^{(3)}\rangle + i\,|D_5^{(2)}\rangle)$$

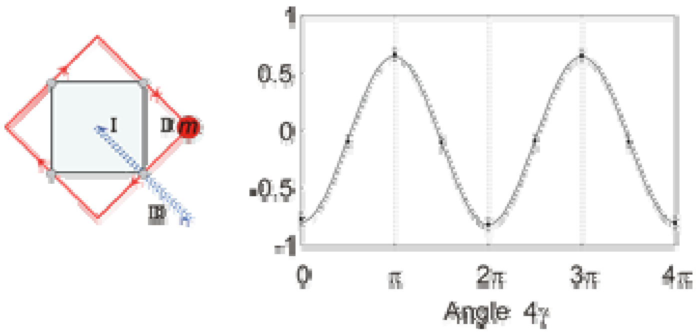
projection onto

$$1/\sqrt{2}(\langle H| - i\langle V|)$$

$$1/\sqrt{2}(|D_4^{(1)}\rangle + |D_4^{(3)}\rangle) = \mathcal{H}^{\otimes 4}|GHZ_4^-\rangle$$

$$|GHZ_4^-\rangle = 1/\sqrt{2}(|HHHH\rangle - |VVVV\rangle)$$

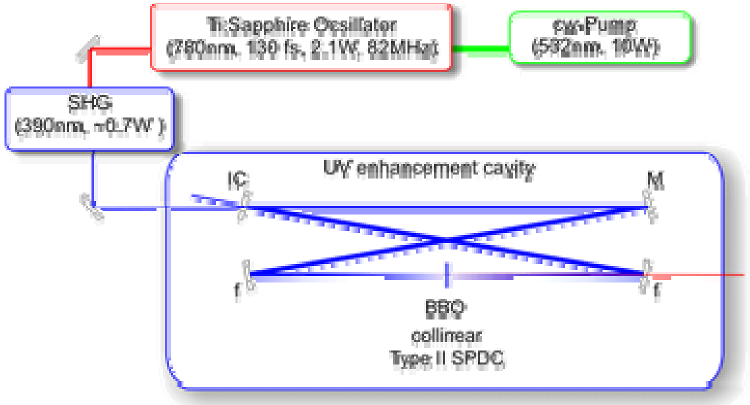
Summary



Four-photon entanglement

Anyonic features based on toric code

Pachos et al., quant-ph 0710.0895 (2007)



Six-photon entanglement

UV enhancement cavity
6 qubit Dicke state as resource

Wieczorek et al., Phys. Rev. A **79** *(2009)*

Wieczorek et al., quant-ph 0903.2213 (2009)