Quantum Hall Hierarchy and the 2nd Landau Level

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work done in collaboration with Joost Slingerland, Adrian Feiguin, and Gunnar Möller
arXiv:0711.3204 (PRB '08) and arXiv:0901.4965

Introduction

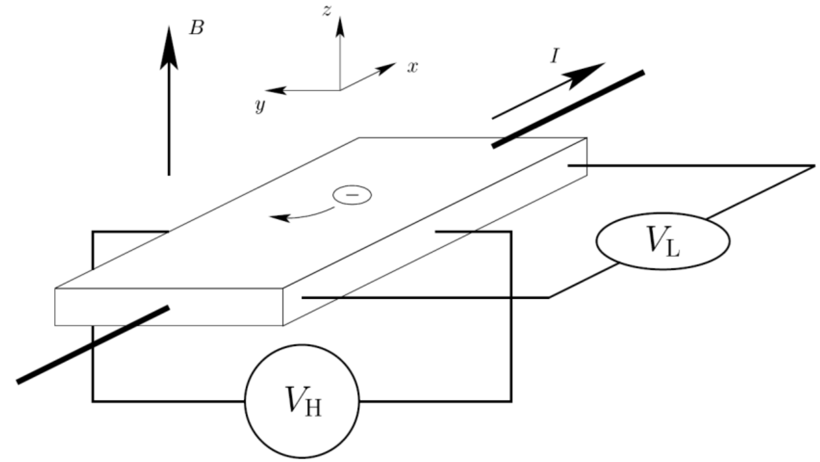
- 2^{nd} Landau level physics is perplexing.
- Experimental determination of the nature of the observed FQH states is just now being obtained!
- Viable proposals for the observed states are desirable. (So far, there is: Laughlin, HH, MR, and RR.)
- Moore-Read (`91) is expected (from numerics and now experiments) to describe $\nu=5/2$ and is relatively well understood.

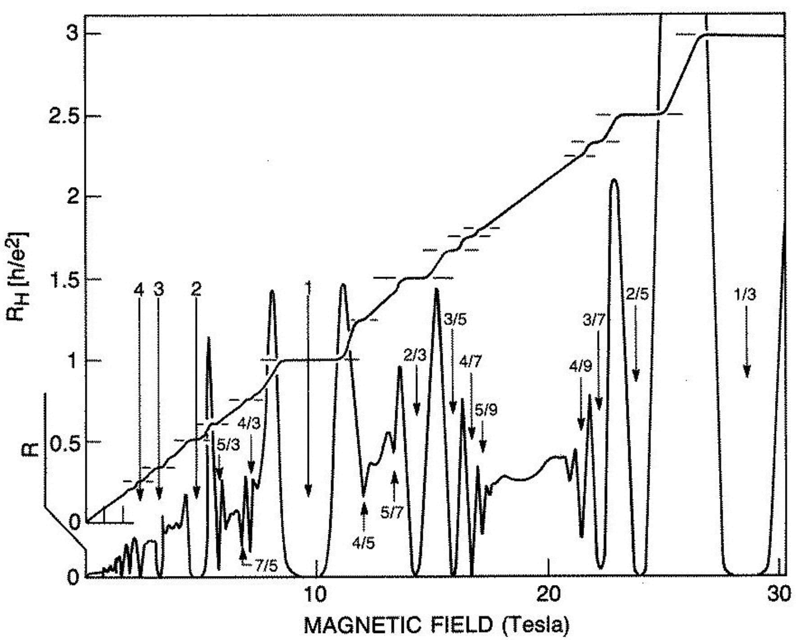
  This leads us to believe that non-Abelian anyons emerge in the FQHE!
- What about the rest of the 2^{nd} LL?

Fractional Quantum Hall Effect

- 2DEG
- large B field (~ 10T)
- low temp (< 1K)
- gapped (incompressible)
- quantized filling fractions

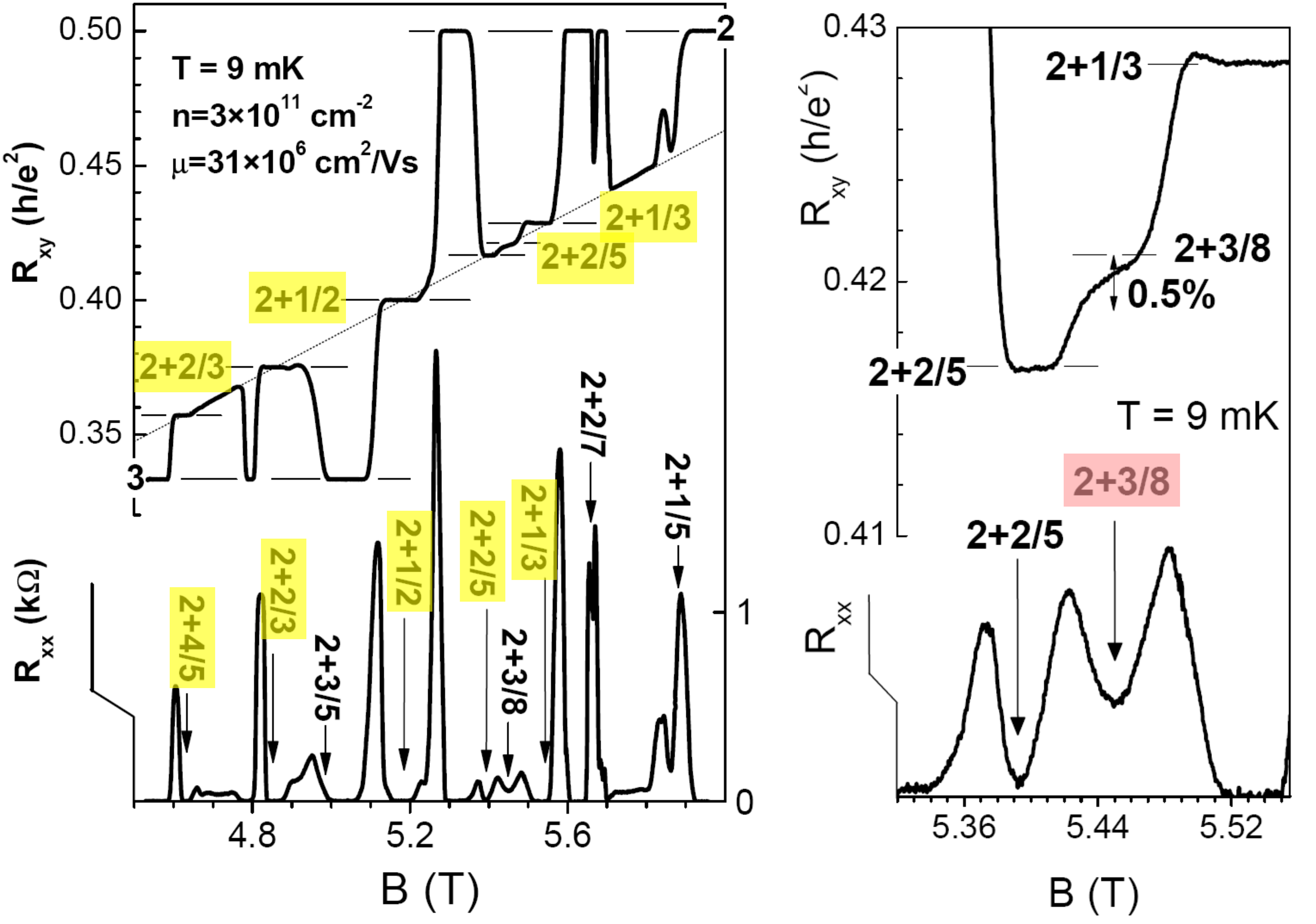
$$\nu = \frac{n}{m}, \quad R_{xy} = \frac{1}{\nu}\frac{h}{e^2}, \quad R_{xx} = 0$$





- fractionally charged quasiparticles
- Abelian anyons at most filling fractions $\theta = \pi \frac{p}{m}$
- non-Abelian anyons in 2^{nd} Landau level, e.g. ν= 5/2, 12/5, …?

Xia *et al.* `04



T = 9 mK
n=3×10^11 cm^-2
μ=31×10^6 cm^2/Vs
Rxy (h/e^2)
0.50
0.45
0.40
0.35
2
3
2+1/3
2+2/5
2+1/2
2+2/3
Rxx (kΩ)
2+4/5
2+2/3
2+3/5
2+1/2
2+2/5
2+3/8
2+1/3
2+2/7
2+1/5
1
0
4.8
5.2
5.6
B (T)
0.43
0.42
0.41
Rxy (h/e^2)
2+1/3
2+3/8
0.5%
2+2/5
T = 9 mK
2+3/8
2+2/5
Rxx
5.36
5.44
5.52
B (T)

ν	$\frac{7}{3}$	$\frac{12}{5}$	$\frac{5}{2}$	$\frac{8}{3}$	$\frac{14}{5}$
Δ_{10}	100	$\star$	110	55	
Δ_{55}	$\star$		310	$\star$	$\star$
Δ_{11}	~ 600	70	$\star$	$\star$	$\star$
Δ_{56}	584	$\star$	544	562	252
Δ'_{56}	206		272	150	≤ 60
Δ_{57}	110		130	60	
Δ_{12}	590	$\star$	450	290	$\star$
Δ_{58}	225		262	64	149

Pan *et al*. \`99,\`08
Eisenstein *et al*. \`02
Xia *et al*. \`04
Choi *et al*. \`08
Miller *et al*. \`07
Dean *et al*. \`08

- ν=5/2 is the strongest state in the 2nd LL.
- Correlation functions for $7/3 \leq \nu \leq 8/3$ have non-Laughlin pairing/clustering character similar to ν=5/2. Outside this region is Laughlin-like. (Wojs \`01)
- The Haldane-Halperin hierarchy (\`83,\`84) seems to work quite well in the lowest Landau level.
- Can we construct a similar hierarchy built on MR?

Generalized Hierarchy Picture

PB and Slingerland `08

1) start with a QH state

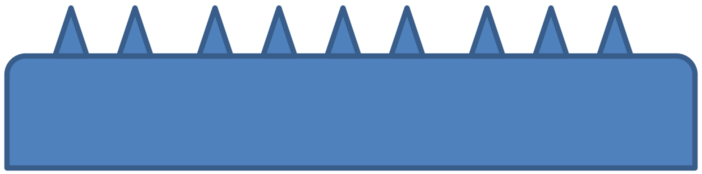
(can be non-Abelian)

$\Psi_{\nu}\left(z_i\right)$

2) add quasiparticles

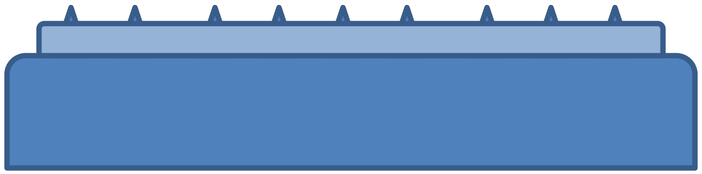
changes density, but in a localized manner

$\Psi_{\nu+\mathrm{qps}}\left(z_i;w_j\right)$



3) project qps onto a QH state

delocalizes and produces uniform incompressible gas at different filling



$$\Psi_{\nu'}\left(z_i\right)=\int\prod_{\alpha}d^2w_{\alpha}\Psi_{\nu+\mathrm{qps}}\left(z_i;w_j\right)\Phi^*\left(w_j\right)$$

e.g. Haldane-Halperin states

1) start with Laughlin $\quad U(1)_3$

$$\Psi_{1/3} = \prod_{i<j} (z_i - z_j)^3$$

2) add qps

$$\Psi_{1/3+\text{qes}} = \prod_{i<j} (w_i - w_j)^{1/3} \prod_{i,j} \left(w_i - 2\frac{\partial}{\partial z_j} \right) \prod_{i<j} (z_i - z_j)^3$$

3) project onto a QH state $\quad U(1)_K \quad K = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

$$\Psi_{2/5} = \int \prod_{\alpha} d^2 w_\alpha \Psi_{1/3+\text{qes}} (z_i; w_j) \prod_{i<j} (w_i - w_j)^{5/3}$$

$$\mathrm{MR} = \mathrm{Ising} \times \mathrm{U}(1)_2\Big|_{\mathcal{C}}$$ (Moore and Read `91)

$$a_{\mathrm{I}} = I, \psi, \sigma$$

$$a_0 \in \tfrac{1}{2} Z$$ number of fluxes

$$\mathcal{C} = \left\{ (I, n), (\psi, n), \left(\sigma, n + \tfrac{1}{2}\right) \right\}$$ where $n \in Z$

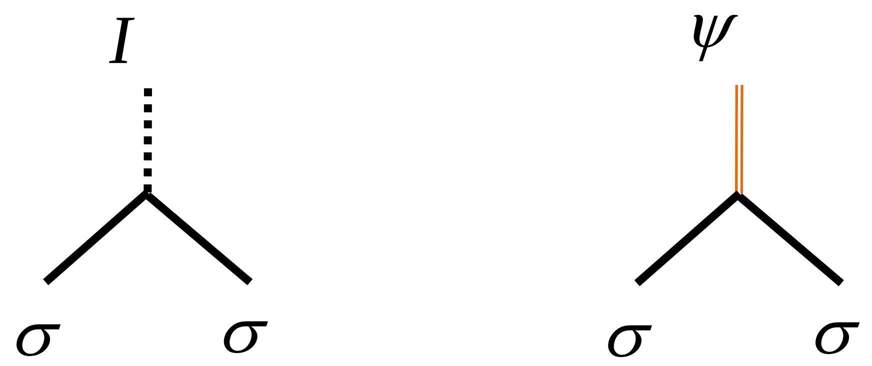
$$e^- = (\psi, 2)$$

$$\Psi_{1/2} = \mathrm{Pf}\left\{ \frac{1}{z_i - z_j} \right\} \prod_{i<j} (z_i - z_j)^2$$

Build hierarchy on MR

(PB and Slingerland `08)

- Form a FQH state with qps and project qps into a new FQH state
- MR e/4 quasiholes $\left(\sigma, \frac{1}{2}\right)$ are non-Abelian, and have two fusion channels:



Form gas of MR e/4 qps? $\left(\sigma, \frac{1}{2}\right)$

- Mathematically well-defined by forming trivial representation of the braid group.
- But physically…
- resulting filling fraction are experimentally non-existent
- next layer "particles" must be non-Abelian,

  i.e. carry conjugate of Ising σ charge
- gas of qps are strongly interacting and not well-separated, non-Abelian degeneracy cannot be preserved
- resulting states have same universality class as other previously constructed Abelian states

Build hierarchy on MR

(PB and Slingerland `08)

- Form a FQH state with qps and project qps into a new FQH state
- MR quasiholes $\left(\sigma, \frac{1}{2}\right)$ are non-Abelian, and have two fusion channels:

- Pair into preferred fusion channel I and form a gas of bound pairs, i.e. e/2 quasiparticles $\left(I, 1\right)$

$$\mathrm{MR} = \mathrm{Ising} \times \mathrm{U}(1)_2\Big|_{\mathcal{C}}$$

Add e/2 quasiparticles $(I,1)$

$$\Psi_{1/2+\mathrm{e}/2\ \mathrm{qhs}} = \prod_{i<j}(u_i - u_j)^{1/2}\prod_{i,j}(u_i - z_j)$$
$$\times \mathrm{Pf}\left\{\frac{1}{z_i - z_j}\right\}\prod_{i<j}(z_i - z_j)^2$$

Now project paired-qhs onto new FQH state.
(Forms hierarchy on charge sector of MR.)

$$\mathrm{BS}_{2/5} = \mathrm{Ising} \times \mathrm{U}(1)_K\big|_{\mathcal{C}} \quad \text{with} \quad K = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{aligned} \Psi_{2/5} &= \int \prod_{\alpha} d^2 u_\alpha \Psi_{1/2+\mathrm{e}/2\ \mathrm{qhs}}(z_i; u_j) \prod_{i<j} \left(u_i^* - u_j^*\right)^{5/2} \\ &= \int \prod_{\alpha} d^2 u_\alpha \prod_{i<j} \left(u_i^* - u_j^*\right)^2 \left|u_i^* - u_j^*\right| \\ &\qquad \times \prod_{i,j} (u_i - z_j)\, \mathrm{Pf}\left\{\frac{1}{z_i - z_j}\right\} \prod_{i<j} (z_i - z_j)^2 \end{aligned}$$

$$\mathrm{BS}_{2/3} = \mathrm{Ising} \times \mathrm{U}(1)_K \big|_{\mathscr{C}} \quad \text{with} \quad K = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{aligned} \Psi_{2/3} &= \int \prod_{\alpha} d^2 u_\alpha \Psi_{1/2+\mathrm{e}/2\ \mathrm{qes}} (z_i; u_j) \prod_{i<j} (u_i - u_j)^{3/2} \\ &= \int \prod_{\alpha} d^2 u_\alpha \prod_{i<j} (u_i^* - u_j^*)^2 \\ &\quad \times \prod_{i,j} \left(u_i^* - 2 \frac{\partial}{\partial z_j} \right) \mathrm{Pf} \left\{ \frac{1}{z_i - z_j} \right\} \prod_{i<j} (z_i - z_j)^2 \end{aligned}$$

$$\mathrm{BS}_{1/3} = \text{particle-hole conjugate of } \mathrm{BS}_{2/3}$$

$$\mathrm{BS}^{\psi}_{1/3} = \mathrm{Ising} \times \mathrm{U}(1)_K \Big|_{\mathscr{C}} \quad \text{with } K = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

with quasiholes condensed in the ψ-channel

$$\begin{aligned} \Psi_{1/3} &= \int \prod_{\alpha} d^2 u_\alpha \Psi_{1/2+\mathrm{e}/2\ \psi\ \mathrm{qhs}}(z_i; u_j) \prod_{i<j} (u_i^* - u_j^*)^{3/2} \\ &= \int \prod_{\alpha} d^2 u_\alpha \prod_{i<j} (u_i^* - u_j^*) \left| u_i^* - u_j^* \right| \\ &\quad \times \prod_{i,j} (u_i - z_j) \, \mathrm{Pf}\left\{ \frac{1}{w_i - w_j} \right\} \prod_{i<j} (z_i - z_j)^2 \end{aligned}$$

$w = z, u$

Other filling fractions?

$$\mathrm{BS}_K = \mathrm{Ising} \times \mathrm{U}(1)_K \big|_{\mathscr{C}}$$

$$\text{with } K = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \Rightarrow \nu = 3/8$$

$$\text{with } K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \Rightarrow \nu = 4/5$$

Composite Fermion Picture of BS States

$$\Psi^{(\text{BS-CF})}_{\frac{n}{3n-1}} = \mathcal{P}_{LLL}\left\{\mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]\chi_1^3\chi_{-n}\right\} \qquad \nu = \frac{n}{3n-1} = \frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{4}{11}, \dots$$

$$\Psi^{(\text{BS-CF})}_{\frac{n}{n+1}} = \mathcal{P}_{LLL}\left\{\mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]\chi_1\chi_n\right\} \qquad \nu = \frac{n}{n+1} = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$$

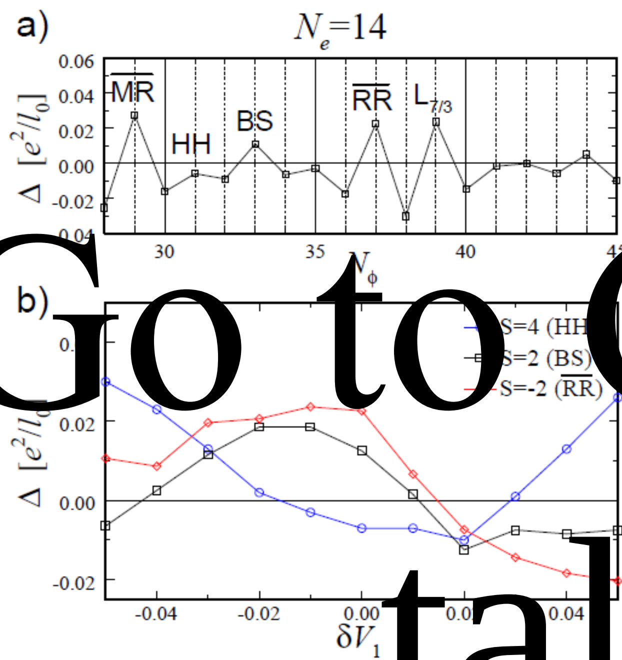
χ_n n filled Landau levels

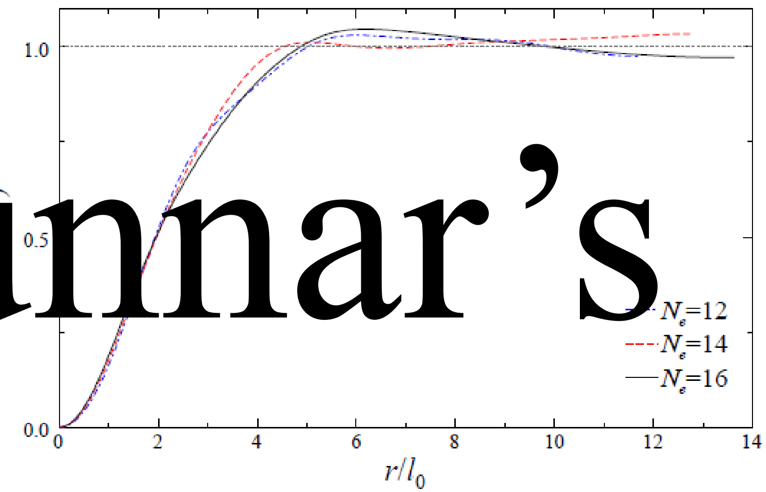
Attachment of $SU(2)_2$ and $U(1)$ CS flux to IQH.

How will we know?

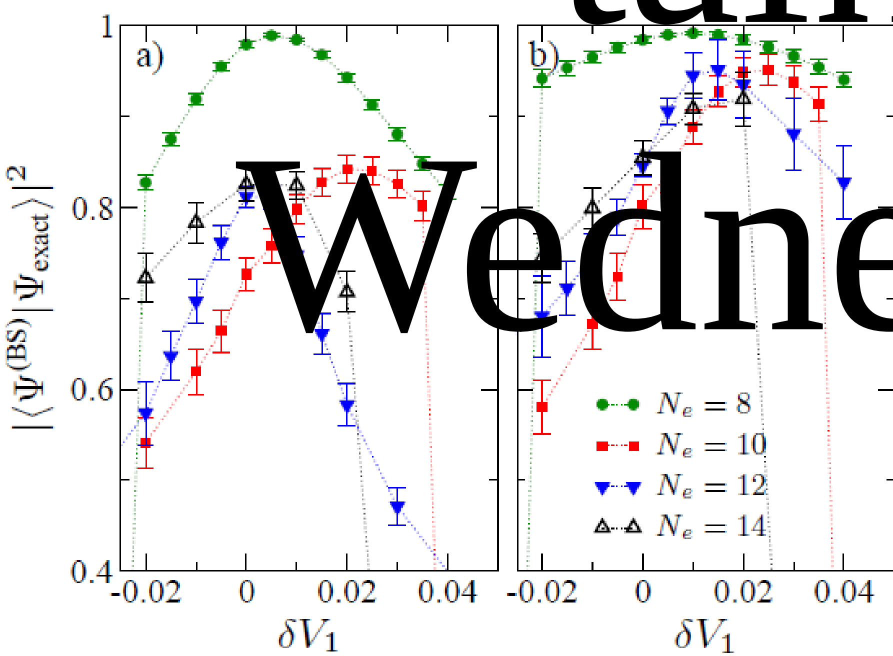
- Numerical evidence…

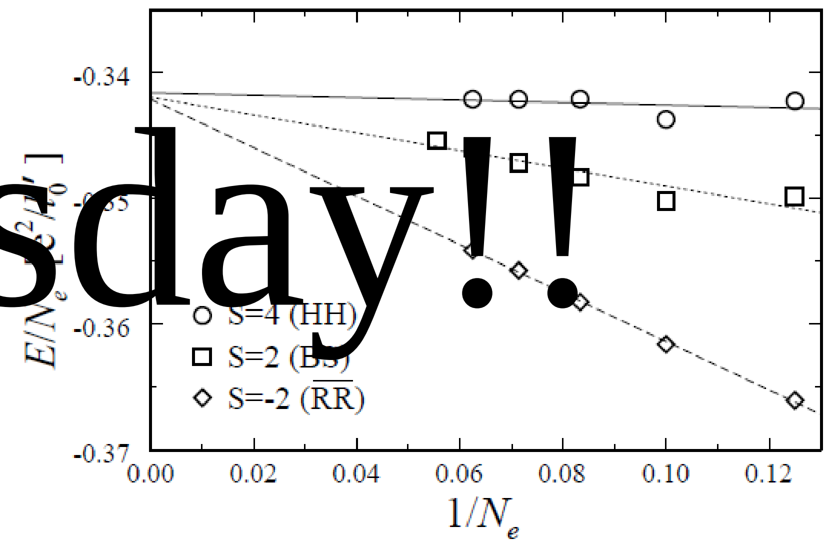
Morf *et al*., Rezayi *et al*., Wojs *et al*.,
Feiguin *et al*., **Möller** *et al*., Peterson *et al*.





Go to Gunnar's talk on Wednesday!!





PB, Feiguin, Möller, Slingerland `09

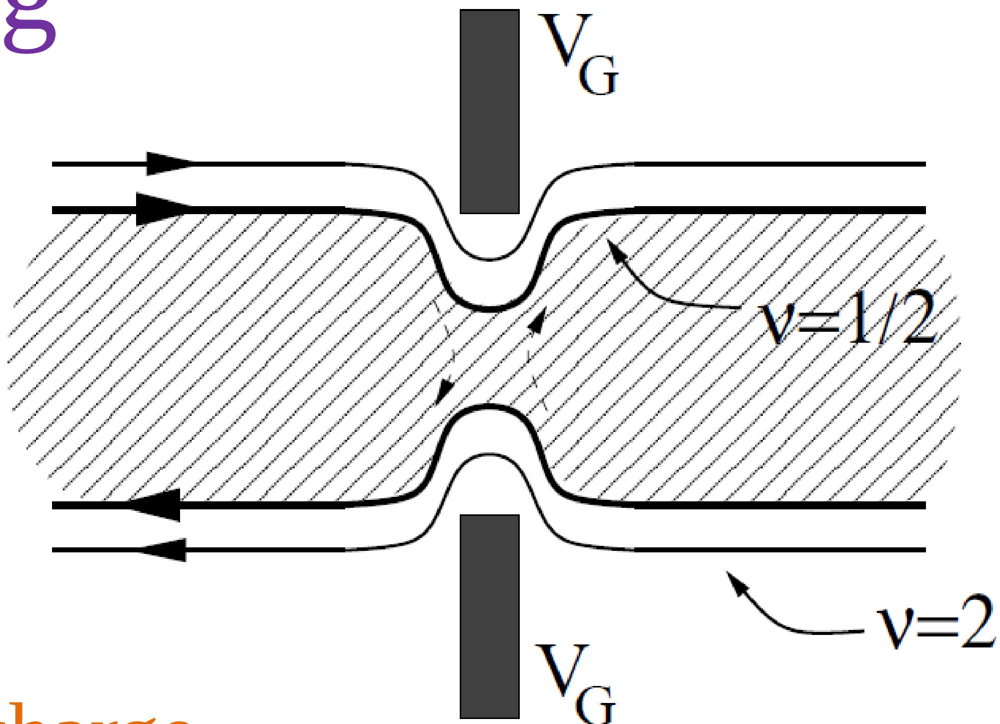
How will we know?

- Numerical evidence.

  Morf *et al.*, Rezayi *et al.*, Wojs *et al.*, **Feiguin** *et al.*, **Möller** *et al.*, Peterson *et al.*

- Experiments must determine:
  - Quasiparticle electric charge tells us something (though not nearly enough).
  - Braiding statistics (determined e.g. by interferometry) tells us almost everything.
  - Scaling relations from tunneling tells us practically everything else.

1PC Tunneling

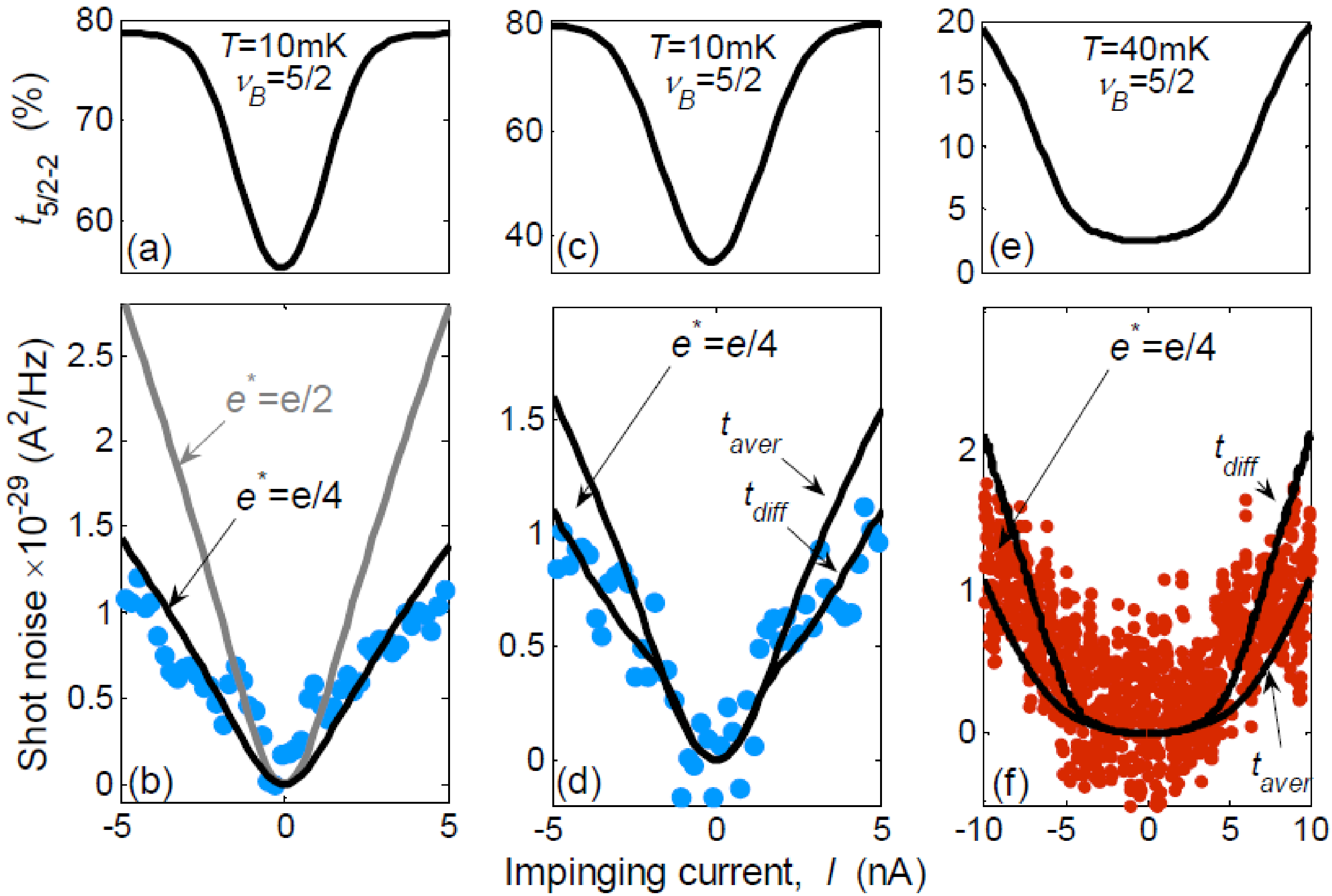


Shot noise gives qp charge.

Scaling relations:

$$I_b^{(qp)} \propto \begin{cases} T^{2g-2}\,V & \text{for small } eV \ll k_B T \\ V^{2g-1} & \text{for small } eV \gg k_B T \end{cases}$$

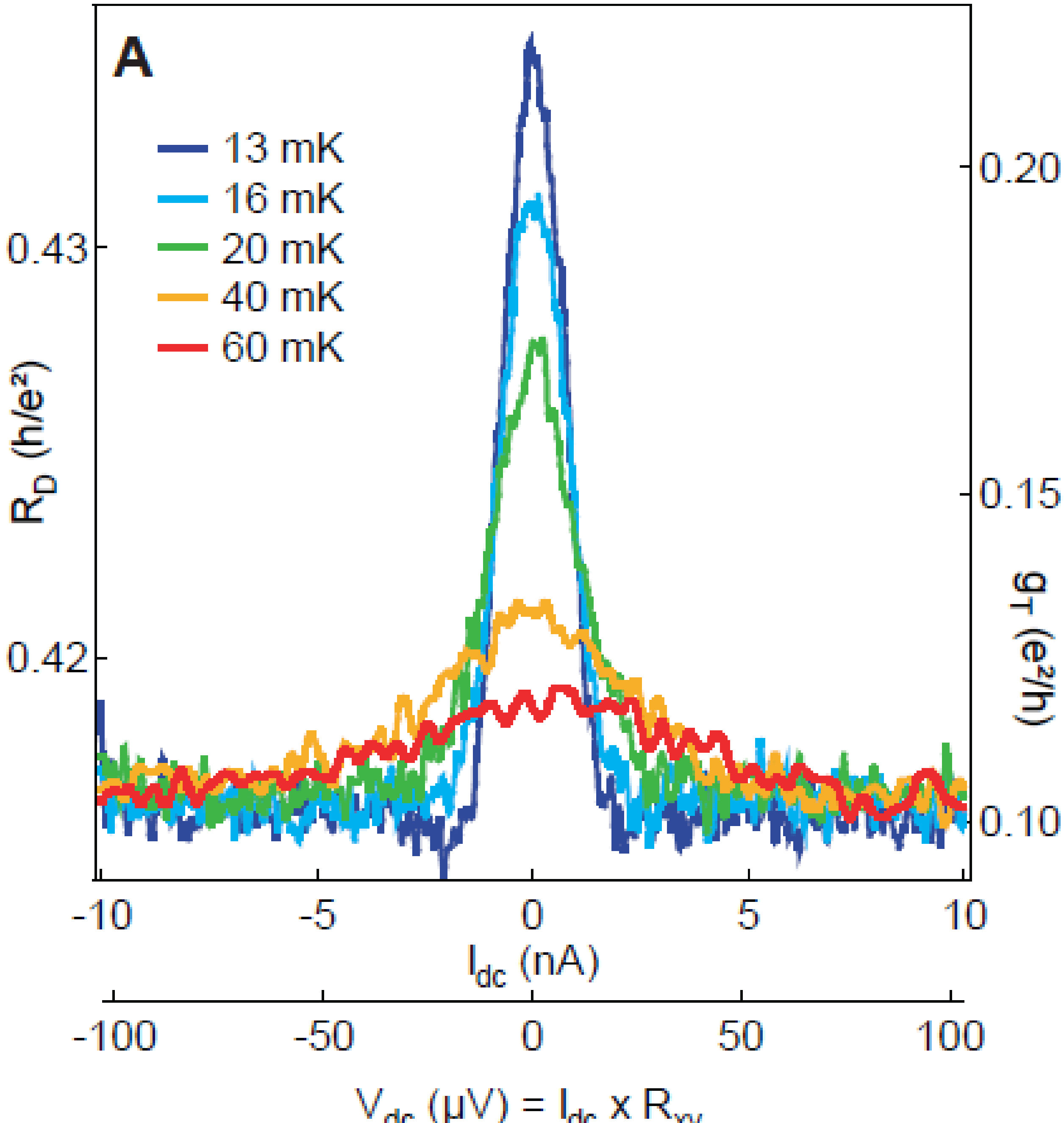
Dolev *et al.* `08
for ν=5/2



e*=e/4

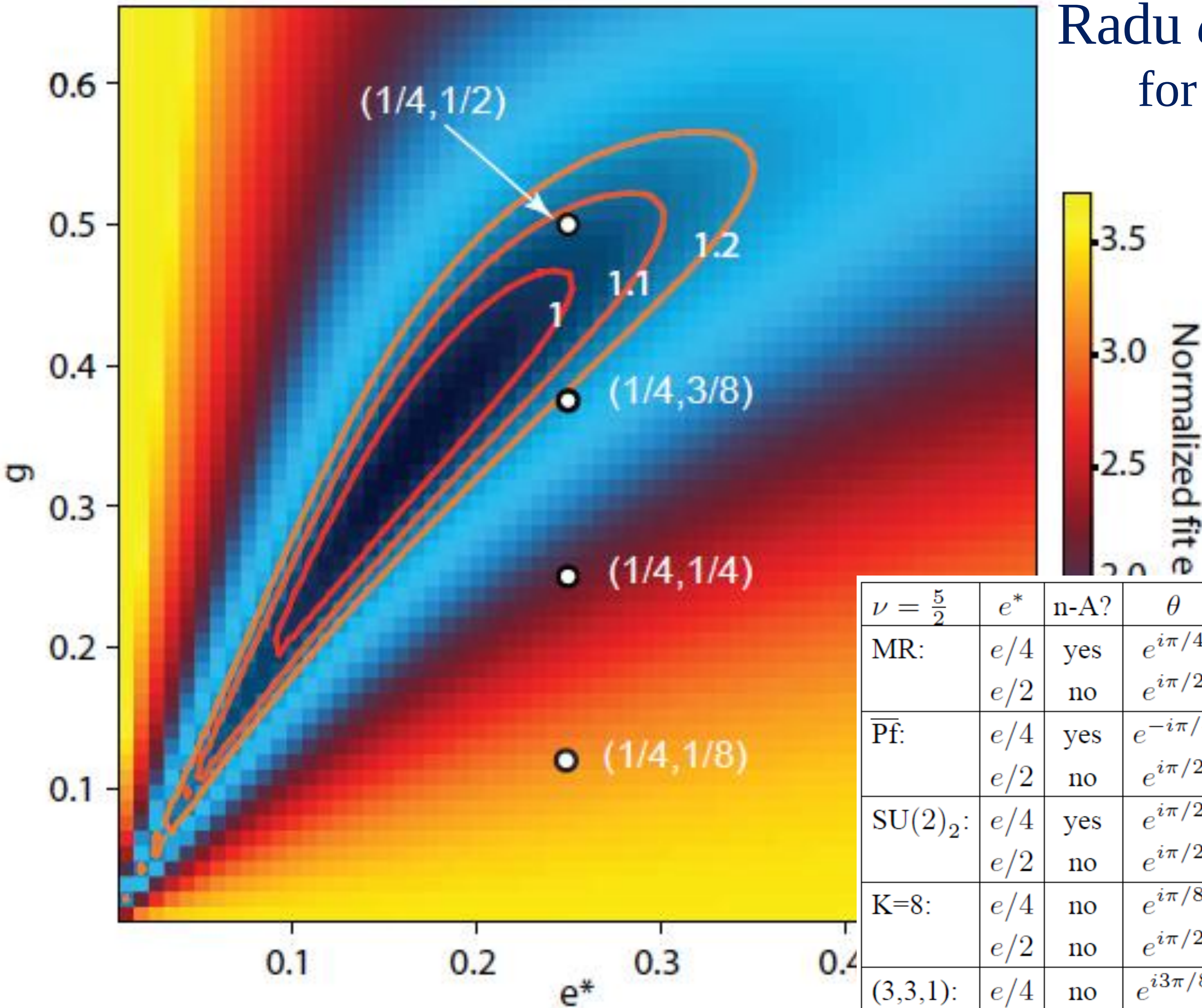
also, for ν=8/3 they find e*=e/3

Radu *et al.* `08
for ν=5/2



Radu *et al.* `08

for ν=5/2



$\nu=\frac{5}{2}$	e^*	n-A?	θ	g_c	g_n	g
MR:	$e/4$	yes	$e^{i\pi/4}$	$1/8$	$1/8$	$1/4$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
$\overline{\text{Pf}}$:	$e/4$	yes	$e^{-i\pi/4}$	$1/8$	$3/8$	$1/2$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
$\text{SU}(2)_2$:	$e/4$	yes	$e^{i\pi/2}$	$1/8$	$3/8$	$1/2$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
K=8:	$e/4$	no	$e^{i\pi/8}$	$1/8$	0	$1/8$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
(3,3,1):	$e/4$	no	$e^{i3\pi/8}$	$1/8$	$1/4$	$3/8$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$

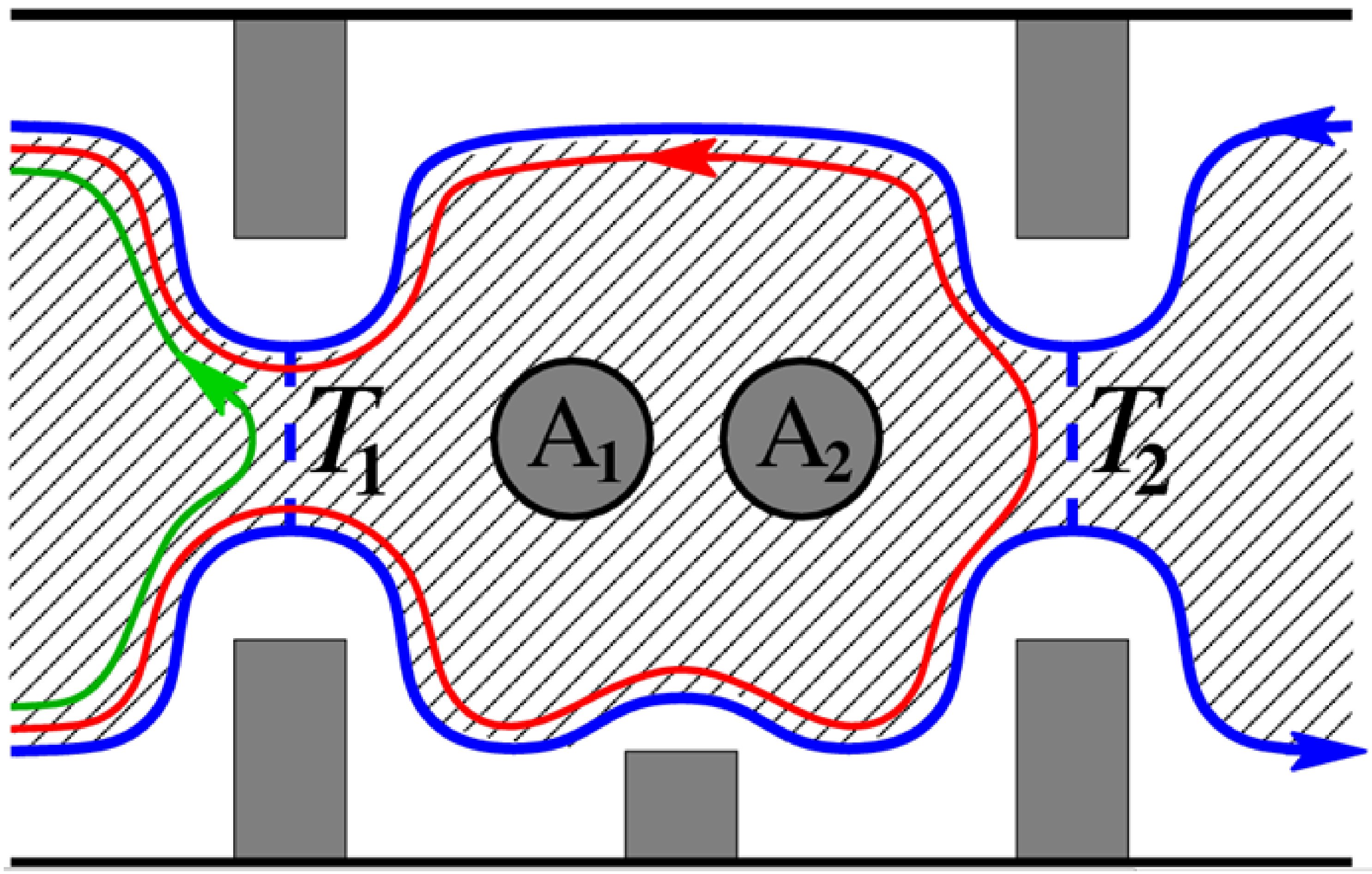
Other filling fractions

$\nu = \frac{7}{3}$	e^*	n-A?	θ	g_c	g_n	g
$\mathrm{L}_{1/3}$:	$e/3$	no	$e^{i\pi/3}$	$1/3$	0	$1/3$
$\overline{\mathrm{BS}}_{2/3}$:	$e/3$	yes	$e^{-i7\pi/24}$	$1/3$	$5/8$	$23/24$
	$e/3$	no	$e^{i\pi/3}$	$1/3$	0	$1/3$
$\mathrm{BS}^{\psi}_{1/3}$:	$e/3$	yes	$e^{i5\pi/24}$	$1/3$	$3/8$	$17/24$
	$e/3$	no	$e^{i\pi/3}$	$1/3$	0	$1/3$
$\overline{\mathrm{RR}}_{k=4}$:	$e/6$	yes	$e^{-i\pi/6}$	$1/12$	$1/4$	$1/3$
	$e/3$	no	$e^{i\pi/3}$	$1/3$	0	$1/3$
	$e/2$	yes	$e^{i\pi/2}$	$3/4$	$1/4$	1

$\nu = \frac{8}{3}$	e^*	n-A?	θ	g_c	g_n	g
$\overline{\mathrm{L}}_{1/3}$:	$e/3$	no	$e^{-i\pi/3}$	$1/3$	$1/3$	$2/3$
	$2e/3$	no	$e^{i2\pi/3}$	$2/3$	0	$2/3$
$\mathrm{BS}_{2/3}$:	$e/3$	yes	$e^{i7\pi/24}$	$1/6$	$1/8$	$7/24$
	$e/3$	no	$e^{i2\pi/3}$	$1/3$	$1/3$	$2/3$
	$2e/3$	no	$e^{i2\pi/3}$	$2/3$	0	$2/3$
$\overline{\mathrm{BS}}^{\psi}_{1/3}$:	$e/3$	yes	$e^{-i5\pi/24}$	$1/6$	$3/8$	$13/24$
	$e/3$	no	$e^{i2\pi/3}$	$1/6$	$1/2$	$2/3$
	$2e/3$	no	$e^{i2\pi/3}$	$2/3$	0	$2/3$
$\mathrm{RR}_{k=4}$:	$e/6$	yes	$e^{i\pi/6}$	$1/24$	$1/8$	$1/6$
	$e/3$	yes	$e^{i\pi/3}$	$1/6$	$1/6$	$1/3$
	$e/2$	yes	$e^{i\pi/2}$	$3/8$	$1/8$	$1/2$
	$2e/3$	no	$e^{i2\pi/3}$	$2/3$	0	$2/3$

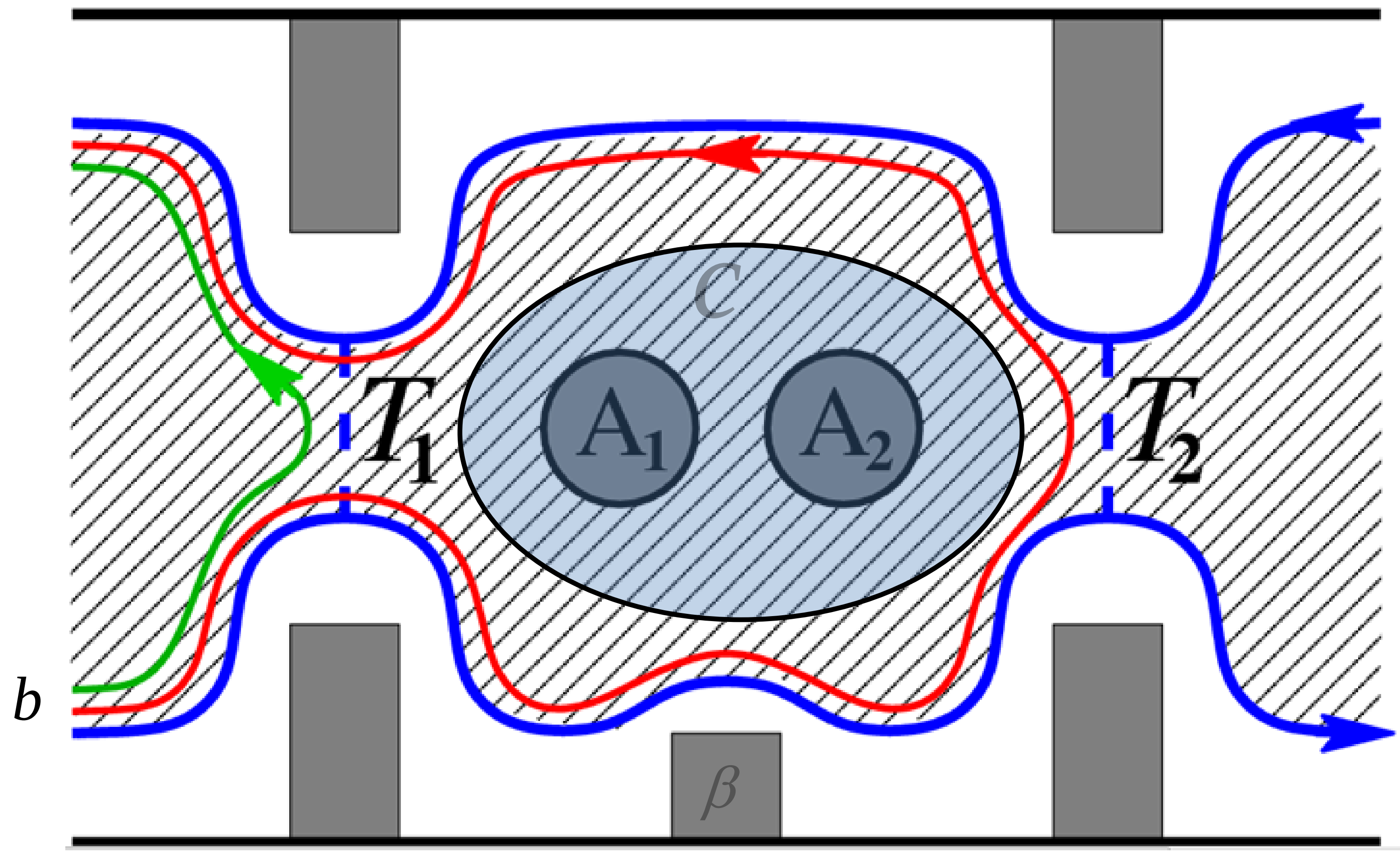
$\nu = \frac{12}{5}$	e^*	n-A?	θ	g_c	g_n	g
$\mathrm{HH}_{2/5}$:	$e/5$	no	$e^{i3\pi/5}$	$1/5$	$2/5$	$3/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$
$\mathrm{BS}_{2/5}$:	$e/5$	yes	$e^{i9\pi/40}$	$1/10$	$1/8$	$9/40$
	$e/5$	no	$e^{-i2\pi/5}$	$1/10$	$1/2$	$3/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$
$\overline{\mathrm{BS}}^{\psi}_{3/5}$:	$e/5$	yes	$e^{-i11\pi/40}$	$1/10$	$3/8$	$19/40$
	$e/5$	no	$e^{-i2\pi/5}$	$1/10$	$1/2$	$3/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$
$\overline{\mathrm{RR}}_{k=3}$:	$e/5$	yes	$e^{-i\pi/5}$	$1/10$	$3/10$	$2/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$

2PC Interferometer



$$\sigma_{xx} \propto |t_1|^2 + |t_2|^2 + 2|t_1 t_2||M_{bc}|\cos(\beta + \theta_{bc})$$

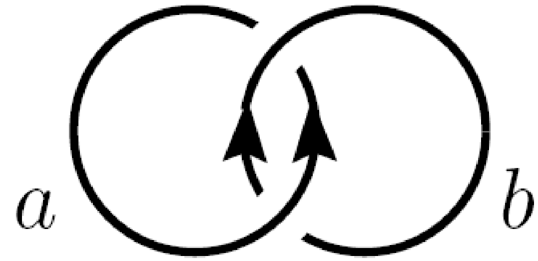
$\beta = \alpha + \arg(t_2 / t_1)$ is a parameter that can be experimentally varied and includes the A-B phase: $\alpha = q\Phi$



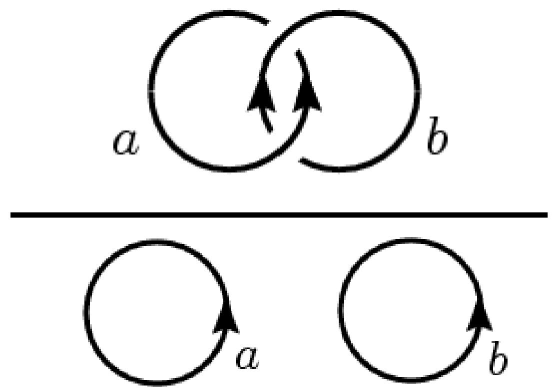
But what is M_{bc} and what are its properties?

Topological S-matrix

$$S_{ab} = \frac{1}{D}$$



$$M_{ab} = \frac{S_{ab} S_{II}}{S_{Ia} S_{Ib}} =$$



$M_{ab} = e^{i2\theta}$ corresponds to Abelian braiding and

$|M_{ab}| < 1$ iff the braiding is non-Abelian.

Smoking gun!

Ising anyons or $SU(2)_2$

Particle types: $I,\ \sigma,\ \psi$

$$\text{Monodromy:}\quad M=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -1\\ 1 & -1 & 1\end{bmatrix}$$

$SU(2)_3$ or $\mathrm{Fib}\times Z_2$

Particle types: $0,\frac{1}{2},1,\frac{3}{2}$

$$\text{Monodromy:}\quad M=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & \phi^2 & -\phi^2 & -1\\ 1 & -\phi^2 & -\phi^2 & 1\\ 1 & -1 & 1 & -1\end{bmatrix}$$

$\phi^{-2}\approx .38$

$SU(2)_4$

Particle types: $0,\frac{1}{2},1,\frac{3}{2},2$

$$\text{Monodromy:}\quad M=\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 1 & \frac{1}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} & -1\\ 1 & 0 & \frac{1}{2} & 0 & 1\\ 1 & \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & -1\\ 1 & -1 & 1 & -1 & 1\end{bmatrix}$$

$\nu = 5/2$ MR and $\overline{\mathrm{Pf}}$ are $\mathrm{U}(1)\times$ Ising

U(1) is a familiar Abelian factor due to charge/flux

quasiholes carry anyonic charge : $(e/4, \sigma)$

electrons carry anyonic charge : $(-e, \psi)$

n quasiholes carry anyonic charge : $(ne/4, \sigma)$ for n odd

$(ne/4, I \text{ or } \psi)$ for n even

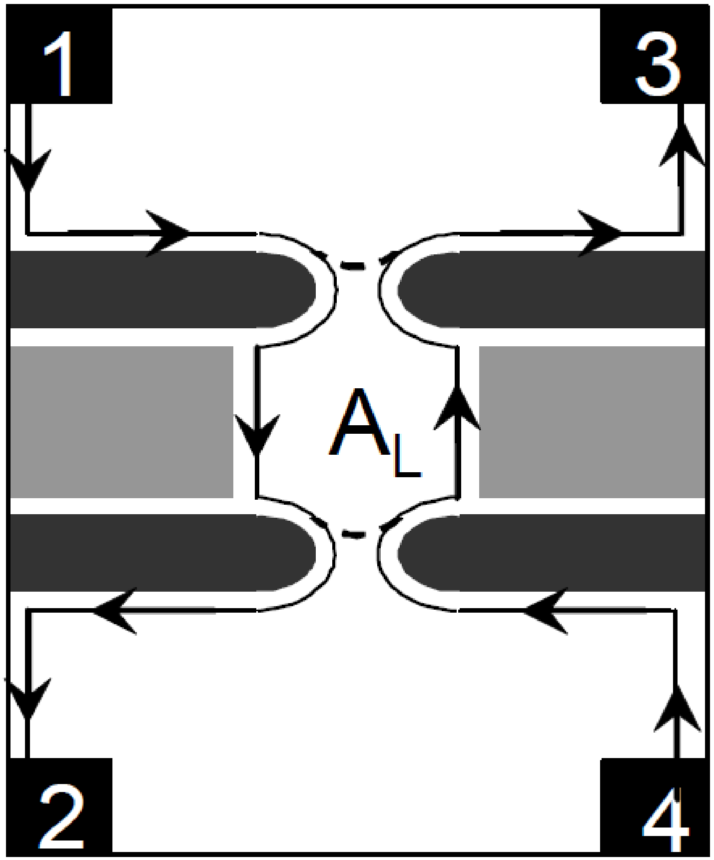
Das Sarma, Freedman, Nayak `05
Stern, Halperin `06
PB, Kitaev, Shtengel `06

$$n \text{ odd}: \quad \sigma_{xx} \propto |t_1|^2 + |t_2|^2$$

$$n \text{ even}: \quad \sigma_{xx} \propto |t_1|^2 + |t_2|^2 + 2|t_1 t_2| \cos\left(\tfrac{e\Phi}{4} \mp n\tfrac{\pi}{4} + N_\psi \pi\right)$$

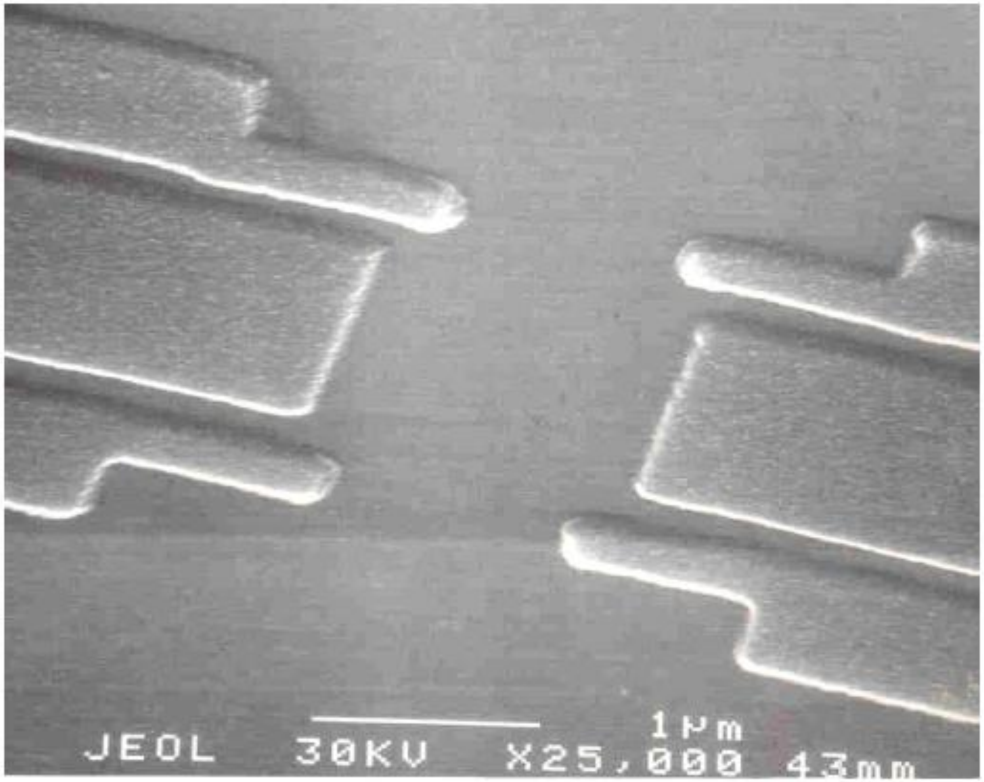
where $N_\psi = 0$ for I and $N_\psi = 1$ for ψ

FQH interferometer

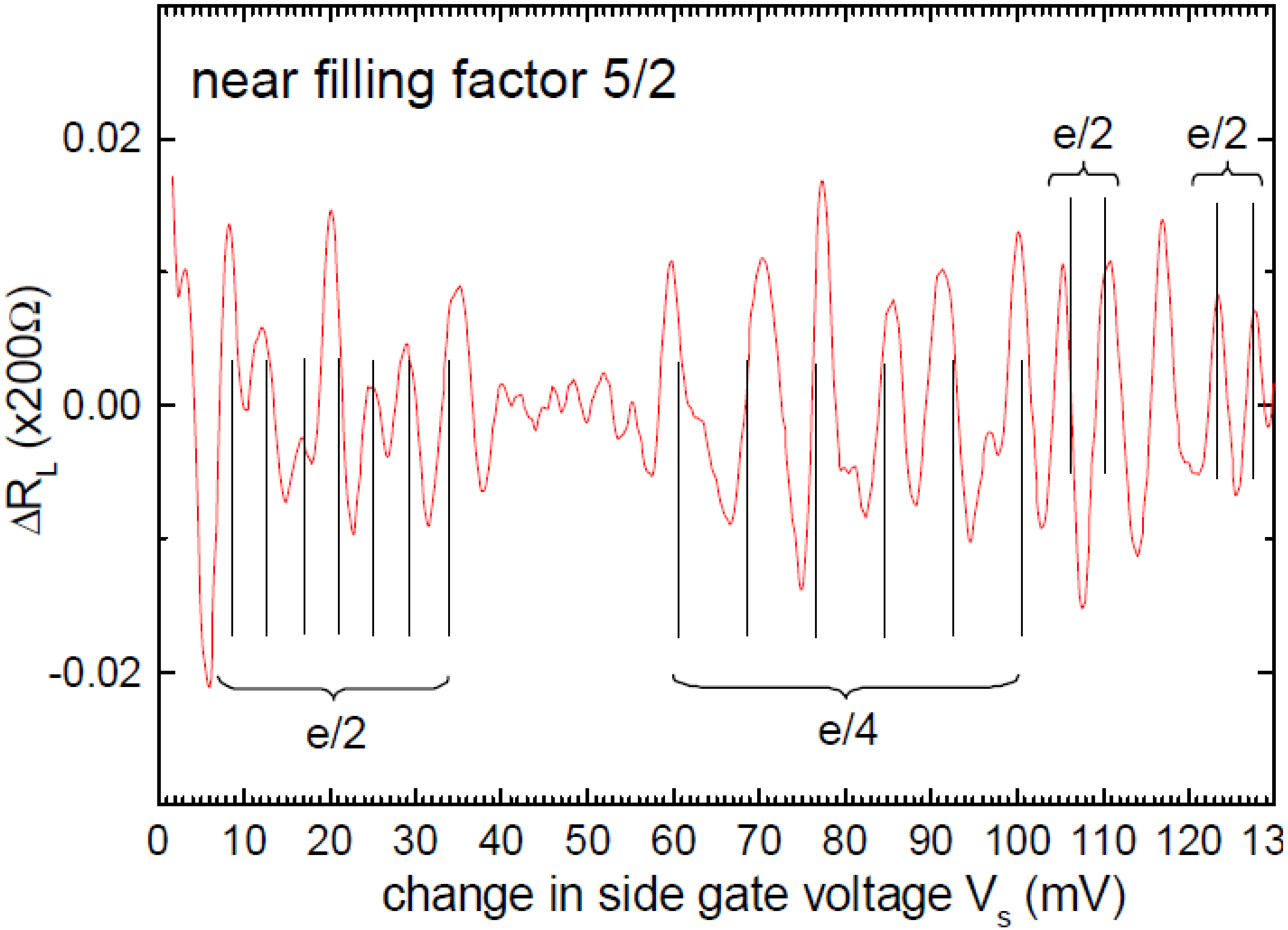


Willett *et al*. `08
for ν=5/2

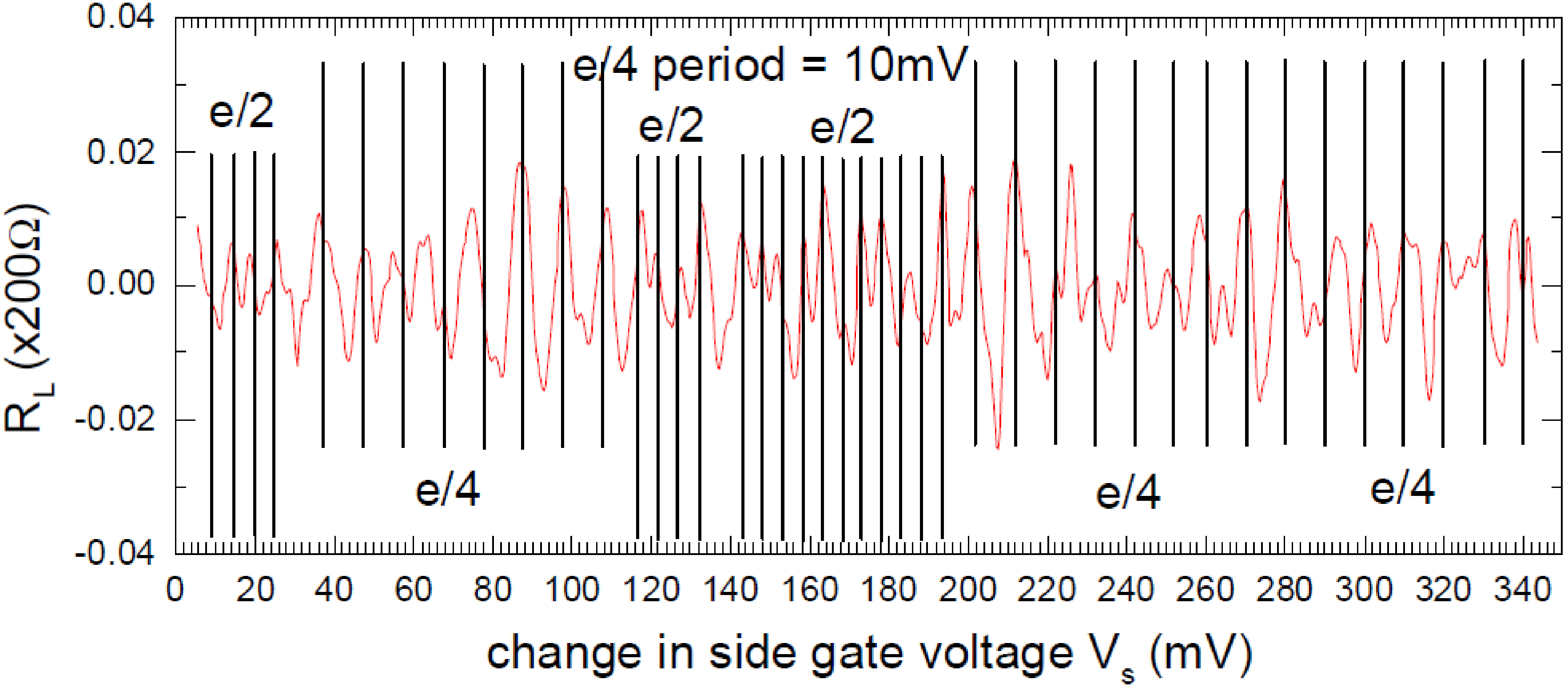
(also progress by: Marcus, Eisenstein, Kang, Heiblum, Goldman, etc.)



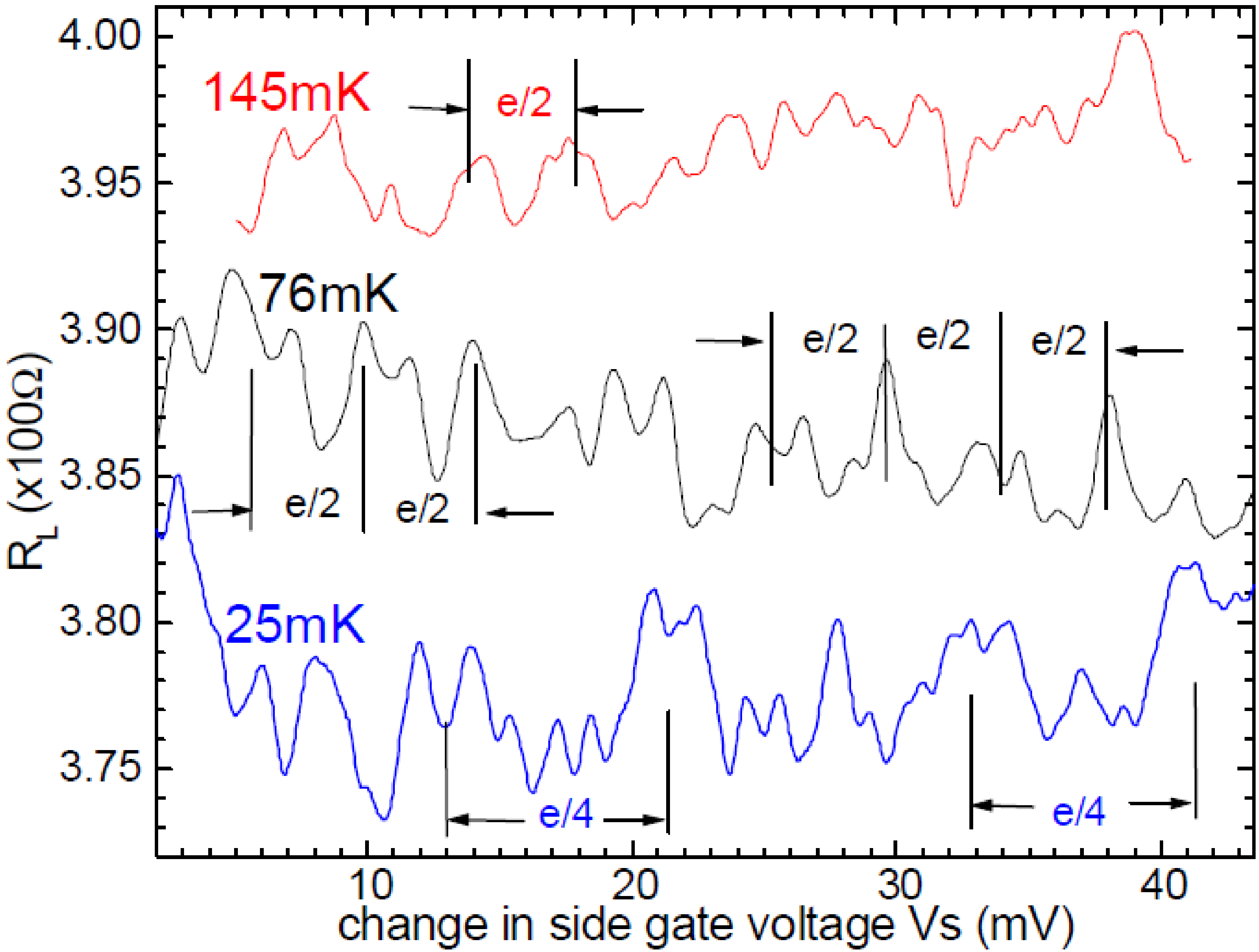
Even/Odd Alternation



Even/Odd Alternation



Temperature Dependence



Why are there e/2 oscillations?

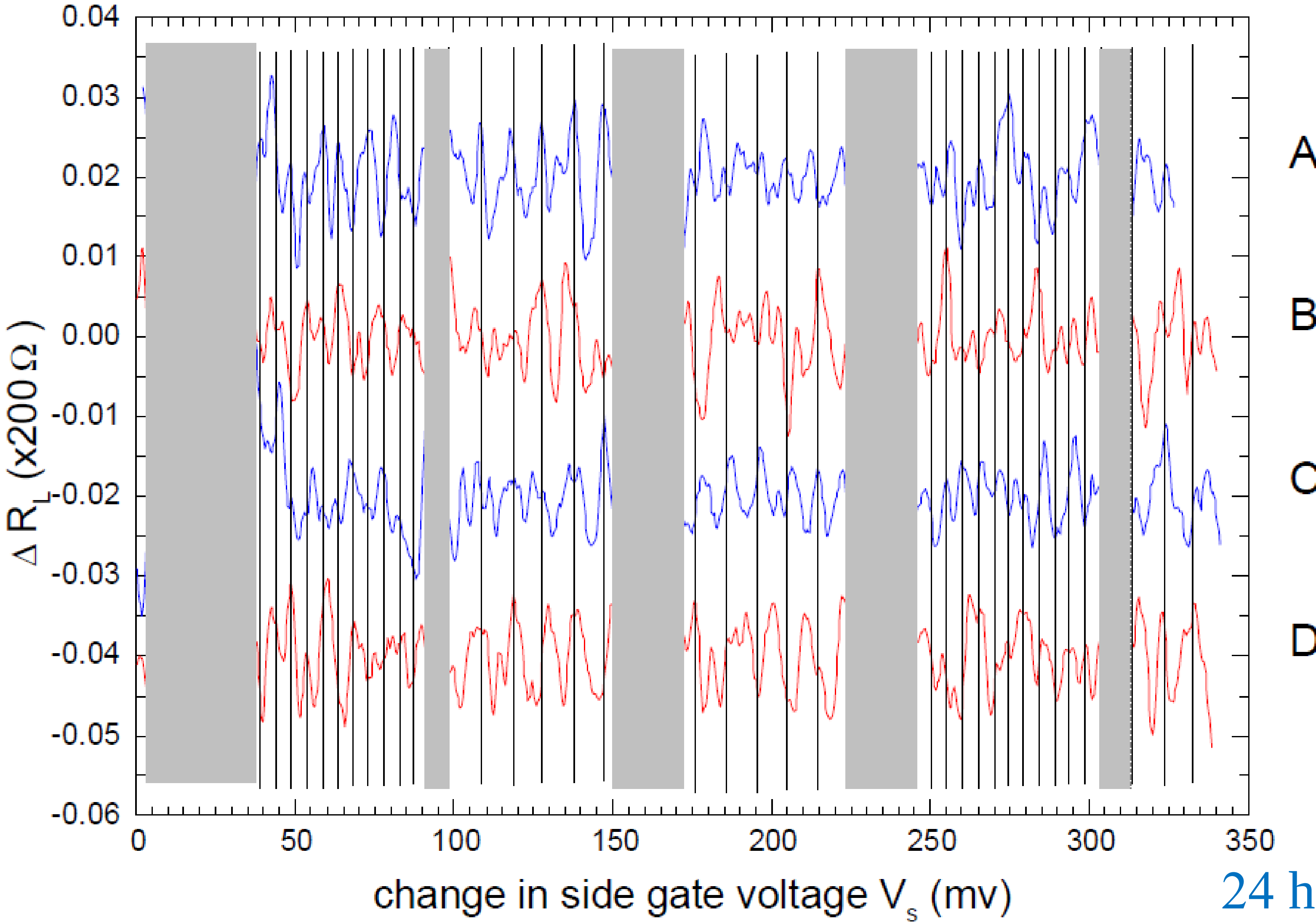
- Tunneling of e/2 quasiparticles
  - Abelian: can be treated as inert background for TQC
- Also <u>very</u> small contribution from double passes of e/4 quasiparticles
  - have order t^2 relative amplitude suppression from additional tunneling
  - have coherence length and temperature exponential suppression
  - temperature dependence also indicates e/2 qps is the source

PB, Shtengel, Slingerland `07
Bishara, Nayak `08
Bishara, PB, Nayak, Shtengel, Slingerland `09

$$\exp\left[-2\pi TL\left(\frac{g_c}{v_c}+\frac{g_n}{v_n}\right)\right]$$

$e/4$	MR	$\overline{\mathrm{Pf}}/\mathrm{SU}(2)_2$	K=8	(3,3,1)	e/2
L^* in μm	1.4	0.5	19	0.7	4.8
T^* in mK	36	13	484	19	121

Reproducibility



24 hrs/run

$\tau_{error} \sim 1$ week!!

Conclusion

- Hierarchy of states that takes the $\nu=5/2$ pairing as its fundamental physics.
- Produces states at **all the observed 2^{nd} Landau level** filling fractions, with $\nu=7/3$, $12/5$, $8/3$ occurring at the first level of hierarchy.
- HH and RR have competition from BS at $\nu=12/5$.
- Appears that **TQC will be achievable**!!! ☺
- No RR at $\nu=12/5$ would be unfortunate for TQC. ☹
- First attempts to experimentally determine the nature of 2^{nd} LL FQH states are currently under way, so we should know more soon!