Topologically protected quantum computation

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Introduction

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- Non-Abelian anyons have non-trivial fusion spaces.

- Fusion information is stored non-locally.

- This makes non-Abelian anyons a natural choice to encode quantum information fault-tolerantly.

- We realize equivalent properties in Abelian models (non-locality, non-Abelian fusion rules).

- Non-topological operations are introduced to allow universal quantum computation.

The anyonic model

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- Many Abelian models can be used. We consider $D(Z_6)$.
- Square lattice with a six level spin on each edge.
- The basis states of the spins, $|g\rangle_i$, are labelled by elements $g \in Z_6$.
- Generalized Pauli operators,

$$\sigma_i^z = \sum_g e^{i\pi g/3} |g\rangle_i \langle g|, \quad \sigma_i^x = \sum_g |g+1\rangle_i \langle g|.$$

The anyonic model

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- Anyons are defined by plaquette and vertex projectors,

$$P_{e_g}(v), \quad P_{m_h}(p).$$

[A. Kitaev, Annals Phys. 303 (2003), 2-30]

- Five non-trivial charge anyons can exist on vertices,

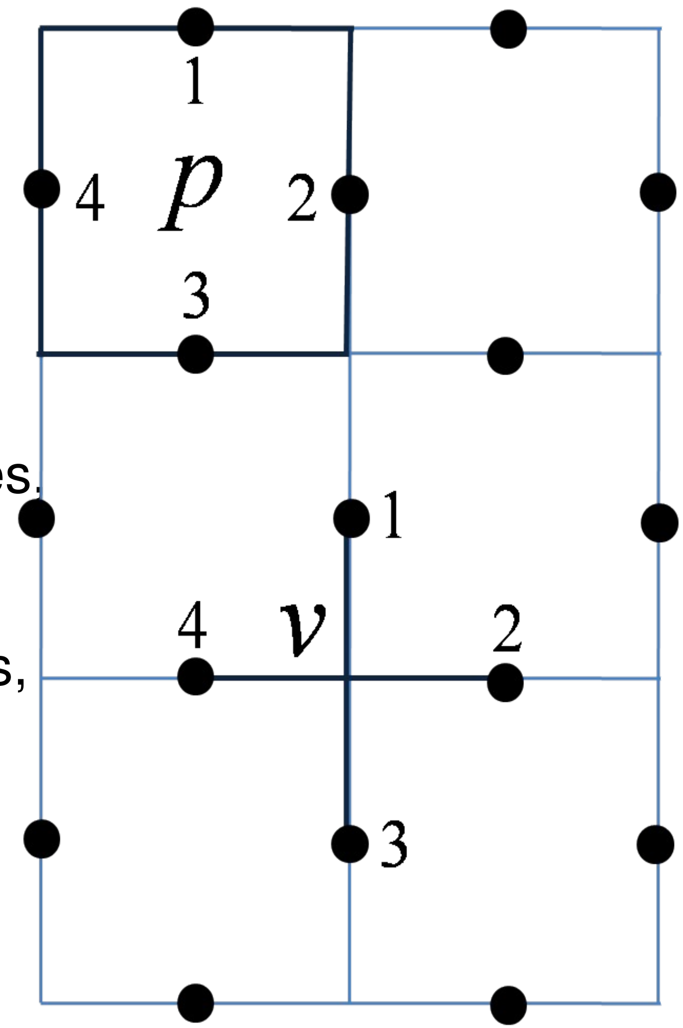
$$e_1, \ldots, e_5$$

- Five non-trivial flux anyons can exist on plaquettes,

$$m_1, \ldots, m_5$$

- The Hamiltonian assigns equal energy to all anyons,

$$H = -\sum_v P_1(v) - \sum_p P_1(p)$$



The anyonic model

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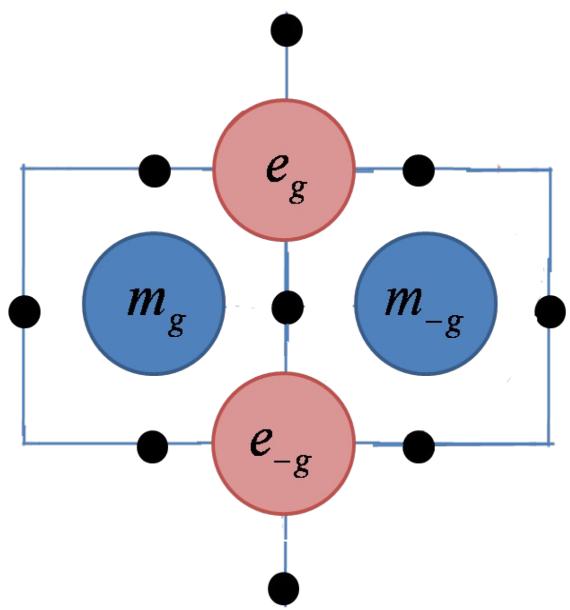
- The application of $(\sigma_i^z)^g$ on a spin i creates an e_g, e_{-g} pair on neighbouring vertices.

- The application of $(\sigma_i^x)^g$ creates a m_g, m_{-g} pair on neighbouring plaquettes.



The quasiparticles

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- Use these anyons to construct new quasiparticles ϕ, $\overline{\phi}$ and λ .
- These are defined by the projectors,

$$P_{\phi}(v)=P_{e_1}(v)+P_{e_4}(v),$$
$$P_{\overline{\phi}}(v)=P_{e_2}(v)+P_{e_5}(v),$$
$$P_{\lambda}(v)=P_{e_3}(v).$$

- The fusion rules are,

$$\phi\times\overline{\phi}=1+\lambda \quad \phi\times\lambda=\phi, \quad \overline{\phi}\times\lambda=\overline{\phi}, \quad \lambda\times\lambda=1.$$

The quasiparticles

- A λ pair can be created on vertices either side of a spin, i, by

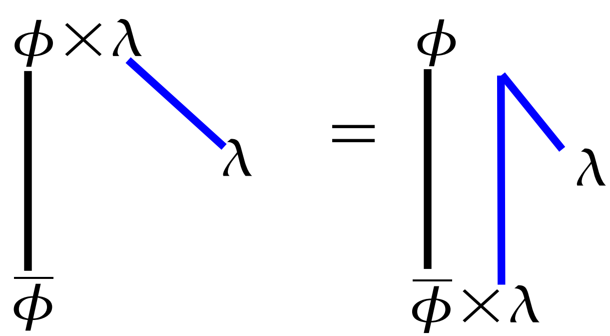
$$W_i^{\lambda} = (\sigma_i^z)^3 .$$

- A $\phi , \overline{\phi}$ pair can be created with,

$$W_i^{\phi} = \sigma_i^z + (\sigma_i^z)^4 = \sigma_i^z [1 + (\sigma_i^z)^3].$$

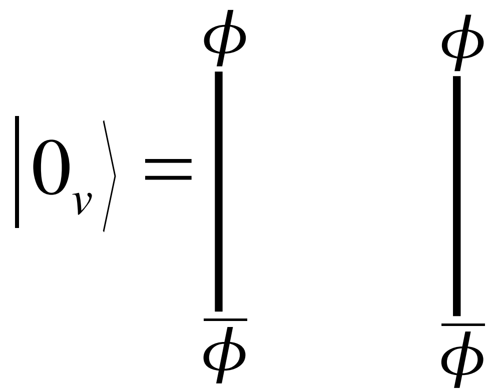
- The non-local nature of the fusion is provided by,

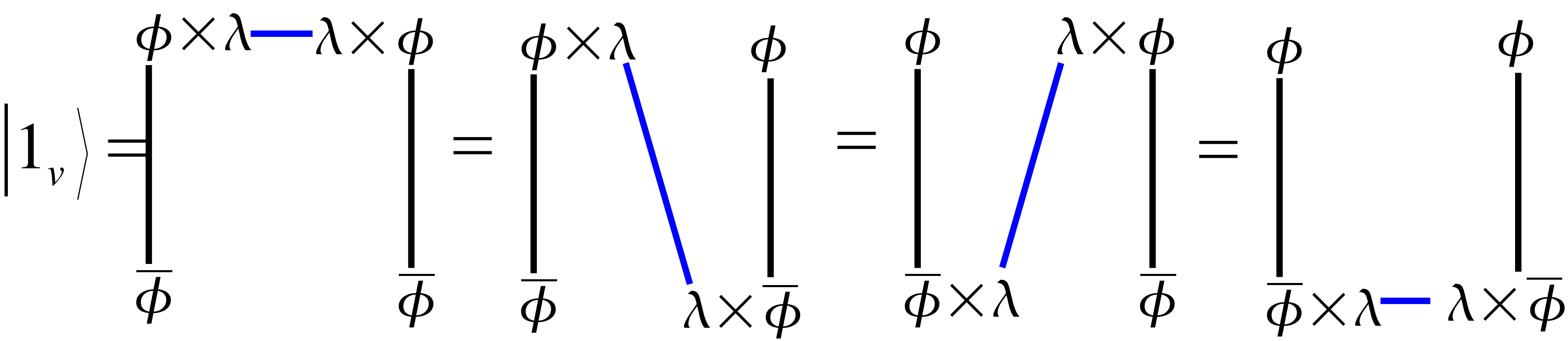
$$W_i^{\phi} W_i^{\lambda} = \sigma_i^z [1 + (\sigma_i^z)^3](\sigma_i^z)^3 = W_i^{\phi} .$$



Qubit encoding

- We can use the ϕ quasiparticles to store, protect and manipulate quantum information.





Qubit encoding

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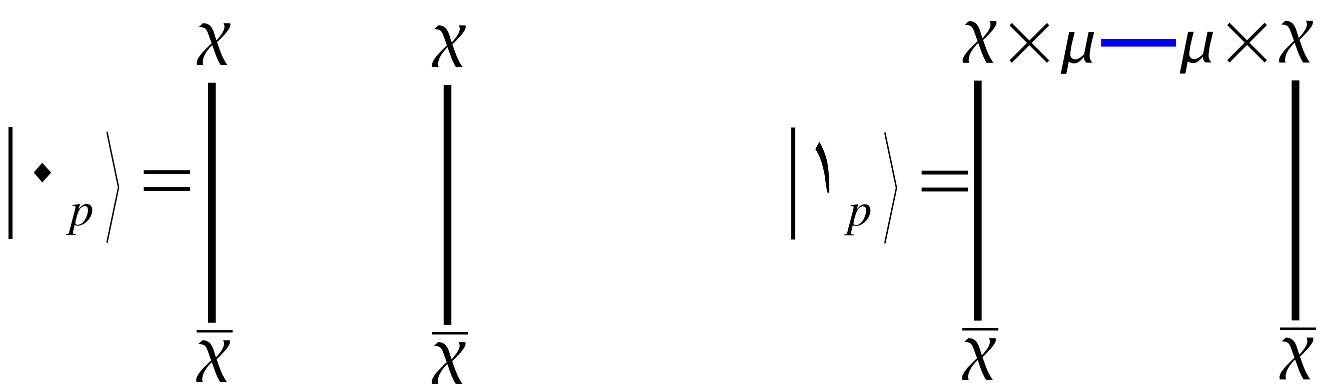
- Similar particles, χ and μ can be defined on plaquettes,

$$P_\chi(p)=P_{m_{۱}}(p)+P_{m_{۴}}(p),\quad P_{\bar{\chi}}(p)=P_{m_{۲}}(p)+P_{m_{۵}}(p),$$

$$P_\mu(p)=P_{m_{۳}}(p).$$

$$W_i^\mu=(\sigma_i^x)^{۳},\quad W_i^\chi=\sigma_i^x+(\sigma_i^x)^{۴}=\sigma_i^x[۱+(\sigma_i^x)^{۳}].$$



Logical Pauli rotations

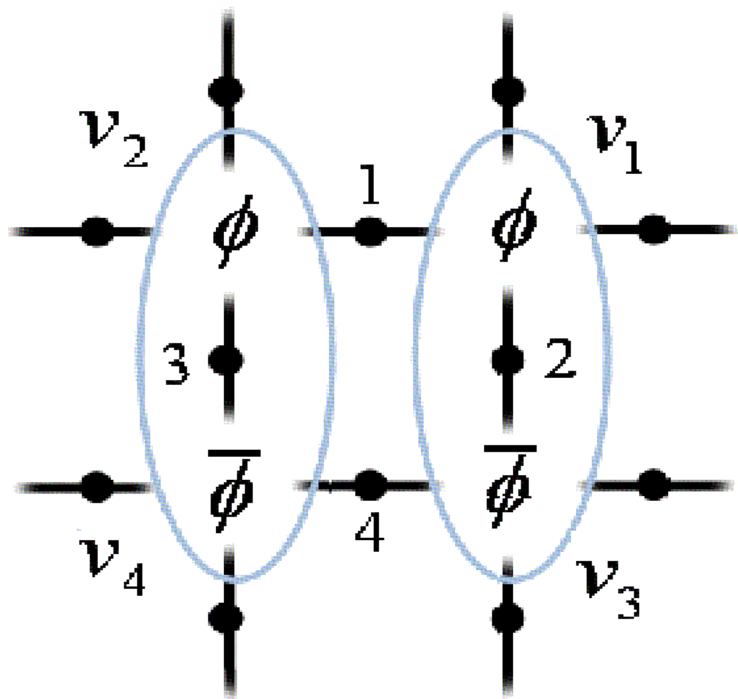
- The logical Z operators correspond to vertex operators,

$$Z_v = A^{۳}(v_{۱})A^{۳}(v_{۳}) \text{ or } A^{۳}(v_{۲})A^{۳}(v_{۴}), \quad A^{۳}(v) = ۱ - ۲P_\lambda(v)$$

- The logical X operators correspond to spin operators,

$$X_v = (\sigma^z_{۱})^{۳} \text{ or } (\sigma^z_{۴})^{۳}.$$

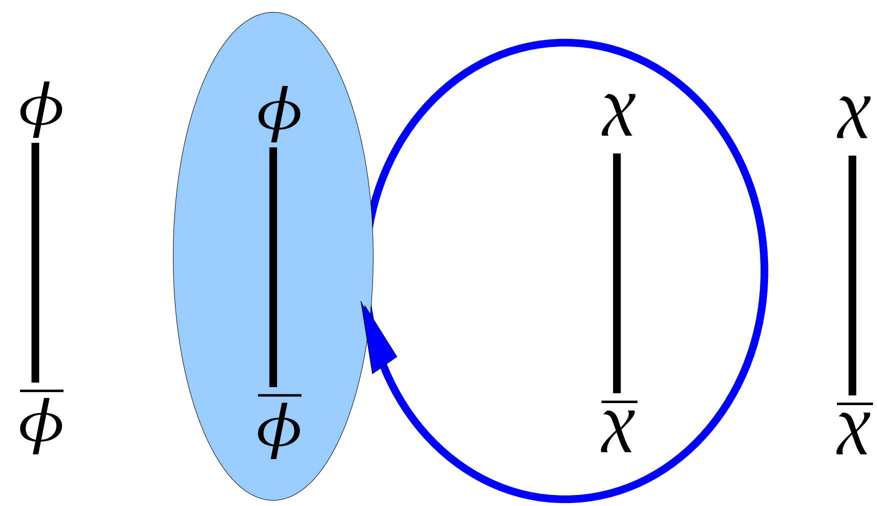
- Corresponding logical Paulis exist for qubits stored on plaquettes.



Controlled-phase gate

- Braiding a λ around a μ gives a phase of -1.

- This may be used to realized a controlled-phase between logical qubits.



$$\left|\text{٠}_v\,\text{٠}_p\right\rangle \rightarrow \left|\text{٠}_v\,\text{٠}_p\right\rangle,$$

$$\left|\text{٠}_v\,\text{١}_p\right\rangle \rightarrow \left|\text{٠}_v\,\text{١}_p\right\rangle,$$

$$\left|\text{١}_v\,\text{٠}_p\right\rangle \rightarrow \left|\text{١}_v\,\text{٠}_p\right\rangle,$$

$$\left|\text{١}_v\,\text{١}_p\right\rangle \rightarrow -\left|\text{١}_v\,\text{١}_p\right\rangle.$$

Spin measurements

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- Since,

$$X_v = (\sigma^z_{۱})^{۳} \text{ or } (\sigma^z_{۴})^{۳},$$

when ϕ and $\overline{\phi}$ are neighbours, single spin measurements on the lattice can realize logical X measurements.

- Single spin measurements may be used to apply the projector,

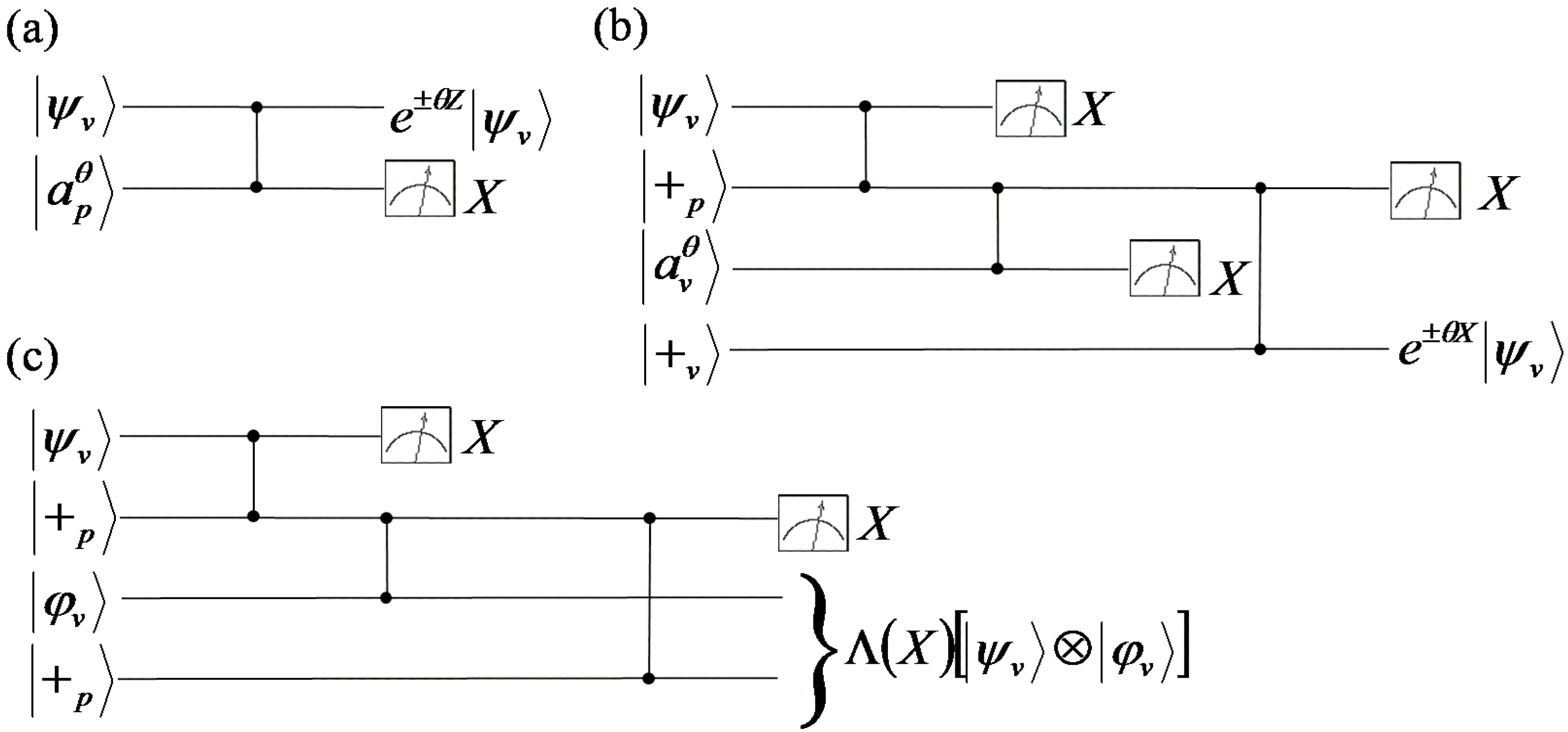
$$\Pi_\theta = \frac{۱}{۲}\left(۱ + \cos\theta\,(\sigma^x_{۱})^{۳} + i\sin\theta\,(\sigma^z_{۱})^{۳}(\sigma^x_{۱})^{۳}\right).$$

- This allows us to create the state,

$$\left|a^\theta_v\right\rangle = \frac{۱}{\sqrt{۲}}\left(\cos\theta\left|٠_v\right\rangle - i\sin\theta\left|۱_v\right\rangle\right).$$

Universal quantum computation

- Universal quantum computation:



- Another gate set can be used without braids that employs entangling operations on the lattice spins.

Fault-tolerance

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- Our encoding depends upon degeneracies within the fusion space.

- These degeneracies may be lifted by perturbations in the Hamiltonian, which causes errors.

- Most perturbations are suppressed by the gap. The dangerous perturbations are those that form strings around or touching the quasiparticles.

Fault-tolerance

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- The smallest dangerous perturbation is any $\sigma_i^{x/z}$ acting on a spin neighbouring a quasiparticle.

- This can cause the quasiparticles to split, and so disturb the encoding space. This does not cost energy, and so is not protected by the gap.

- We can invert the coupling of the plaquettes and vertices on which the quasiparticles reside. This assigns energy to an absence of anyons.

- This creates an additional gap to suppress such perturbations.

- Perturbations that stretch between the quasiparticles may lift the degeneracy. These are suppressed by separation.

Fault-tolerance

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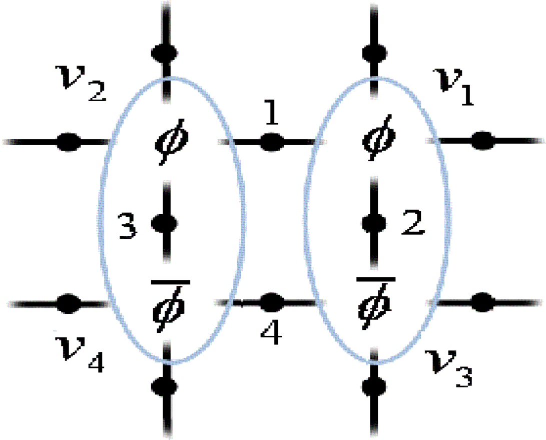
- Perturbations may lift the degeneracy of the stored information if they stretch between the pairs.

- An example would be a perturbation

$$\delta A^3(v_1).$$

- To protect against these, note that the creation operator for a $\phi, \bar{\phi}$ pair contains a projection,

$$W_i^\phi = \sigma_i^z [1 + (\sigma_i^z)^3].$$



- An additional single spin term, $B\,(\sigma_i^x)^3$, will therefore not affect the encoding space.

- This term suppresses the perturbations by $(\delta/B)^2$.

Conclusions

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- Abelian models are simpler to implement experimentally.
- We can achieve the same encoding as in non-Abelian models.
- Quantum information is manipulated by braiding and single spin measurements.
- Fault-tolerance is enhanced by external magnetic fields.
- The method we use is general, and may be applied to many abelian models.