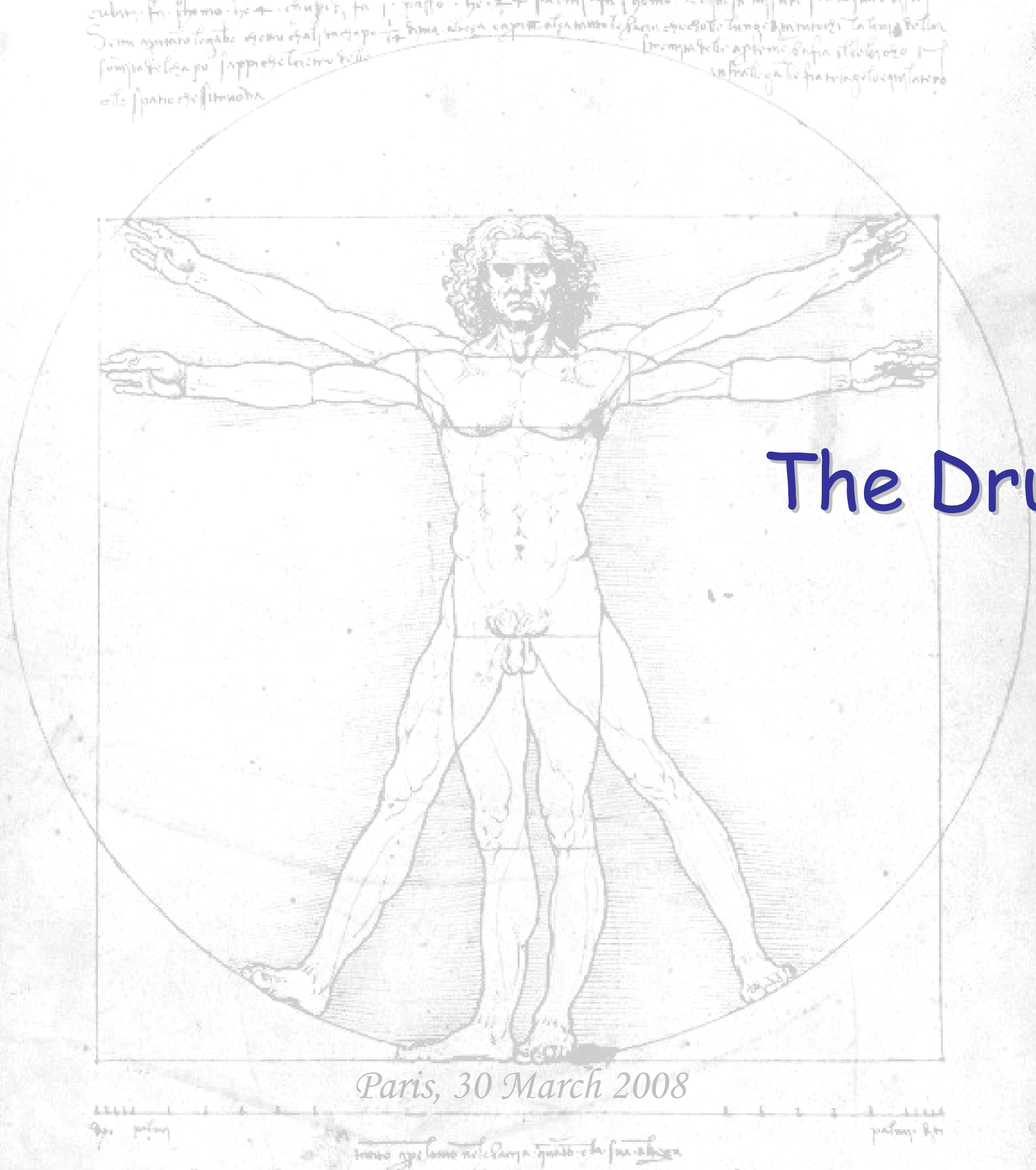


The Drunken Slalom

Gavin Brennen

Viv Kendon

JKP

Paris, 30 March 2008

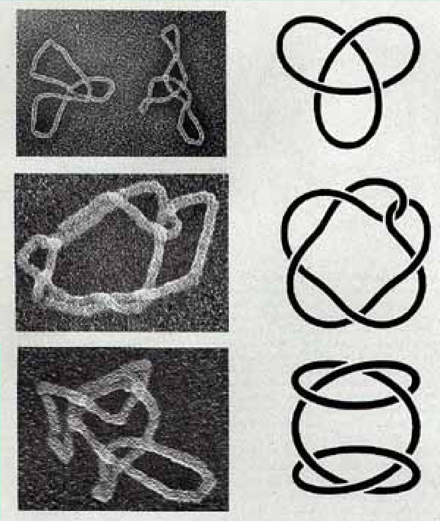
UNIVERSITY OF LEEDS

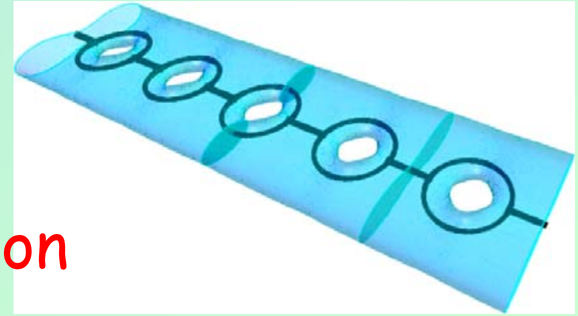
THE ROYAL SOCIETY

Motivation

Random evolutions of topological structures arise in:

- Statistical physics:
  - **Entropy** of ensembles of extended object
- Plasma physics:
  - Vortex **dynamics**
- Polymer physics:
  - **Diffusion** of polymer chains
- Molecular biology:
  - DNA **folding**
- Cosmic strings
- **Kinematic** Golden Chain

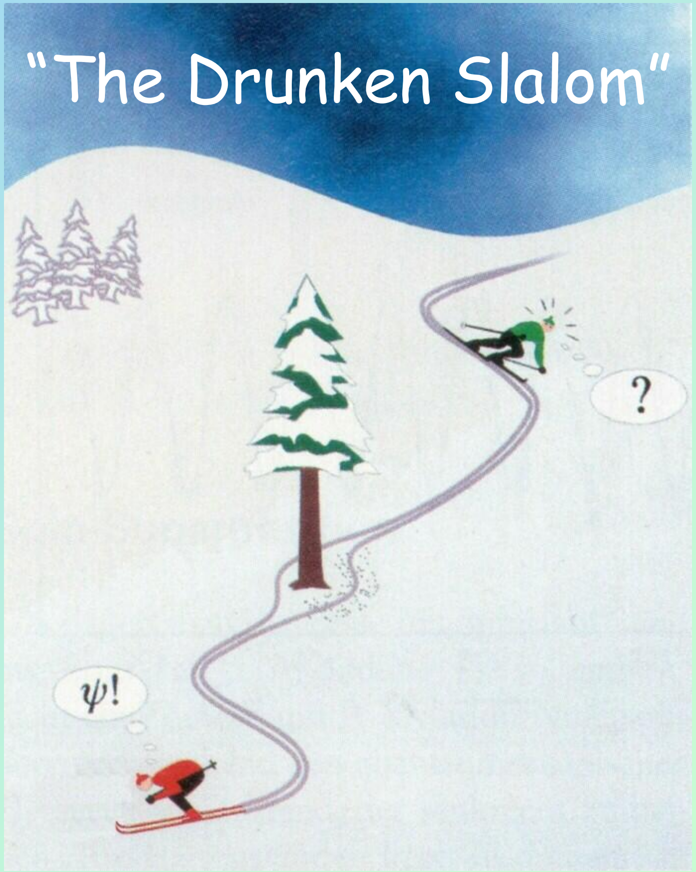




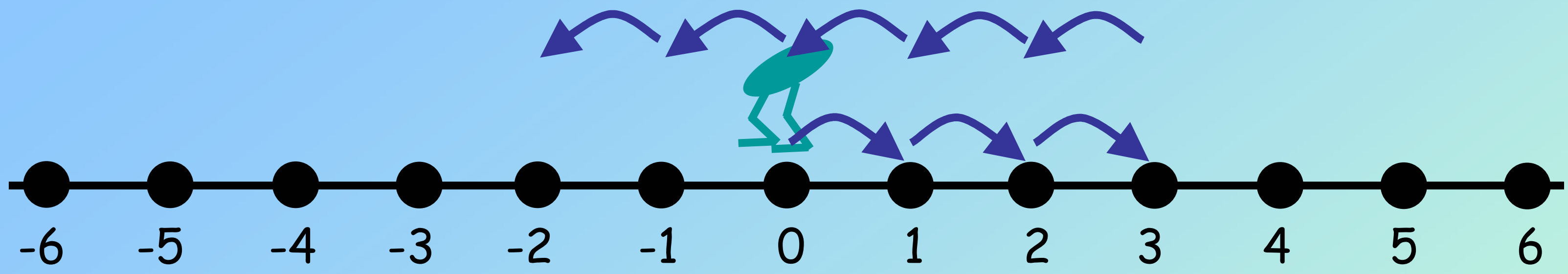
Quantum simulation

Overview

- Classical random walks
- Quantum walks
- **Decoherence** and **multiple coins** in quantum walks
- Anyon models
- Quantum walks with **Abelian anyons**
- Quantum walks with **non-Abelian anyons**
- Conclusions and open questions



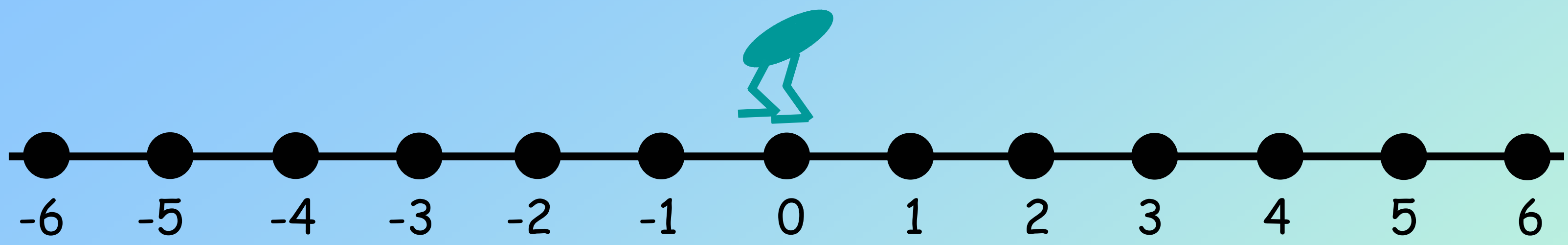
Classical random walk on a line



- Recipe:
  1) Start at the origin
  2) Toss a fair coin: HEADS or TAILS
  3) Move one unit: Right for HEADS and Left for TAILS
  4) Repeat steps (2) and (3) T times
  5) Measure position of the walker
  6) Repeat steps (1) to (5) many times

The resulting probability distribution P(x,T) is a **binomial** with standard deviation: $\langle x^2 \rangle = T$

Quantum walk on a line



- Recipe:

  1) Start at the origin

  2) Toss a quantum coin (qubit): $H|0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$

  3) Move left **and** right: $S|x,0\rangle = |x-1,0\rangle, \quad S|x,1\rangle = |x+1,1\rangle$

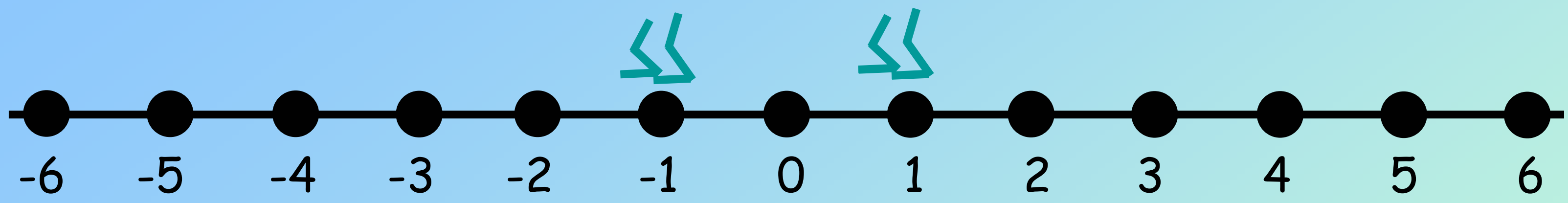
  4) Repeat steps (2) and (3) T times

  5) Measure position of walker

  6) Repeat steps (1) to (5) many times

  Probability distribution P(x,T):...

Quantum walk on a line



- Recipe:
  1) Start at the origin
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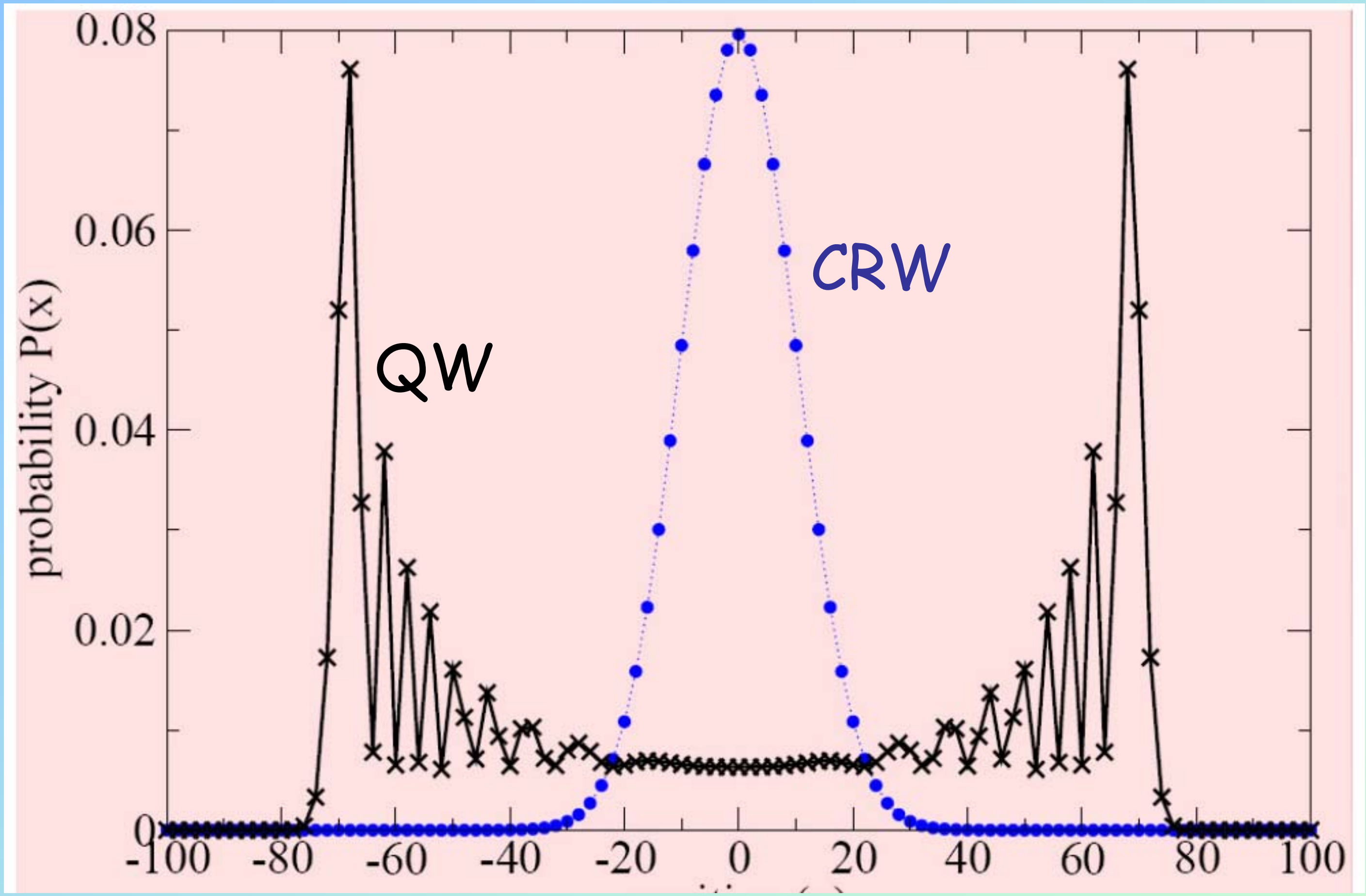
  4) Repeat steps (2) and (3) T times
  5) **Measure** position of walker
  6) Repeat steps (1) to (5) many times

  Probability distribution P(x,T):...

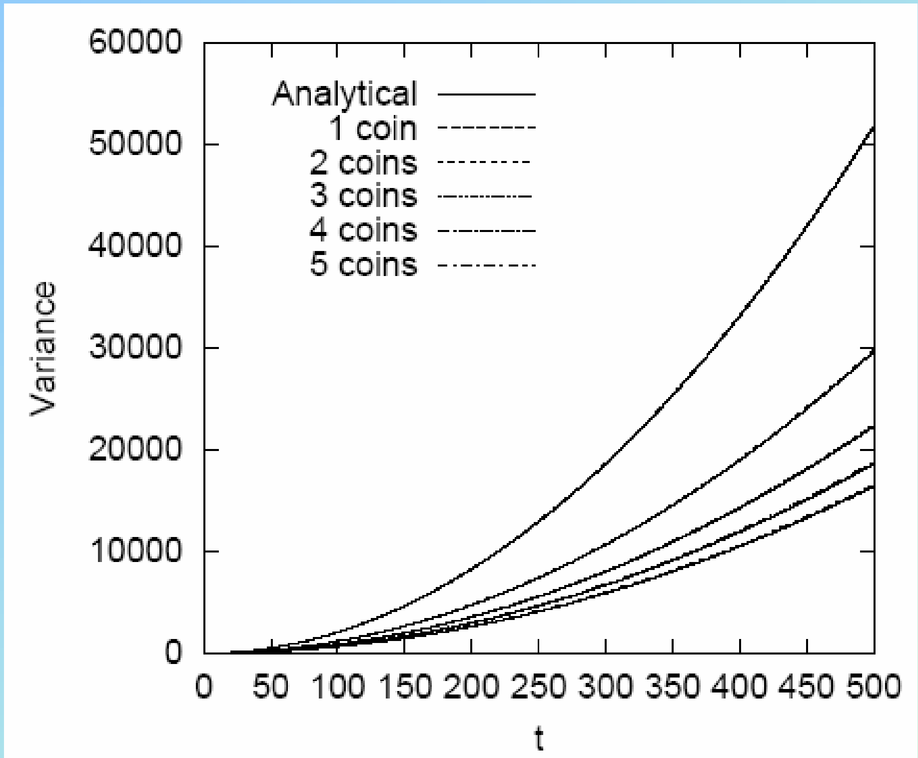
CRW vs QW



Quantum spread $\langle x^2 \rangle - \langle x \rangle^2 \sim T^2$, classical spread~T

[Nayak, Vishwanath, quant-ph/0010117;
Ambainis, Bach, Nayak, Vishwanath, Watrous, *STOC'01* pp60-69 (2001)]

QW with more coins



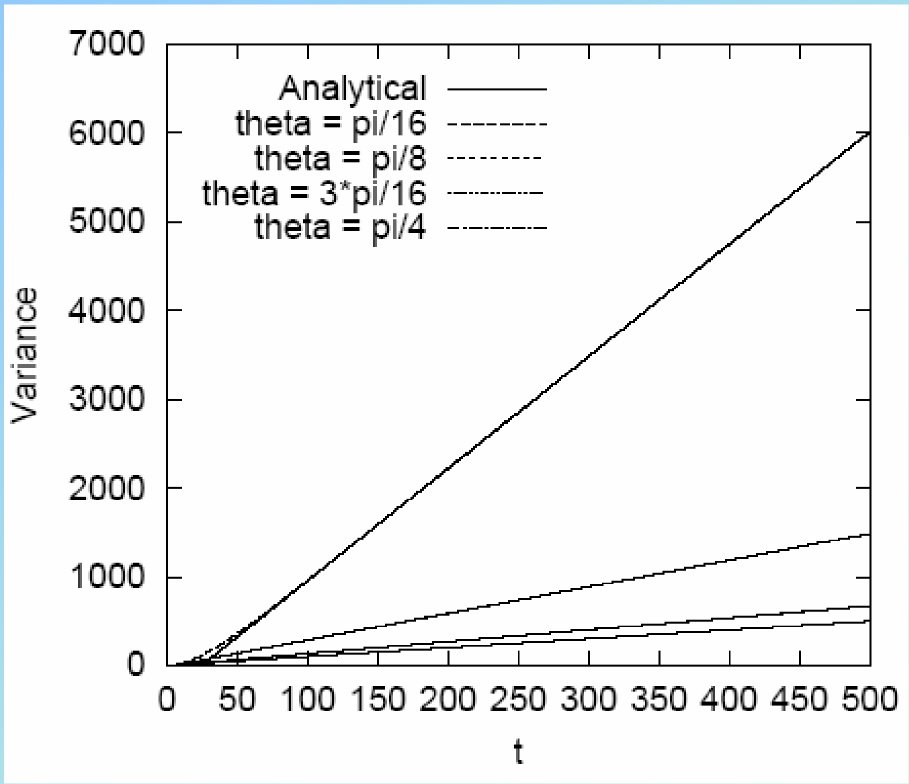
Variance =aT^2

More (or larger) coins dilute the effect of interference (smaller a)

A new coin at each step would destroy speed up

[T. A. Brun, *et al.*, PRL 91, 130602 (2003)]

QW with decoherence



Variance =aT

Quantum speed up is completely lost due to decoherence

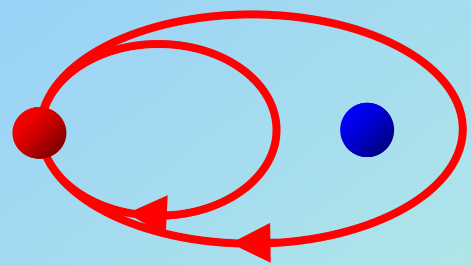
Environment is like an extra qubit at each site

[T. A. Brun, *et al.*, PRL 91, 130602 (2003)]

Topological Quantum Systems: Anyons

- Two dimensional systems
- Dynamically trivial (H=0), interested only in statistics

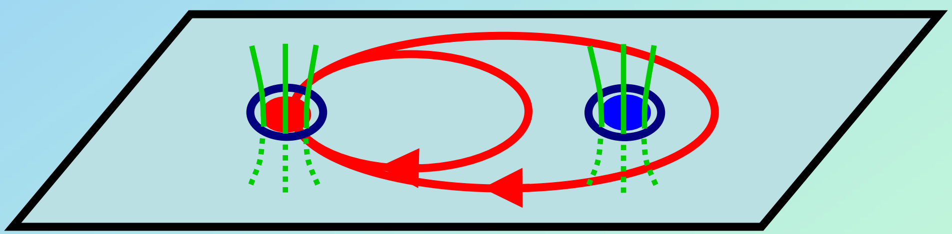
3D



Bosons $\left|\Psi\right\rangle \rightarrow \left|\Psi\right\rangle$

Fermions $\left|\Psi\right\rangle \rightarrow e^{i2\pi}\left|\Psi\right\rangle$

2D



$$\left|\Psi\right\rangle \rightarrow e^{i2\varphi}\left|\Psi\right\rangle$$

$$\left|\Psi\right\rangle \rightarrow U\left|\Psi\right\rangle$$

Anyons

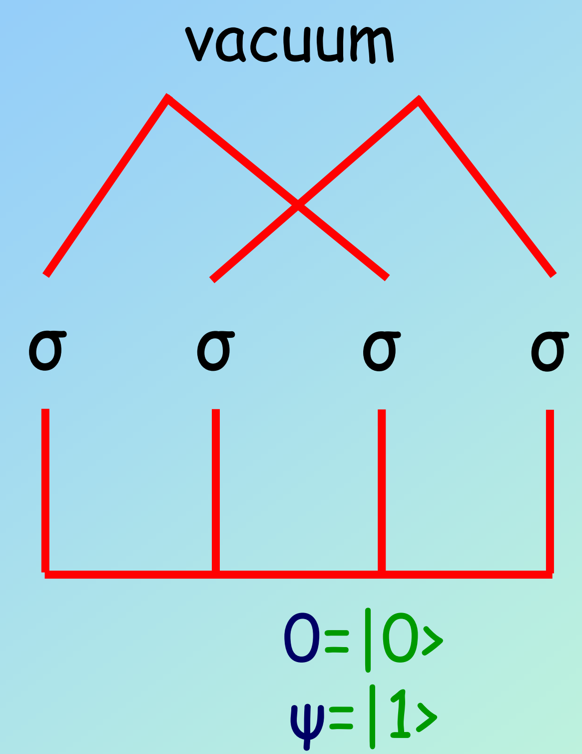
Composite particles of **fluxes** and **charges**

Phase is like the **Aharonov-Bohm** effect

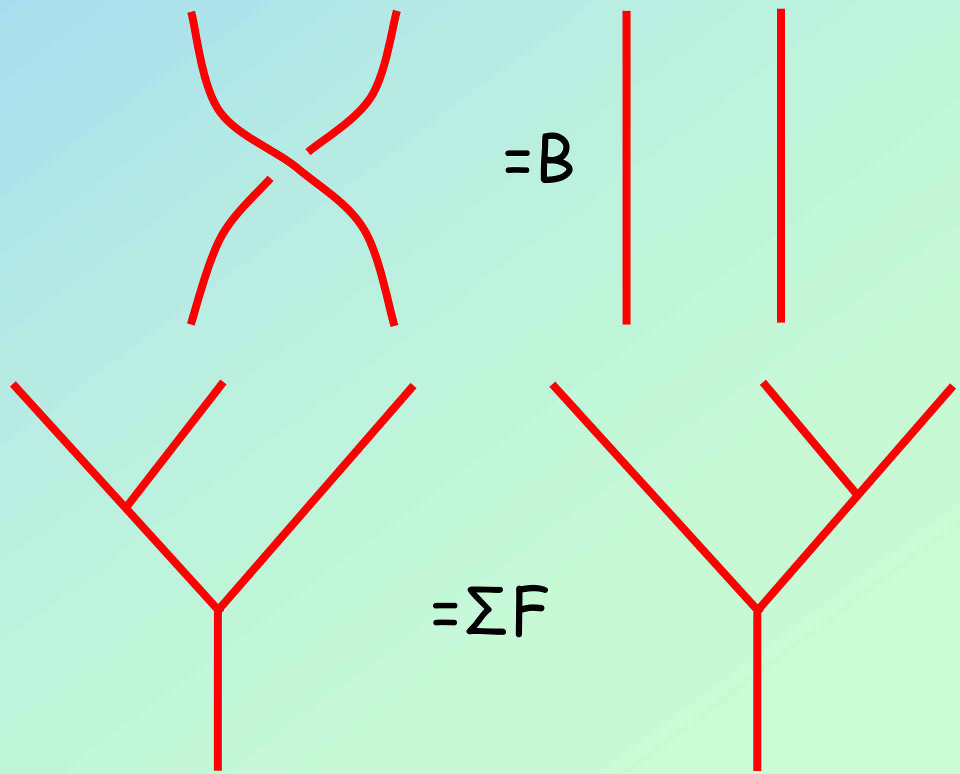
Ising Anyons

Fusion rules: $\sigma \times \sigma = 1 + \psi, \psi \times \sigma = \sigma, \psi \times \psi = 1$

Qubit encoding:

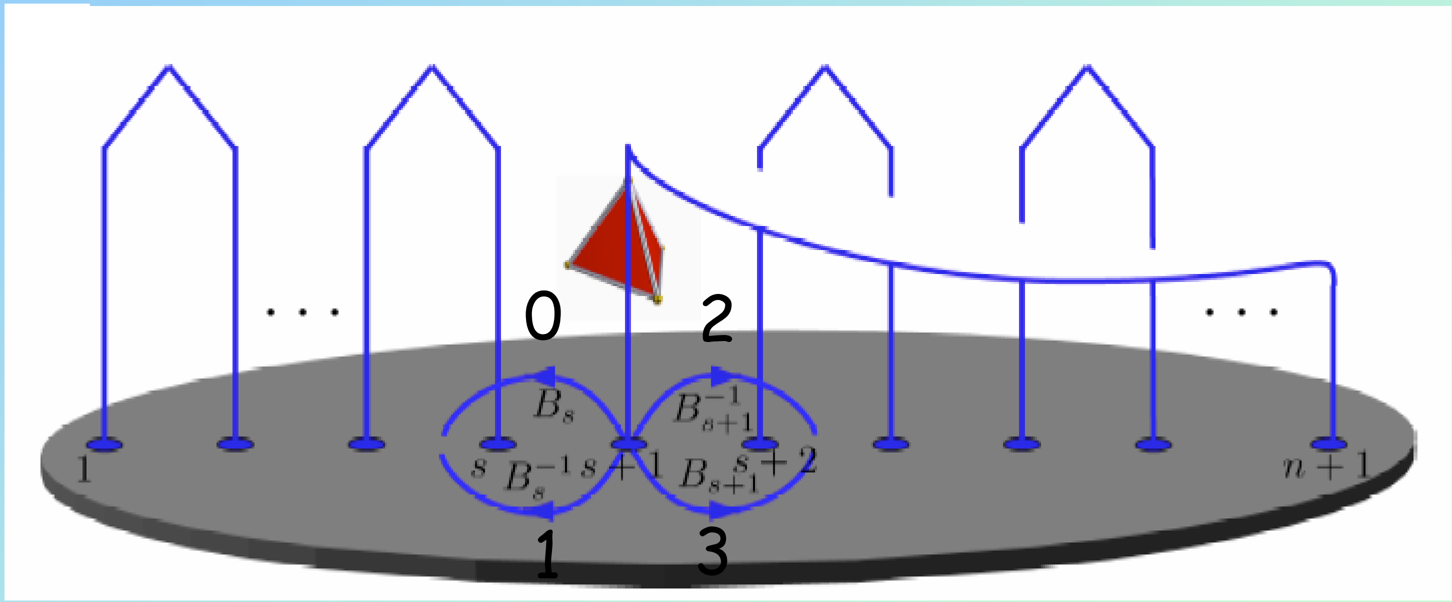


Topological operations:



Anyonic quantum walk

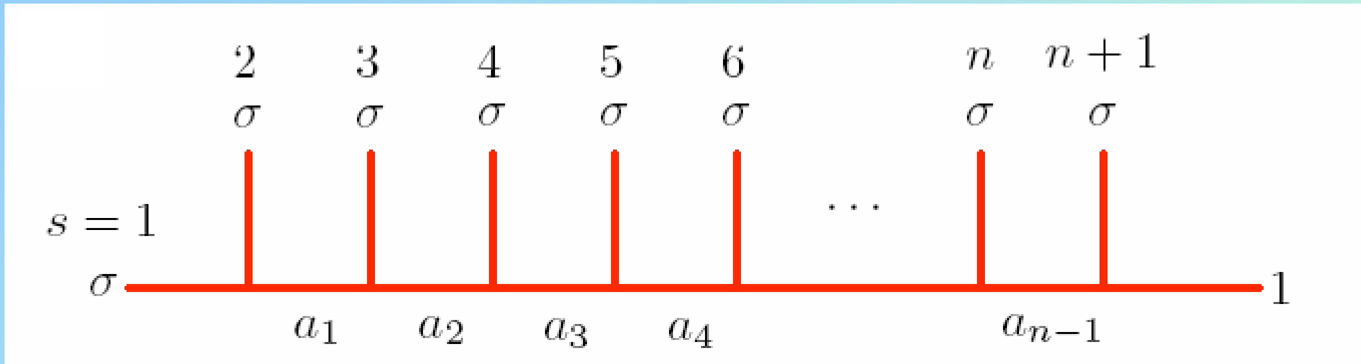
Quantum walk of an **anyon with a die** by **braiding it with other anyons** of the same type fixed on a line



Anyonic quantum walk

Basis states: $\left|\Psi(a_1, a_2, \ldots, a_{n-1})\right\rangle$

Initial state: $\left|\Psi_0\right\rangle = B_n B_{n-1} \ldots B_{(n+3)/2} \left|\Psi(1,\sigma,1,\sigma,\ldots,1,\sigma)\right\rangle$



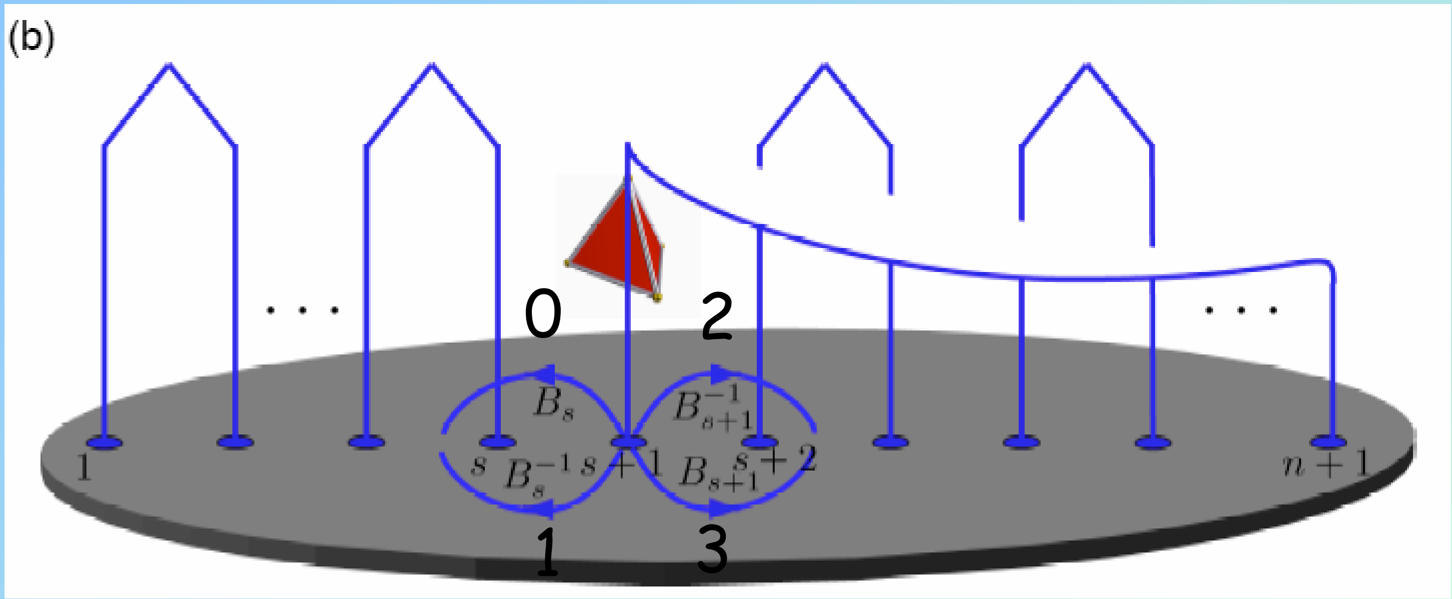
Hilbert space:

$$H(n) = H_{anyons}(n) \otimes H_{die}(n)$$

$$H_{anyons}(n) = H_{local}(n) \otimes H_{fusion}(n), \;\; \dim(H_{fusion}(n)) = \sum_{a_1,\ldots,a_n} N_{\sigma\sigma}^{a_1} \ldots N_{a_{n-1}\sigma}^{1}$$

$$H_{die}(n) = H_{space}(n) \otimes H_{spin}, \quad \dim(H_{spin}) = 4$$

Anyonic quantum walk

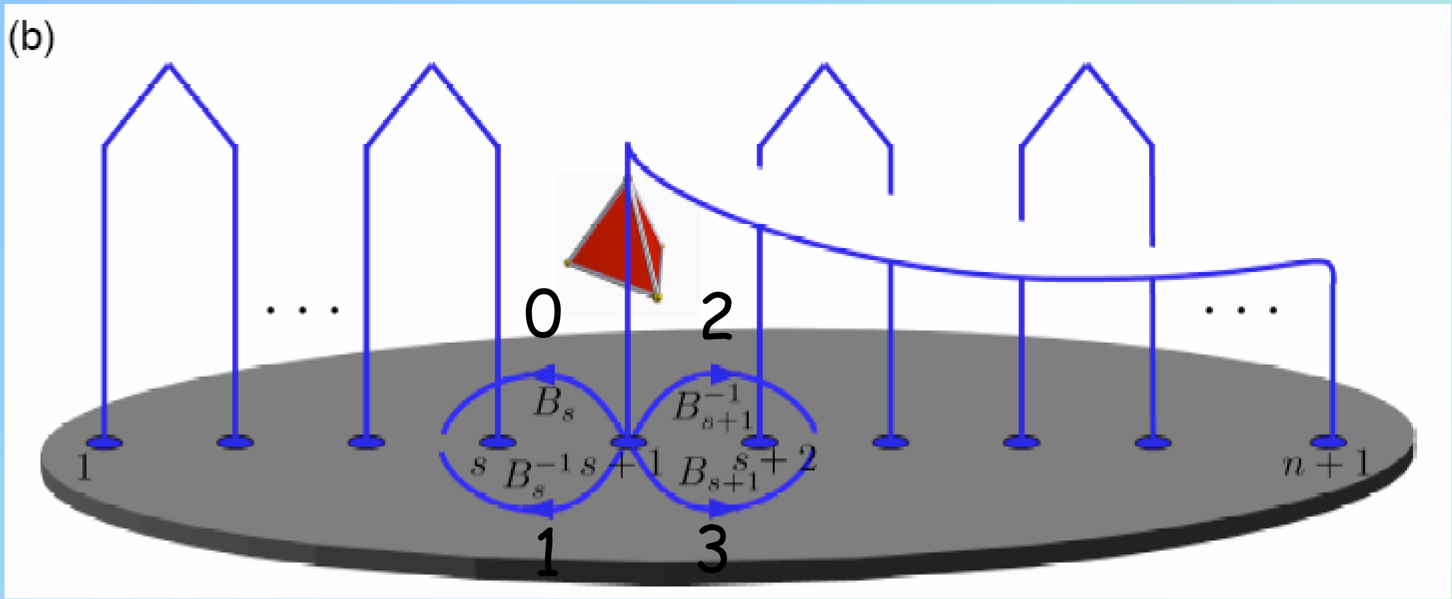


1) Start at state: $\left|\Phi(0)\right\rangle = \left|\Psi_0\right\rangle_{fusion} \left|\frac{n+1}{2}\right\rangle_{space} \left|\psi\right\rangle_{spin}$

2) Die toss: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

3) Move left-up, left-down, right-up, right-down by braidings

Abelian anyonic quantum walk

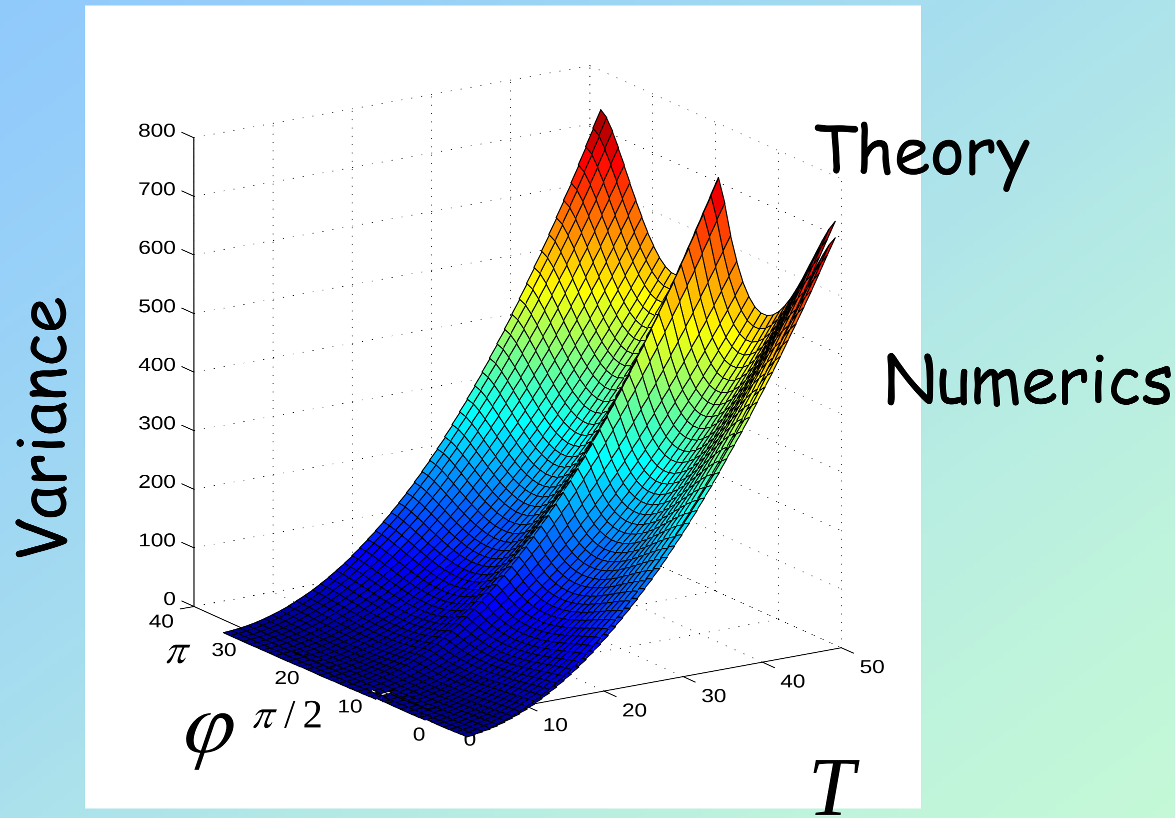


Phases from braiding: $e^{i\varphi\sigma^z\otimes\sigma^z}$

Choose initial state with equal populations of die in order to have left-right symmetric evolution

Can find **analytical solution** of this evolution

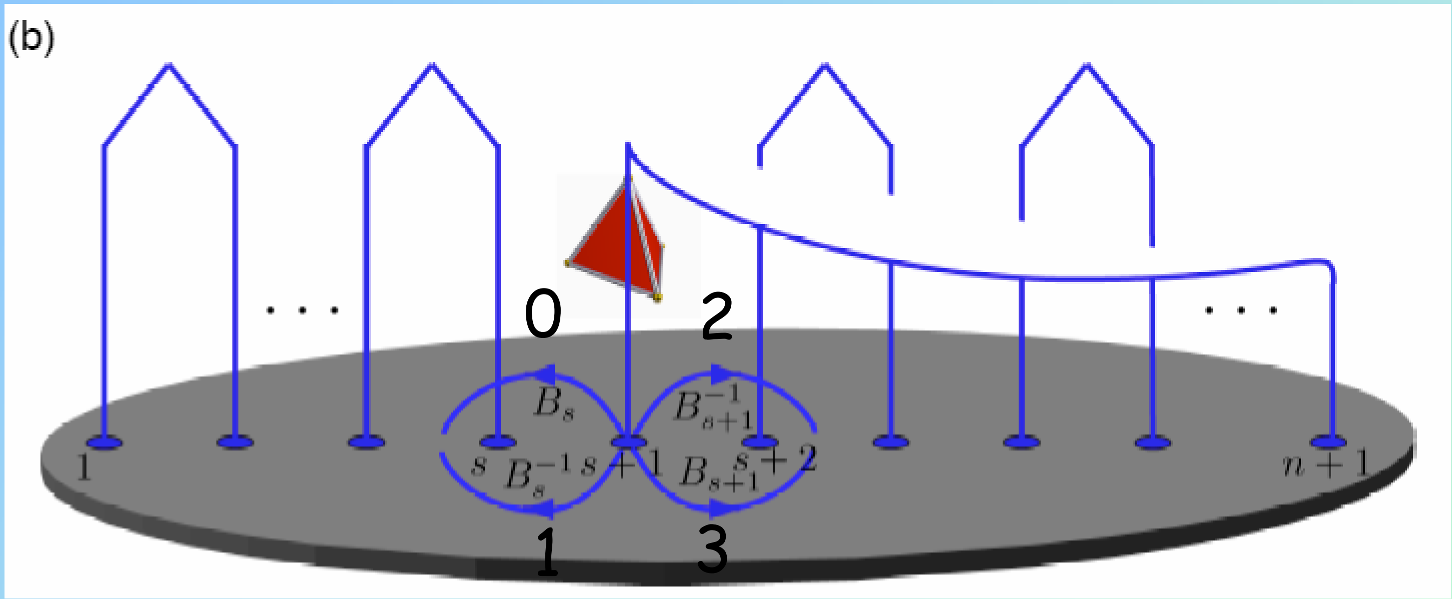
Abelian anyonic quantum walk



Quantum speedup. For $\varphi \neq 0, \pi/2, \pi, \ldots$ the coin is four dimensional. Otherwise the evolution is the tensor product of two QW each with two dimensional coins

$$e^{i\varphi\sigma^z \otimes \sigma^z} = 1\cos\varphi + i\sin\varphi\,\sigma^z \otimes \sigma^z$$

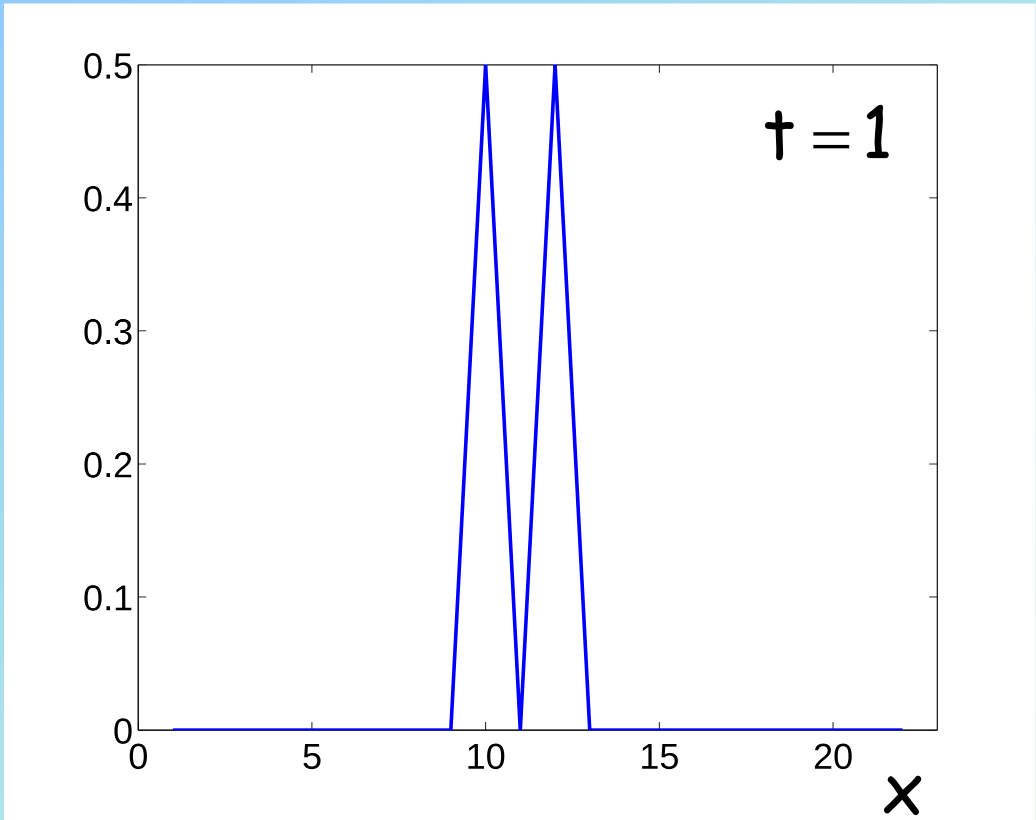
Ising anyonic quantum walk



$$R_{\sigma\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad F^{\sigma}_{\sigma\sigma\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

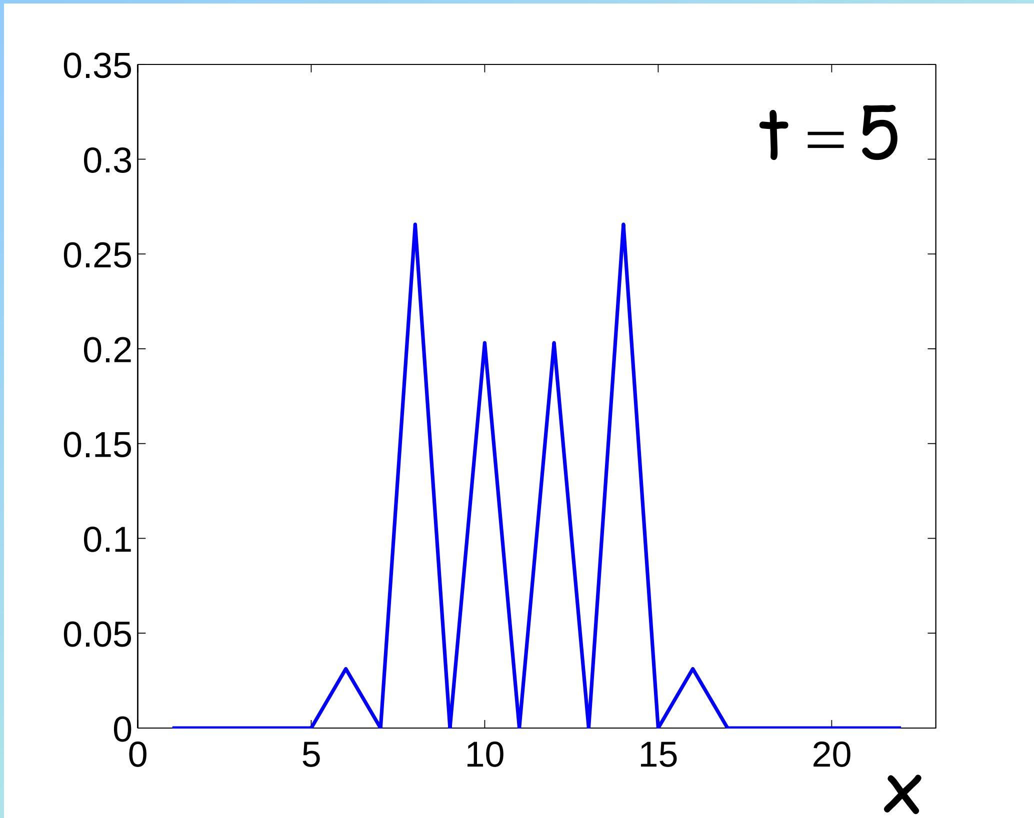
From these calculate the general braiding element B_s

Ising anyonic quantum walk



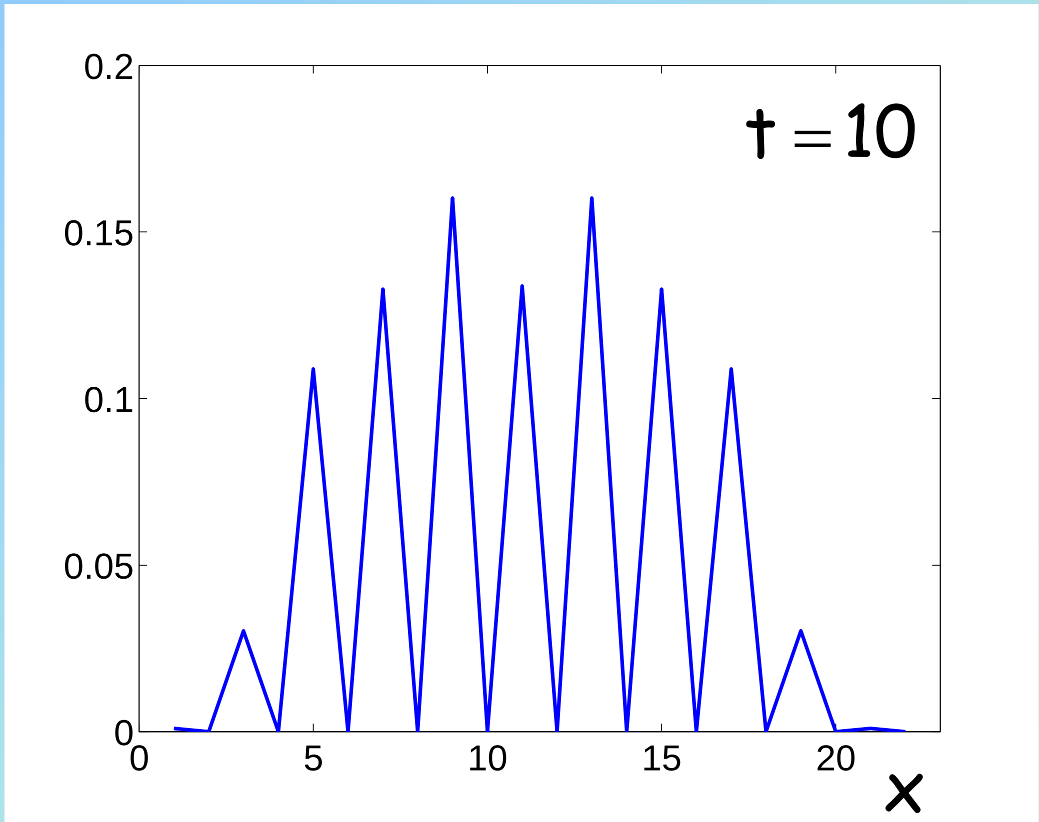
Populations $P(x,t) = |\Psi(x,t)|^2$

Ising anyonic quantum walk



Populations $P(x,t) = |\Psi(x,t)|^2$

Ising anyonic quantum walk

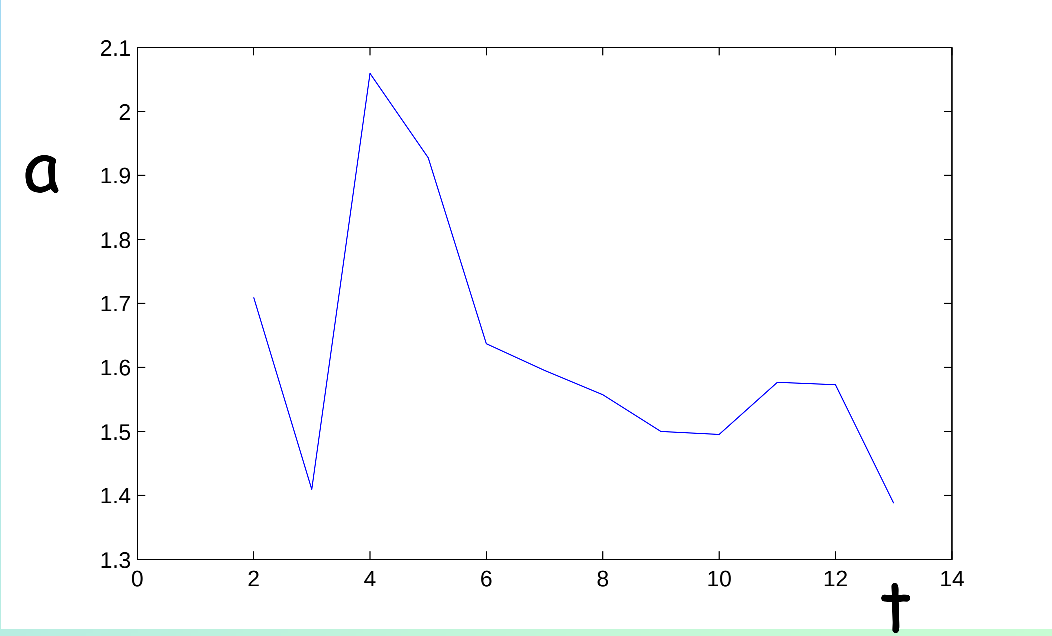


Populations $P(x,t) = |\Psi(x,t)|^2$

Ising anyonic quantum walk

For a variance of the form $P \propto T^{a}$ we can evaluate

$$a \approx \frac{\log P(t+1) - \log P(t)}{\log(t+1) - \log t}$$

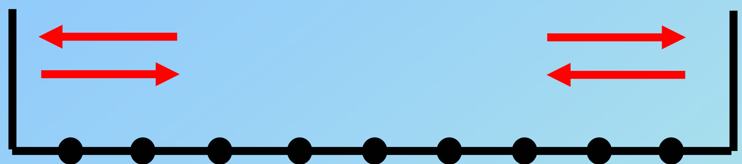


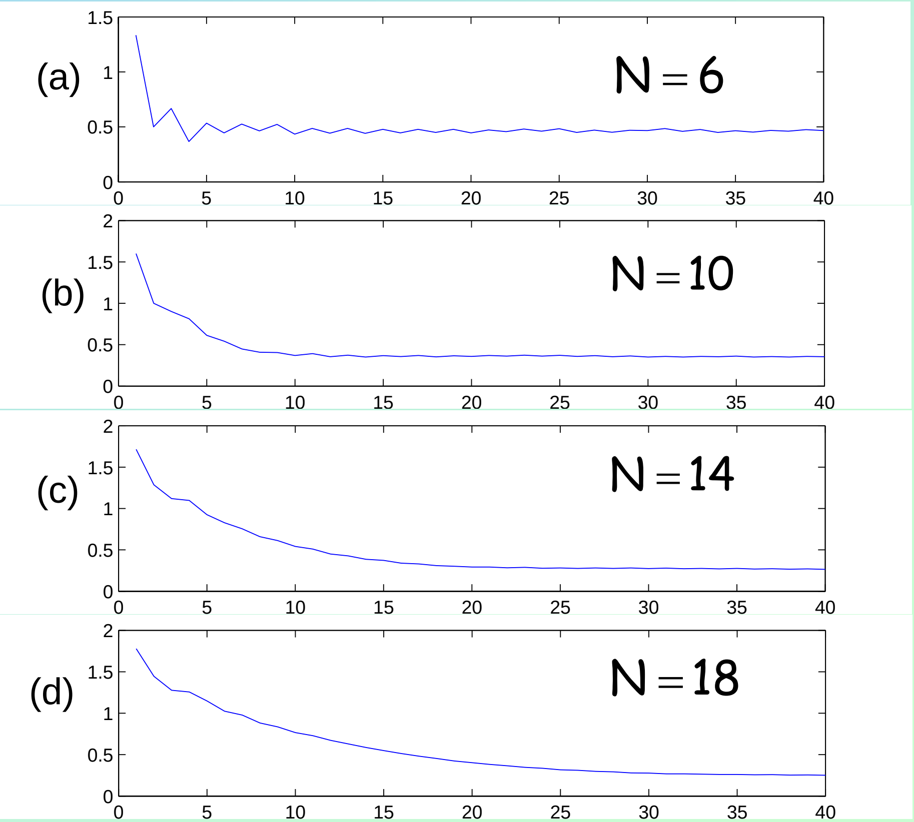
No quadratic speedup
Seems to grow
faster than classical…

Analytics is very hard, as **coin** is now **position dependent**
(this quantum walk problem is **not** solved)

Bounded Ising quantum walk

Consider an Ising QW with **reflective boundaries**
When is the system going into **equilibrium**?





The larger the chain the more steps needed to reach equilibrium

It does not reach the uniform distribution

Conclusions

We have studied an anyonic walker braiding with static anyons positioned on a line

Abelian anyons preserve the quadratic speedup
Equivalent to increasing finitely the dimension of the coin

Non-Abelian anyons seem to spread as classical QW, due to the exponential increase in the coin dimension
This dramatically reduces the effect of interference
No quadratic speedup (**Fibonacci** is similar)

Analytics on non-Abelian anyons very hard (but we try...:-)
Possible: **quantum simulations** with photons, atoms...

Thank you for your attention!