Bell Tests with Anyons

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Foreword

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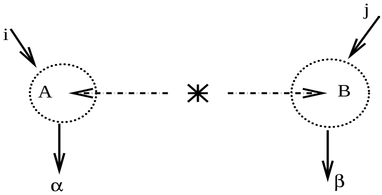
N.B. Project started two "TQC symposia ago", Leeds, April 2008.

Plan

- Local hidden variables and Bell inequalities
- Some features of $(2+1)$-dimensional physics.
- Bell tests with Ising anyons
- Conclusions and open problems

Local hidden variables



Measurement type: M_i^A (left side), M_j^B (right side).

Measurement outcome: $m_{i,\alpha}^a$ (left side), $m_{j,\beta}^B$ (right side).

$$\Pi(m_{i,\alpha}^a, m_{j,\beta}^B | M_i^A, M_j^B) = \int d\lambda \; p(\lambda) \; \pi(m_{i,\alpha}^a, m_{j,\beta}^B | M_i^A, M_j^B, \lambda) \; ?$$

In words: Can we always explain correlations in terms of a local preparation?

N.B. The question is not specific to quantum mechanics.

Bell inequalities

The assumption

$$\Pi(m^{a}_{i,\alpha}, m^{B}_{j,\beta}|M^{A}_{i}, M^{B}_{j}) = \int d\lambda\, p(\lambda)\, \pi(m^{a}_{i,\alpha}, m^{B}_{j,\beta}|M^{A}_{i}, M^{B}_{j}, \lambda)$$

can be *tested*. For example, if A_1, A_2, B_1, B_2 are binary outcome observables, then

$$\mathcal{B} \equiv \langle A_1 B_1 + A_2 B_2 + A_2 B_1 - A_1 B_2 \rangle \leq 2.$$

The most interesting feature of this relation is that it is *inconsistent* with quantum mechanics.

An experiment with two qubits (I)

$$\mathcal{B} \equiv \langle A_1 B_1 + A_2 B_2 + A_2 B_1 - A_1 B_2 \rangle \leq 2. \text{ (LHV)}$$

Consider $|\Psi_{AB}\rangle = |0,0\rangle + |0,1\rangle + |1,0\rangle - |1,1\rangle$ and

$$A_1 = \sigma^z; A_2 = \sigma^x,$$

$$B_1 = \frac{1}{\sqrt{2}}(\sigma^x + \sigma^z); B_2 = \frac{1}{\sqrt{2}}(\sigma^x - \sigma^z).$$

Quantum mechanics predicts $\mathcal{B} = 2\sqrt{2}$.

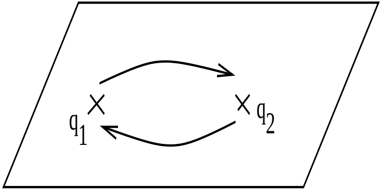
An experiment with two qubits (II)

qubit A	qubit B	consistent with LHV	consistent with QM
photon polarisation	photon polarisation	no	yes
photon polarisation	electron spin in atom	no	yes
⋮	⋮	⋮	⋮

It is interesting to perform Bell tests with different media.
Different systems give rise to different *loopholes*.

We here study the possiblity to perform Bell tests with anyons.

Physics in $2+1$ dimensions



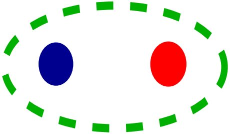
Let R: exchange operator (topological interaction).

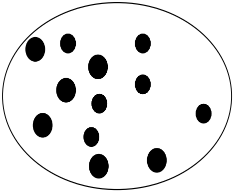
Unlike what happens with bosons and fermions, it can be the case that $R\,\Psi_{12} \neq \pm\Psi_{12}$. Two classes of anyons:

$$R = e^{i\theta_{12}}\mathbf{1} \qquad (AA)$$

$$R \neq e^{i\theta_{12}}\mathbf{1} \qquad (NA)$$

Local and non-local degrees of freedom





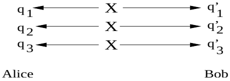
Fusion rules	Hilbert Space
$q_a \times q_b = N_{ab}^c q_c$	$H[q_1 \ldots q_n] = H_{q_1} \otimes \ldots \otimes H_{q_n} \otimes M_{q_1 \ldots q_n}$

We are interested in the fusion space $M_{q_1 \ldots q_n}$.

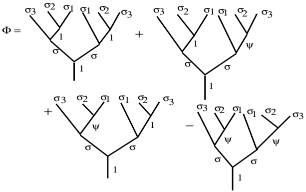
Bell tests with Ising anyons (I)

$$\psi \times \psi = 1, \quad \psi \times \sigma = \sigma, \quad \sigma \times \sigma = 1 + \psi. \tag{1}$$

Create three pairs from the vacuum. Each half goes to one side.



Using a sequence of F-moves, we find that



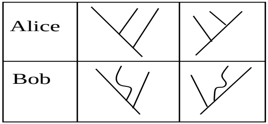
This is a "Bell" state:

$|0,0\rangle + |0,1\rangle + |1,0\rangle - |1,1\rangle$ (see before).

Bell tests with Ising anyons (II)

Pairs of non-commuting observables:

Alice	σ^x, σ^z
Bob	$\frac{1}{\sqrt{2}}(\sigma^x \pm \sigma^z)$



Bob's measurements are achieved by getting the quasiparticles close for a fixed amount of time (unprotected operation) before performing a left or right measurement.

Maximal violation is obtained: $2\sqrt{2} - 2$.

More Bell violations

Anyon type	$SU(2)_2$	$SU(2)_k$	$D(S_3)$
Bell violation	$2\sqrt{2}-2$	$\mathcal{B}(k)-2$	$2.0512-2$

$$\mathcal{B}(k) = \pm \sec^2(\frac{\pi}{k+2})\sqrt{4\cos(\frac{2\pi}{k+2}) + \frac{1}{2}\cos(\frac{4\pi}{k+2}) + \frac{5}{2}}.$$

N.B. There is room for improvement.

Conclusions and open questions

- Bell tests could be achieved with non-Abelian anyons.
- Schemes with $SU(2)_k$ anyons and with quantum double models.
- Is an experiment possible? Ising anyons? What would be the difficulties? For instance, no "single shot coincidence".
- What would be the Loopholes?
- Find schemes that are fully contained in the fusion space.
- Which non-Abelian anyons require to go out of the fusion space for violation? Is there a relation with the possibility to perform universal quantum computation?
- More general question: relation gate set / Bell violation.