Quasielectrons in the Moore-Read state and how to condense them

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Motivation and main results

Write QH wave function as CFT correlators
Natural description for electrons and quasiholes, but not for *quasielectrons*

Construction of a quasielectron operator (nearly equivalent to quasihole)

- general definition
- correct charge
- quasilocal
- statistics hidden in the Berry phase

Condensates of non-Abelian quasielectrons (in the sense of Haldane/Halperin hierarchy)

- filling factors
  - $\nu = 4m/(4m-1)$ bosonic
  - $\nu = 4m/(8m-1)$ fermionic
- fundamental quasholes have charge e/q for filling fraction $\nu = p/q$ are non-Abelian, but not of Ising type

Example: $\nu = 1 \rightarrow \nu = 4/3$: $su(2)_2 \rightarrow su(3)_2$ (Fibonacci $\times$ Abelian)

Generalization to a hierarchy of non-Abelian quasielectron states

QH wave functions as CFT correlators

Laughlin's wave functions "look like" correlation functions in a Conformal Field Theory.

(Fubini, Moore&Read, Wen)

- QH wave functions are the conformal blocks of a RCFT
- Electrons and quasiholes are represented as local, chiral operators
- Statistics coded in the braiding properties of the corresponding operators
- Generalization to other FQH states (Jain, Moore-Read, *etc.*)

consider a massless scalar field: $\langle \varphi(z)\varphi(w)\rangle = -\ln(z-w)$

particles are described by vertex operators: $V_\alpha(z) \quad = \quad e^{i\alpha\varphi(z)}$

$\langle V_\alpha(z)V_\beta(w)\rangle \sim (z-w)^{\alpha\beta} \longrightarrow$ fermionic statistics for α^2 odd integer

Laughlin's fractions: $\nu = 1/m$

ground state:

$$\Psi_L = \langle \prod_{i=1}^{N} V_e(z_i) \mathcal{O}_{bg} \rangle = \prod_{i<j} (z_i - z_j)^m e^{-\sum_{i=1}^{N} |z_i|^2/4\ell^2}$$

→ Electron operator $V_e(z) = e^{i\sqrt{m}\varphi(z)}$

quasihole state:

$$\Psi_L^{1qh}(\eta) \equiv \langle H(\eta) \prod_{i=1}^{N} V(z_i) \mathcal{O}_{bg} \rangle$$

$$= e^{-\frac{|\eta|^2}{4m\ell^2}} \prod_{i=1}^{N} (\eta - z_i) \prod_{i<j} (z_i - z_j)^m e^{-\sum_{i=1}^{N} \frac{|z_i|^2}{4\ell^2}}$$

→ Quasihole operator $H(\eta) = e^{\frac{i}{\sqrt{m}}\varphi(\eta)}$

local
charge e/m
statistics encoded in operators

Different hole operators give the same electronic wave function

Monodromy vs. Berry's phase

Arovas et. al.

Laughlin fractions

- statistical phase = monodromy + Berry phase
- wave functions not normalized

$$\Psi^{2qh}_{1/m} = \mathcal{N}(\eta,\bar{\eta}) \prod_{i,a}(z_i - \eta_a) \prod_{i<j}(z_i - z_j)^m$$

- original proposal of Laughlin
- no monodromy
- statistical phase totally in Berry phase

$$\Psi^{2qh}_{1/m} = \mathcal{N}' \cdot (\eta_1 - \eta_2)^{1/m} \prod_{i,a}(z_i - \eta_a) \prod_{i<j}(z_i - z_j)^m$$

- explicit monodromy
- zero Berry phase
- "natural" from CFT construction

free interpolation between different monodromies by introducing a second uncharged bosonic field

$$H(\eta) = e^{\frac{i}{\sqrt{m}}\varphi(\eta)}$$

$$H'(\eta) = e^{\frac{i}{\sqrt{m}}\varphi(\eta) + i\alpha\varphi'(\eta)}$$

- $\alpha = \sqrt{\frac{2m-1}{m}}$ bosonic representation
- $\alpha = \sqrt{\frac{m-1}{m}}$ fermionic representation

What about quasi*electrons*?

several proposals!

Laughlin's Quasielectron: $\Psi_L^{(qe)}(\bar{\eta}; z_1, \ldots, z_N) = e^{-\frac{|\eta|^2}{4m\ell^2}} \prod_i (2\partial_i - \bar{\eta}) \prod_{i<j} (z_i - z_j)^m e^{-\sum_i \frac{|z_i|^2}{4\ell^2}}$

- localized charge
- but problems with anyonic statistics

What about quasi*electrons*?

several proposals!

Jain's CF Quasielectron: $\Psi_J^{(qe)}(0; z_1, \ldots, z_N) = \sum_i (-1)^i \prod_{\substack{j<k \\ j,k\neq i}} (z_j - z_k)^m \partial_i \prod_{l\neq i} (z_l - z_i)^{m-1} e^{-\sum_i \frac{|z_i|^2}{4\ell^2}}$

- localized charge
- numerical calculations show anyonic exchange phase

more complicated than quasiholes

- It is not obvious that it corresponds to a localized -*e*/*m* charge
- The statistical phase cannot be computed analytically

Is there a CFT operator for the quasielectron with these properties manifest?

Goal: quasielectron operator $\langle \mathcal{P}(\eta) \prod_{i=1}^{N} V_e(z_i) \mathcal{O}_{bg} \rangle$

CFT operator for quasi-electron analogous to the quasi-hole operator

wanted features

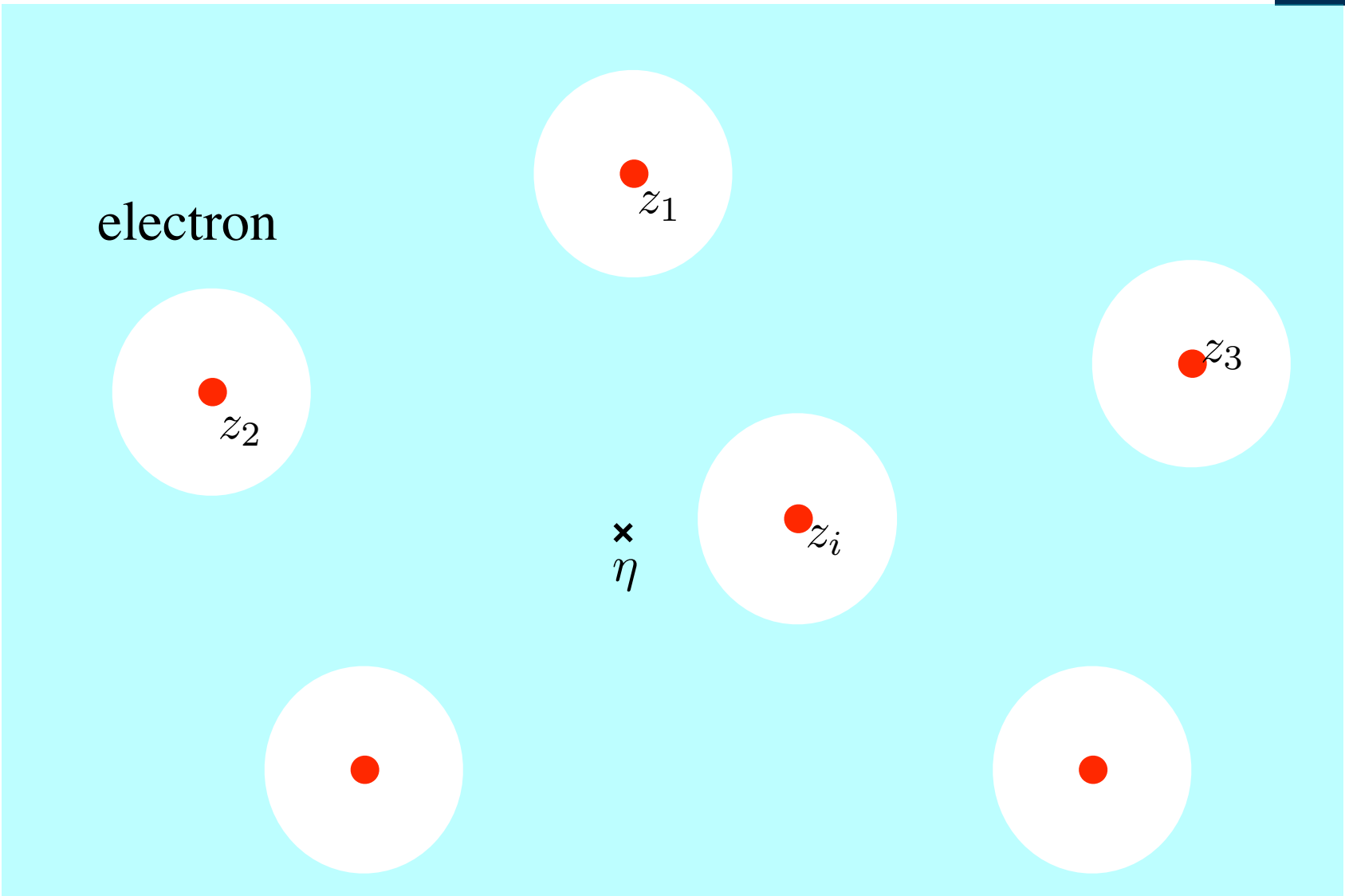
- local operator
- charge -e/m
- statistics manifest in the operators
- create multi-qe states by repeated insertion of qe operators

naive guess: $\mathcal{P}(\eta) = H^{-1}(\eta)$

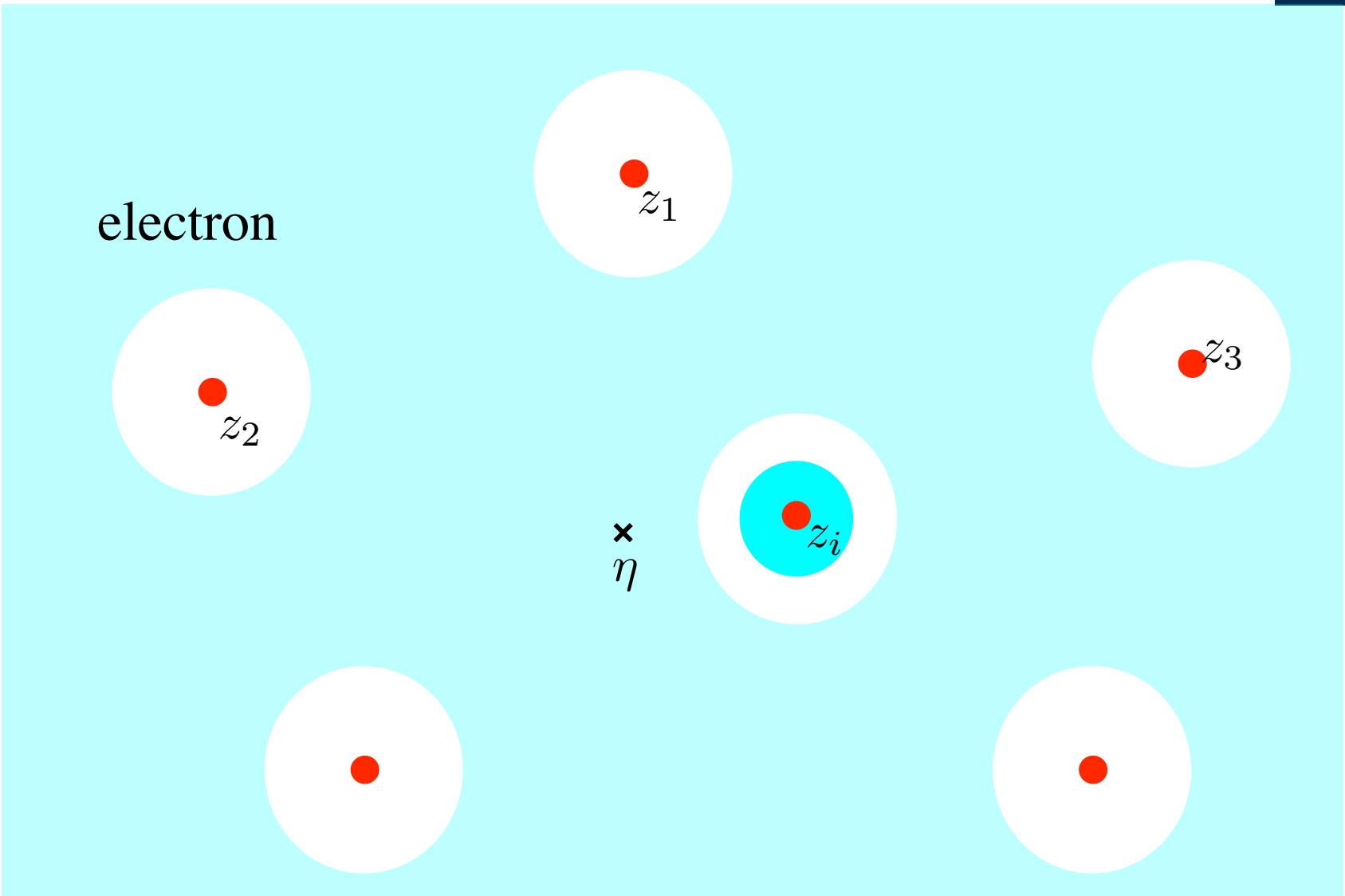
- local
- correct charge
- expected statistics

But: does not yield valid wave functions $\langle \mathcal{P}(\eta) \prod_{i=1}^{N} V(z_i) \mathcal{O}_{bg} \rangle \sim \prod_{i=1}^{N} (\eta - z_i)^{-1}$

The QH quasielectron



The QH quasielectron



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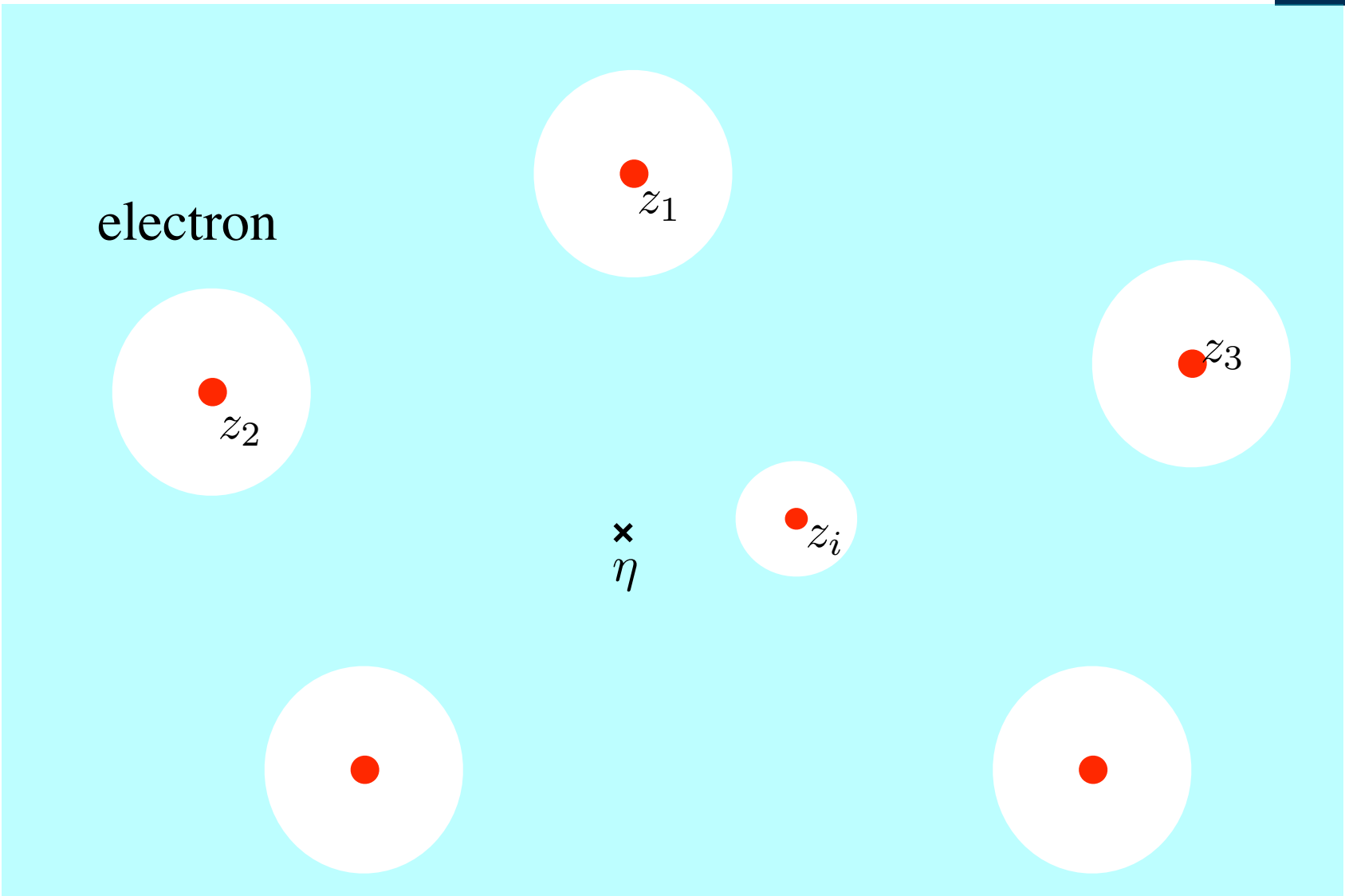

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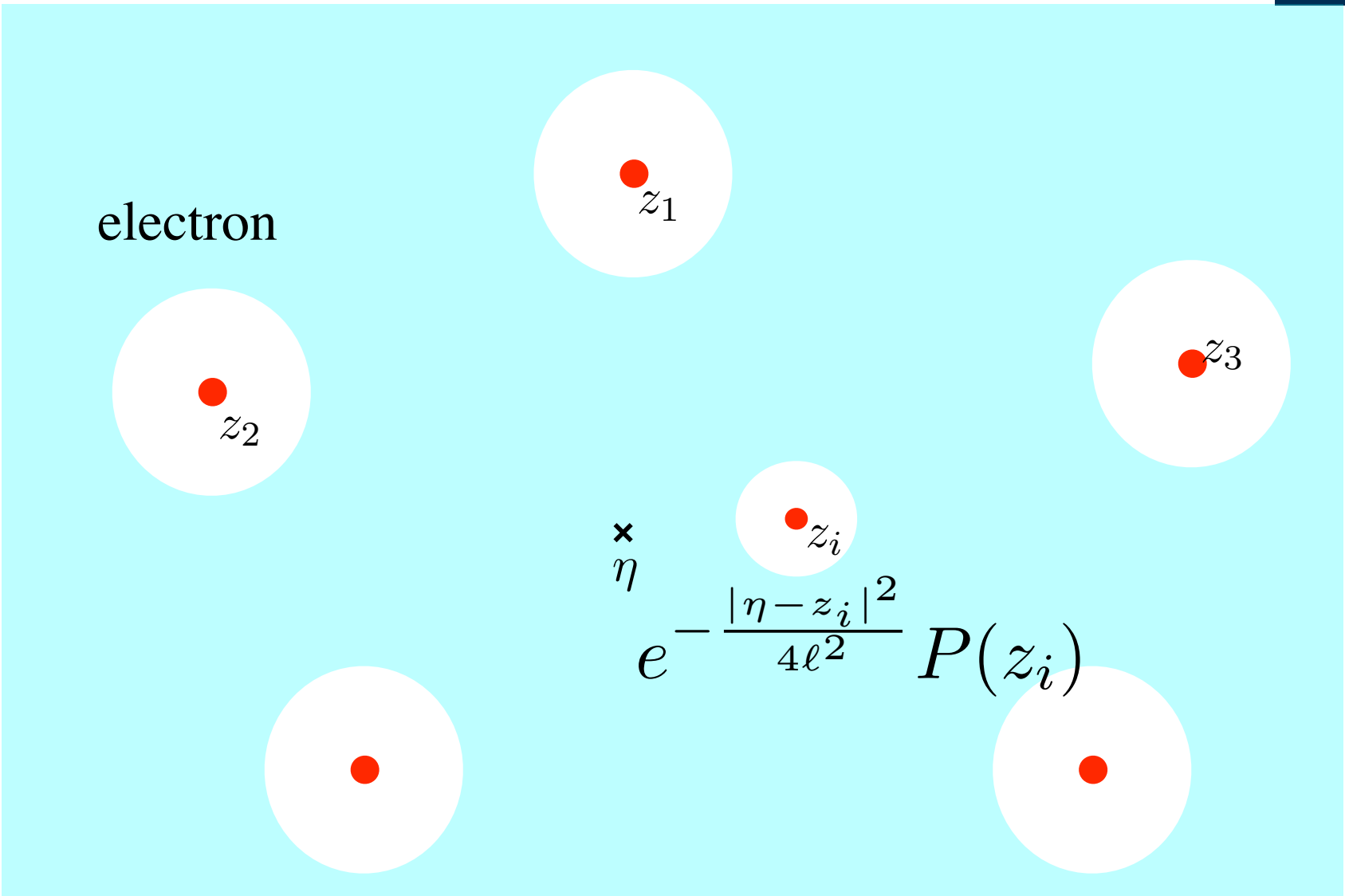


$$e^{-\frac{|\eta - z_i|^2}{4\ell^2}} P(z_i)$$

The quasielectron operator

$$\mathcal{P}(\eta) \quad = \quad \int d^2w \; e^{-\frac{q_h}{4\ell^2}(|w|^2 - 2w\bar{\eta} + |\eta|^2)} \quad \left(H^{-1}\bar{\partial}J\right)_n (w)$$

Results:

- **quasi-local on magnetic length scale**
- **same charge and conformal dimension as $H^{-1}(\eta)$**
- **multiple insertion gives multi-quasielectron states**
- **straightforward generalization to other states (Moore-Read, Jain, Read-Rezayi)**
- **construction completely in the lowest LL (no need for projection)**

Comments:

- insert *fermionic* or *bosonic* inverse quasihole to avoid branch cuts
- **no monodromy, statistics completely in the Berry phase**
- $(...)_n$ is a (generalized) normal ordering $\left(H^{-1}V\right)_n \equiv \partial : H^{-1}V : (z)$

Moore-Read ν=1/2

Moore&Read, Nayak&Wilczek

$$\begin{aligned} \psi \times \psi &= \mathbf{1} \\ \sigma \times \psi &= \sigma \\ \sigma \times \sigma &= \mathbf{1} + \psi \end{aligned}$$

Ising representation

$$\begin{aligned} V(z) &= \psi(z) e^{i\sqrt{2}\varphi(z)} \\ H(\eta) &= \sigma(\eta) e^{\frac{i}{2\sqrt{2}}\varphi(\eta)} \end{aligned}$$

1:1 ⟷

331 representation

$$\begin{aligned} V(z) &= \cos\phi(z) e^{i\sqrt{2}\varphi(z)} \\ H_\pm(\eta) &= e^{\pm i\phi(\eta)/2} e^{\frac{i}{2\sqrt{2}}\varphi(\eta)} \end{aligned}$$

Ground state is an antisymmetrized two layer state: 331 state

$$\Psi_{gs} \equiv \mathcal{A}\left[\prod_{\alpha<\beta}(z_\alpha - z_\beta)^3 \prod_{a<b}(z_a - z_b)^3 \prod_{\alpha,a}(z_\alpha - z_a)\right]$$

$$\begin{aligned} \alpha, \beta &= 1, \ldots, \frac{N}{2} \\ a, b &= \frac{N}{2} + 1, \ldots, N \end{aligned}$$

quasihole states are obtained by inserting $H_+ H_-$ pairs

- equal number of H_+ and H_-
- states are labeled by orderings of H_+ and H_-
- for $2n$ quasiholes there are 2^{n-1} linear independent orderings

Quasielectrons in the Moore-Read state

Take 331 representation

$$\begin{aligned} V(z) &= \cos\phi(z) e^{i\sqrt{2}\varphi(z)} \\ H_\pm(\eta) &= e^{\pm i\phi(\eta)/2} e^{\frac{i}{2\sqrt{2}}\varphi(\eta)} \end{aligned}$$

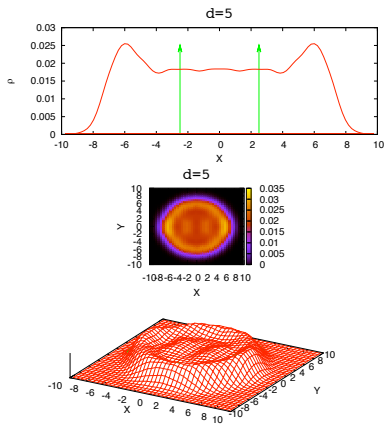
And fermionize the quasihole:

$$H_\pm(\eta) = e^{\frac{i}{\sqrt{8}}\varphi(\eta)} e^{\pm\frac{i}{2}\phi(\eta)} e^{-i\sqrt{\frac{3}{8}}\chi_1(\eta)\mp\frac{i}{2}\chi_2(\eta)}$$

Leads to two different operators

$$\mathcal{P}_\pm(\bar\eta) = \int d^2w\, e^{-\frac{q_h}{4m\ell^2}(|w|^2-2\bar\eta w+|\eta|^2)} \left(H_\pm^{-1}\,\bar\partial J\right)_n (w)$$

describes localized e/4 charges



Different orderings of '+' and '-' span the Hilbert space for the multi quasielectron state

4 qe:

$$\langle \mathcal{P}_+(\eta_1)\mathcal{P}_+(\eta_2)\mathcal{P}_-(\eta_3)\mathcal{P}_-(\eta_4)V(z_1)\ldots V(z_N)\rangle$$
$$\langle \mathcal{P}_+(\eta_1)\mathcal{P}_-(\eta_2)\mathcal{P}_+(\eta_3)\mathcal{P}_-(\eta_4)V(z_1)\ldots V(z_N)\rangle$$

- **wave function does not correspond to a specific fusion channel of the quasielectrons**
- **non-Abelian statistics is hidden entirely in the Berry phase**

Condensation of quasielectrons

Haldane/Halperin hierarchy:

$$\Psi_{n+1}(z_1 \dots z_N) = \int d^2\vec{R}_1 \dots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \dots \vec{R}_M)\Psi_n(\vec{R}_1 \dots \vec{R}_M; z_1 \dots z_N)$$

multi-qe wave function

$$\langle \mathcal{P}_+(\vec{R}_1) \dots \mathcal{P}_+(\vec{R}_{M/2})\mathcal{P}_-(\vec{R}_{M/2+1}) \dots \mathcal{P}_-(\vec{R}_M) V_e(z_1) \dots V_e(z_N) \mathcal{O}_{bg} \rangle$$

pseudo wave function

$$\Phi^*(\vec{R}_1 \dots \vec{R}_M) = \prod_{\substack{i<j \\ i,j\in\{+\}}} (\bar{\eta}_i - \bar{\eta}_j)^{2k+1} \prod_{\substack{i<j \\ i,j\in\{-\}}} (\bar{\eta}_i - \bar{\eta}_j)^{2k+1} e^{-\frac{q_h}{4\ell^2}\sum_{i=1}^{M}|\eta_i|^2}$$

Unique trial wave function

- Integration can be performed analytically
- Integration over quasielectron position amounts to an average over different fusion channels

Fermionic ν = 1/2 $\longrightarrow$ $\nu = \dfrac{4(k+1)}{8(k+1)-1}$

Bosonic ν = 1 $\longrightarrow$ $\nu = \dfrac{4(k+1)}{4(k+1)-1}$

The bosonic state at ν = 4/3

k=0 and M=N/2 yields trial wave function for fermionic ν = 4/7 or bosonic ν = 4/3

$$\Psi_{4/3} = \mathcal{S}\left[\left\{\partial_1\ (1-1)^2(2-2)^2(1-2)\right\} \times \left\{\partial_3\ (3-3)^2(4-4)^2(3-4)\right\}\right]$$

$$\partial_j = \prod_{a \in I_j} \partial_a$$

$$(j-j) = \prod_{\substack{a<b \\ a,b \in I_j}} (z_a - z_b)$$

$$(j-k) = \prod_{\substack{a \in I_j \\ b \in I_k}} (z_a - z_b)$$

The fundamental quasihole has

charge $q_h = e/3$

$su(3)_2$ fusion rules

comments:

Derivatives come from the quasielectrons and are needed to obtain a non-vanishing result

Natural interpretation as a symmetrized two layer state with ν=2/3 bosonic Jain states in the layers

Generalization to a hierarchy of non-Abelian quasielectron condensates

Conclusions

Construction of a quasielectron operator (nearly on the same footing as quasiholes)

- correct charge
- quasilocal
- statistics hidden in the Berry phase

Condensates of non-Abelian quasielectrons (Haldane/Halperin hierarchy)

- filling factors
  - $\nu = 4m/(4m-1)$ bosonic
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