Temps de cohérence d'un gaz condensé à température non nulle

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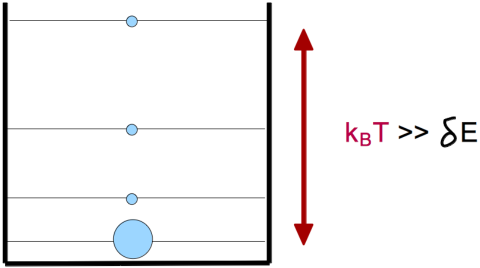
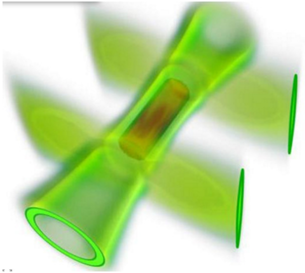
Plan

Bose-Einstein condensate

Bosons $T < T_c$: macroscopic population of a single particle state

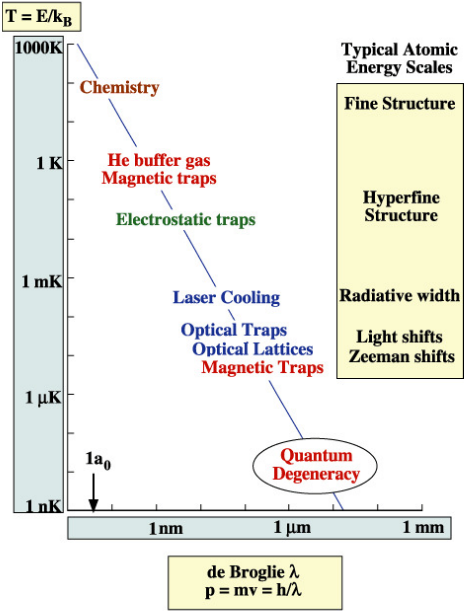




box potentials now available in experiments !

$\Rightarrow$ **Macroscopic coherence properties : spatial and temporal**

BEC : Energy scales



Typical numbers for a BEC

Size: $\Delta x = 50\,\mu$m

Number of atoms: $N = 10^6$

Temperature $T = 100$ nK
$(\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}})$

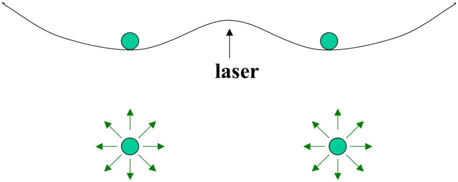
Density $\rho = 10^{19}$ at/m^3
$(\rho\lambda_{th}^3 > 1)$

Lifetime $\tau = 100$ s

Figure from Burnett et al., Nature (2002)

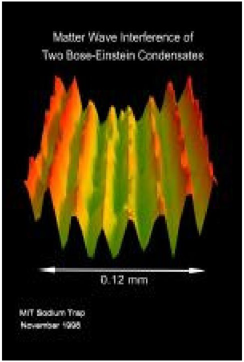
First evidence of phase coherence

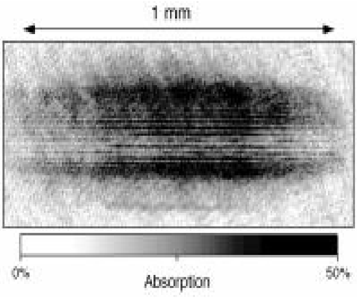
Interference of two BEC, MIT 1997



Set-up :

Result :



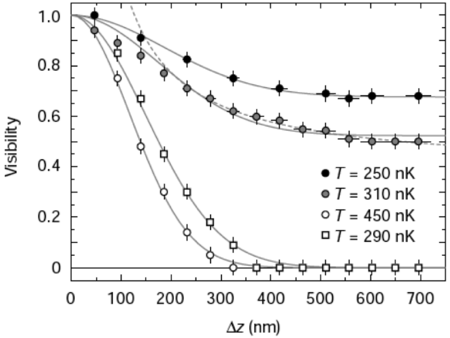


BEC Spatial Coherence : g_1 correlation function

Spatial coherence of a single Bose-condensed gas

$$g_1(\mathbf{r}) = \langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(0)\rangle\rangle \overset{r\to\infty}{\sim} \phi_0^*(\mathbf{r})\phi_0(0)\langle \hat{a}_0^\dagger \hat{a}_0\rangle; \qquad \hat{\psi}(\mathbf{r}) = \sum_\alpha \phi_\alpha(\mathbf{r})\hat{a}_\alpha$$

Spatial coherence



$T < T_c$

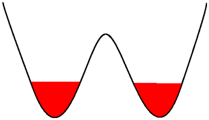
$T > T_c$

(for different N)

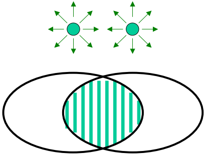
Figure from Bloch, Hänsch, Esslinger, Nature (2000)

BEC Time Coherence

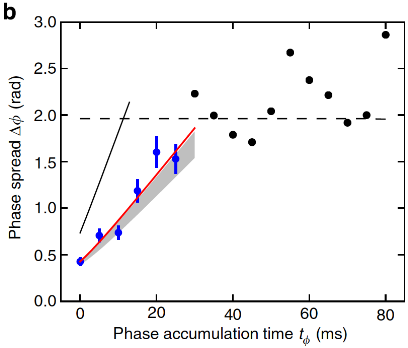
Two BEC with a well-defined relative phase at time $t = 0$



Interferometric measurement of the relative phase at time t



Temporal coherence



For how long do the condensates remember their (relative) phase ?

Figure from Tarik Berrada et al. Nat. Comm. (2013)

Fluctuations of N : $T = 0$ effect already measured

Case of a pure condensate, $T = 0$, simplest one mode model :

$$H = \frac{g}{2}\frac{N^2}{V} \qquad \text{and} \qquad \mu = \frac{gN}{V}$$

condensate phase derivative :

$$[N, \theta] = i \qquad \dot{\theta} = \frac{1}{i\hbar}[\theta, H] = -\frac{gN}{\hbar V} = -\frac{\mu(N)}{\hbar}.$$

N **is a conserved quantity**.

If N fluctuates around $\bar{N}$ $\Rightarrow$ ballistic spreading of the phase

$$\mathrm{Var}\,[\theta(\mathbf{t}) - \theta(\mathbf{0})] = \frac{t^2}{\hbar^2}\left(\frac{d\mu(\bar{N})}{dN}\right)^2 \mathrm{Var}N$$

Sols (1994); Walls; You, Lewenstein (1996); Castin, Dalibard (1997)
Seen in experiments on $\langle a_0^\dagger(t) b_0(t)\rangle$ (two-component condensates, equal time) rather than $\langle a_0^\dagger(t) a_0(0)\rangle$ (one component different times).
T. Berrada, Nat. Comm. (2013)

Fluctuations of E : $T \neq 0$ effect not yet measured

Here we fix N. On the other hand $T \neq 0$, many modes.

condensate phase derivative under ergodic assumption :

$$\dot{\theta} = \frac{1}{i\hbar}[\theta, H] = -\frac{\mu(E)}{\hbar}$$

$\mu(E)$=microcanonical chemical potential of the gas.

If the system is isolated during evolution, E is a conserved quantity.

If E fluctuates around $\bar{E}$ (canonical ensemble) $\Rightarrow$ ballistic spreading of the phase

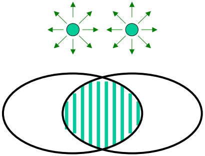
$$\mathrm{Var}\left[\theta(\mathbf{t}) - \theta(\mathbf{0})\right] \sim \frac{t^2}{\hbar^2}\left(\frac{d\mu(\bar{E})}{dE}\right)^2 \mathrm{Var}E$$

$\Rightarrow$ The spreading effect will be given by thermal fluctuations of μ at fixed N.

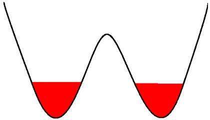
Temporal coherence of a BEC

A fundamental property of BEC, useful for applications

Macroscopic population of a single particle state → macroscopic coherence



Well-defined relative phase at time $t = 0$: How long do the BECs remember their (relative) phase ?



Spatial coherence of a single condensed gas

$$g_1(r) = \langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(0)\rangle \overset{r\to\infty}{\sim} \phi_0^*(\mathbf{r})\phi_0(0)\langle \hat{a}_0^\dagger \hat{a}_0\rangle;$$

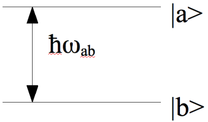
Temporal coherence of a single condensed gas

$$g_1(t,0) = \langle \hat{\psi}^\dagger(\mathbf{r},t)\hat{\psi}(\mathbf{r},0)\rangle \overset{t\to\infty}{\sim} |\phi_0(\mathbf{r})|^2 \langle \hat{\mathbf{a}}_0^\dagger(\mathbf{t})\hat{\mathbf{a}}_0(\mathbf{0})\rangle$$

System in equilibrium, isolated, homogeneous, condensed

BEC versus thermal gas

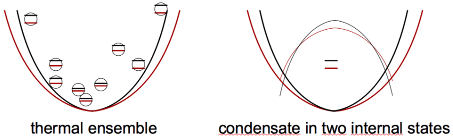
We all know atoms are useful oscillators



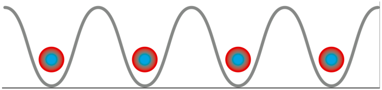
Example : atomic clocks

What about the BEC ?

- Absence of inhomogenous broadening
- BEC as a localized probe



- Use manybody physics : Mott to suppress the collisional shift
- Engineer correlations for “quantum metrology”



D. Kajtoch, E. Witkowka, A. Sinatra EPL (2018)

The condensate phase operator $\hat{\theta}_0$

We introduce the following representation for a bosonic mode ϕ

$$\hat{a}_\phi = \hat{A}_\phi \sqrt{\hat{n}_\phi} \qquad \hat{n}_\phi = \hat{a}_\phi^\dagger \hat{a}_\phi \qquad \hat{A}_\phi = \frac{1}{\sqrt{\hat{n}_\phi + 1}} \hat{a}_\phi \qquad (af(n) = f(n+1)a)$$

$$\hat{A}_\phi |n:\phi\rangle = |n-1:\phi\rangle \quad \text{for} \quad n > 0 \qquad \text{and} \quad \hat{A}_\phi |0:\phi\rangle = 0$$

$$\hat{A}_\phi^\dagger |n:\phi\rangle = |n+1:\phi\rangle \quad \text{for} \quad n \in \mathbb{N}$$

$\hat{A}_\phi$ **is "almost unitary" :** $\hat{A}_\phi \hat{A}_\phi^\dagger = 1 \qquad \hat{A}_\phi^\dagger \hat{A}_\phi = 1 - |0:\phi\rangle\langle 0:\phi|$

For a macroscopically populated mode ϕ_0 , we approximate $\hat{A}_{\phi_0} \simeq e^{i\hat{\theta}_0}$ **with $\hat{\theta}_0$ an hermitian operator**

MODULUS-PHASE REPRESENTATION OF CONDENSATE OPERATOR $\hat{a}_0$

$$\hat{a}_0 = e^{i\hat{\theta}_0} \sqrt{\hat{N}_0} \quad , \quad [\hat{n}_0, \hat{\theta}_0] = i$$

Hamiltonian on a lattice (spinless bosons)

$$\hat{H} = b^3 \sum_{\mathbf{r}} \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\Delta_{\mathbf{r}}\right) \hat{\psi}(\mathbf{r}) + g_0 b^3 \sum_{\mathbf{r}} \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})$$

Space discretisation step b, consequent cut-off in $\mathbf{k} \in \mathcal{D} \equiv [-\frac{\pi}{b}, \frac{\pi}{b}[^3$

Commutators $[\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')] = \frac{\delta_{\mathbf{r},\mathbf{r}'}}{b^3}$; **Kinetic energy** $\Delta_{\mathbf{r}}\langle\mathbf{r}|\mathbf{k}\rangle = -k^2\langle\mathbf{r}|\mathbf{k}\rangle$

Contact Interaction potential $V = g_0 \frac{\delta_{\mathbf{r},\mathbf{0}}}{b^3}$ **with** $g_0 \neq g = \frac{4\pi\hbar^2 a}{m}$

g_0 **adjusted to obtain scattering length a on the lattice**

$$\frac{1}{g_0} = \frac{1}{g} - \int_{\mathcal{D}} \frac{d^3k}{(2\pi)^3}\frac{m}{\hbar^2 k^2} \qquad \int_{\mathcal{D}} \frac{d^3k}{(2\pi)^3}\frac{m}{\hbar^2 k^2} = \frac{C}{g}\left(\frac{a}{b}\right)$$

$\xi = \sqrt{\frac{\hbar^2}{m\rho g}}$ **Bogoliubov :** $\frac{a}{\xi} \propto \sqrt{\rho a^3} \to 0$ **Lattice :** $\frac{b}{\xi} \overset{\textbf{fixed}}{=} \eta < 1$ **(Born)**

Ground state energy expansion in powers of a/ξ at fixed η :

$$\frac{E_0}{N} = \frac{\rho g}{2}\left(1 + C_\eta^{(1)}\frac{a}{\xi} + \dots\right)$$ Then $\eta \to 0$ in the coeff (no divergence)

Bogoliubov theory (homogeneous $\phi_0 = 1/\sqrt{V}$)

Splitting of the field operator

$$\hat{\psi}(r) = \frac{\hat{a}_0}{\sqrt{V}} + \hat{\psi}_\perp(\mathbf{r}) \qquad \hat{N} = \hat{N}_0 + \sum_{\mathbf{r}} |\hat{\psi}_\perp(\mathbf{r})|^2$$

Orthogonal number-conserving field $\hat{\Lambda}$

$$\hat{\Lambda}(\mathbf{r}) = e^{-i\hat{\theta}_0}\hat{\psi}_\perp(\mathbf{r}) \qquad [\hat{\Lambda}, \hat{\theta}_0] = [\hat{\Lambda}^\dagger, \hat{\theta}_0] = 0, \qquad [\hat{N}, \hat{\theta}] = i$$

Elimination of the condensate variables from the Hamiltonian

$$H(\hat{\psi}, \hat{\psi}^\dagger) \to H(\hat{N}, \hat{\Lambda}, \hat{\Lambda}^\dagger) \qquad \hat{N}_0 = \hat{N} - \sum_{\mathbf{r}} |\hat{\Lambda}(\mathbf{r})|^2$$

The Bogoliubov Hamiltonian is quadratic in $\hat{\Lambda}$ and $\hat{\Lambda}^\dagger$

$$H_{\mathrm{Bog}}(\hat{N}) = \frac{g_0 \hat{N}^2}{2V} + \sum_{\mathbf{r}} b^3 \left[\hat{\Lambda}^\dagger \left(h_0 + \frac{g_0 \hat{N}}{V}\right)\hat{\Lambda} + \frac{g_0 \hat{N}}{2V}\left(\hat{\Lambda}^2 + \hat{\Lambda}^{\dagger\, 2}\right)\right]$$

Explicit calculation : coarse-grain time average $\overline{\frac{\mathrm{d}\hat{\theta}_0}{\mathrm{d}t}}^t$

Heisenberg picture : $i\hbar\frac{\mathrm{d}\hat{\theta}_0}{\mathrm{d}t} = -i\frac{\partial H_{\mathrm{Bog}}(\hat{N},\hat{\Lambda},\hat{\Lambda}^\dagger)}{\partial N}|_{\Lambda,\Lambda^\dagger}$

$$\frac{\mathrm{d}\hat{\theta}_0}{\mathrm{d}t} = -\frac{1}{\hbar}\left\{\frac{g_0\hat{N}}{V} + \frac{g_0}{V}\sum_{\mathbf{r}} b^3\left[\hat{\Lambda}^\dagger\hat{\Lambda} + \frac{1}{2}\left(\hat{\Lambda}^2 + \hat{\Lambda}^{\dagger\,2}\right)\right]\right\}$$

Expansion over eigenmodes of linear equations of motion for $\Lambda, \Lambda^\dagger$

$$\begin{pmatrix}\hat{\Lambda}(\mathbf{r})\\ \hat{\Lambda}^\dagger(\mathbf{r})\end{pmatrix} = \sum_{\mathbf{k}\neq\mathbf{0}}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{V^{1/2}}\left[\begin{pmatrix}U_k\\ V_k\end{pmatrix}\hat{b}_{\mathbf{k}} + \begin{pmatrix}V_k\\ U_k\end{pmatrix}\hat{b}^\dagger_{-\mathbf{k}}\right]$$

Coarse-grain time average of $\mathrm{d}\hat{\theta}_0/\mathrm{d}t$

$$-\hbar\overline{\frac{\mathrm{d}\hat{\theta}_0}{\mathrm{d}t}}^t = \mu_0(\hat{N}) + \sum_{\mathbf{k}\neq\mathbf{0}}\frac{\partial\epsilon_k}{\partial N}\hat{n}_{\mathbf{k}}$$

with $\mu_0(\hat{N}) = \frac{\mathrm{d}E_0(N)}{\mathrm{d}N}$ **and** $E_0(N) = \frac{g_0 N^2}{2V} - \sum_{\mathbf{k}\neq\mathbf{0}}\epsilon_k V_k^2$

$U_k \pm V_k = \left(\frac{E_k}{\epsilon_k}\right)^{\pm 1/2}$; $\epsilon_k = \sqrt{E_k(E_k + 2\rho g_0)}$; $E_k = \frac{\hbar^2 k^2}{2m}$; $\rho = \frac{N}{V}$

Physical interpretation of $\overline{\frac{\mathrm{d}\hat\theta_0}{\mathrm{d}t}}^t$: contribution of thermal excitations

Canonical ensemble

$$\hat\sigma_{\mathrm{can}} = \frac{e^{-\beta \hat H_{\mathrm{Bog}}}}{Z} \qquad \textbf{with} \qquad \hat H_{\mathrm{Bog}} = E_0(N) + \sum_{\mathbf{k}\neq\mathbf{0}} \epsilon_k \hat n_{\mathbf{k}} \qquad \bar n_k = \frac{1}{e^{\beta\epsilon_k}-1}$$

Free energy of ideal bose gas (Bogoliubov quasi particles)

$$F = E_0(N) + k_B T \sum_{\mathbf{k}} \ln(1-e^{\beta\epsilon_k})$$

$$\mu_{\mathrm{can}} = \left(\frac{dF}{dN}\right)_{V,T} = \mu_0(\hat N) + \sum_{\mathbf{k}\neq\mathbf{0}} \frac{\partial\epsilon_k}{\partial N}\bar n_k = -\hbar\left\langle \overline{\frac{\mathrm{d}\hat\theta_0}{\mathrm{d}t}}^t \right\rangle_{\mathrm{can}}$$

PHASE DERIVATIVE ↔ "CHEMICAL POTENTIAL OPERATOR"

$$\overline{\frac{d\hat\theta_0}{dt}}^t = -\frac{\hat\mu}{\hbar}$$

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Bosons

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Fermions

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Time coherence and condensate phase

- Time correlation of condensate amplitude $\langle a_0^\dagger(t)a_0(0)\rangle$:

thermally popolated modes = noisy environnement $\rightarrow$ decay of the correlation function, spreading of the condensate phase

- Equivalently, we can consider the variance of the condensate phase difference $\mathrm{Var}\,[\theta(t)-\theta(0)]$ as a function of time t.

Modulus-phase representation : $a_0 = e^{i\theta}\sqrt{N_0}$, $[N_0,\theta]=i$

For a Gaussian probability distribution of $\theta(t)-\theta(0)$ and weak fluctuations of N_0

$$|\langle a_0^\dagger(t)a_0(0)\rangle| \simeq \langle N_0\rangle \exp\left\{-\frac{1}{2}\mathrm{Var}\,[\theta(\mathbf{t})-\theta(\mathbf{0})]\right\}$$

Intrinsic sources of the condensate phase spreading

First source : shot to shot fluctuations of conserved quantities, as N (total number), or E (total energy) in the canonical ensemble.

It gives rise to :

- **Ballistic spreading** of the phase difference $\mathrm{Var}\,[\theta(\mathbf{t})-\theta(\mathbf{0})] \sim At^2$
- Gaussian decay of $\langle a_0^\dagger(t)a_0(0)\rangle$
- Coherence time scaling as $\sqrt{N}$ in a finite system.

Second source (even for fixed E and N) : fluctuations of quasiparticle numbers that perturbing the the condensate phase.

It gives rise to :

- **Diffusive spreading** of the phase difference $\mathrm{Var}\,[\theta(\mathbf{t})-\theta(\mathbf{0})] \sim 2Dt$
- Exponential decay of $\langle a_0^\dagger(t)a_0(0)\rangle$
- Coherence time scaling as N in a finite system.

Condensate correlation function

The system state : $\hat{\rho} = \sum_\lambda \Pi_\lambda |\psi_\lambda\rangle\langle\psi_\lambda|$ **with** $\hat{H}|\psi_\lambda\rangle = E_\lambda|\psi_\lambda\rangle$

Correlation function in a many-body eigenstate ψ_λ

$$g_1^\lambda(t) \simeq \bar{N}_0\langle e^{-i\hat{\theta}_0(t)}e^{i\hat{\theta}_0(0)}\rangle = \bar{N}_0 e^{iE_\lambda t/\hbar}\langle\psi_\lambda|e^{-i\hat{H}_\theta t/\hbar}|\psi_\lambda\rangle$$

with $\hat{H}_\theta = e^{-i\hat{\theta}_0}\hat{H}e^{-i\hat{\theta}_0}$ where we used $e^{\xi\hat{A}}F(\hat{B})e^{-\xi\hat{A}} = F(e^{\xi\hat{A}}\hat{B}e^{-\xi\hat{A}})$

Then we write : $\hat{H}_\theta = \hat{H} + \hat{W}$ and

$$\hat{W} \equiv \mathrm{e}^{-\mathrm{i}\hat{\theta}_0}\hat{H}\mathrm{e}^{\mathrm{i}\hat{\theta}_0} - \hat{H} = \underbrace{-\mathrm{i}[\hat{\theta}_0,\hat{H}]}_{O(\hat{N}^0)}\underbrace{-\frac{1}{2}[\hat{\theta}_0,[\hat{\theta}_0,\hat{H}]]}_{O(\hat{N}^{-1})} + \ldots$$

Correlation function in a many-body eigenstate ψ_λ

$$g_1^\lambda(t) \simeq \bar{N}_0 e^{iE_\lambda t/\hbar}\langle\psi_\lambda|e^{-i(\hat{H}+\hat{W})t/\hbar}|\psi_\lambda\rangle \qquad \hat{W} = \hbar\frac{d\hat{\theta}_0}{dt} + O(\frac{1}{N})$$

For a large system, $\hat{W} \ll \hat{H} = O(\hat{N})$

Link with the problem of a state weakly coupled to a continuum (Resolvant and projectors method)

- $g_1^\lambda(t) \propto$ **probability amplitude that the system, initially prepared in state $|\psi_\lambda\rangle$, is still there at time t for the perturbed evolution of Hamiltonian $\hat{H} + \hat{W}$**

- **In the thermodynamic limit (quasi-continuous spectrum), the perturbation has two effects:**
  - **energy shift: angular frequency** $(E_\lambda + \langle\psi_\lambda|\hat{W}|\psi_\lambda\rangle + O(N^-1))/\hbar$
  - **exponential decay with a rate (Fermi golden rule) :**

$$\gamma_\lambda = \frac{\pi}{\hbar}\sum_{\mu\neq\lambda} |\langle\psi_\mu|\hat{W}|\psi_\lambda\rangle|^2\,\delta(E_\lambda - E_\mu)$$

CORRELATION FUNCTION IN A MANY-BODY EIGENSTATE ψ_λ

$$g_1^\lambda(t) \simeq \bar{N}_0\,\mathrm{e}^{-\mathbf{i}\mathbf{t}\langle\psi_\lambda|\hat{\mathbf{W}}|\psi_\lambda\rangle/\hbar}\,\mathrm{e}^{-\gamma_\lambda \mathbf{t}} \qquad \hat{W} = \hbar\frac{d\hat{\theta}_0}{dt} + O(\frac{1}{N})$$

Gaussian decay of $g_1^\lambda(t)$ (dominant contribution)

COARSE GRAIN PHASE DERIVATIVE : FOR $\hbar/\epsilon_k^{\mathrm{th}} \ll t \ll \gamma_{\mathrm{coll}}^{-1}$

Generalization - to the quantum case and $T \neq 0$ - of the second Josephson equation

$$-\hbar\frac{d\hat\theta}{dt} = \mu_0(\hat N) + \sum_k \frac{d\epsilon_k}{dN}\,\hat n_k \equiv \hat\mu$$

Indeed $\hat\mu$ is the adiabatic derivative of the H_{Bog} with respect to N

$$H_{\mathrm{Bog}} = E_0(\hat N) + \sum_k \epsilon_k\,\hat n_k$$

$$\langle\psi_\lambda|\hat W|\psi_\lambda\rangle \underset{\mathrm{ETH}}{\simeq} -\mu_{\mathrm{mc}}(E_\lambda, N_\lambda)$$

- **we now have to average $g_1^\lambda(t) \simeq \bar N_0\,\mathrm{e}^{-\mathrm{i}t\langle\psi_\lambda|\hat W|\psi_\lambda\rangle/\hbar}$ over $|\psi_\lambda\rangle$**

Coherence time for ballistic spreading

- **Linearizing** $\mu_{\rm mc}(E_\lambda, N_\lambda)$ around $(\bar{E}, \bar{N})$,
- **The averaging** $g_1^\lambda(t) \simeq \bar{N}_0\, \mathrm{e}^{-\mathrm{i}\mu_{\rm mc}(E_\lambda,N_\lambda)t/\hbar}$ **over** $|\psi_\lambda\rangle$ **gives :**

Time correlation function

$$g_1(t) \simeq \bar{N}_0 \mathrm{e}^{\mathrm{i}\mu_{\rm mc}(\bar{E},\bar{N})t/\hbar} \mathrm{e}^{-\mathbf{t}^2/2\mathbf{t}_{\rm br}^2}$$

Spreading of the condenstate phase

$$\mathrm{Var}\,[\hat{\theta}(t) - \hat{\theta}(0)] \overset{t\to\infty}{=} \mathbf{At^2} + \ldots$$

- **Characteristic time**

Blurring time for ballistic spreading

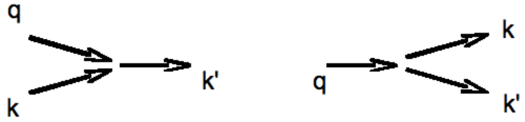
$$\frac{1}{2\mathbf{t}_{\rm br}^2} = \mathbf{A} = \mathrm{Var}\left(N\frac{\partial\mu_{\rm mc}}{\partial N}(\bar{E},\bar{N}) + E\frac{\partial\mu_{\rm mc}}{\partial E}(\bar{E},\bar{N})\right)$$

Physical origin of ergodicity

The system is described by weakly interacting quasi particles

$$H = E_0 + \sum_{\mathbf{k}\neq 0} \epsilon_k b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \text{cubic terms} + \text{quartic terms}$$

Interactions among Bogoliubov modes ensure ergodicity



Landau and Beliaev processes

The populations $n_{\mathbf{k}}$ fluctuate $\rightarrow$ the condensate phase spreads

PHASE DERIVATIVE $\dot{\theta}(t)$:

$$\dot{\theta}(t) \simeq -\mu_\Phi/\hbar + \sum_{\mathbf{k}\neq 0} \left(\partial_N \epsilon_k\right) n_{\mathbf{k}}$$

Phase diffusion (subdominant)

- On can rewrite

$$\gamma_\lambda = \int_0^{+\infty} \mathrm{d}t \left[\mathrm{Re} \left\langle \frac{\mathrm{d}\hat{\theta}_0(t)}{\mathrm{d}t} \frac{\mathrm{d}\hat{\theta}_0(0)}{\mathrm{d}t} \right\rangle_\lambda - \left\langle \frac{\mathrm{d}\hat{\theta}_0}{\mathrm{d}t} \right\rangle_\lambda^2 \right]$$

- thus this is the phase diffusion coefficient :

$$\gamma_\lambda \underset{\mathrm{ETH}}{=} D(E_\lambda, N_\lambda) \simeq D(\bar{E}, \bar{N})$$

Main result after ensemble average

$$g_1(t) \simeq \bar{N}_0 \mathrm{e}^{2\mathrm{i}\mu_{\mathrm{mc}}(\bar{E},\bar{N})t/\hbar} \mathrm{e}^{-t^2/2t_{\mathrm{br}}^2} \mathrm{e}^{-D(\bar{E},\bar{N})t}$$

$$\mathrm{Var}\,[\hat{\theta}(t) - \hat{\theta}(0)] \overset{t\to\infty}{=} At^2 + 2Dt$$

Phase diffusion occurs even in the absence of energy fluctuations

Equation for $\frac{\mathrm{d}\hat{\theta}_0(t)}{\mathrm{d}t}$ + kinetic equations describing the quasi-particles collisions allow us to calculate γ_λ

The system

- **Harmonically trapped Bosons with zero-range interactions.**
  $a =$ s**-wave scattering length**
- **The gas is in equilibrium in the deeply condensed regime**
- **State of the system (generalized ensemble) :**

$$\hat{\sigma} = \sum_{\lambda} P_{\lambda} |\psi_{\lambda}(N_{\lambda}, E_{\lambda})\rangle\langle\psi_{\lambda}(N_{\lambda}, E_{\lambda})|$$

  $Var(E) = O(\bar{E})$ and $Var(N) = O(\bar{N})$ in the thermodynamic limit.

- **Thermodynamic limit in the trap :**

$$N \to \infty, \quad \textbf{with} \quad \mu_{GP} \quad \textbf{and} \quad \mathrm{T} \quad \textbf{fixed} \quad \to \quad \omega_{\alpha} \propto 1/N^{1/3}$$

  Considered regime : $\hbar\omega_{\alpha} \ll \mu_{GP}, k_B T$

- **Collisionless regime for the quasi-particles (opposite to hydrodynamic regime)**

$$\gamma_{\mathrm{coll}} \ll \omega_{\alpha} \quad \text{where} \quad \gamma_{\mathrm{coll}}^{-1} = \text{collision time between thermal Bogolibov QP}$$

Ballistic phase spreading coefficient A

$$\mathrm{Var}\,[\hat{\theta}(t) - \hat{\theta}(0)] \overset{t\to\infty}{=} At^2 + \ldots$$

- **For a statistical mixture of canonical ensembles (same T, different N) with Poissonian fluctuations of N**

$$A = \underbrace{A_{\mathrm{Pois}}}_{\mathbf{A_{T=0}}(1+\mathbf{O}(\mathbf{f}_{\mathrm{nc}}))} + \underbrace{A_{\mathrm{can}}}_{\mathbf{A}_{\mathrm{can}}(\mathbf{T}) = \mathbf{O}(\mathbf{f}_{\mathrm{nc}})}$$

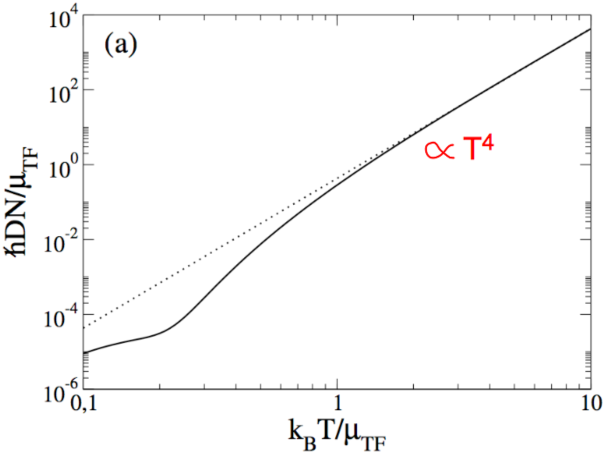
- **Unless fluctuations of N are reduced/suppressed, the $T=0$ contribution to the ballistic coefficient dominates**

For Poissonian fluctuations of N

$$A_{\mathrm{Pois}} = \bar{N}\left(\frac{\partial_N \mu_{\mathrm{TF}}}{\hbar}\right)^2 \qquad \frac{A_{\mathrm{can}}(T)}{A_{\mathrm{Pois}}} \underset{k_B T \gg \mu_{\mathrm{TF}}}{\sim} \frac{3\zeta(3)}{4\zeta(4)}\left(\frac{T}{T_c^{(0)}}\right)^3$$

Phase diffusion coefficient in a trap

$$\mathrm{Var}\,[\hat{\theta}(t) - \hat{\theta}(0)] \stackrel{t\to\infty}{=} At^2 + 2Dt + \ldots$$



Independent of the ensemble and of the trap frequencies

Conclusions

- We calculate the intrinsic coherence time of a condensate in thermal equilibrium.
- Coherence time $\leftrightarrow$ phase dynamics, and $d\hat{\theta}_0/dt \propto$ "chemical potential operator" including pair-breaking and pair-motion excitations.
- As $\hat{\theta}_0(t) \simeq -\mu_{\rm mc}(E)t/\hbar$, energy fluctuations from one realization to the other $\rightarrow$ Gaussian decay of the coherence $t_{\rm br} \propto N^{1/2}$.
- In the absence of energy fluctuations, the coherence time scales as N due to the diffusive motion of $\hat{\theta}_0$.

$$\mathrm{Var}\,[\hat{\theta}(t) - \hat{\theta}(0)] \overset{t\to\infty}{=} At^2 + 2D(t - t_0) + o(1)$$

- Calculated A, D and t_0 in a harmonic trap, for $\omega_\alpha \ll \mu_{\rm TF}, k_B T$, collisionless regime $\gamma_{\rm coll} \ll \omega_\alpha$ and ergodic motion of quasiparticles (anisotropic trap)
- If properly rescaled, $A \propto 1/N$, $D \propto 1/N$ and t_0 are universal functions of $k_B T/\mu_{\rm TF}$