Amortissement Landau-Khalatnikov des phonons dans un superfluide

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Plan

Universal low-T limit for a superfluid

A superfluid with short-range interactions (with a gapless, phonic excitation branch) in three dimensions

$\Downarrow$ At low temperature $\Downarrow$

weakly interacting gas of phonons

Irrespectively of :

- Statistics : bosons or paired fermions
- Interaction strength : gas or liquid

Phonon damping $\leftrightarrow$ phonon-phonon interactions

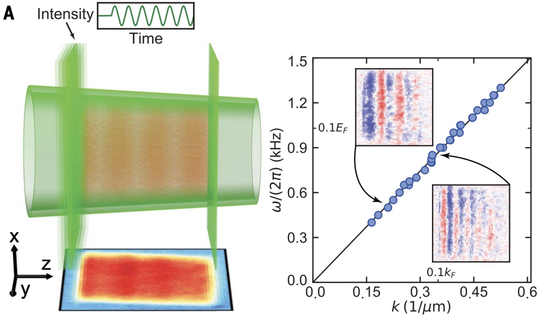
- Transport properties
- Intrinsic coherence time of the condensate

The two physical systems we have in mind

- **Liquid Helium 4 spinless bosons → Bose-Einstein condensation → superfuidity** ($\rho \sim 10^{30}$ **atoms**$/m^3$**,** $T_c \sim 2K$)

- **Ultracold atomic gases unpolarized spin-1/2 fermions → condensation of bound ↑ − ↓ pairs → superfuidity** ($\rho \sim 10^{19}$ **atoms**$/m^3$**,** $T_c \sim \mu K$)

Sound propagation in a flat-bottom potential
M. Zwierlein, Science 370, (2020)

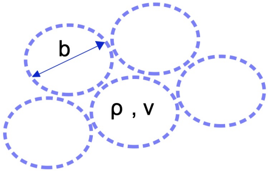


Effective low energy theory: quantum hydrodynamics, Landau-Khalatnikov (1949)

Two canonically conjugated quantum fields:

- local density $\hat{\rho}(r)$
- local phase $\hat{\phi}(r) \rightarrow$ local velocity $\hat{\mathrm{v}}(\mathrm{r}) = \frac{\hbar}{m}\nabla\hat{\phi}(\mathrm{r})$

Coarse-grained (large scale) description of the fluid:



$$\rho^{-1/3}\,,\ \xi = \frac{\hbar}{mc} \ll \mathrm{b} \ll \lambda_{\mathrm{th}}\,,\ q_{\mathrm{th}}^{-1}$$

each grain is homogeneous, in local ground state, with overall velocity v.

$$\hat{H} = \int d^3r\,\left[\frac{1}{2}m\,\hat{\mathrm{v}}\cdot\hat{\rho}\,\hat{\mathrm{v}} + e_0(\hat{\rho})\right]$$

$e_0(\rho)$ = ground state energy density at uniform density ρ

Normal modes and expansion of the hamiltonian

Weak spatial fluctuations at low temperature

$$\hat{\rho}(r,t) = \hat{\rho}_0 + \delta\hat{\rho}(r,t) \quad \textbf{and} \quad \hat{\phi}(r,t) = \hat{\phi}_0 + \delta\hat{\phi}(r,t)$$

- **Fourier expansion and diagonalization of linearized equations**

$$\delta\hat{\rho}_{\mathrm{q}} \propto \sqrt{q}\,(\hat{b}_{\mathrm{q}} + \hat{b}^{\dagger}_{-\mathrm{q}}) \qquad \delta\hat{\phi}_{\mathrm{q}} \propto \frac{-i}{\sqrt{q}}\,(\hat{b}_{\mathrm{q}} - \hat{b}^{\dagger}_{-\mathrm{q}}) \quad \textbf{with} \quad [\hat{b}_{\mathrm{q}}, \hat{b}^{\dagger}_{\mathrm{q}}] = 1$$

- **An ensemble of weakly coupled harmonic oscillators**

$$H_2 = \sum_{q\neq 0} \hbar\omega_q b_q^{\dagger} b_q$$

- **Phonon dispersion relation :**

$$\hbar\omega_q = \hbar c q \quad \textbf{with} \quad mc^2 = \rho\frac{\mathrm{d}\mu}{\mathrm{d}\rho} \quad \textbf{and} \quad \mu = \frac{\mathrm{d}e_0}{\mathrm{d}\rho}$$

- **Expansion of the hamiltonian**

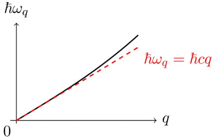
$$H = E_0 + H_2 + H_3 + H_4 + \dots$$

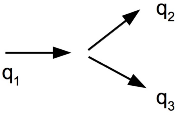
Phonon-phonon interactions

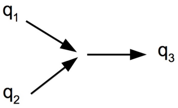
Correction to phonon dispersion relation $q \mapsto \omega_{\mathrm{q}}$ close to $q = 0$

$$\hbar\omega_{\mathrm{q}} \underset{q\to 0}{=} \hbar c q \left[1 + \frac{\gamma}{8}\left(\frac{\hbar q}{mc}\right)^2 + \ \ldots \ \right]$$

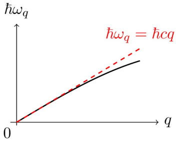
- $\gamma > 0$: **Beliaev and Landau 3-phonon processes dominant**

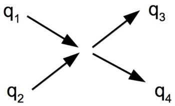






- $\gamma < 0$: **3-phonon processes forbidden by energy conservation, Landau-Khalatnikov 4-phonon processes are dominant**

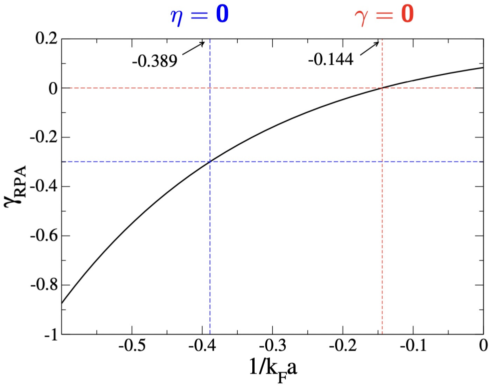




Tuning γ in a strongly interacting Fermi gas

RPA calculation : **H. Kurkjian, Y. Castin, A. Sinatra, PRA (2016)**

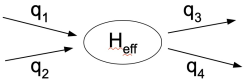
$$\hbar\omega_{\mathbf{q}} \underset{q\to 0}{=} \hbar c q \left[1 + \frac{\gamma}{8}\left(\frac{\hbar q}{mc}\right)^2 + \frac{\eta}{16}\left(\frac{\hbar q}{mc}\right)^4 + O\left(\frac{\hbar q}{mc}\right)^6\right]$$



N.B. γ can also be tuned in liquid ^{4}He by changing the pressure

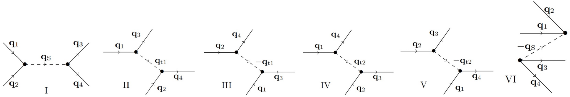
Effective coupling for $2 \leftrightarrow 2$ process $(\gamma < 0)$

$$H = E_0 + H_2 + H_3 + H_4 + \dots$$



$$\langle f|\hat{H}^{\mathrm{eff}}|i\rangle = \langle f|\hat{H}_4|i\rangle + \sum_{\lambda} \frac{\langle f|\hat{H}_3|\lambda\rangle\langle\lambda|\hat{H}_3|i\rangle}{E_i - E_\lambda}$$

Six intermediate states $|\lambda\rangle$. Inclusion of $\gamma \neq 0$ to avoid divergences



$$\hat{H}_{\mathrm{eff}} = \frac{mc^2}{\rho L^3} \sum_{\substack{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4 \\ \mathbf{q}_1+\mathbf{q}_2=\mathbf{q}_3+\mathbf{q}_4}} \mathcal{A}_{\mathrm{eff}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4) \quad \hat{b}^\dagger_{\mathbf{q}_3}\hat{b}^\dagger_{\mathbf{q}_4}\hat{b}_{\mathbf{q}_1}\hat{b}_{\mathbf{q}_2}$$

$\mathcal{A}_{\mathrm{eff}}$ depends on the angles between the vectors, the energies (also of the intermediate states) and the thermodynamic quantities :

$$\Sigma_{\mathrm{F}} \equiv \frac{\rho^3}{mc^2}\frac{\mathrm{d}^3\mu}{\mathrm{d}\rho^3} \qquad \Lambda_{\mathrm{F}} \equiv \frac{\rho}{3}\frac{\mathrm{d}^2\mu}{\mathrm{d}\rho^2}\left(\frac{\mathrm{d}\mu}{\mathrm{d}\rho}\right)^{-1}$$

Landau-Khalatnikov damping rate

- **The damping rate calculated with the Fermi Golden rule in the so called collisionless regime $\omega_q \gg \Gamma_{\rm th}$**

$$\frac{d}{dt}\delta n_q = -\Gamma_q \delta n_q$$

$$\Gamma_q \propto \int d^3q_2 d^3q_3 |\mathcal{A}_{\rm eff}|^2 \delta(\omega_3+\omega_4-\omega_2-\omega_q)[\bar{n}_2(1+\bar{n}_3)(1+\bar{n}_4)-(1+\bar{n}_2)\bar{n}_3\bar{n}_4]$$

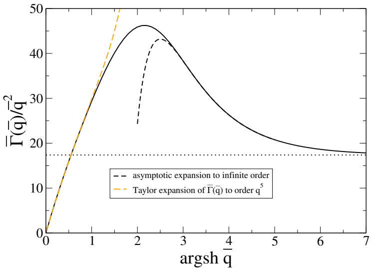
- **We take the limit $\epsilon = k_B T/mc^2 \to 0$ at fixed $\tilde{q} = \hbar c q/k_B T$**

Landau-Khalatnikov damping $(\gamma < 0)$

$$\frac{\hbar\Gamma_{\rm q}}{mc^2} \underset{T\to 0}{\sim} \frac{81(1+\Lambda_{\rm F})^4}{256\pi^4|\gamma|} \left(\frac{mc}{\hbar\rho^{1/3}}\right)^6 \left(\frac{k_{\rm B}T}{mc^2}\right)^7 \tilde{\Gamma}(\tilde{q})$$

$\tilde{\Gamma}$ (double integral) is a universal function of $\tilde{q} = \hbar c q/k_B T$ with simple asymptotic behaviors

Landau-Khalatnikov damping rate



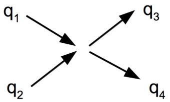
$\bar{\Gamma}(\bar{q}) = \frac{4\pi}{3\bar{q}n_q}\int_0^{+\infty} \mathrm{d}\bar{q}' \int_0^{\bar{q}+\bar{q}'} \mathrm{d}\bar{k}\, \bar{q}'\bar{k}\bar{k}'(1+n_{q'})n_k n_{k'}[\min(q,q',k,k')]^3$ **avec** $k+k'=q+q'$

Behavior of $\tilde{\Gamma}$ for small $\tilde{q}$

$$\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q}\to 0}{=} \frac{16\pi^5}{135}\tilde{q}^3 + \dots \qquad \tilde{\Gamma}(\tilde{q}) \underset{\tilde{q}\to\infty}{=} \frac{16\pi\zeta(5)}{3}\tilde{q}^2 + \dots$$

- **Landau-Khalatnikov only calculated small-$\tilde{q}$ and large-$\tilde{q}$ limits**
- **They included a single diagram supposed to be dominant. We disagree, our result begin sub-leading by two orders in $\tilde{q}$ in both limits.**

Scaling with temperature of Landau-Khalatnikov damping rate



$$\Gamma_q \simeq \int d^3q_2\, d^3q_3 |\mathcal{A}|^2 \delta(\omega_q + \omega_2 - \omega_3 - \omega_4)$$

- Wave vectors $q = \tilde{q}\frac{k_B T}{\hbar c}$ scale as T
- Integral dominated by almost aligned wave vectors, $\theta = \epsilon|\gamma|^{1/2}\tilde{\theta}$ with $\epsilon = \frac{k_B T}{mc^2}$. Hence solid angle $\propto T^2$
- Energy denominators in $\mathcal{A}$ scale as $q^3 \propto T^3$
- $(\omega_q + \omega_2 - \omega_3 - \omega_4)$ vanishes for a linear spectrum (momentum conservation) and it is $\propto q^3 \propto T^3$ for a curved spectrum

$$\Gamma \sim \left[T^3 \times \underset{\text{solid angle}}{T^2} \right]^2 \left| \frac{T^{3/2} \times T^{3/2}}{T^3} \right|^2 \frac{1}{T^3} = T^7$$

H. Kurkjian, Y. Castin, A. Sinatra, EPL (2016)

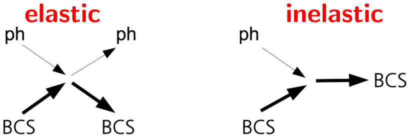
H. Kurkjian, Y. Castin, A. Sinatra, Annalen der Physik (2017).

Can one observe Landau-Khalatnokov damping ?

- Concave dispersion relation otherwise Landau-Beliaev damping is dominant at low temperature

$$\Gamma_q^{\mathrm{LK}} \underset{T\to 0}{\sim} \propto q^3 T^4 \qquad \text{and} \qquad \Gamma_q^{\mathrm{LB}} \underset{T\to 0}{\sim} \propto q T^4$$

- Negligible damping by gapped excitations (rotons in helium or BCS pair-breaking excitation in Fermi gases)



- We calculated the damping by gapped excitations

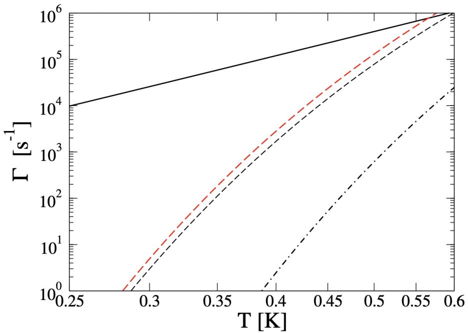
$$\Gamma_q^{\mathrm{rot}} \sim Bq^4 T^{1/2} e^{-\Delta/k_B T}$$

Our expression of B disagrees with Landau-Khalatnikov and with Nicolis-Penco PRB (2018).

Y. Castin, A. Sinatra, H. Kurkjian PRL (2017)+erratum

Landau-Khalatnikov damping in liquid ^{4}He

Phonons $\omega_q = 2\pi \times 165$ GHz ($q = 0.3 Å^{-1}$) ; $P = 20$ bar ($\gamma = -6.9$)

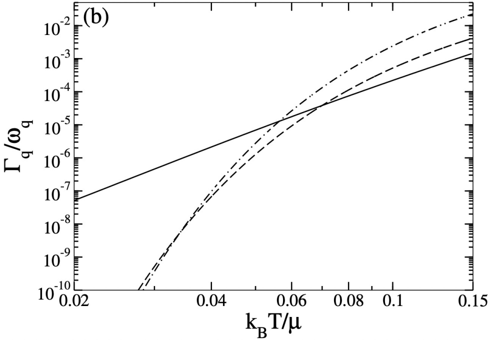


Solid : Landau-Khalatnikov 4-phonon damping.
Dashed : elastic ph-roton processes (Red - original Landau-Khalatnikov)
Dash-dotted : inelastic ph-roton processes

$\Delta/k_B = 7.44$K, $k_0 = 2.05 Å^{-1}$, $c = 346.6$ m/s The very low values of $\frac{\hbar q}{mc} = 0.13$ and of $\frac{k_B T}{mc^2} < 10^{-2}$ justify the use of quantum hydrodynamics

Landau-Khalatnikov damping in a strongly interacting Fermi gas (BCS side)

Phonons $q = mc/2\hbar$ in a Fermi gas on the BCS side $1/k_F a^{-1} = -0.389$, $\gamma \simeq -0.30 < 0$. Parameters of the phonons and the fermionic quasiparticles have been estimated from BCS theory $\mu/\epsilon_F \simeq 0.809$



Solid : Landau-Khalatnikov 4-phonon damping.
Dashed : elastic ph-BCS processes
Dash-dotted : inelastic ph-BCS processes

Implications for the condensate coherence time

Small system isolated from the environnement (ultracold fermionic atoms, microcanonical ensemble)

- **Finite coherence time due to condensate phase spreading**

$$\langle \hat{a}_0^\dagger(t)\hat{a}_0(0)\rangle \propto \langle e^{-i[\hat{\theta}(t)-\hat{\theta}(0)]}\rangle$$

- **Quantum version of 2^{nd} Josephson equation with q-hydro**

$$\hat{\theta} = 2\hat{\phi}_0 \quad ; \qquad -\frac{\hbar}{2}\frac{d\hat{\theta}}{dt} = \mu_0(\hat{N}) + \sum_{q\neq 0}\frac{d\hbar\omega_q}{dN}\hat{b}_q^\dagger\hat{b}_q \equiv \hat{\mu}$$

- **Phonon collisions decorrelate the occupation numbers**
  - **Convex ($\gamma > 0$)** $\rightarrow$ $\mathrm{Var}[\hat{\theta}(t)-\hat{\theta}(0)] \propto \frac{T^4 t}{N}$ **diffusive**

    A. Sinatra, Y. Castin, E. Witkowska PRA (2009)
  - **Concave ($\gamma < 0$)** $\rightarrow$ $\mathrm{Var}[\hat{\theta}(t)-\hat{\theta}(0)] \propto \frac{T^{20/3}t^{5/3}}{N}$ **super-diffusive**

    Y. Castin CRP (2019)

Conclusions

- **We calculated the purely phononic damping rate in a superfluid with concave dispersion relation ($2 \leftrightarrow 2$ processes).**
- **In the limit $\epsilon = k_B T/mc^2 \to 0$ at fixed $\tilde{q} = \hbar cq/k_B T$ we find**

$$\hbar\Gamma \propto (k_B T)^7 \, \tilde{\Gamma}(\tilde{q})$$

where $\tilde{\Gamma}$ is a universal function of $\tilde{q}$.

- **We correct the original Landau-Khalatnikov result in the large-$\tilde{q}$ and small-$\tilde{q}$ limit.**

$$\text{e.g. small-}\tilde{q}\text{ limit}: \quad \hbar\Gamma_q \propto q^3 T^4 \quad (\text{Landau}: \hbar\Gamma_q \propto qT^6)$$

- **We apply the theory to liquid helium and Fermi gases, including competing processes (sub-leading in the $T \to 0$ limit) where the phonons interact with gapped excitations.**
- **The phonon damping has implications for the condensate coherence time.**