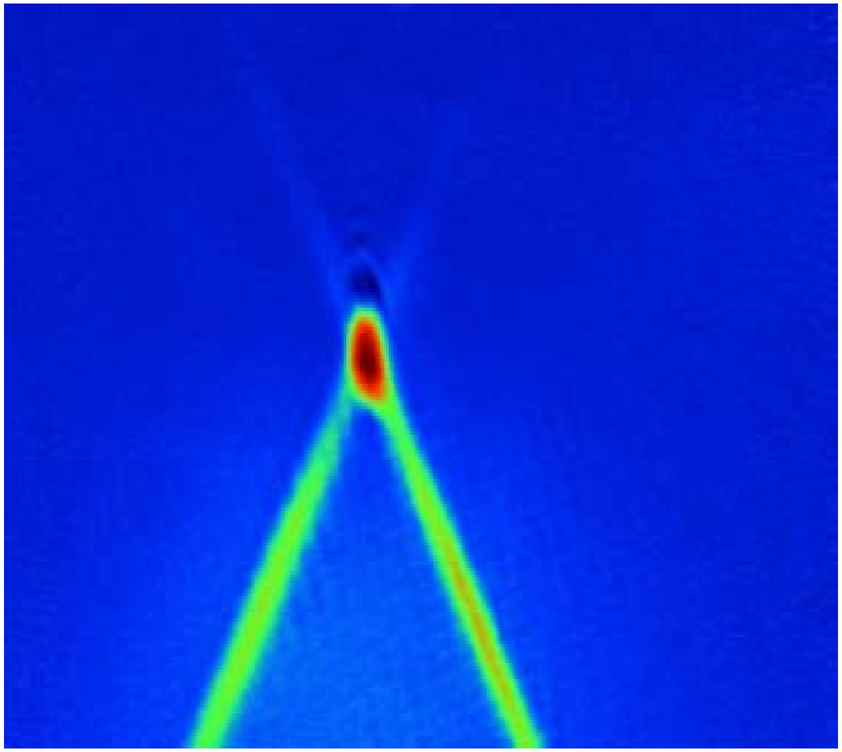
Lecture 2
Bose-Einstein Condensation and Clocks



C. Salomon
Laboratoire Kastler Brossel, Ecole Normale Supérieure, Paris
BIPM, July 24, 2003

Outline

1) Coherence of Bose-Einstein condensate

   Atom lasers

2) The variety of frequency standards

   Frequency stability, accuracy

3) Cold atom fountains

   Current performances

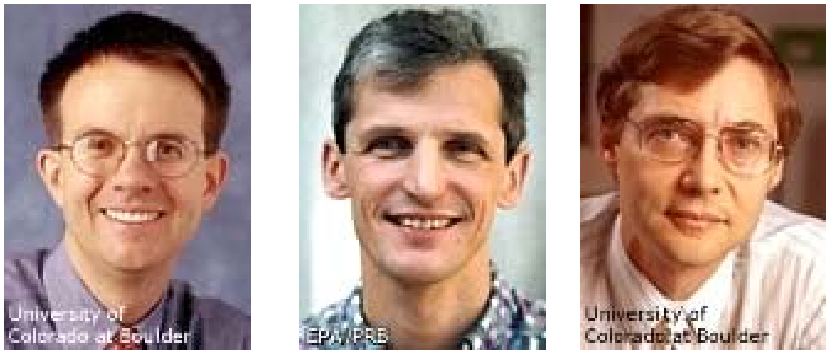
   Limits

4) Applications of ultra-stable clocks

   and perspectives: lecture 3

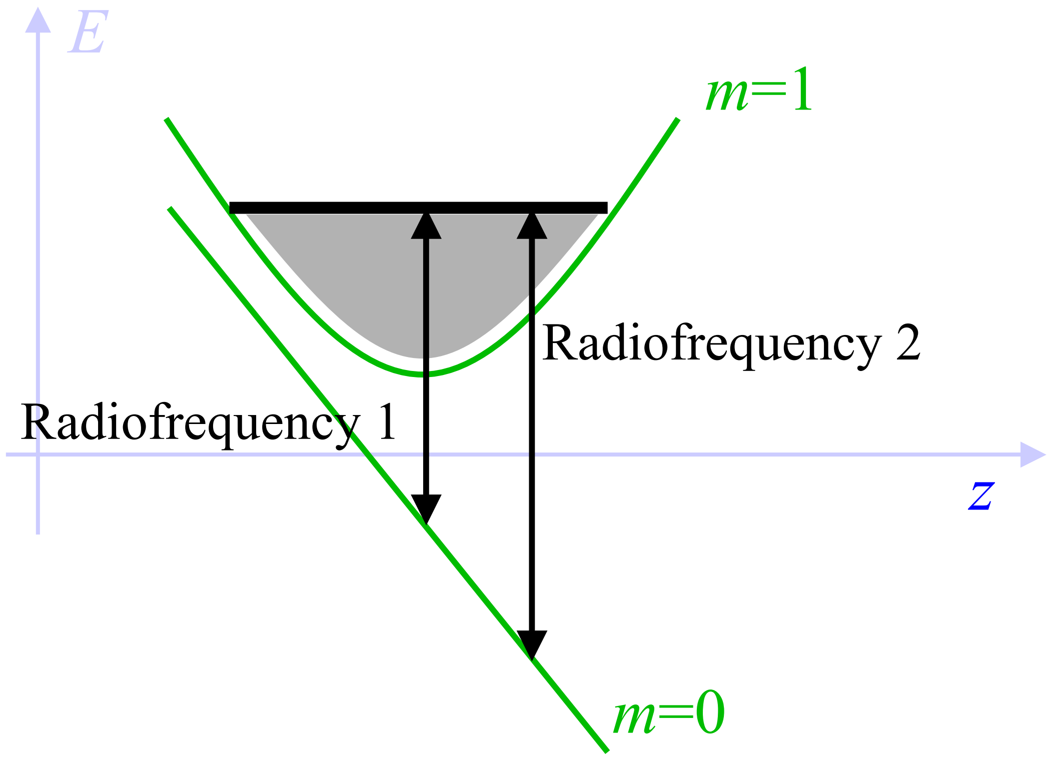
1997 Nobel prize in physics
S. Chu, C. Cohen Tannoudji, W. Phillips
Laser manipulation of atoms

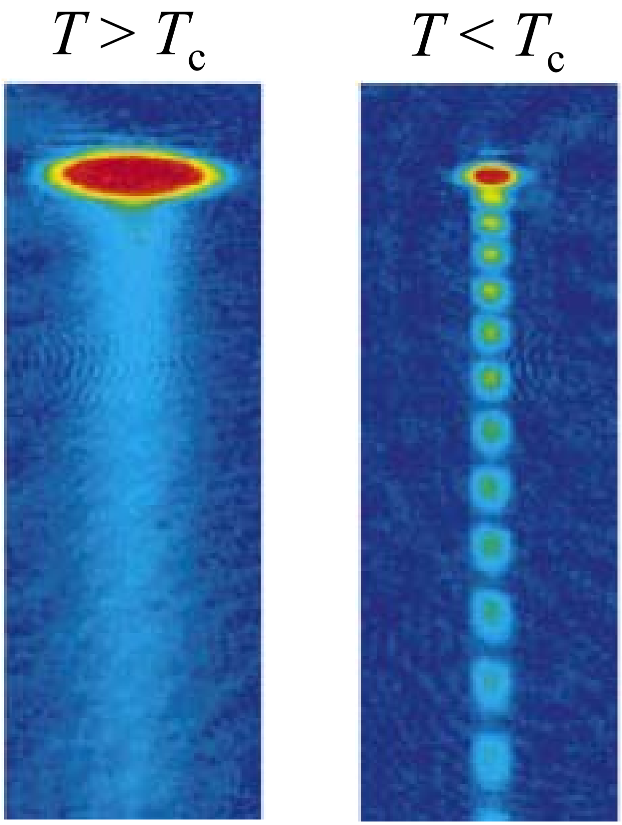
2001 Nobel prize in physics
E. Cornell, W. Ketterle, C. Wieman
Bose-Einstein Condensation



Coherence of Bose-Einstein condensates

Young slit experiment, Munich





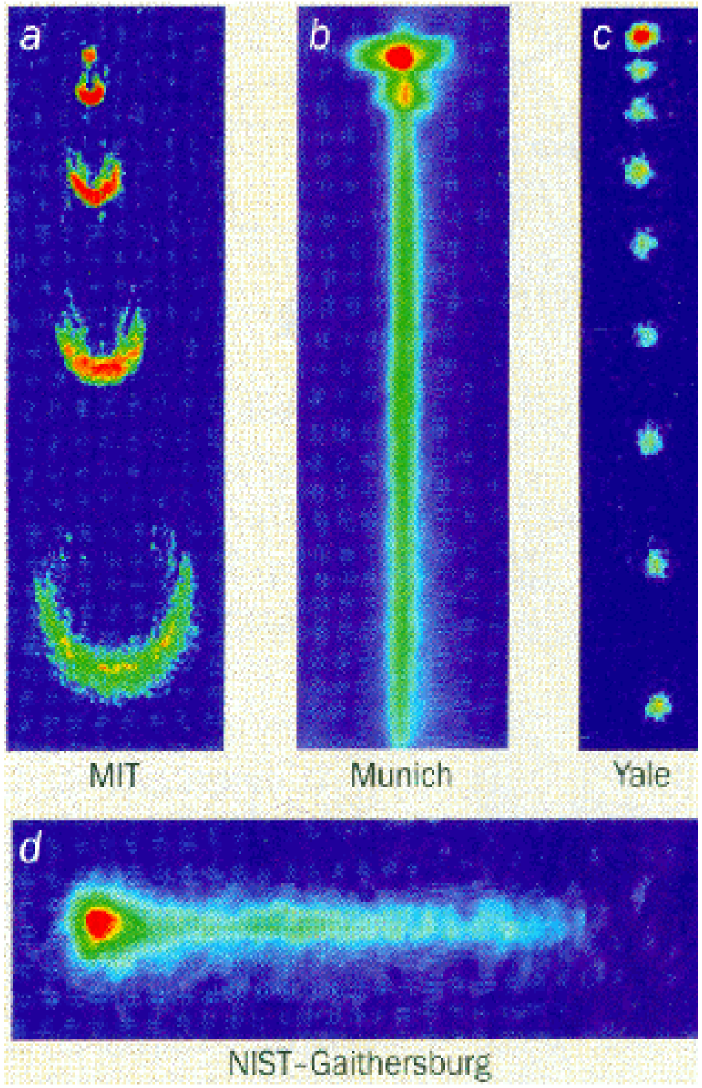
$$n_{out}(z) = \left|\psi_{\text{out}}(z - z_1) + \psi_{\text{out}}(z - z_2)\right|^2$$

$$\propto \frac{1}{\sqrt{z}}\left\{2 + 2\cos\left(q\sqrt{z} + (\omega_1 - \omega_2)t\right)\right\}$$

$$q = m(z_2 - z_1)\sqrt{2g}\,/\,\hbar$$

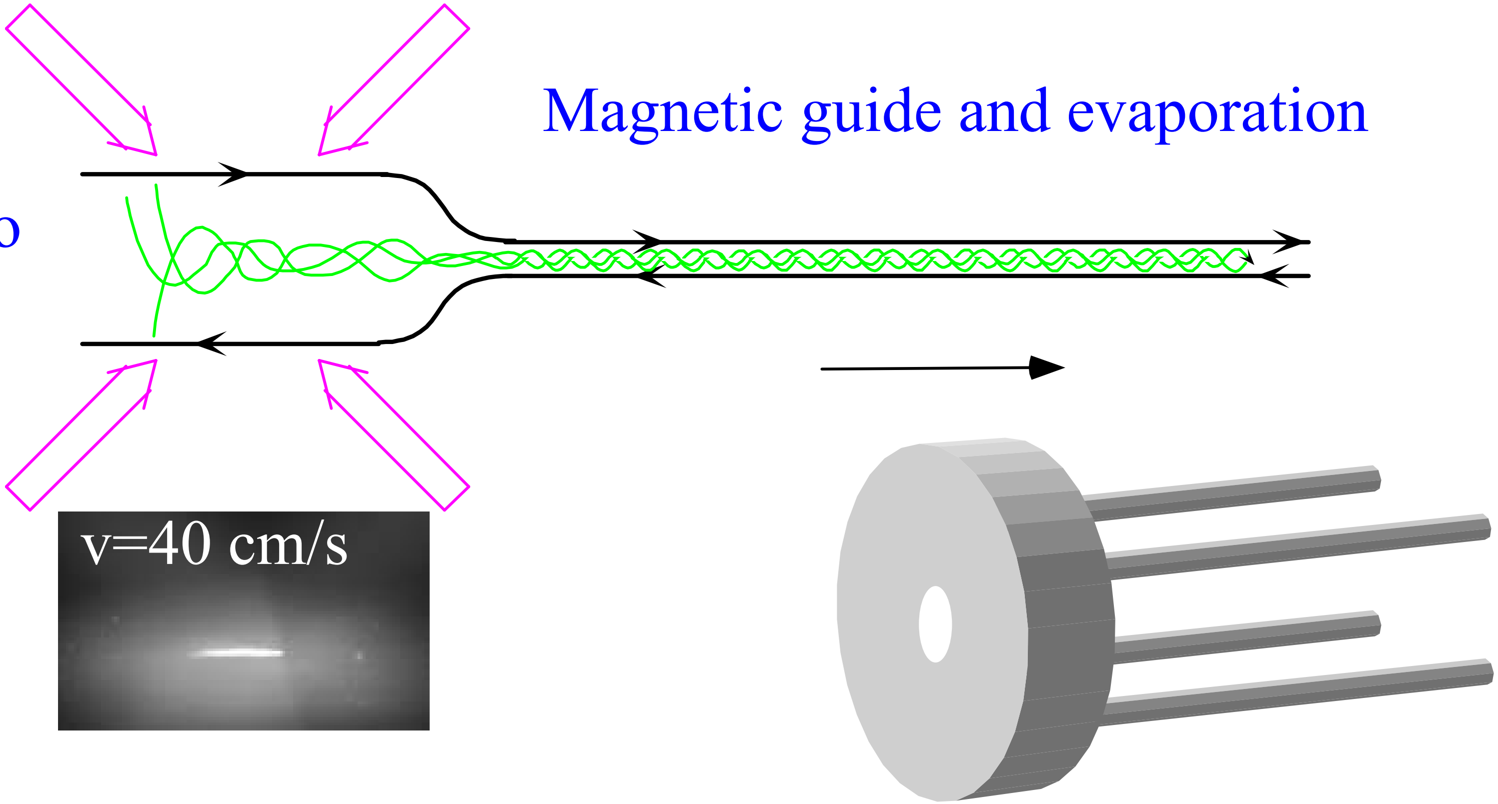
High contrast reveals macroscopic occupation of single quantum state

Atom lasers



But limited flux:
less than 10^6 /second

Towards a continuous atom laser



Magnetic guide and evaporation

2D magneto
-optic trap

v=40 cm/s

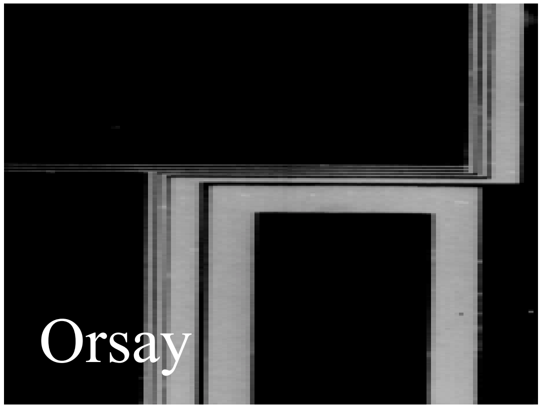
Expected flux: 10^7 condensed atoms /second

In progress at ENS

Condensates on a chip

All wires for magnetic trapping and manipulation are integrated on a chip with a few cm^2 area


Munich, 2001


Orsay

Gold wires ($10\mu m$) on silicon substrate

Greatly simplifies vacuum requirements, power requirements,
But so far, condensates with small numbers ($< 10^5$)
Application to atom interferometry on a chip
Quantum sensors in space
See C. Bordé lectures

Perspectives on BEC

Exploit the quantum coherence:

Can one build a continuous atom laser with high output flux?

Atom interferometry, metrology with coherent matter waves lithography

Study of these new macroscopic quantum systems

Quantum correlated systems, Mott insulator transition

Low dimension BEC : 1D or 2D (Tonks gas, quantum Hall effect)

Molecular BEC

Fermions: towards superfluid transition in Fermi systems

Cold atoms and atomic clocks

Oscillators and Atomic Clocks

Variety of stable periodic phenomena:

1) Macroscopic phenomena:
    - Pendulum, planet orbital periods, binary pulsar emission

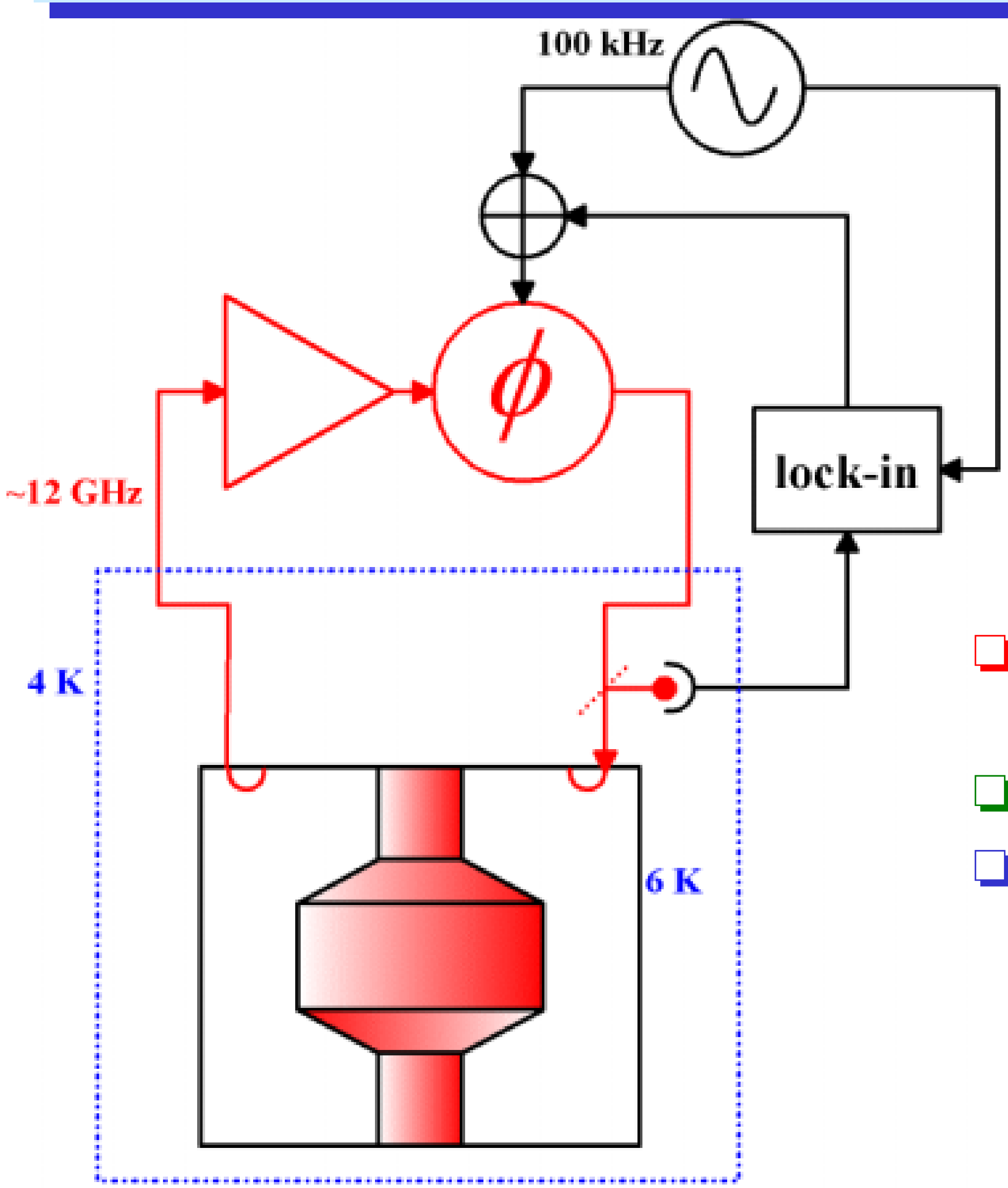
2) Electromagnetic radiation:
    - Solid-state oscillators
    - Atomic clocks:
    - Active or passive oscillators
    - Intrinsic stability of atomic energy levels
    - Control of atomic motion:
    - Trapping and cooling: long interaction times

Variety of transitions in trapped ions and neutral atoms

From microwave to optical domain

Easy link with femtosecond laser systems

Solid-state oscillator:
ex: UWA cryogenic saphire oscillator



A. Mann, A. Luiten,
M. Tobar, E. Ivanov

- Whispering gallery mode : **$Q \sim 4\ 10^9$** , ν ~12 GHz
- Extremum of $\nu(T)$
- Excellent short-term stability

$\sigma_y(\tau) \sim 2\times10^{-16}$

for [2 s-100s]

Atomic clock

Definition of the second :

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground electronic state of cesium 133

Intrinsic stability of atomic energy levels

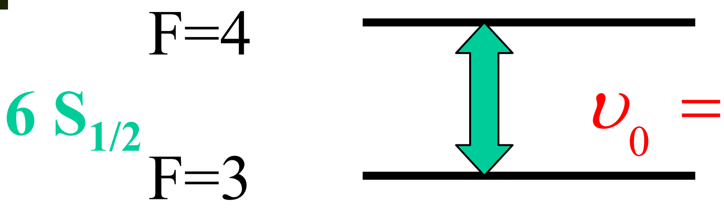
Laser cooling to 1 μK

Corresponding to rms velocity of 7mm/s

1)Fountain geometry

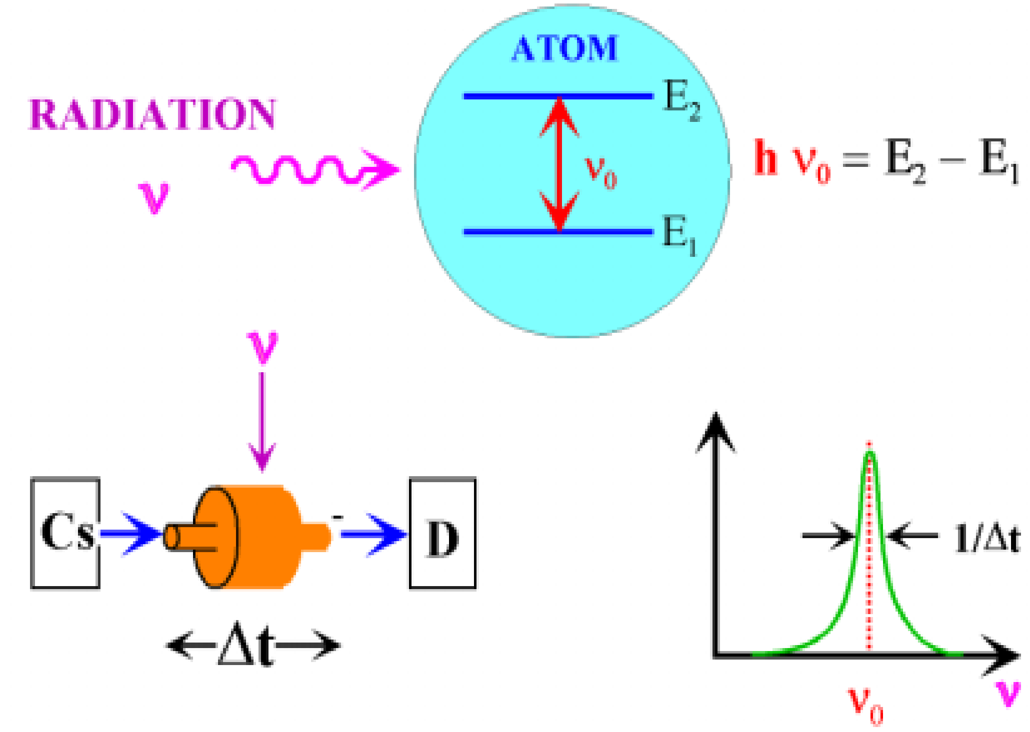
2)Microgravity environment

1. **Stability**
2. **Accuracy**

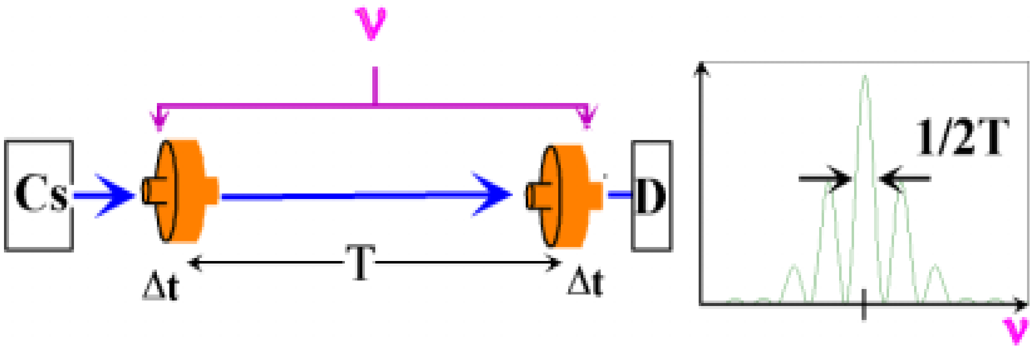


$\upsilon_0 = 9\ 192\ 631\ 770\ \mathbf{Hz}$

Atomic Clock



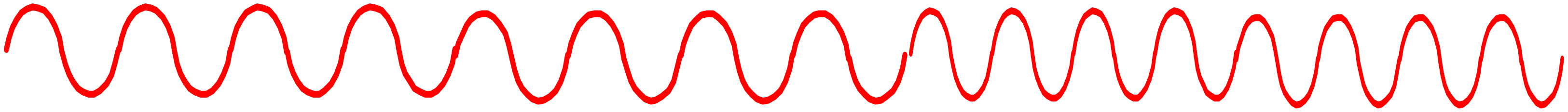
Ramsey method



Conventional Cs atomic beam machines

Accuracy ~ 7 10^{-15}

Frequency stability



$E=E_0\cos(2\pi\nu(t)\ t-kz)$ $\lambda=c/\nu$

$\nu(t)= d\phi/dt$

1) Resonance Quality Factor: $Q = \upsilon_0 / \delta\upsilon$

10^{10} -10^{14}

2) Signal to noise ratio : S/N

Allan standard deviation

$$\sigma_y(\tau) = (1/\pi Q)(N/S)(T_c/\tau)^{1/2}$$

τ : averaging time in *seconds*

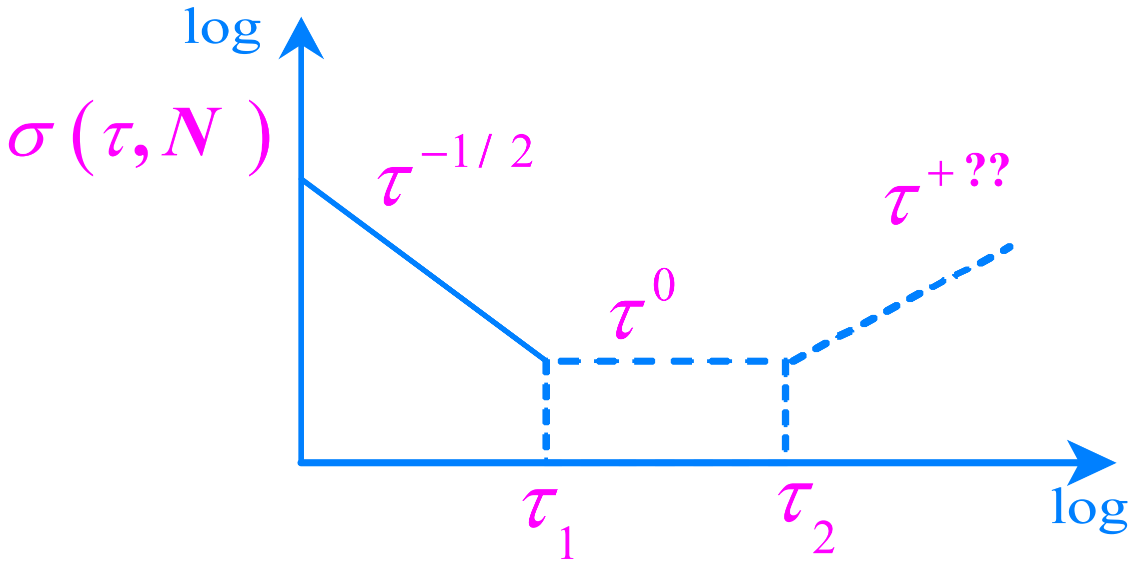
T : duration of one cycle

Frequency stability

Allan variance :

$$\sigma^2(\tau, N) = \frac{1}{2(N-1)} \sum_{k=1}^{k=N-1} \left(\bar{y}_{k+1} - \bar{y}_k\right)^2$$

$$\bar{y}_k = \frac{1}{\upsilon_0 \tau} \int_{t_k}^{t_k+\tau} \upsilon_{\text{clock}}(t)\,dt$$



averaging time τ

Accuracy

To what extent does the clock realize the definition of the second ?

$$\nu_{clock} = \nu_{cesium} + \varepsilon$$

where ν_{cesium} is the frequency of a cesium atom at rest in absence of perturbation

In practice :

Atoms move : Doppler effects, relativistic effects

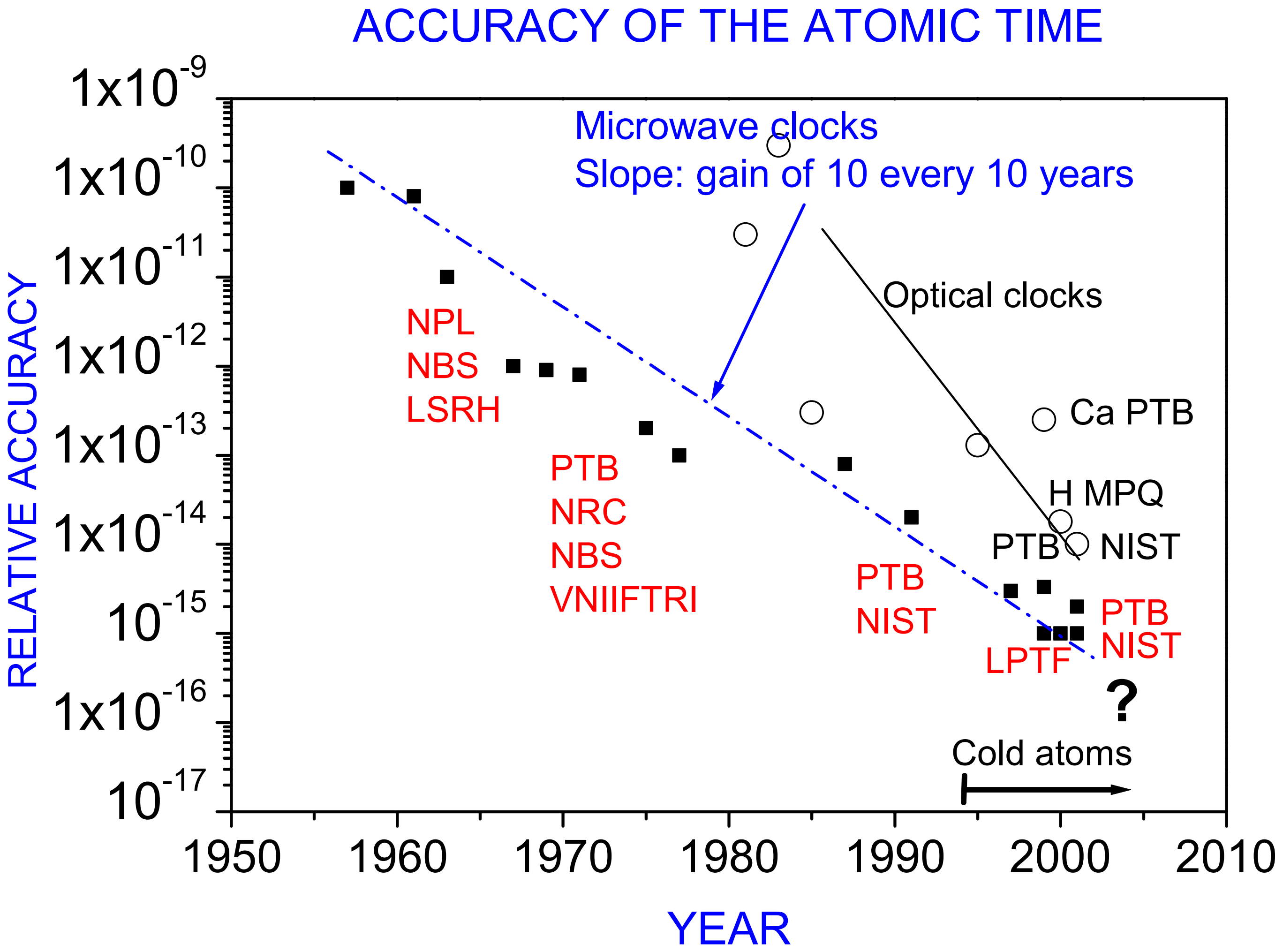
Atoms interact with other atoms or with external world : magnetic field, electric field, blackbody radiation, microwave field servo system,…

Method :

measure and /or calculate all these effects and deduce ε with the highest precision.

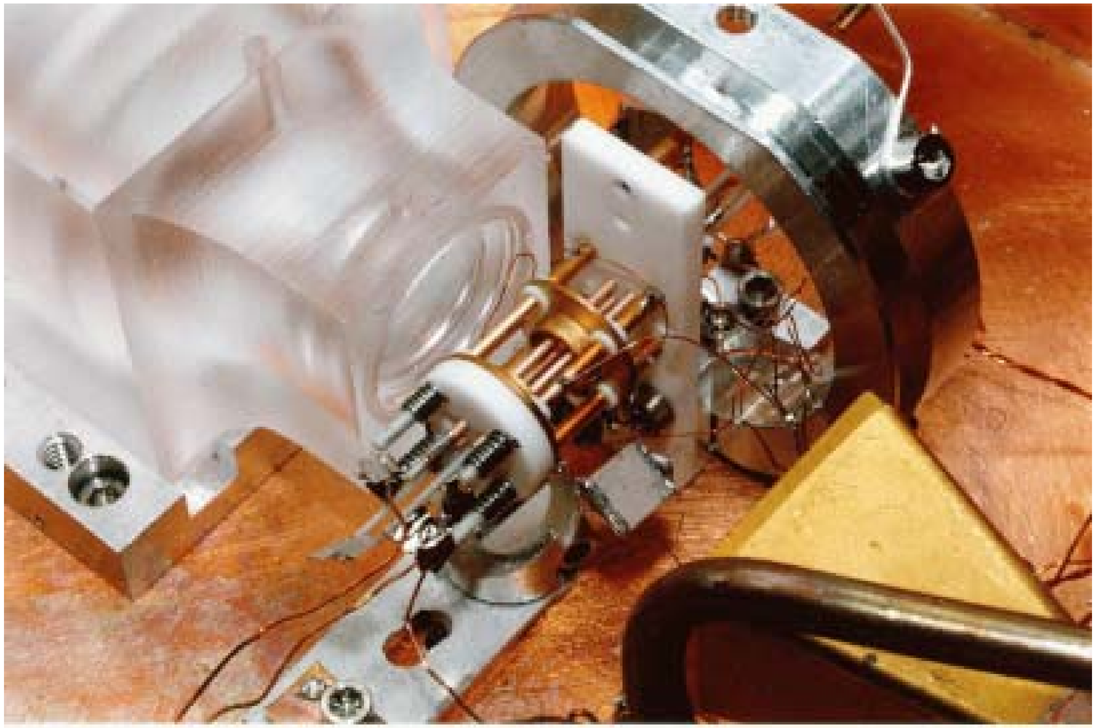
The clock accuracy is the best estimate of ε

Accuracy of the atomic time



Trapped ions or neutral atoms ?

- $Q = \nu/\Delta\nu = 2\,\nu\,T$
- Increase the frequency and increase T:
- Trapped ions :
- T very long but only a few (or one) ions in the same trap. For accuracy they must also be cold.



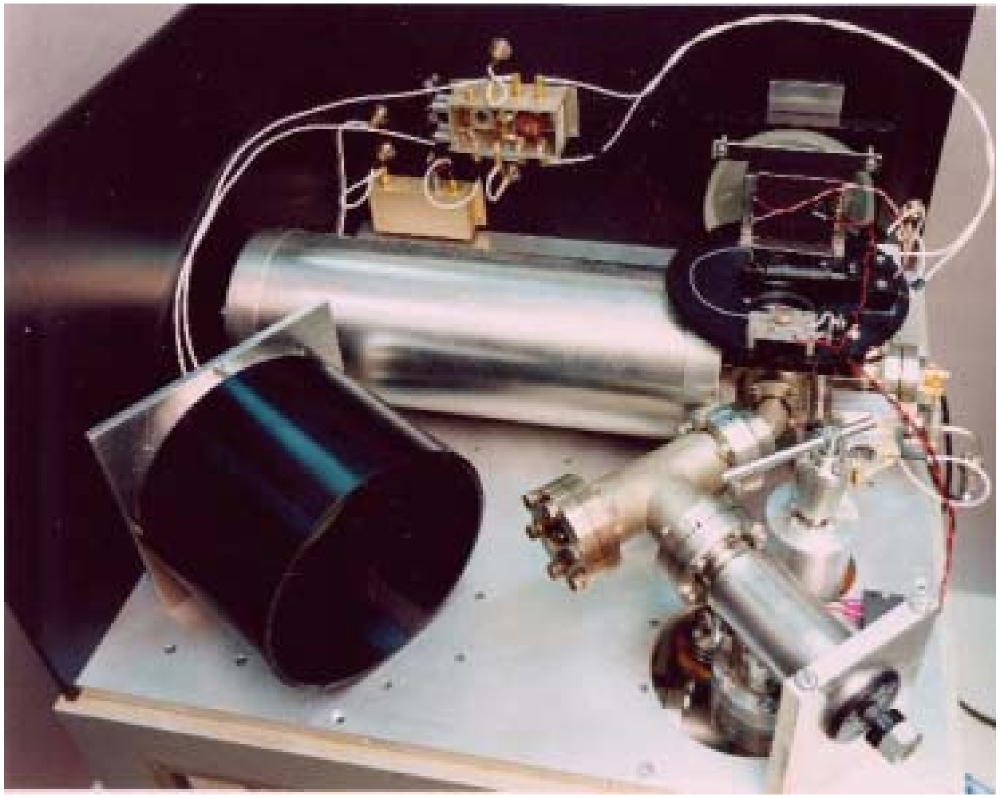
- Hg^+: 40 GHz: NIST 98
- stability: $3.3\ 10^{-13}\ \tau^{-1/2}$
- accuracy $3.3\ 10^{-15}$

Trapped ions (2)

- Hg+: 40 GHz;Linear trap
- Lamped pumped, JPL 90-’02
- stability: $3\ 10^{-14}\ \tau^{-1/2}$

  down to $7\ 10^{-16}$
- Simplicity and compactness,
- continuous operation,
- suitable for space applications
- Yb^{+} : CSIRO
- Optical transitions in

  Ba+, In+, Cd+, S r+ …
- NRC, NPL, PTB...



UV spectroscopy of ^{199}Hg $^{+}$ with an ultrastable laser

R. Rafac, B. Young, J. Beall, W. Itano, D. Wineland and J. Bergquist, PRL **85,** 2462 (2000)

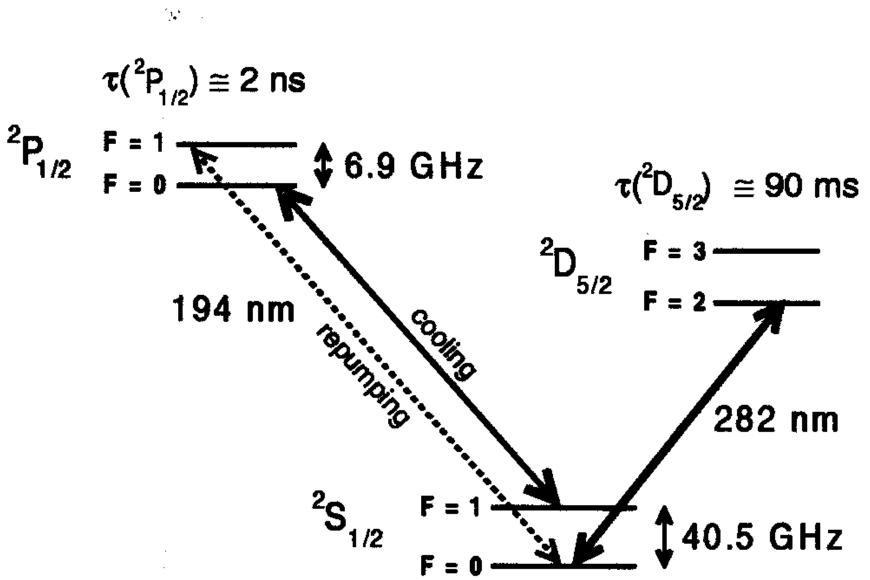
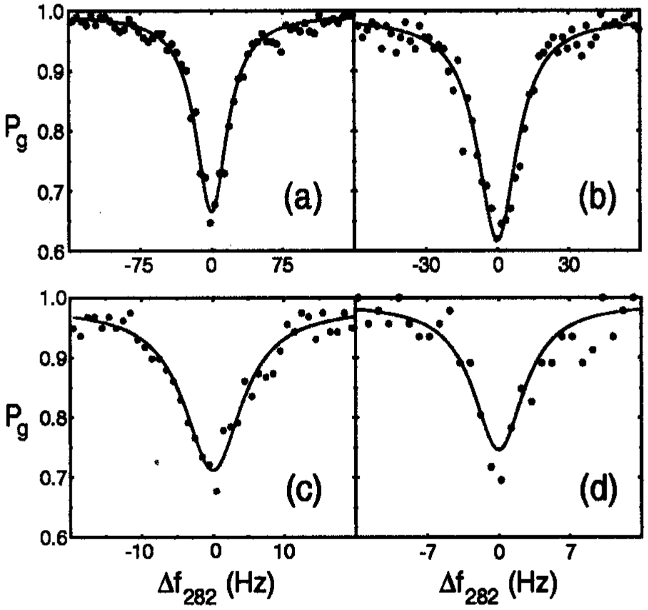


FIG. 1. Partial energy level diagram of ^{199}Hg^{+} with the transitions of interest indicated.



Width

40 Hz

20 Hz

10 Hz

6 Hz

Quality factor: Q = $\nu/\Delta\nu=1.6\ 10^{14}$

Highest Q ever achieved

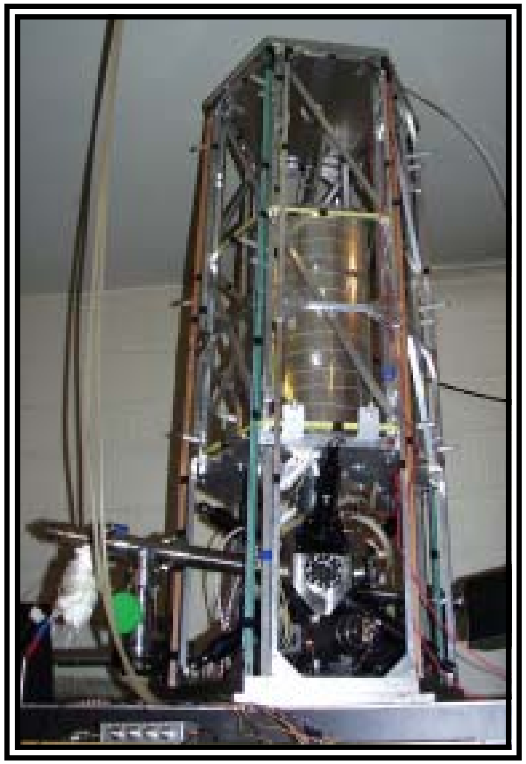
Current accuracy: 10^{-14}

Cold neutral atoms

- On Earth: fountains T~ 1s
- In space: T~10 s
- A lot of atoms at the same time, hence good S/N at short term.
- Microwave domain: Cs, Rb,
- Optical domain…Ca, Mg, Sr, Ag,…

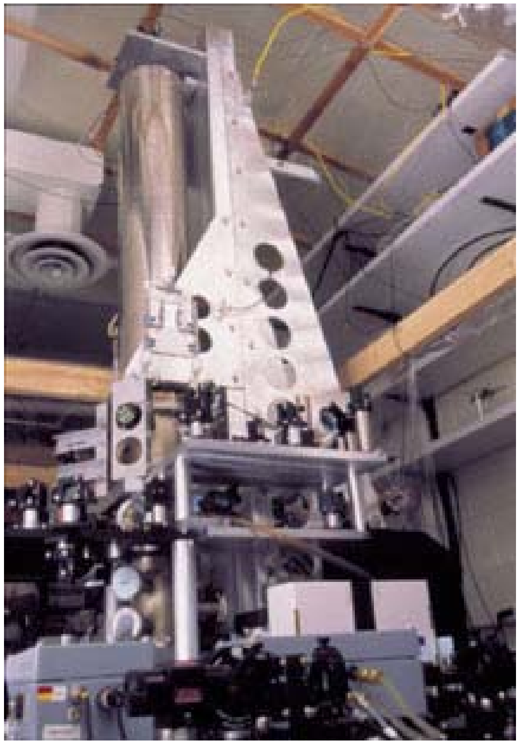
Atomic Fountains

8 fountains in operation at SYRTE, PTB, NIST, USNO, Penn St, IEN, ON. 5 with accuracy at 1 10^{-15} . More than 10 under construction.



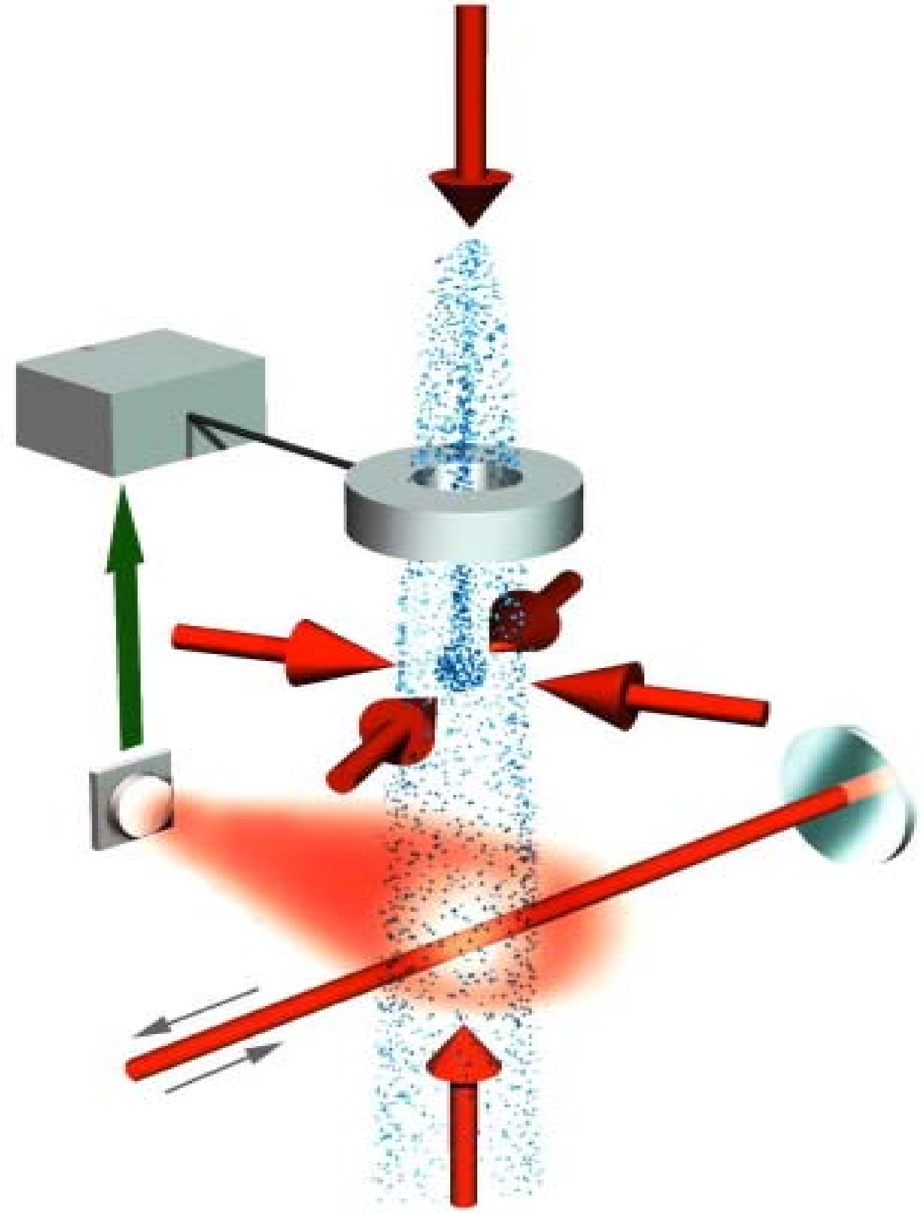
BNM-SYRTE, FR

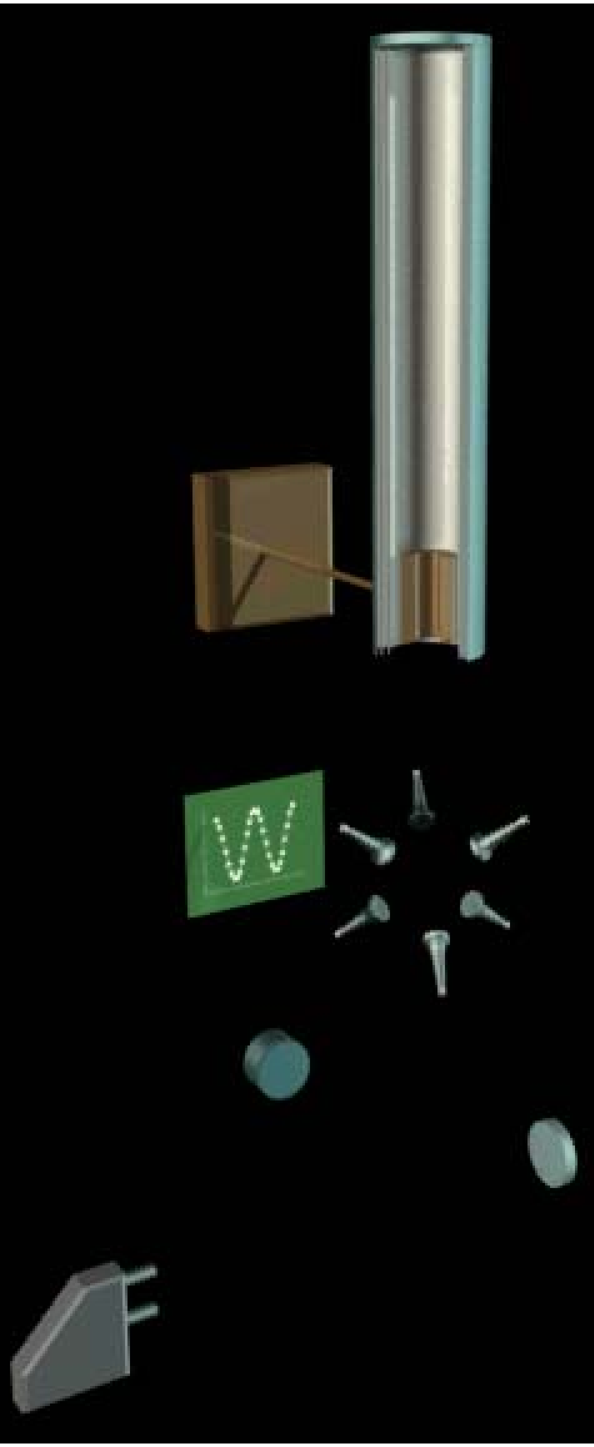
PTB, D



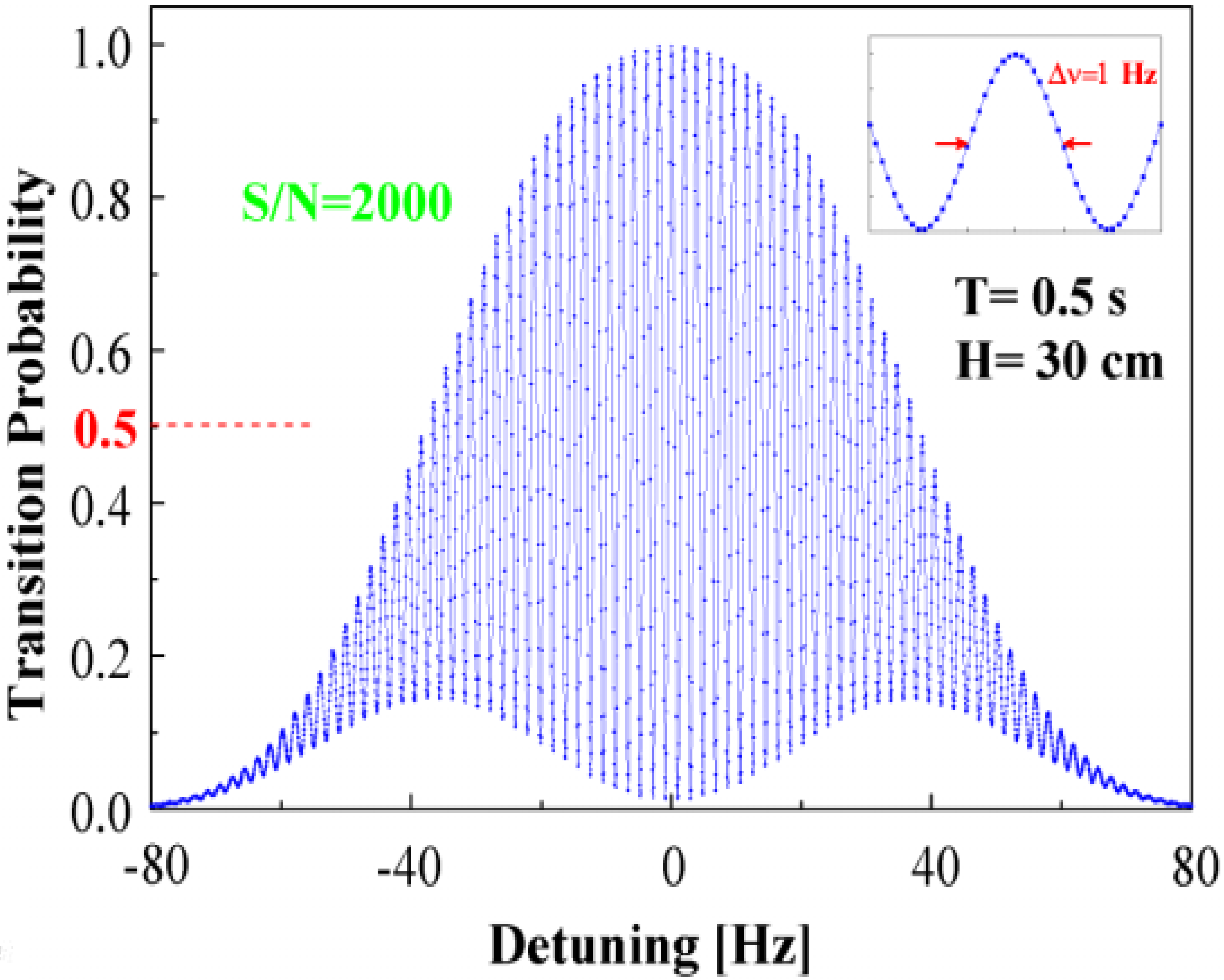
NIST, USA

Atomic Fountain principle





Ramsey fringes in atomic fountain



BNM BUREAU NATIONAL DE MÉTROLOGIE

G. Santarelli et al., PRL 82, 4619 (1999)

The fundamental noise in the fountain : The quantum projection noise

W.Itano et al. (1993)

State of the system : $|\psi>=\alpha|g>+\beta|e>$

The measurement projects the system in |g> or|e>

With probabilities $p=|\alpha|^2$ and $1-|\alpha|^2$

When p=1/2, an atom has equal chances to be found in |g> or|e>

For **N** uncorrelated atoms : p=1/2 to within **$1/N^{1/2}$**

The expected fountain frequency stability is :

$\sigma(\tau)$: Allan standard deviation

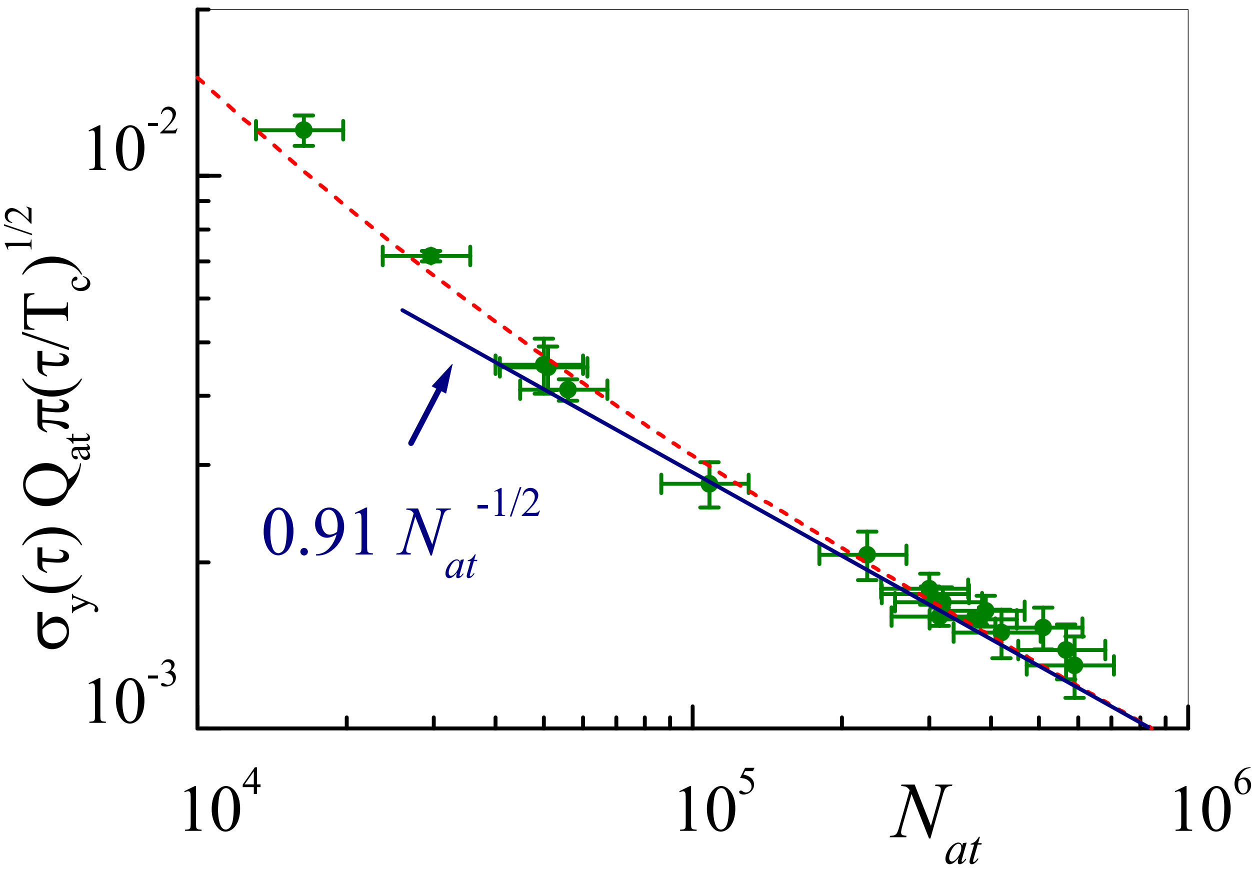
$$\sigma(\tau) = 1/\pi\left(\delta\upsilon/\upsilon_{cs}\right)\left(1/N^{1/2}\right)\left(T_c/\tau\right)^{1/2}$$

Example : $N = 10^6$, $\delta\upsilon = 1 Hz$, $T_c = 1.1 s$, $\upsilon_{cs} = 9.2 GHz$

$$\boxed{\sigma(\tau) = 3\ 10^{-14}\tau^{-1/2}}$$

Quantum projection noise

Quantum Limited Operation of a Cs Fountain up to $N_{at}\sim 6\ 10^5$

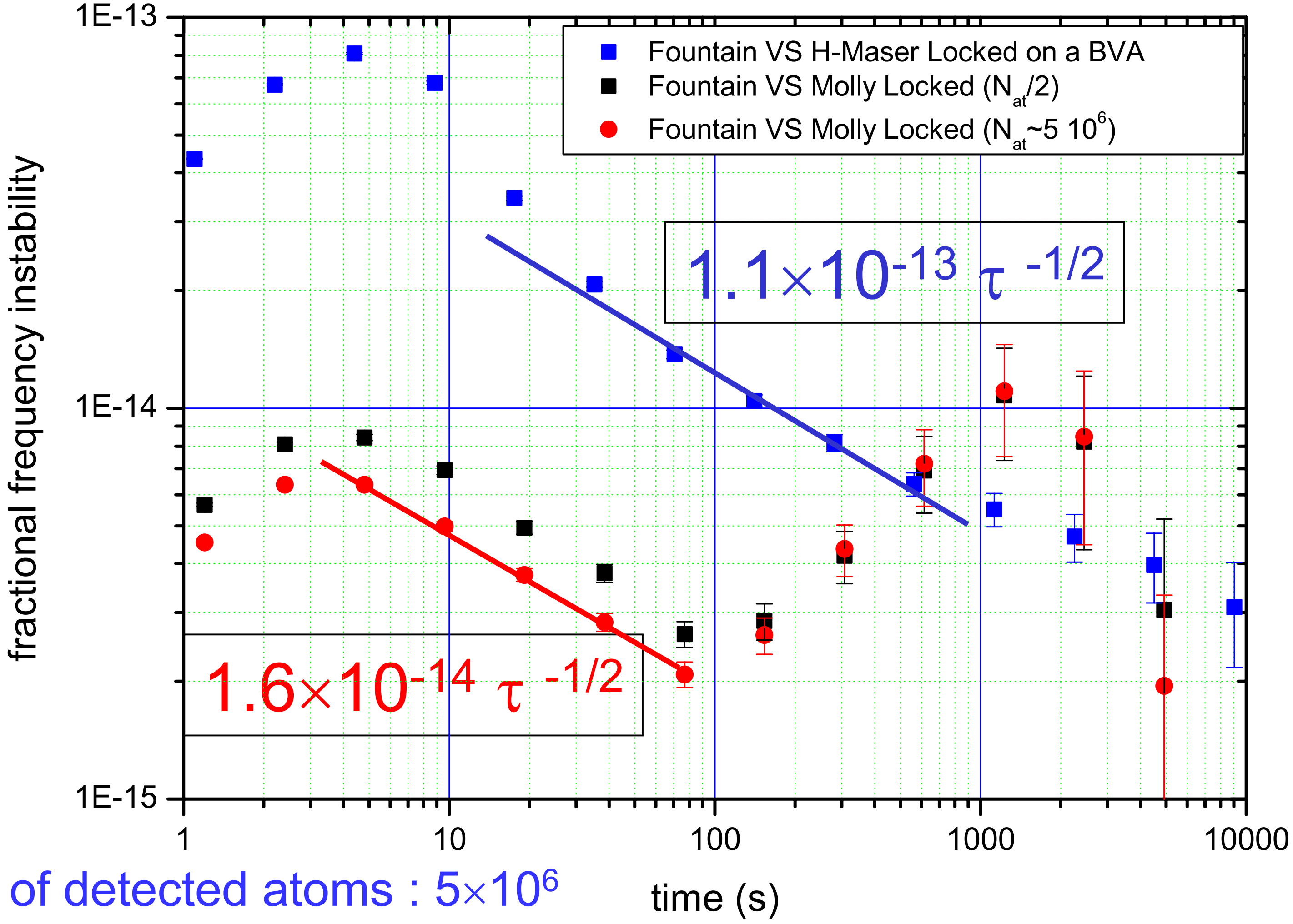


For $N_{at}\sim 6\ 10^5$ atoms, the frequency stability is

$$\sigma_y(\tau) \sim 4\ 10^{-14}\ \ \tau^{-1/2}$$

G.Santarelli et al. PRL,**82**,4619 (1999)

Atomic fountain vs cryogenic oscillator
Improved stability



S. Bize
et al.
ICOLS
2003

Number of detected atoms : 5×10^6

Mostly quantum limited => stability below 10^{-14} at 1s will be obtained by increasing the atom number; interesting with Rb

Accuracy of SYRTE Fountains

Effect	**Correction [10^{-15}]**	**Uncertainty [10^{-15}]**
2nd order Zeeman	-133	<0.1
Blackbody Radiation	17.6	0.3
Microwave spectrum	0	0.2
First Order Doppler	0	<0.4
Microwave leaks	0	0.1
Pulling by other lines	0	0.4
Second Order Doppler and Gravity	~0	<<0.1
Microwave Recoil	0	0.3
Background Gas collisions	0	<0.1
Cold collisions + Cavity pulling Molasses	2.4	0.5
Total		**8×10^{-16}**

Participants

M. Abgrall, S. Zhang, Y. Macsimovic, F. Allard,
J. gruenert, Y. Sortais, H. Marion, L. Cacciapuoti
D. Calonico, F. Pereira, C. Mandache,
S. Bize, P. Lemonde, P. Laurent,
G. Santarelli, P. Rosenbuch
P. Wolf, A. Clairon
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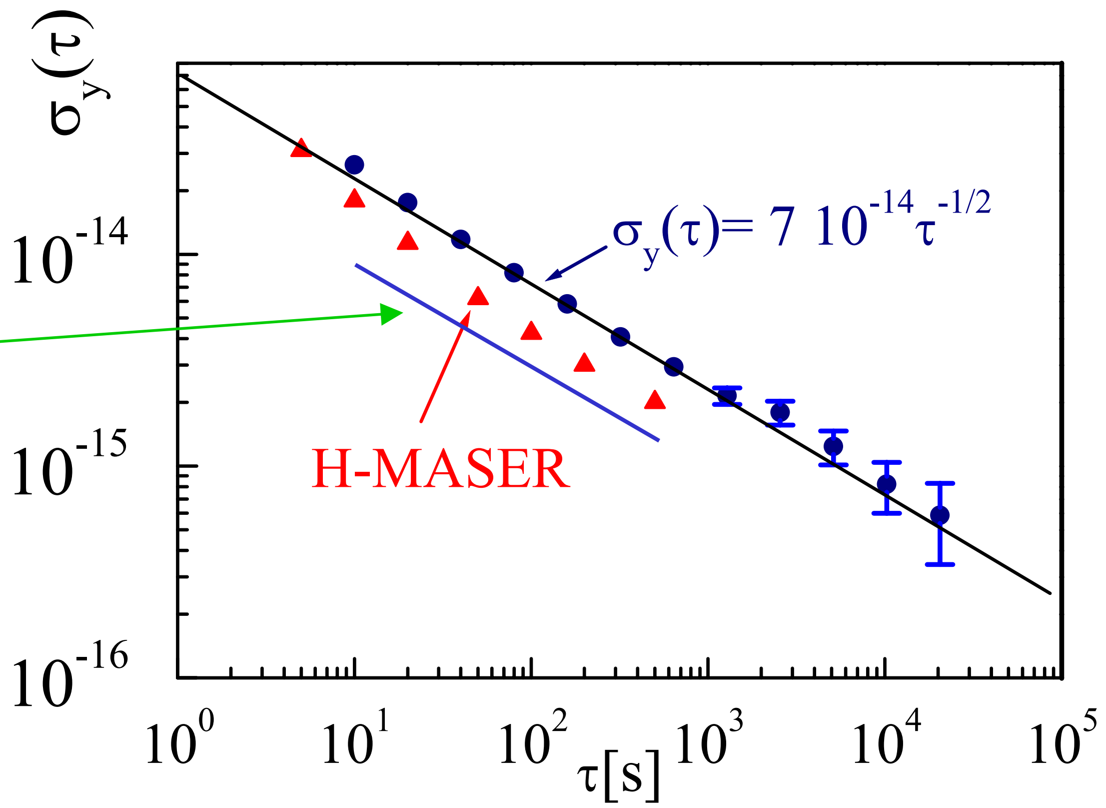
Fountain frequency stability with UWA cryogenic oscillator

Recent result

$3.5\ 10^{-14}\ \tau^{-1/2}$



- ☐ **Fountain intrinsic stability : $5\ 10^{-14}\ \tau^{-1/2}$**
- ☐ **Lowest measured value : $6\ 10^{-16}$ at 20000 s, probably limited by the H-maser flicker floor**
- ☐ **Direct comparisons between fountains are ongoing**