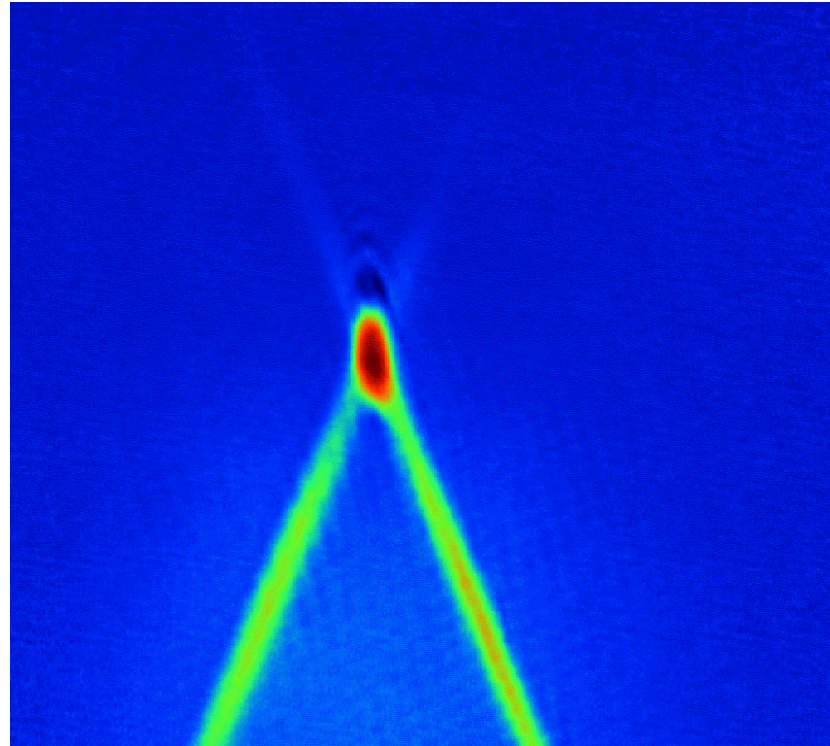


Cold Atoms, Bose–Einstein Condensation Interferometry and Clocks



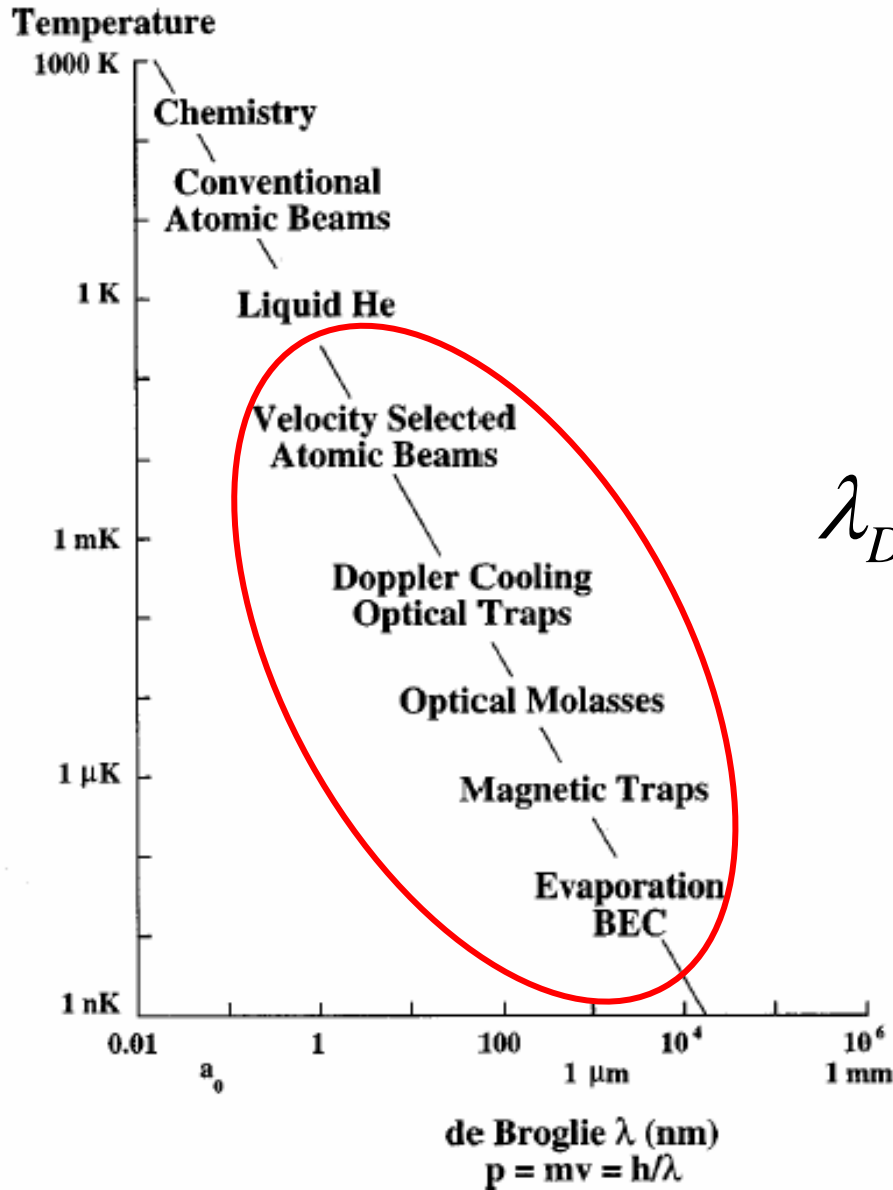
C. Salomon

Laboratoire Kastler Brossel, Ecole Normale Supérieure, Paris
SIGRAV School, Firenze, September 2006

Outline of lectures

- Lecture 1
 - Laser cooling
 - Optical molasses, traps
 - Bose-Einstein condensation and applications
- Lecture 2
 - Atom interferometry
- Lecture 3
 - Atomic clocks: basics
- Lecture 4
 - Fundamental tests and space applications

Temperature scale



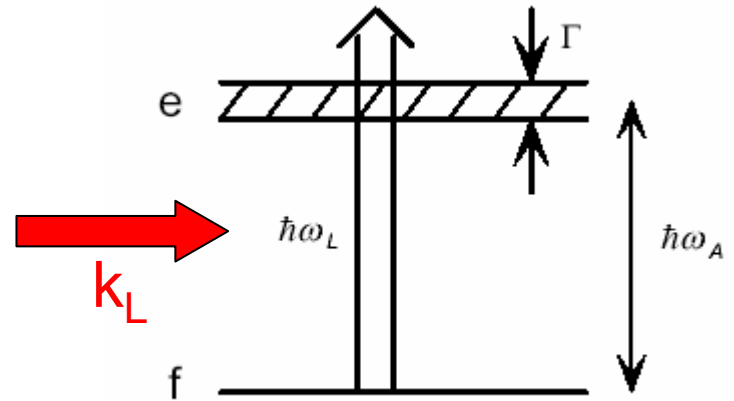
$$\lambda_{DB} = \frac{h}{Mv}$$

Radiation Pressure

Two-level atom

Γ^{-1} lifetime : 32 ns Cesium

$\delta = \omega_L - \omega_A$ detuning



Interaction between atoms and electromagnetic field:
electric **dipole interaction**

Rabi frequency $\hbar\Omega_1(\mathbf{r}) = -(\mathbf{d} \cdot \boldsymbol{\epsilon}_L(\mathbf{r})) \mathcal{E}_L(\mathbf{r})$

Change of velocity due to the absorption of a photon:

$$v_{\text{rec}} = \frac{\hbar k_A}{m} = \frac{h}{m\lambda_A} \quad 3,5 \text{ mm s}^{-1} \quad \text{cesium atoms}$$

<<< thermal velocity

Radiation Pressure

Radiation pressure

$$F_{PR} = \frac{\hbar \mathbf{k}_L \Gamma}{2} \frac{s(\mathbf{R})}{1 + s(\mathbf{R})} = \hbar \mathbf{k}_L \Gamma \sigma_{ee}$$

$\Gamma \sigma_{ee}$ average rate of absorption/spontaneous emission cycles

$$F_{PR} \leq \hbar \mathbf{k}_L \Gamma / 2$$

Each cycle transfers a $\hbar \mathbf{k}_L$ momentum in average, the contribution of spontaneous emission is zero in average.

Spontaneous emission leads to a diffusion in the velocity space

Atomic beam slowing

Resonance condition

$$\omega_L - k_L V(z) = \omega_A(z)$$

$$V^2(z) = V_0^2 - 2a_{\max} z$$

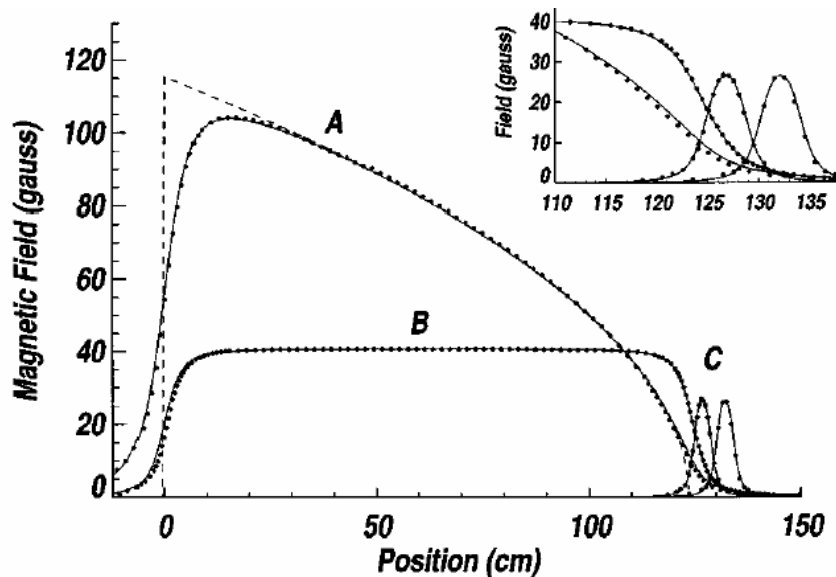
$$z_0 = V_0^2 / (2a_{\max})$$

inhomogeneous magnetic field

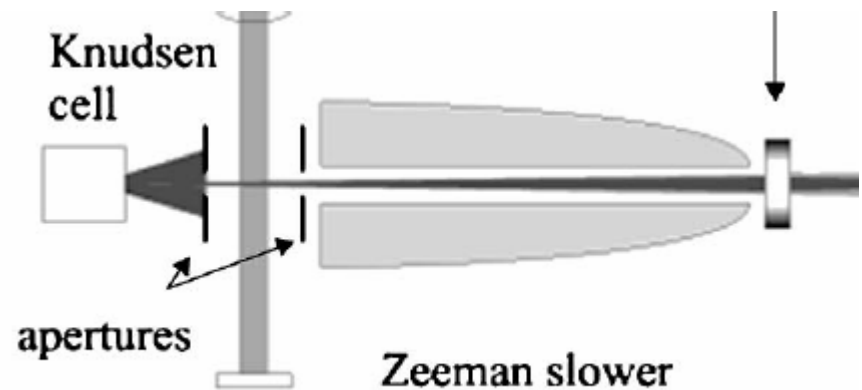
$$F(z, v) = \frac{\hbar k \Gamma}{2} \frac{s_0}{1 + s_0 + 4[\delta + kv - \mu' B(z)/\hbar]^2 / \Gamma^2}$$

$$s_0 \equiv I/I_0$$

$$I_0 = \frac{\Gamma \hbar \omega_0^3}{12 \pi c^2}$$



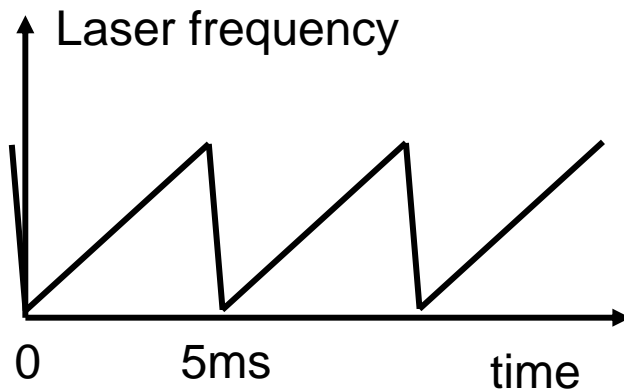
$$B(z) = B_b + B_p \sqrt{1 - z/z_s}$$



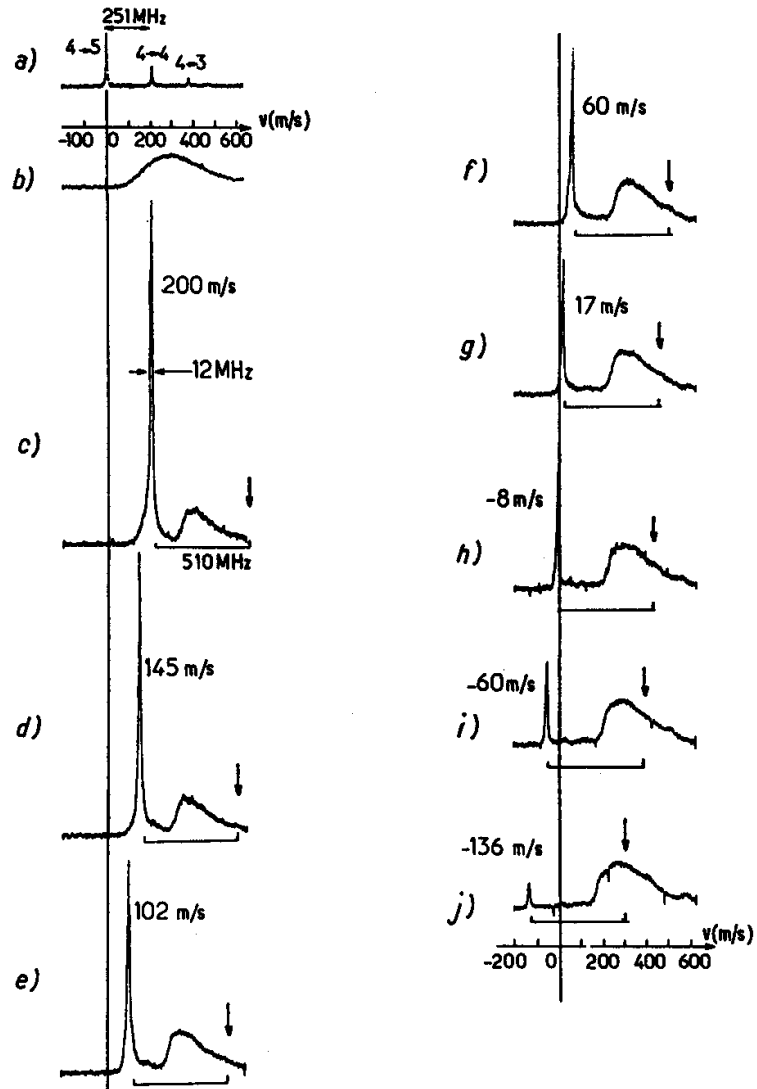
Atomic beam slowing (2)

- Chirped cooling

$$\omega_L(t) - k_L V(t) = \omega_A$$



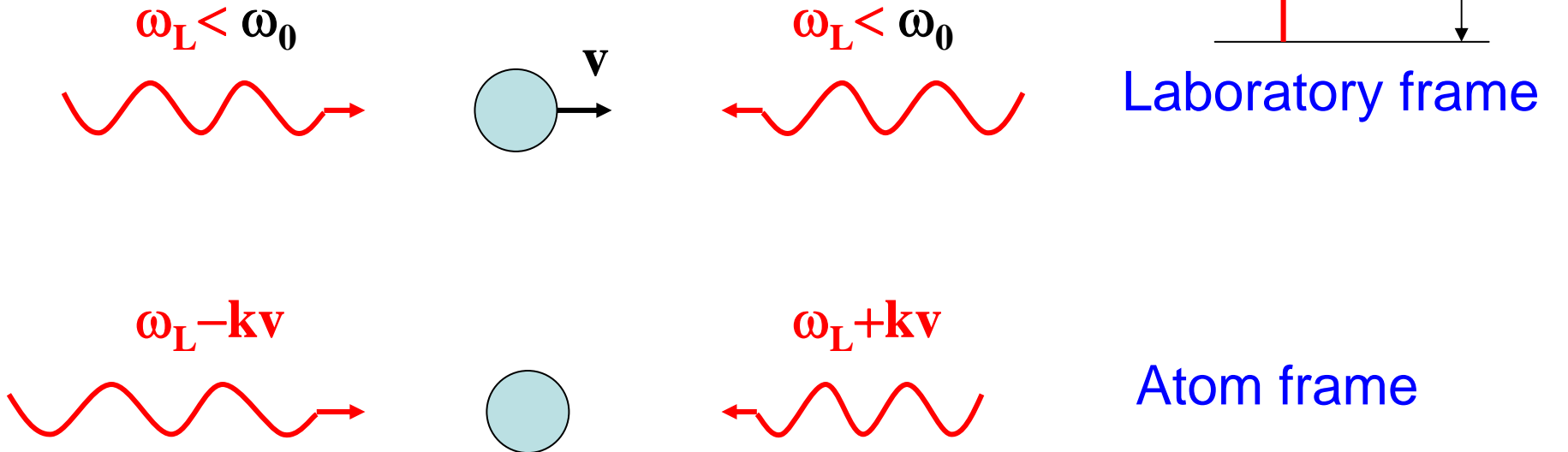
Extremely simple with diode lasers !



Hänsch, Schawlow
Wineland, Dehmelt
1975

Doppler Cooling

Doppler effect



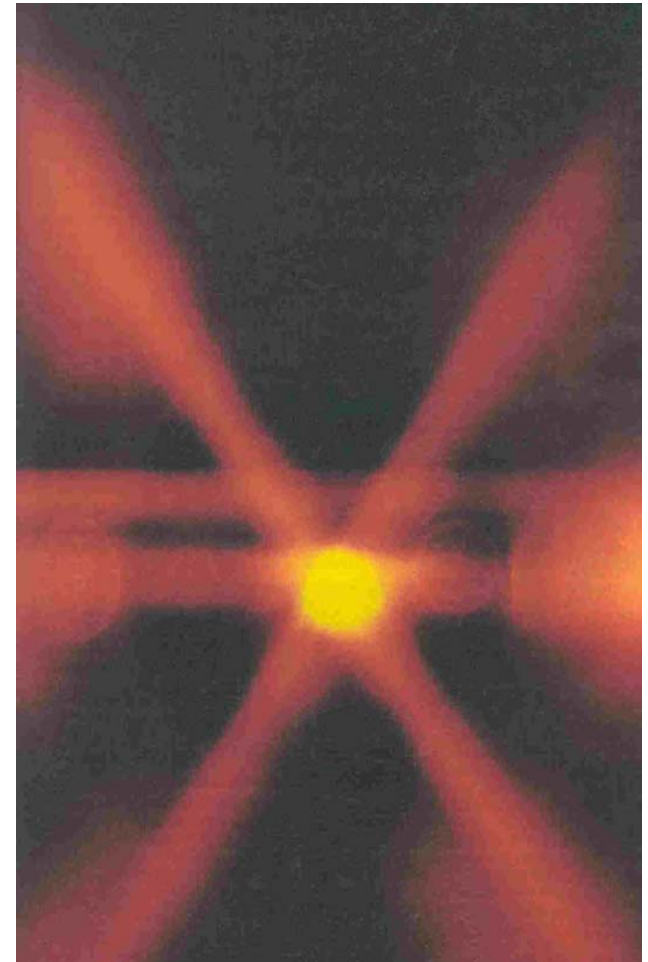
Absorption of the photon $\omega_L + kv$, followed by a spontaneous emission equiprobable in all directions of the space.

Act as: $\mathbf{F} = -\alpha \mathbf{v}$ (friction force) = $m d\mathbf{v}/dt$

Optical molasses

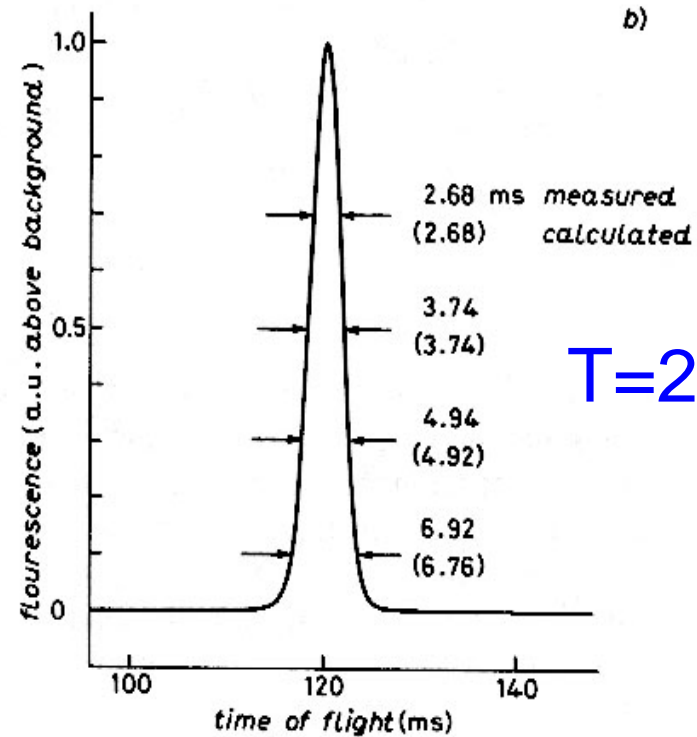
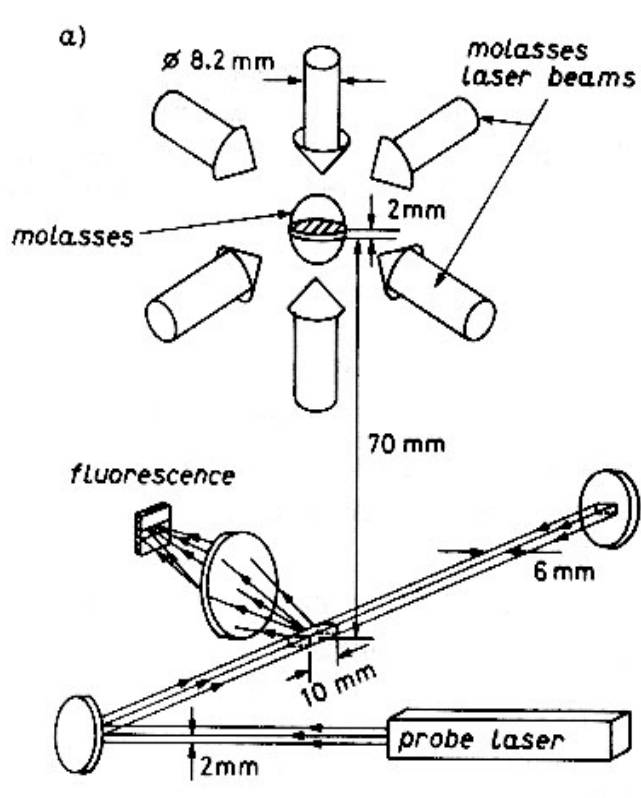


S. Chu, Scientific American, 174, 1992



Na molasses

Temperature measurement



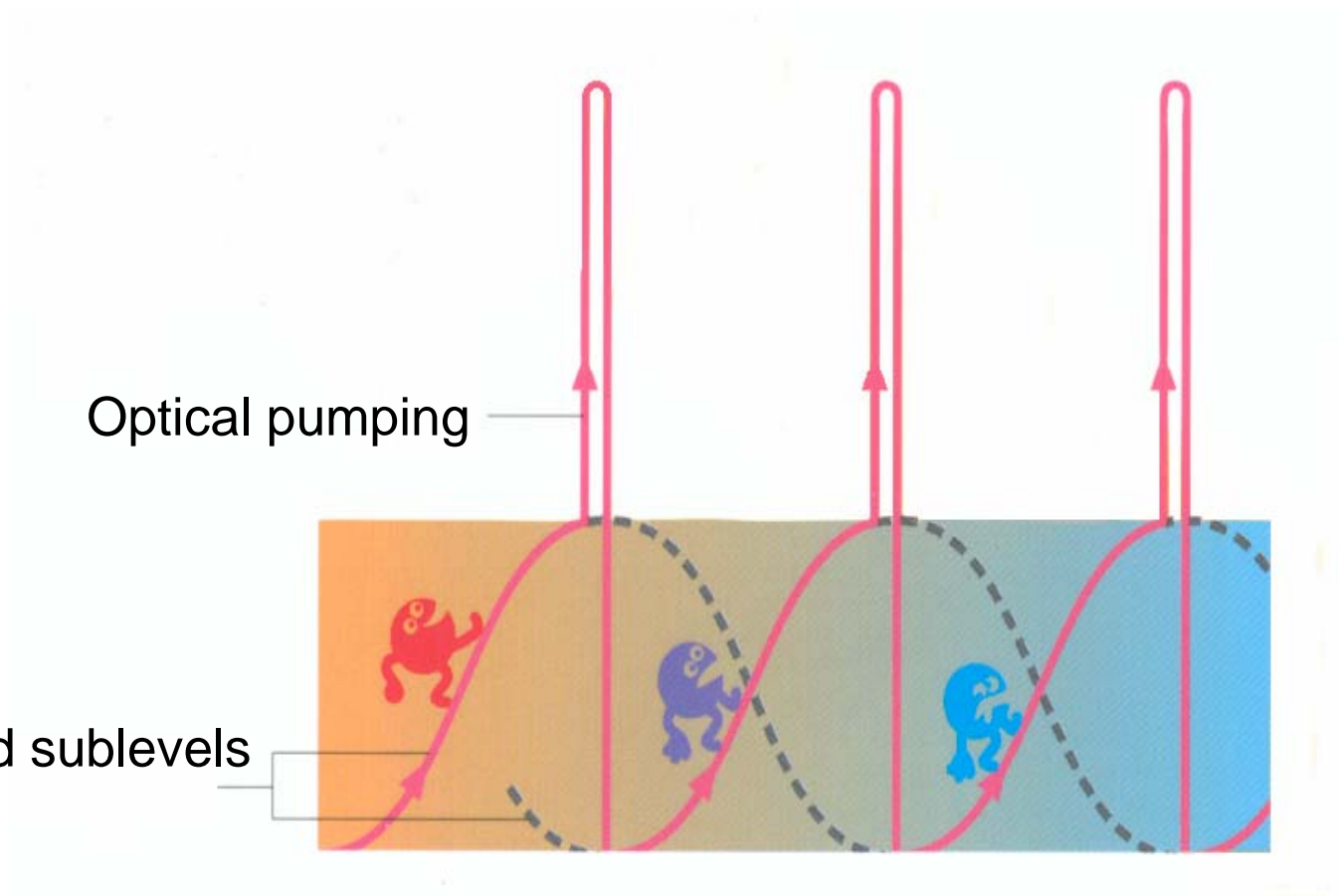
$T = 2.5 \mu\text{K}$

$$\frac{1}{2} k_B T = \frac{1}{2} M V_{rms}^2$$

$$V_{rms} = 12 \text{ mm/s}$$

Sisyphus cooling

J. Dalibard, C. Cohen-Tannoudji

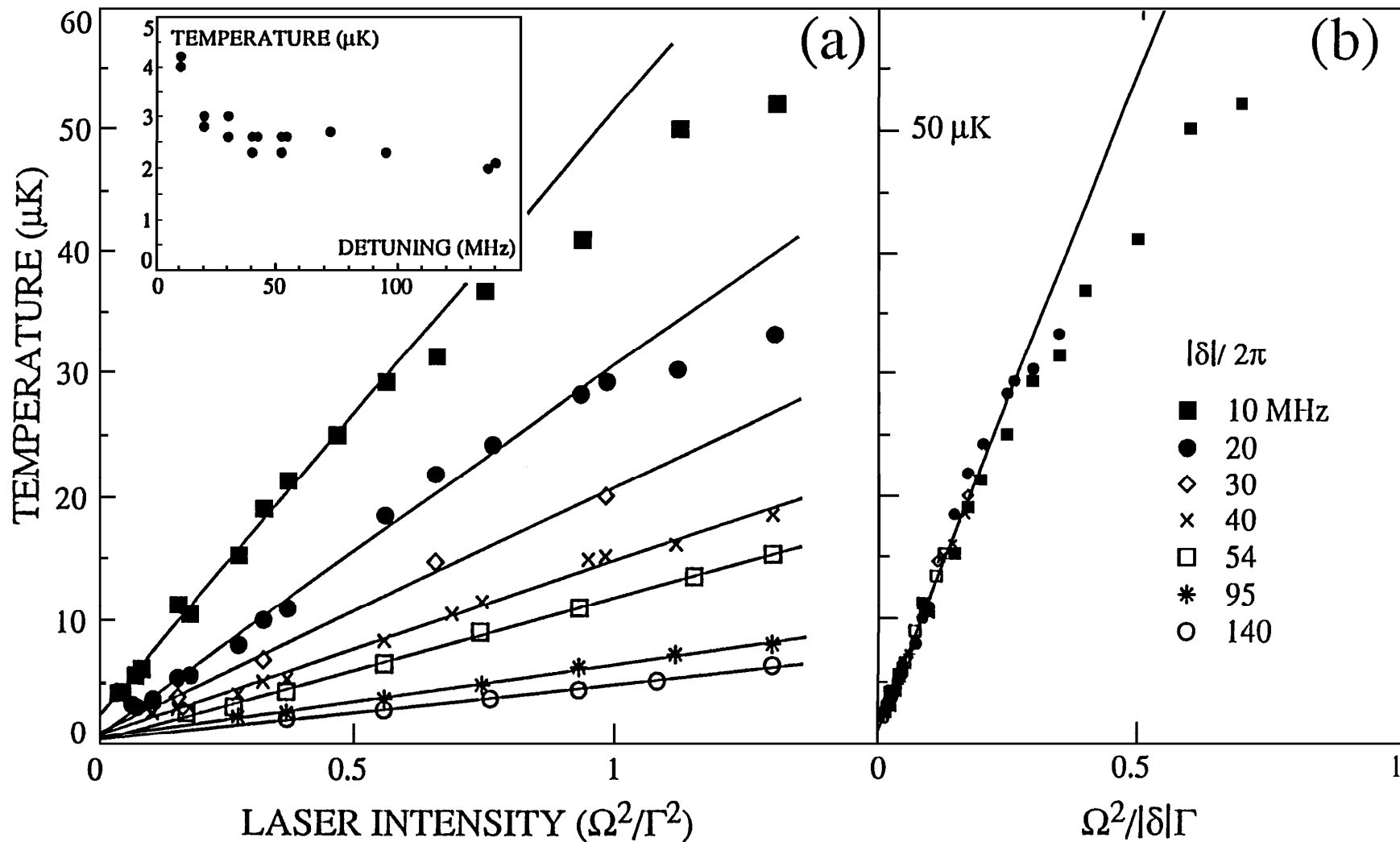


$$k_B T = U_0 / 4$$

Limit Temperature: about 10 times recoil energy

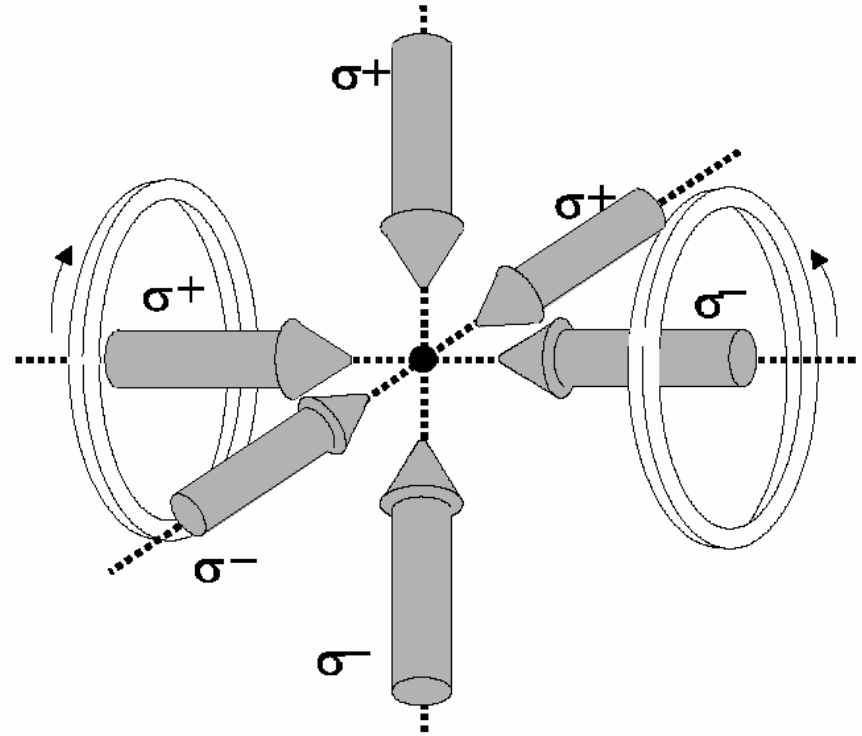
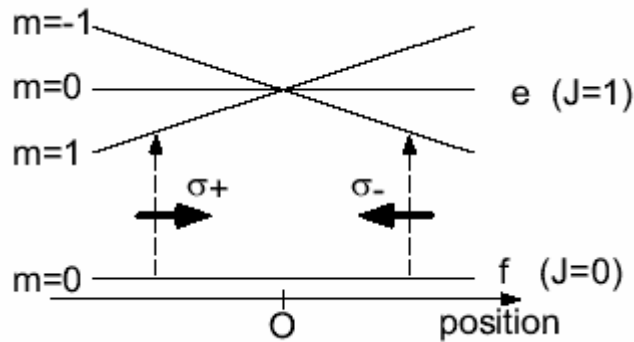
Test of Sisyphus cooling

$T \sim \text{Laser intensity/Laser detuning}$



Magneto-optical trap

J. Dalibard - 1987



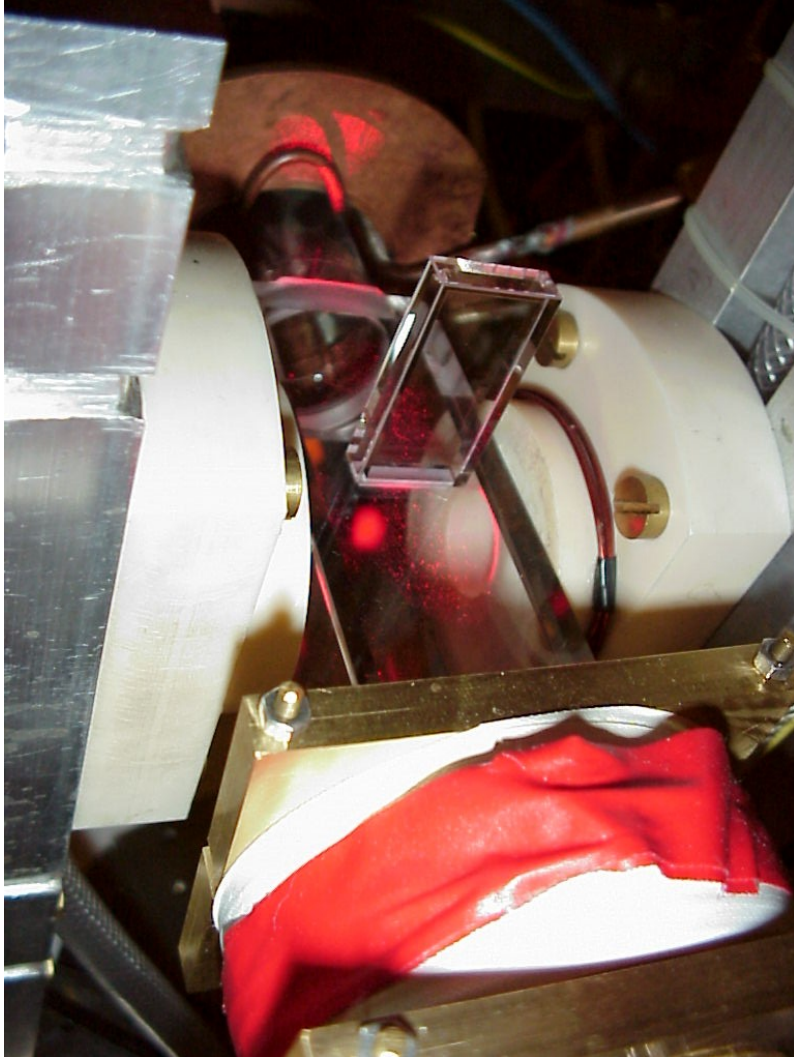
$$\mathbf{F} = -\alpha \mathbf{v} - \mathbf{k} \mathbf{r}$$

3D Molasses
Doppler effect

Trapping
Zeeman effect
Produces imbalance
in rad pressure forces

$b' = 10 \text{ Gauss / cm}$
 $I = \text{a few mW per beam}$

Lithium magneto-optical trap



Trapping:

$$\vec{F} = -\vec{k} \cdot \vec{r}$$

In addition, cooling:

$$\vec{F} = -\vec{k} \cdot \vec{r} - \alpha \vec{v}$$

Doppler cooling

+ sub-doppler cooling

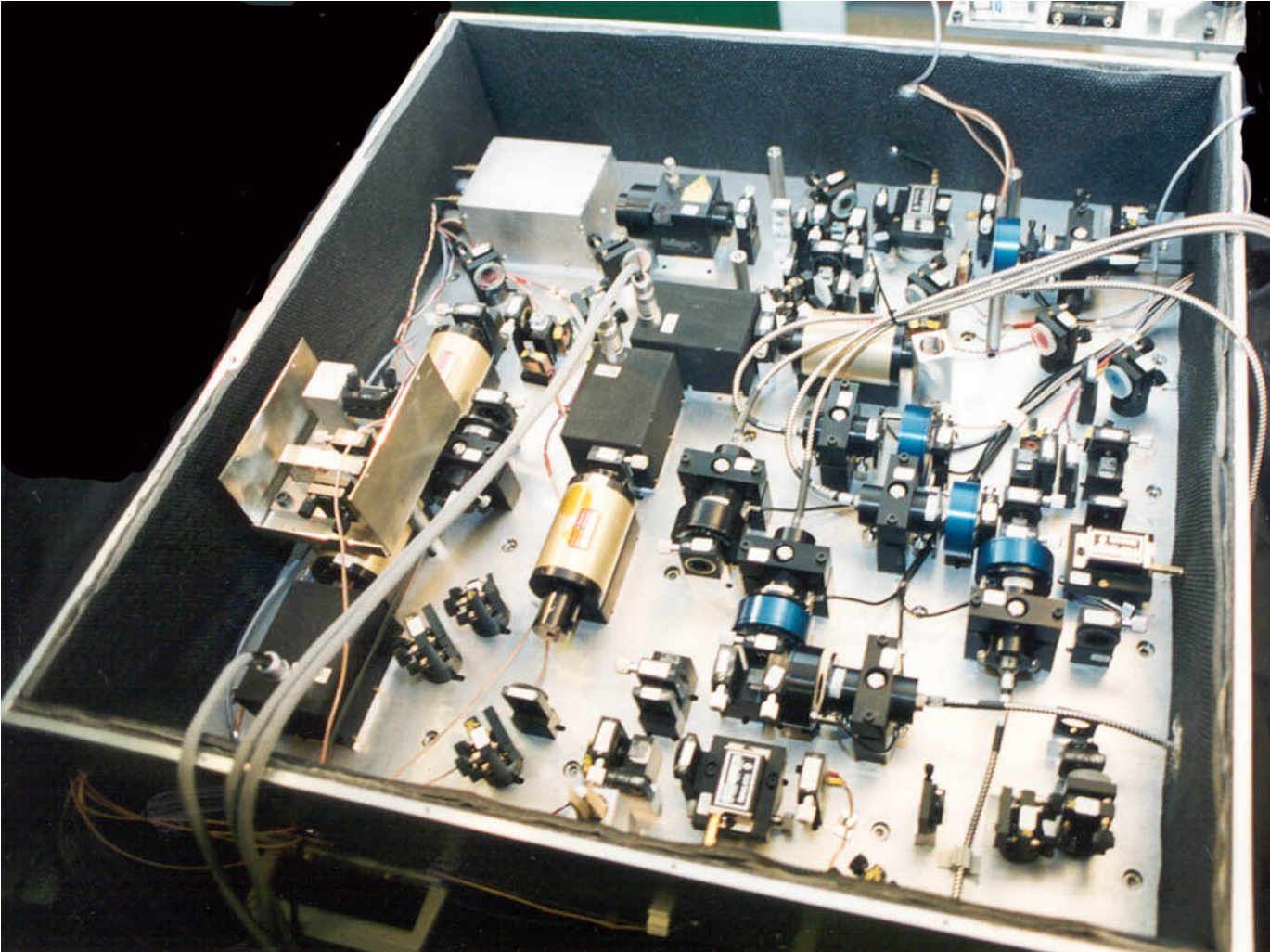
Considerable increase
in phase-space density:

$$n \lambda_{DB}^3 \sim 10^{-6}$$

Up to a few 10^{10} atoms

Density 10^{11} at/cm³

Optical bench for laser cooling



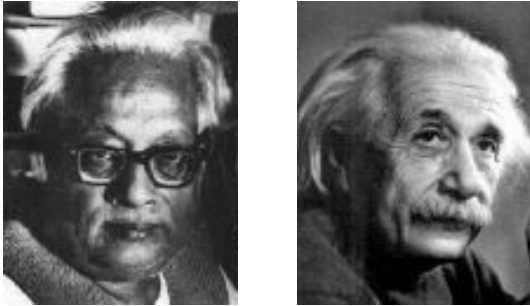
l'Observatoire
de Paris | SYRTE

PHARAO

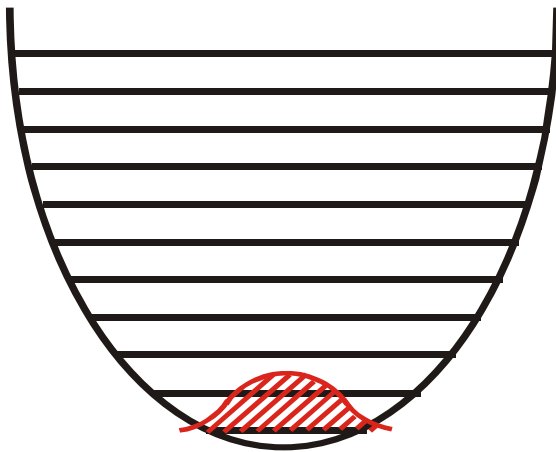
Bose-Einstein condensation and Fermi degeneracy

Quantum statistics in harmonic traps

● Bose-Einstein statistics (1924)



Bose-Einstein condensate



Bose enhancement

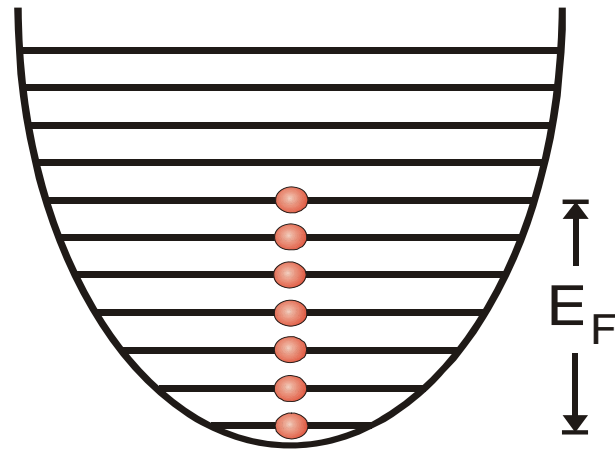
$$T_C = \frac{\hbar\omega}{k_B} (0.83 N)^{1/3}$$

Dilute gases: 1995, JILA, MIT

● Fermi-Dirac statistics (1926)



Fermi sea



Pauli Exclusion

$$T \ll T_F = \frac{\hbar\omega}{k_B} (6 N)^{1/3}$$

Dilute gases: 1999, JILA

Bose-Einstein Condensates: orders of magnitude



Dilute gas of atoms at temperature T confined in harmonic potential :

$$V(\vec{r}) = \frac{1}{2} m \omega^2 r^2$$

Condensation threshold:

$$N = 1.202 \left(\frac{k_B T}{\hbar \omega} \right)^3 \quad \longleftrightarrow \quad n_0 \lambda^3 = 2.612$$

$k_B T \gg \hbar \omega$

$$\left\{ \begin{array}{l} n_0 : \text{central density} \\ \lambda = \frac{h}{\sqrt{2\pi m k_B T}} \end{array} \right.$$

Liquid helium :

$$10^{27} \text{ atoms/m}^3$$

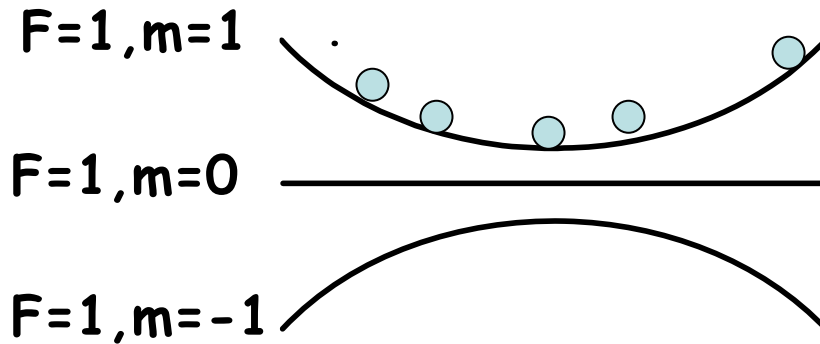
$$n_0^{-1/3} = 10 \text{ \AA} \quad T \sim 1 \text{ K}$$

Gaseous condensate

$$10^{19} \text{ at/m}^3$$

$$n_0^{-1/3} = 0.5 \text{ \mu m} \quad T \sim 1 \text{ \mu K}$$

Magnetic trap



$$E = -\vec{\mu} \cdot \vec{B} = +|\vec{\mu}| |\vec{B}|$$

Local minimum of $|B|$
+ spin polarisation

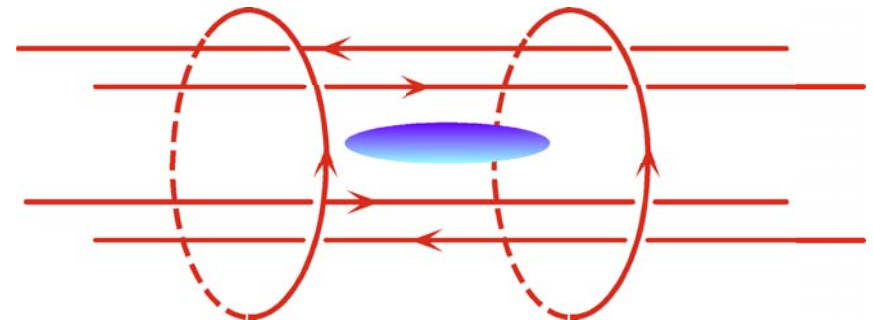
$$V = |\mu| |B|$$

Maxwell's equations:
No max of $|B|$ in vacuum.

Atoms cannot be magnetically trapped in the lowest energy state.

▲ Two-body inelastic collisions

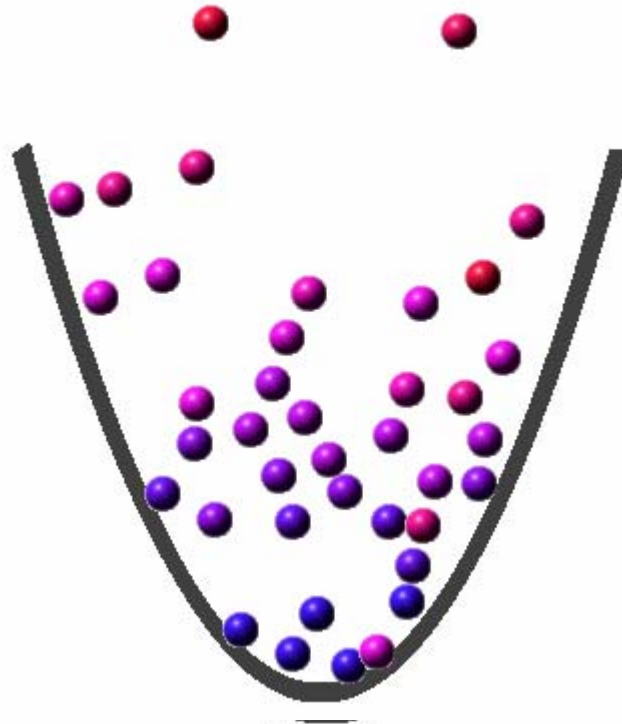
Example: Ioffe-Pritchard trap
Trap depth 1 mK
Loaded with laser cooled atoms
Or cryo-cooled atoms (Harvard)



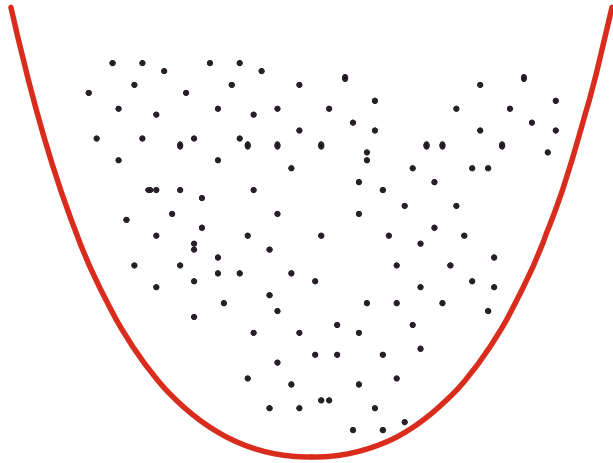
Evaporative cooling

- The only known method to reach quantum degeneracy
 - Remove hot atoms
 - Elastic collisions ensure re-thermalisation

$$\gamma_{elastic} / \gamma_{inelastic} \geq 150$$



Evaporative cooling (2)



$$N \longrightarrow N / 100$$

$$T \longrightarrow T / 1000$$

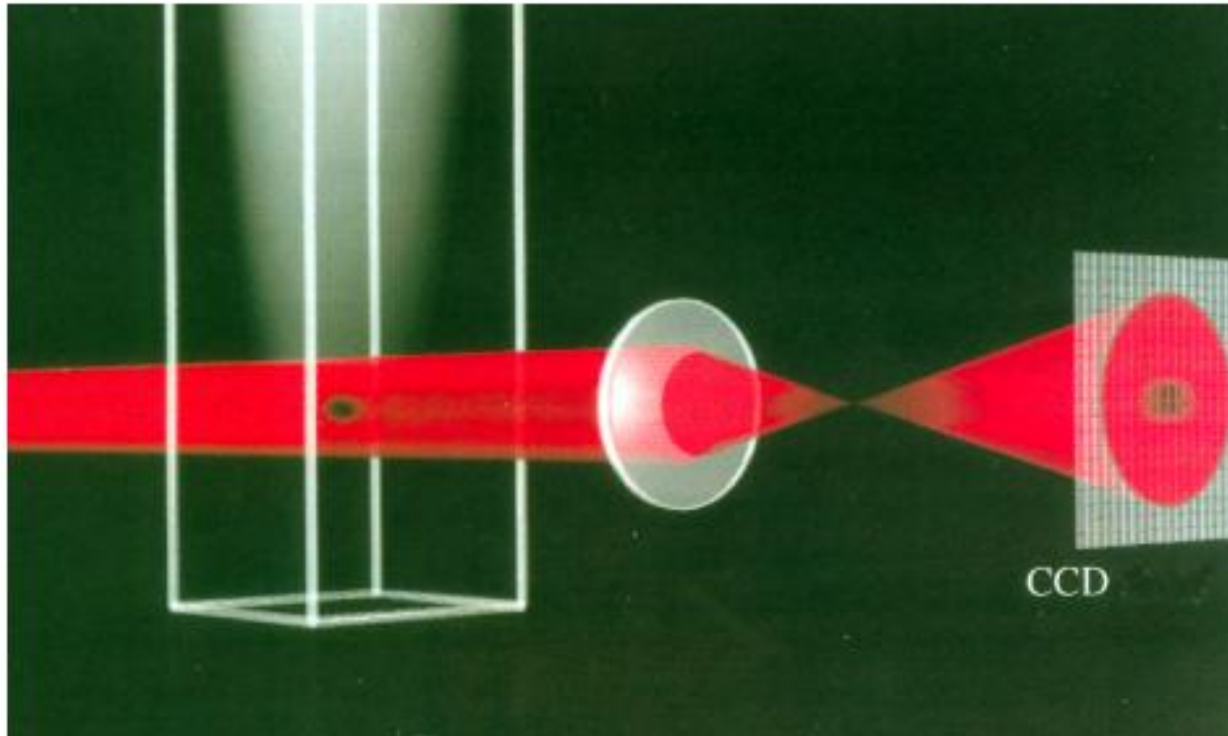


Phase-space density $n\lambda^3$
multiplied by 10^7



Duration : 5 to 30 seconds, $N_f = 10^5$ to 10^7 atoms, $T_f = 0.2$ to $2 \mu\text{K}$

Imaging cold atomic clouds and condensates

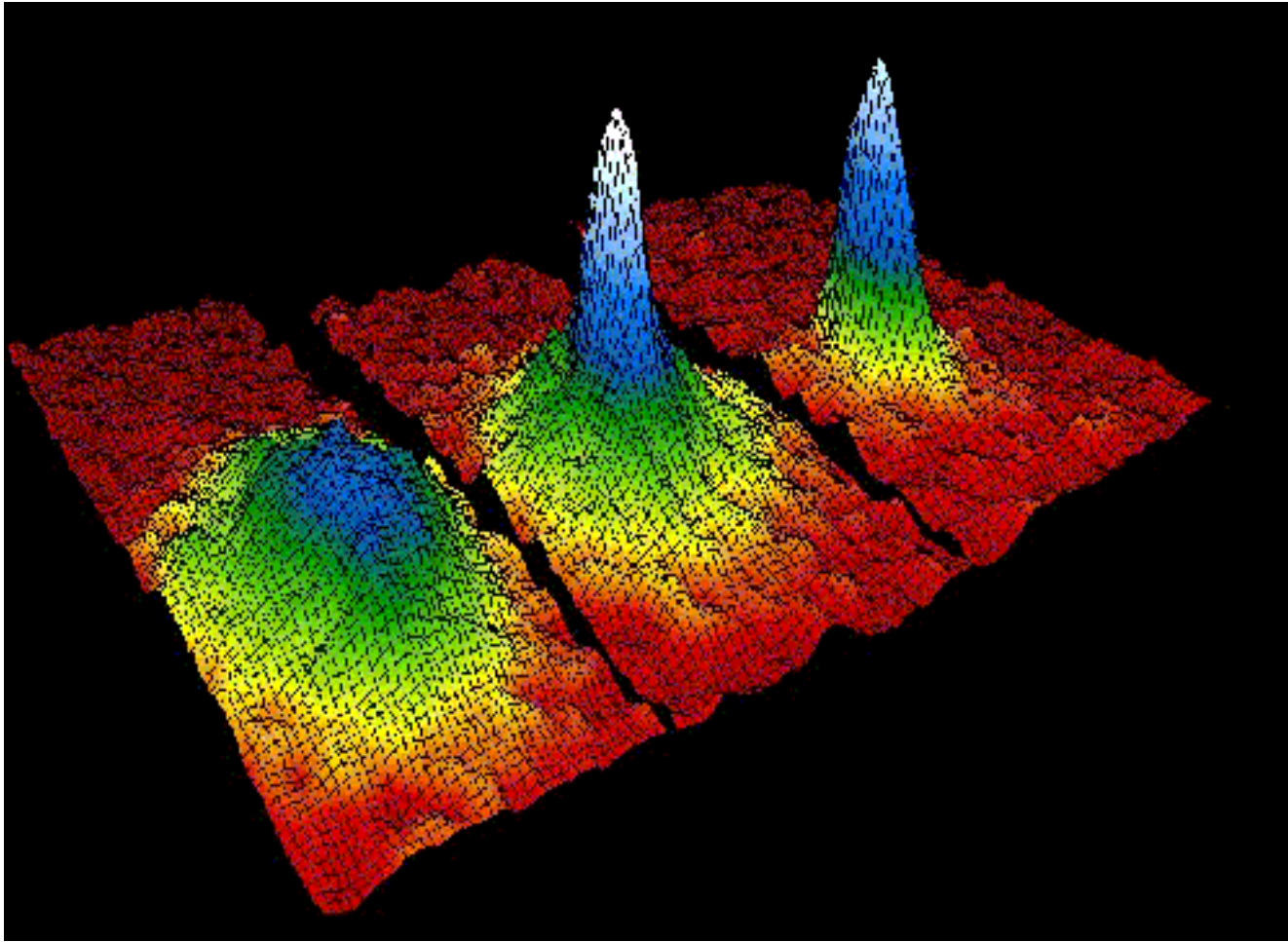


Absorption imaging

- 1) In situ measurement: spatial distribution in the trap
- 2) After time of flight expansion: velocity distribution

Bose-Einstein Condensation in Rubidium 87

JILA - Boulder



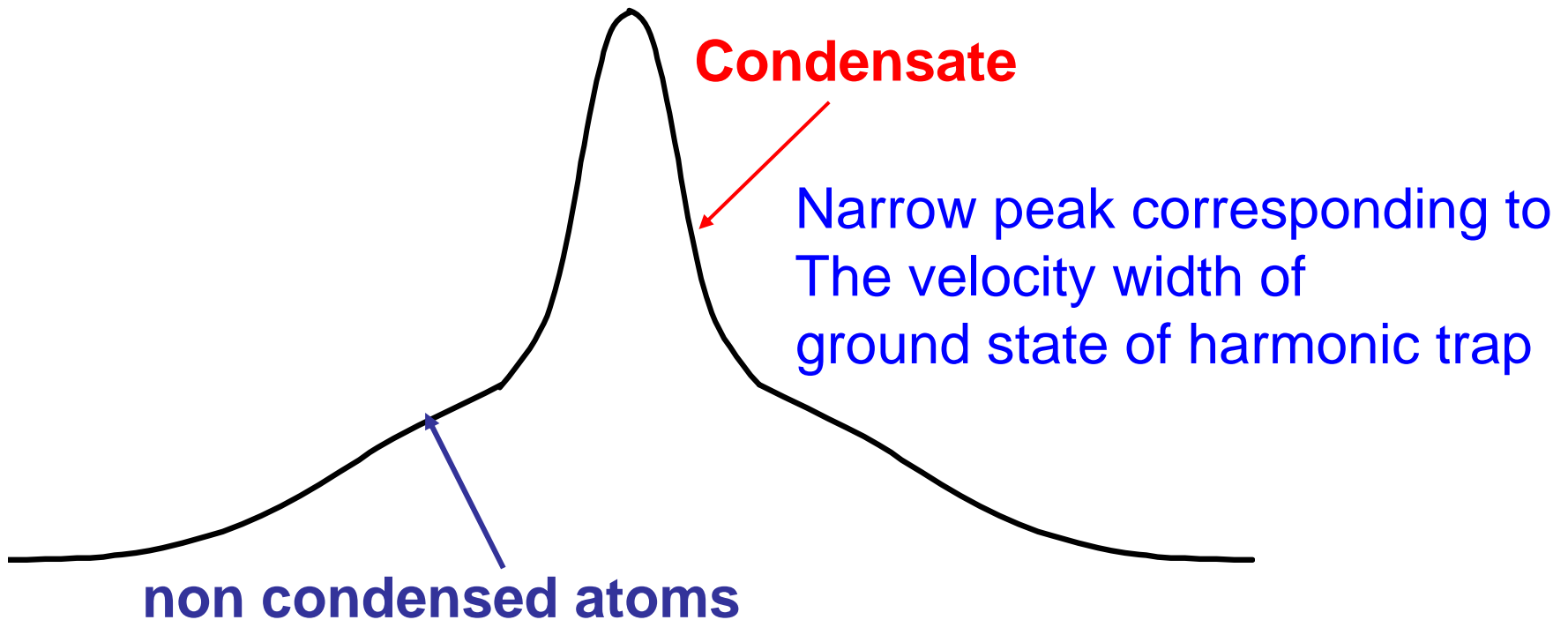
1000 atoms in ground state of magnetic trap.

Remark:
Metastable systems
The true ground state of Rb at $1 \mu\text{K}$ is a piece of solid

Science, 269, 198 (1995)
M. Anderson, E. Cornell and C. Wieman

+ Sodium, Lithium, Hydrogen, Potassium
Helium (2s state), Cesium, Ytterbium

Bimodal distributions

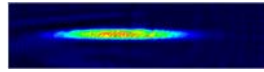


Thermal atoms in excited states: broader distribution

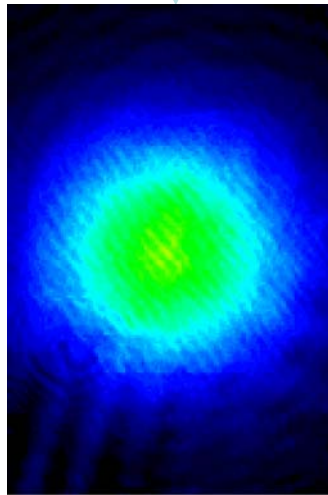
Condensate signature

A few millions atoms in anisotropic magnetic trap

$$T > T_c$$



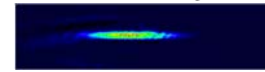
Boltzmann
Gas



isotropic

$$\frac{1}{2}mv_i^2 = \frac{1}{2}kT$$

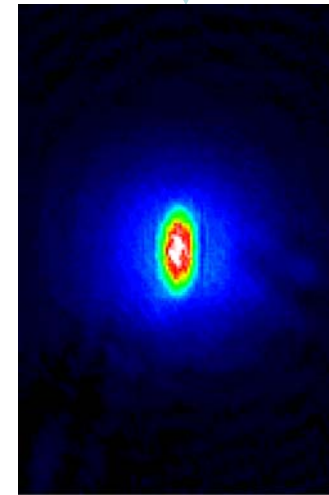
$$T < T_c$$



0,5 to 1 μK

100 μm * 5 μm

Time of flight



anisotropic

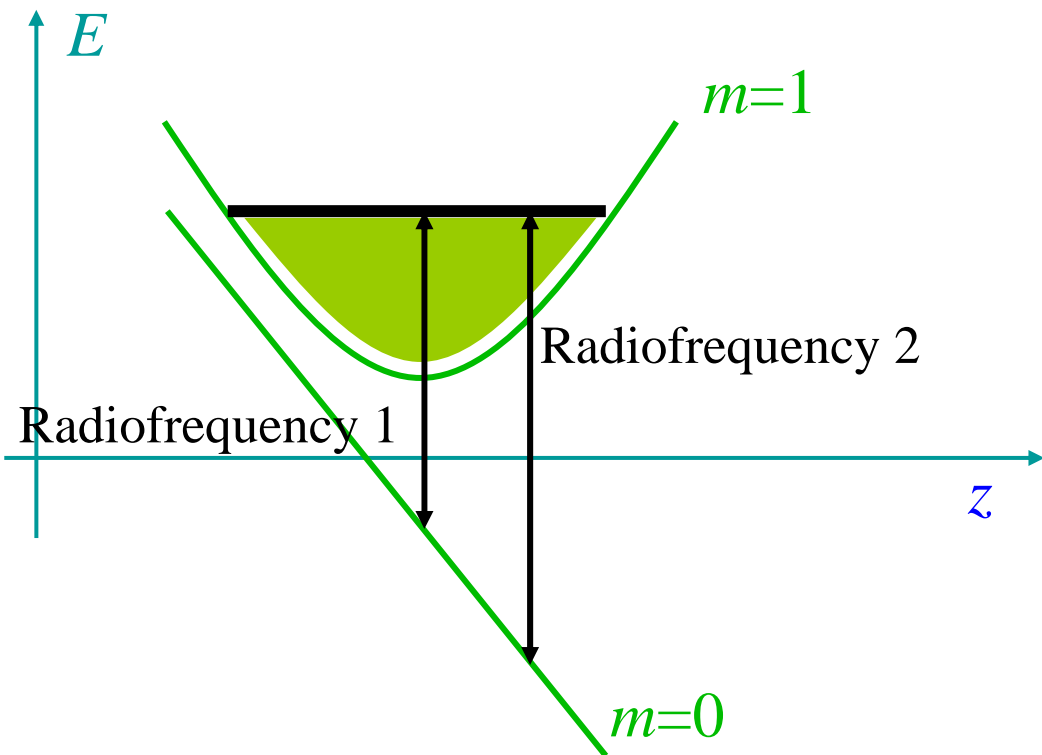
condensate

$$\frac{1}{2}mv_i^2 = \frac{1}{4}\hbar\omega_i$$

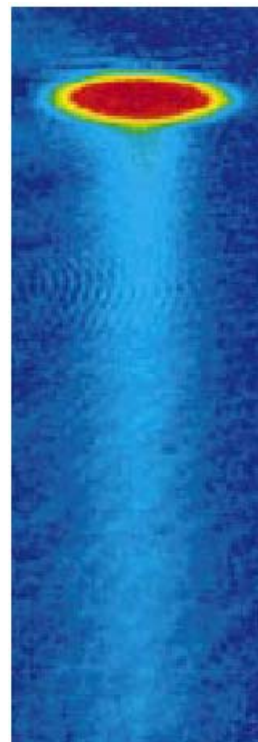
without
interactions

Coherence of Bose-Einstein condensates

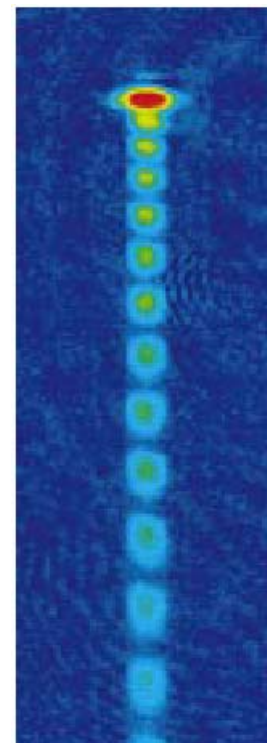
Young slit experiment, Munich



$T > T_c$



$T < T_c$



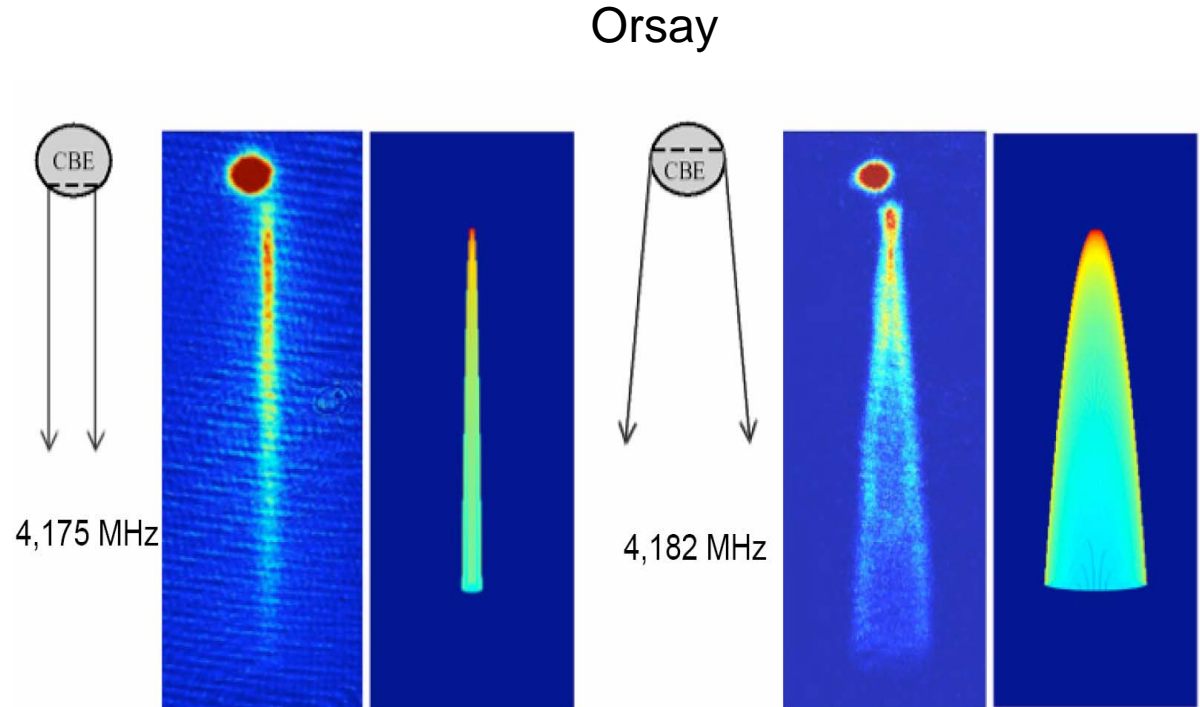
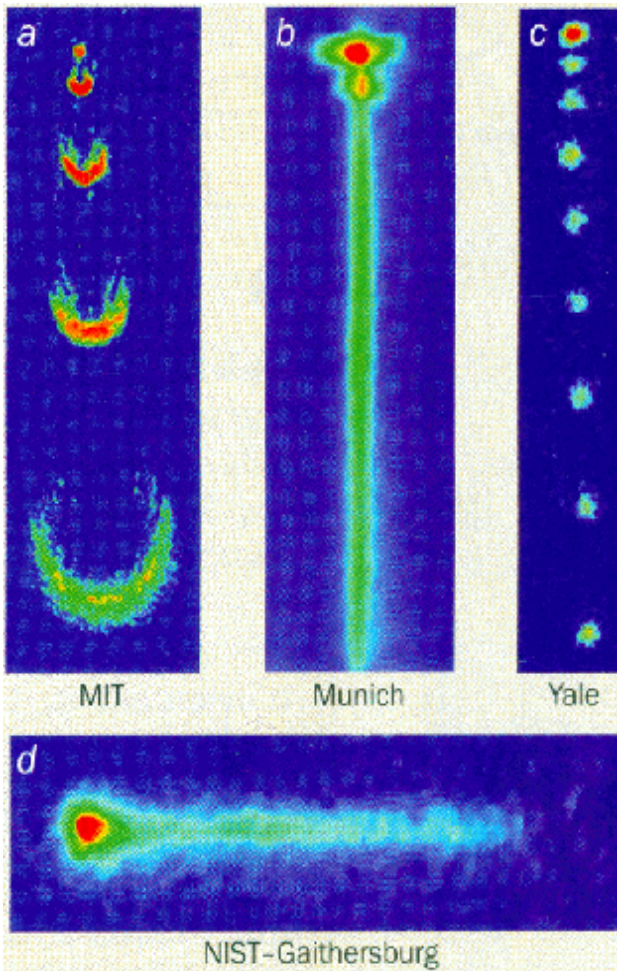
$$n_{out}(z) = |\psi_{out}(z - z_1) + \psi_{out}(z - z_2)|^2$$

$$\sim \frac{1}{\sqrt{z}} \left\{ 2 + 2 \cos \left(q\sqrt{z} + (\omega_1 - \omega_2)t \right) \right\}$$

$$q = m(z_2 - z_1)\sqrt{2g} / \hbar$$

High contrast reveals
macroscopic occupation
of single quantum state

Atom lasers



Influence of atom-atom interactions

So far limited flux:
less than 10^6 /second

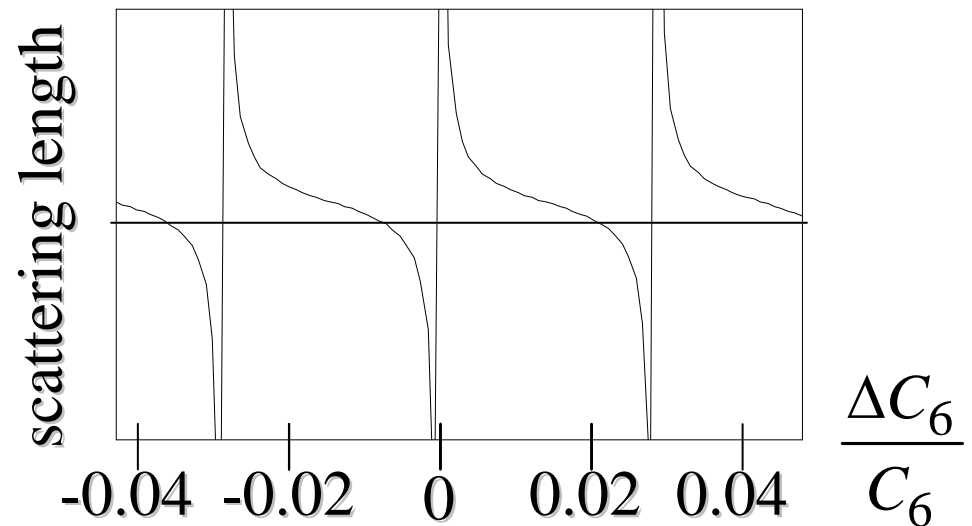
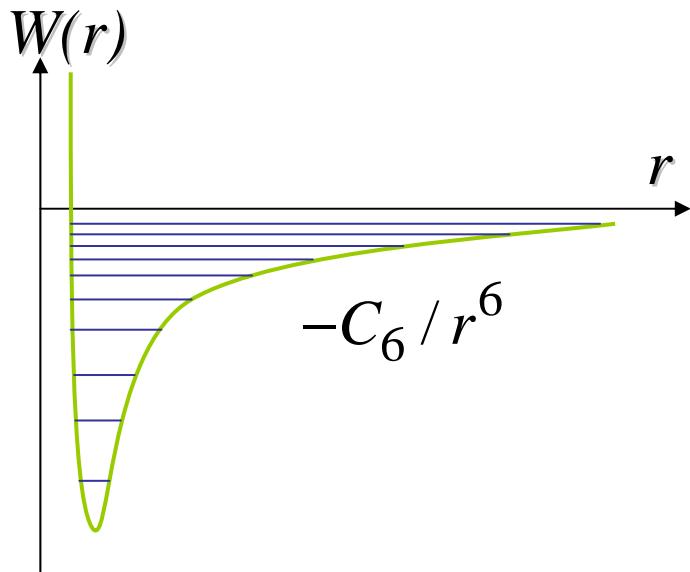
Control of interactions within a condensate

Gross-Pitaevski equation :

$$\left(-\frac{\hbar^2}{2m} \Delta + V(\vec{r}) + Ng |\psi(\vec{r})|^2 \right) \psi(\vec{r}) = \mu \psi(\vec{r})$$

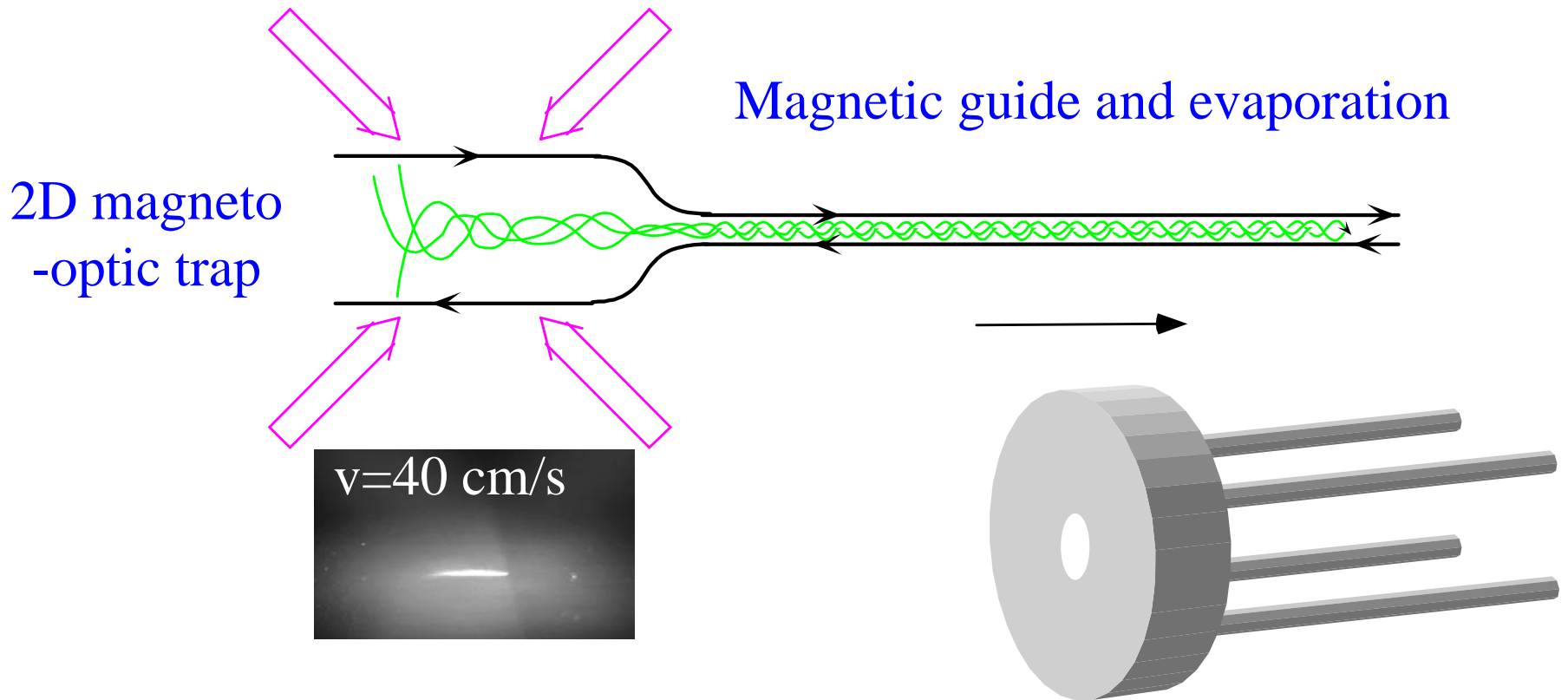
$$g = \frac{4\pi\hbar^2 a}{m}$$

a : scattering length



$|a|$ varies typically between 1 and 100 nanometers (Bohr radius : 0,05 nm)

Towards a continuous atom laser

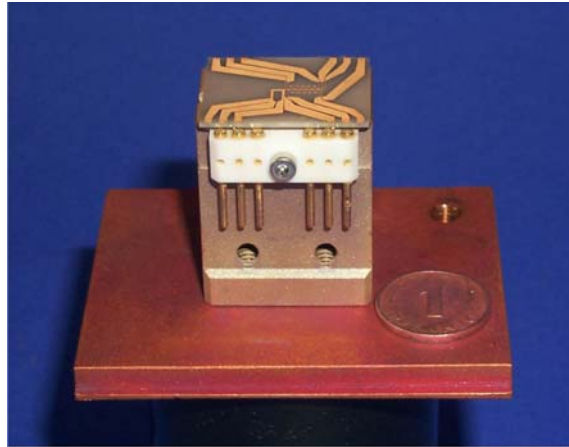


Expected flux: 10^7 condensed atoms /second
Factor 20 gain in phase-space density over 4 meters

Condensates on a chip

All wires for magnetic trapping and manipulation are integrated on a chip with a few cm^2 area

Munich
Reichel et al.



*Gold wires ($10\mu\text{m}$) on
silicon substrate*

Greatly simplifies vacuum requirements, power requirements,
Condensates with a few 10^5 atoms
Application to atom interferometry on a chip
Quantum sensors in space

More than 10 groups with condensates and Fermi gas on chips

FURTHER READING

- **BOSE-EINSTEIN CONDENSATION IN ATOMIC GASES**, Proceedings of the International School of Physics « Enrico Fermi », Course CXL, ed. **M. Inguscio, S. Stringari, C.E. Wieman**, IOS Press, 1999.
- **LASER MANIPULATION OF ATOMS AND IONS**, Proceedings of the International School of Physics « Enrico Fermi », Course CXVIII, ed. **E. Arimondo, W.D. Phillips, F. Strumia**, Elsevier Science, 1992.