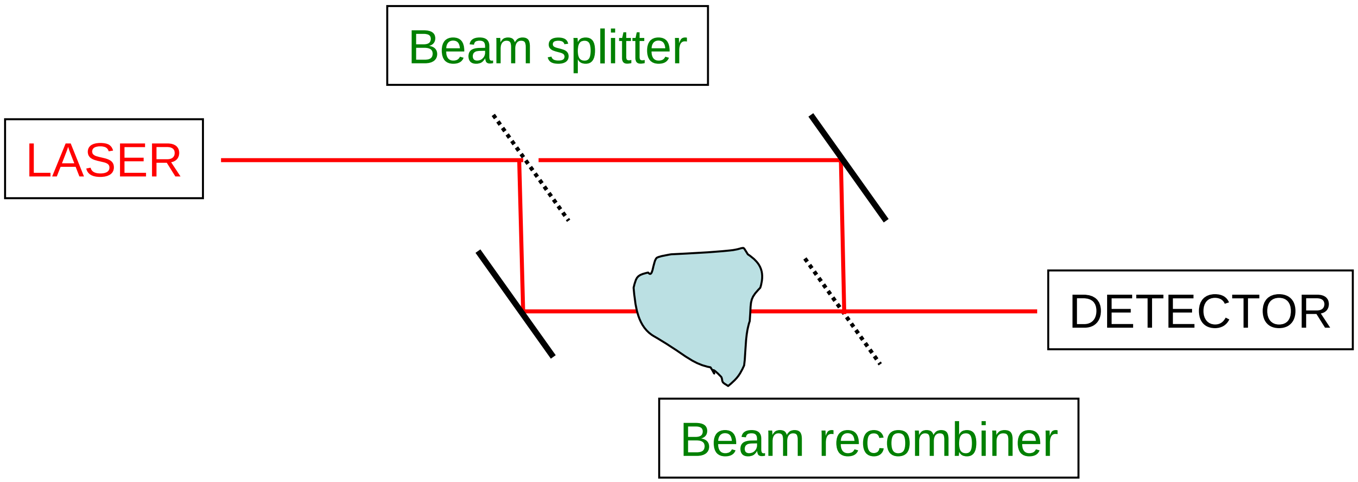
Lecture 2:Matter Wave Interferometry

- Matter wave interferometry: as old as Quantum Mechanics
- Neutron diffraction and electron diffraction are standard investigation tools in solid state physics
- Cold atoms: new possibilities

$$\lambda_{DB} = \frac{h}{Mv} \qquad T = 1\mu K \qquad \lambda_{DB} \sim 1\mu m$$

Rapid advances in precision measurements

Interferometry



Change of optical path,
Length, pressure, temperature,… ⇒ Change of phase of interference pattern

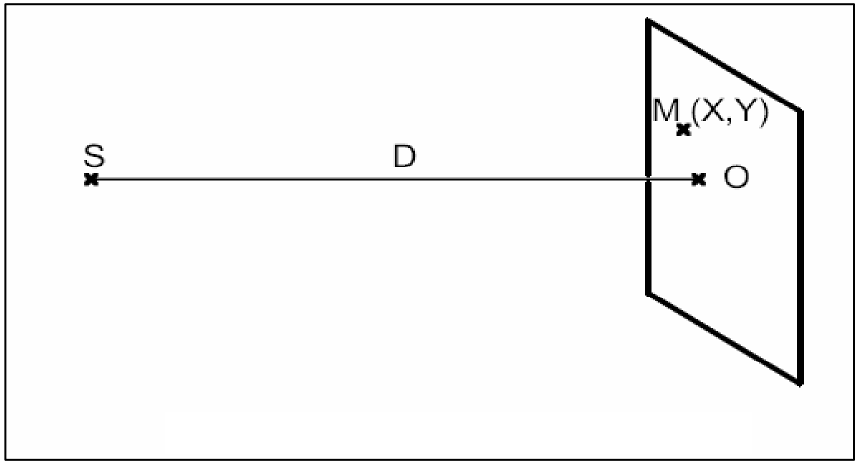
Atom interferometry ⇒ Atomic source
Beam splitters, mirrors
Detectors

Matter wave diffraction
Huygens-Fresnel principle

Monokinetic beam of particles described by $\psi(r,t)$

$$\frac{-\hbar^2}{2m}\Delta\psi = E\psi \qquad \Delta\psi + k^2\psi = 0 \quad \text{with} \quad k^2 = \frac{2mE}{\hbar^2}$$

Analogy with $\Delta \boldsymbol{E} + k^2 \boldsymbol{E} = 0$ for electric field $\boldsymbol{E}(r,t)$ of light beam with $\lambda = \frac{2\pi}{k}$



with

$$T = D / v_0$$
$$\hbar k = m v_0$$

$$\psi(X,Y,D) = \psi_0 \, exp\left(\frac{ik(X^2+Y^2)}{2D}\right) = \psi_0 \, exp\left(\frac{im(X^2+Y^2)}{2\hbar T}\right)$$

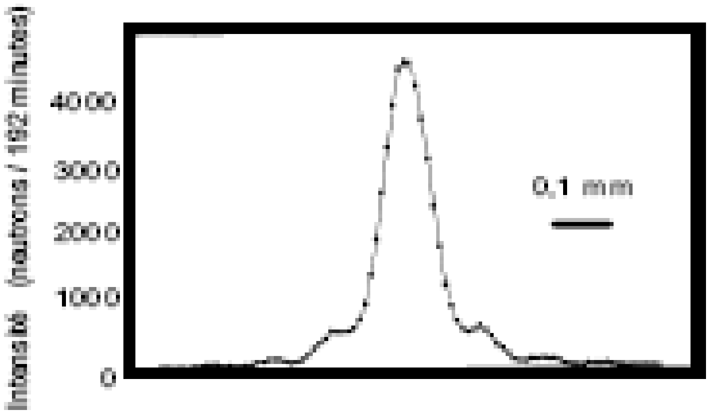
Neutron Diffraction from a slit

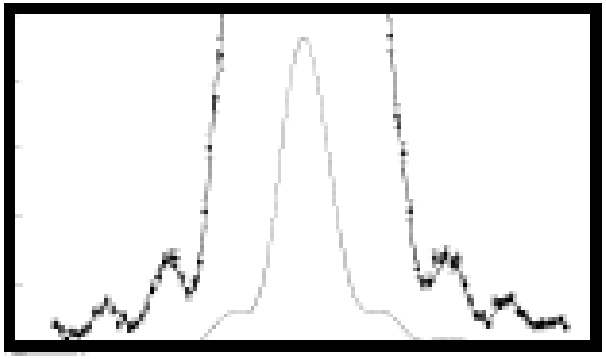
Experiment with slow neutrons: $\lambda = 1.926\ nm$ $v_0 = 206\ m/s$

ILL Grenoble

Neutrons produced at 20 000 km/s slowed in successive steps to T=20K

Detection by nuclear reaction with 100% efficiency producing alpha particles

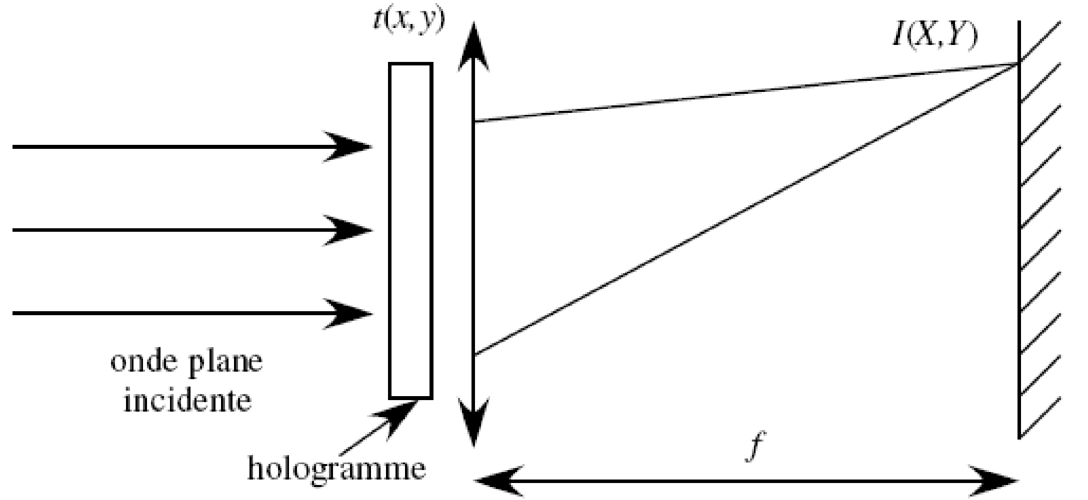




Slit width:
93 microns

Atom holography

Generalization to arbitrary image patterns

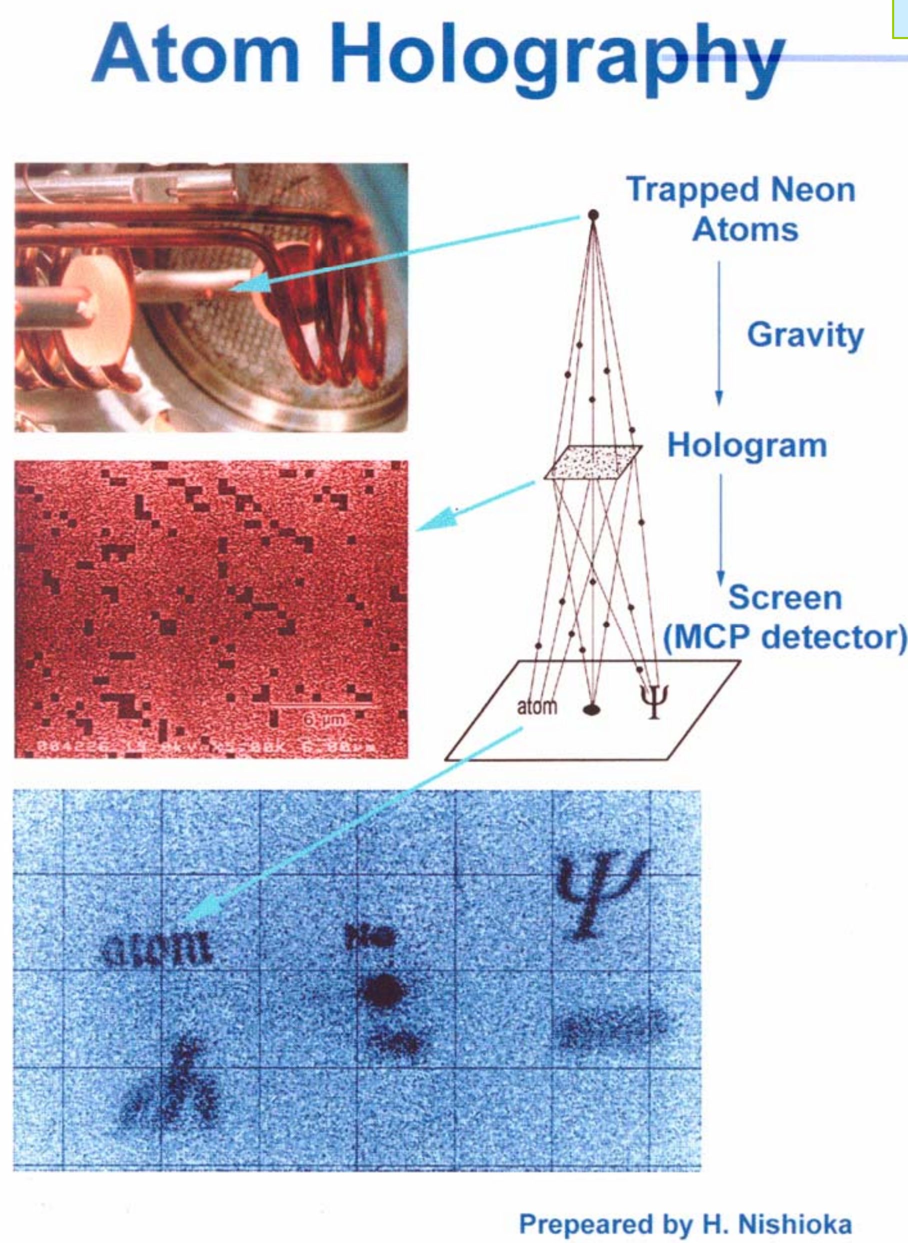


Principle of a transmission hologram

We start from the momentum distribution $I(X,Y)$ that we want to produce on the screen. Its amplitude is $a(X,Y)=\sqrt{I(X,Y)}$ and is the amplitude diffracted in direction k_x, k_y with $k_x = kX/f, \quad k_y = kY/f$

$a(X,Y) = Fourier\ Transform\ of\ t(x,y)$ the transmission function of the diffracting hologram

Tokyo experiment



M. Morinaga et al. PRL, 77, 802 (1996)

$$a(k_x,k_y)=\int e^{i(k_x x+k_y y)}\, t(x,y)\,dx\;dy$$

By inverse Fourier transform we deduce the required *t(x,y)*

But !

In general *t(x,y)* is complex !

In fact, we can only code real values and only 0 or 1 (block or transmit !)

Solution: code: $t(x,y)+t^*(x,y)/2$

This gives two images:

$$I(X,Y)\ and\ I(-X,-Y)$$

One also adds a constant t_0 to the function to be coded

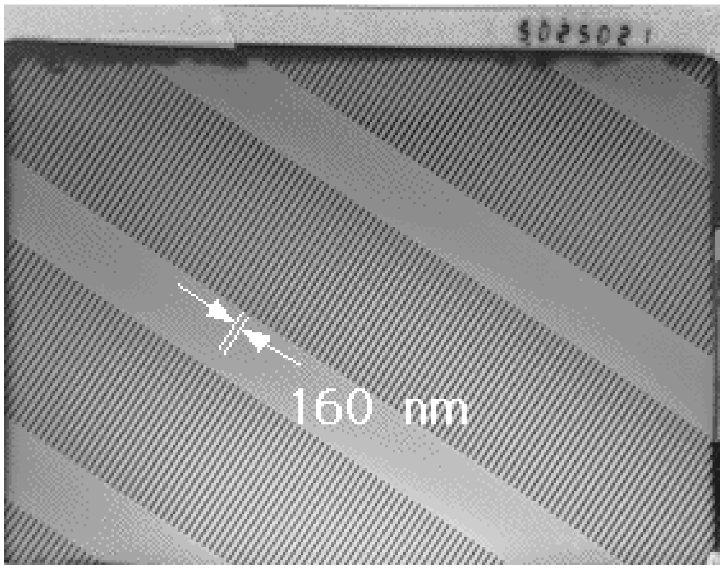
Such that $t_0+t(x,y)+t^*(x,y)/2$

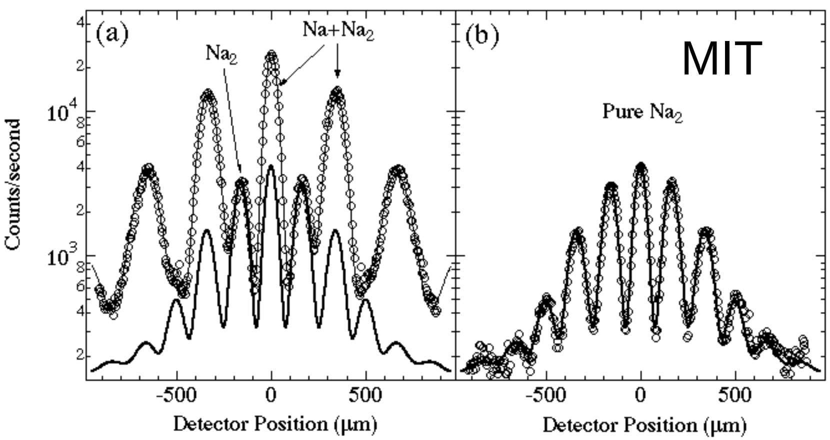
is a positive number between 0 and $t_{\max}$

The transmission 0 or 1 of each pixel is then obtained by comparing $t(x_i,y_i)$ to $\eta\, t_{\max}$ where η is a constant

Matter wave diffraction by periodic structures

Material grating

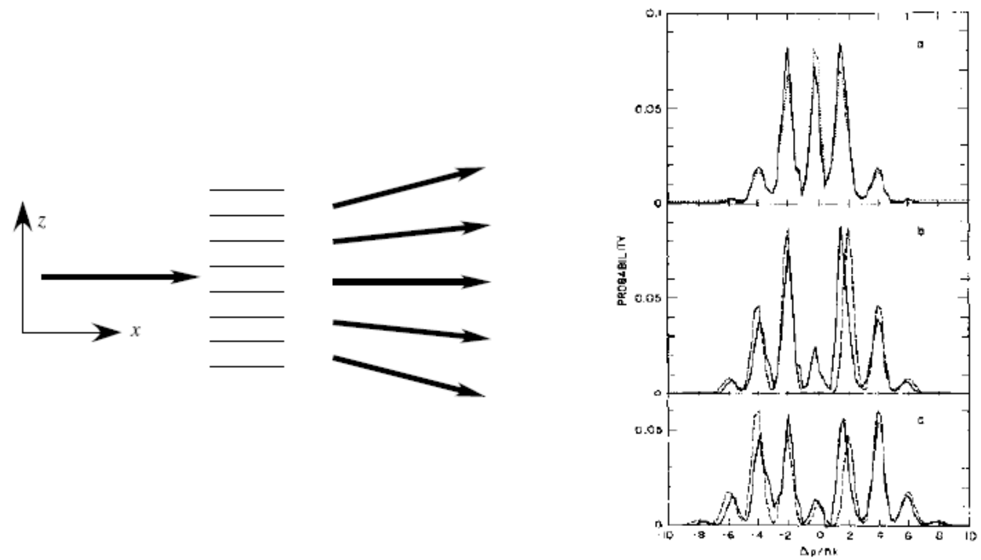




MIT

Laser standing wave

100% transmission

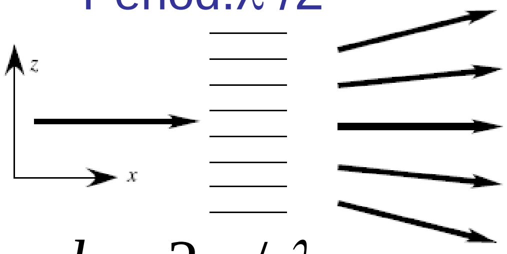


P. Gould et al. PRL 56,827 (1986)

Thin phase grating approximation

Standing wave

Period: λ /2



$$k = 2\pi / \lambda$$

Simple treatment:

1D along z, quasi monochromatic wave packet:

mean momentum p_0, $\Delta p_z \ll \hbar k$

Hamiltonian $$H = \frac{p_z^2}{2m} + U_0 \sin^2 kz = \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{2}\cos 2kz$$

$$= \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{4}\left(e^{2ikz} + e^{-2ikz}\right)$$

$$\psi(T) = e^{-iHT/\hbar}\,|\psi(0)\rangle \; with \; |\psi(0)\rangle = |\,p_0\rangle$$

First we neglect motion along z during T

$$\psi(T) = e^{i(U_0 T/2\hbar)\cos(2kz)}\,|\,p_0\rangle = \left(\sum_{n=-\infty}^{n=+\infty} (i)^n J_n\left(\frac{U_0 T}{2\hbar}\right) e^{-2inkz}\right)|\,p_0\rangle$$

$$= \sum_{n=-\infty}^{n=+\infty} (i)^n J_n\left(\frac{U_0 T}{2\hbar}\right)|\,p_0 - 2n\hbar k\rangle$$

Intensity of diffracted peaks prop. to $J_n^2\left(\frac{U_0 T}{2\hbar}\right)$

3D case and validity of thin grating approx.

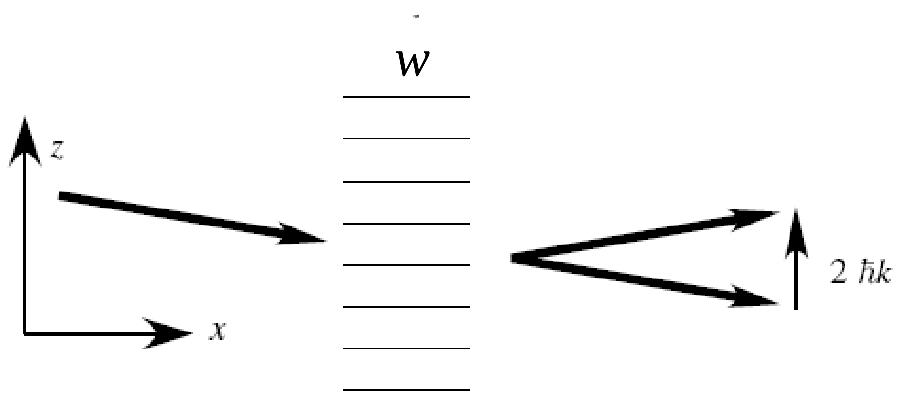
3D motion $T = \frac{w}{v_x}$

Validity: $\left| \frac{p_0^2}{2m} - \frac{(p_0 + 2n_c \hbar k)^2}{2m} \right| \ll U_0$ where n_c is a typical diff. order $n_c \sim U_0 T / 2\hbar$

$$T^2 U_0 E_R / \hbar^2 \ll 1$$

Introducing the classical oscillation period in potential well $\omega = 2\sqrt{U_0 E_R} / \hbar$

Validity: $\omega^2 T^2 \ll 1$ If condition is violated: thick grating and Bragg diffraction



Bragg diffraction

Energy conservation and Bragg regime

Stationary problem: total energy is conserved

For small momenta along z, $p_0 \leq \hbar k$ and n_c~1

The kinetic energy along z changes by $\Delta E_z = \left| \frac{p_z^2}{2m} - \frac{(p_z \pm 2\hbar k)^2}{2m} \right| \sim E_R$

It must be compensated by a change of kinetic energy along x $\Delta E_z = -\Delta E_x$

How does a standing wave along z changes the kinetic energy along x ?

p_x

Finite transit time: standing wave (diameter w) has angular divergence $\theta = \lambda / w$

This enables momentum changes along x of $\hbar k \theta \sim h / w$

and of kinetic energy $\Delta E_x = \frac{(p_x \pm h/w)^2}{2m} - \frac{p_x^2}{2m} \sim \frac{p_x h}{mw} = \frac{h}{T}$

As long as $\frac{h}{T} \geq E_R$ total energy can be conserved and exchanged between x and z

Otherwise, for long times, choose $p_z = -\hbar k$ Diffraction leads to $p_z = +\hbar k$

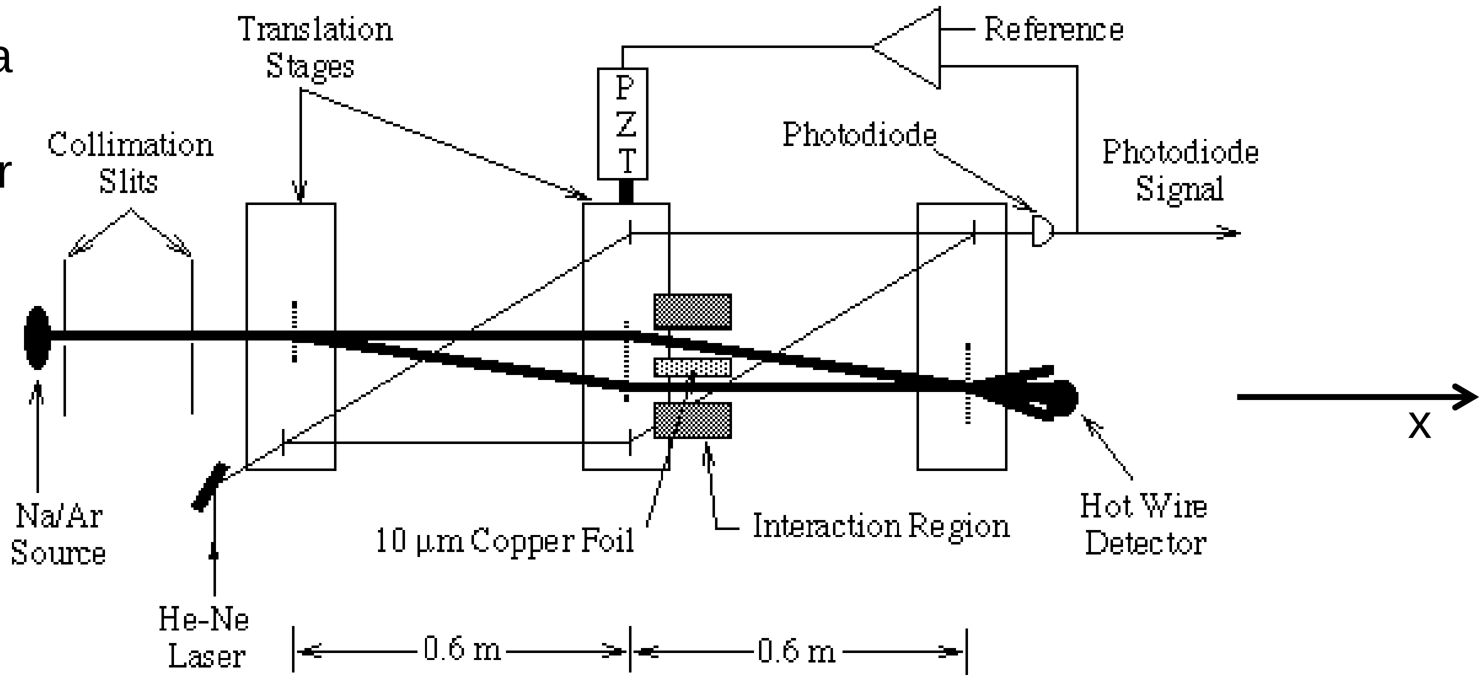
momentum along z is conserved. This is the Bragg regime.

Rabi oscillation between the two states with pulsation $U_0 / 2\hbar$ Adjustable beamsplitter

Three grating Interferometers

MIT, Konstanz
Stanford,Vienna
Yale,Toulouse,
Paris, Hannover

Beam
with
Velocity v_0



Phase shift calculated on classical path (See below) $\phi(k_0) = \int_{\Gamma_1} k_1(x)dx - \int_{\Gamma_2} k_2(x)dx$

where $k_0 = \sqrt{2mE}/\hbar$ and $k(x) = \sqrt{2m(E - V(x)}/\hbar$

For $V \ll E$ and stationary: $\delta\varphi(k_0) = -\frac{k_0}{2}\int_{x_a}^{x_b} \frac{V(x)}{E}dx = -\frac{1}{\hbar v_0}\int_{t_a}^{t_b} V(t)dt$

Example: Na beam with v_0 =1000m/s
x_b-x_a=0.1m, V=6.6 10^{-12} eV, $\delta\phi = 1\,rad$

High precision measurement of electric polarizability, A- B effect, index of refraction for matter waves,…

Path integral formalism

A particle travels from $\boldsymbol{r}_a, t_a$ to $\boldsymbol{r}_b, t_b$ with a probability amplitude given by:

$$K(\boldsymbol{r}_b, t_b; \boldsymbol{r}_a, t_a) = \sum_{\Gamma} e^{iS_\Gamma/\hbar}$$

The sum is over all paths Γ connecting $\boldsymbol{r}_a, t_a$ to $\boldsymbol{r}_b, t_b$

$$\psi(\boldsymbol{r}_b, t_b) = \int K(\boldsymbol{r}_b, t_b; \boldsymbol{r}_a, t_a)\, \psi(\boldsymbol{r}_a, t_a)\, d^3 r_a$$

$$S_\Gamma = \int_{t_a}^{t_b} L(\boldsymbol{r}(t), \dot{\boldsymbol{r}}), t)\, dt$$

is the action calculated along the path Γ

Remarkable result:

For a Lagrangien which is linear or quadratic in position and momentum we have:

$$K(\boldsymbol{r}_b, t_b; \boldsymbol{r}_a, t_a) = F(t_b, t_a)\, e^{iS_{class.}/\hbar}$$

where $F(t_b, t_a)$ is independent of initial r_a and final r_b positions

and where $S_{class.}$ is the action evaluated on the classical path connecting r_a, t_a to r_b, t_b

Examples

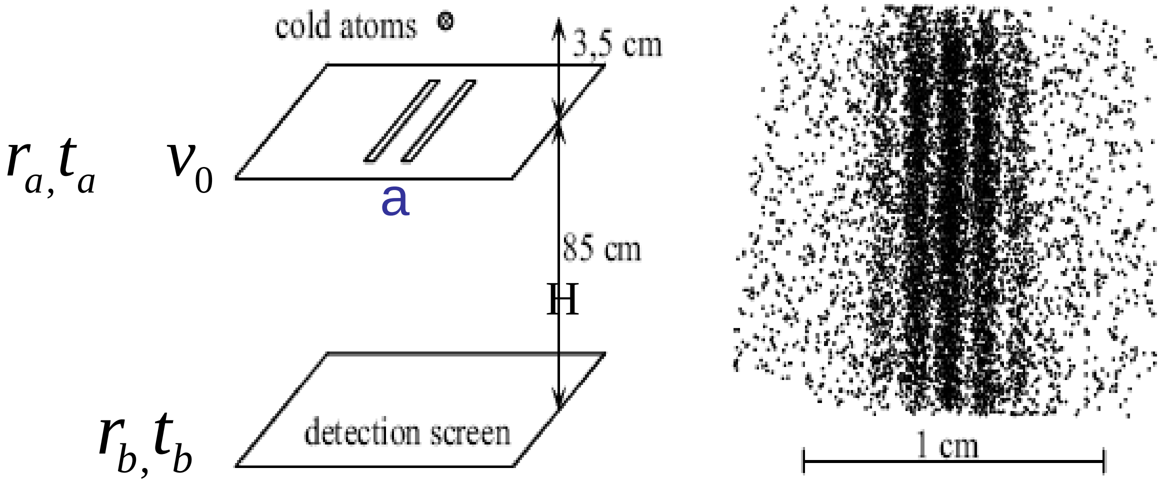
Particle in gravity field $L = m\dot{r}^2/2 - mgz$

Particle in harmonic trap $L = m\dot{r}^2/2 - m\omega^2 r^2/2$

Particle in rotating frame $L = m\dot{\boldsymbol{r}}^2/2 + m\dot{\boldsymbol{r}}.(\boldsymbol{\Omega} \times \boldsymbol{r}) + m(\boldsymbol{\Omega} \times \boldsymbol{r})^2/2$

Young slit experiment with free falling atoms

F. Shimizu et al., Phys. Rev A, 46 R17 (1992)



$$T = \frac{\sqrt{v_0^2 + 2gH} + v_0}{g}$$

$$|\psi(r_b, t_b)|^2 \sim \cos^2(m x_b a / 2\hbar T)$$

a=6 μm, H= 0.85 m

Fringe period $$\delta x_b = \frac{2\pi\hbar T}{ma}$$

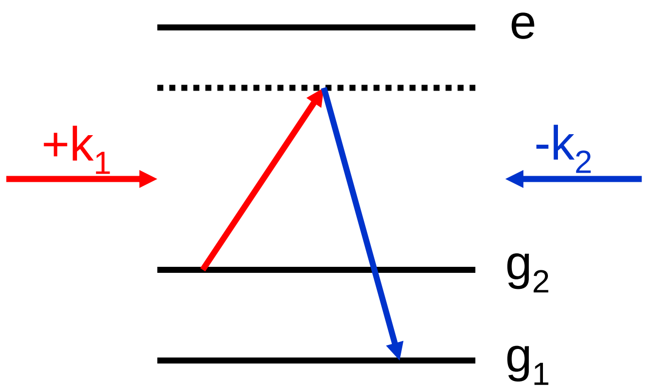
Atom interferometers, Spectroscopy, and Clocks

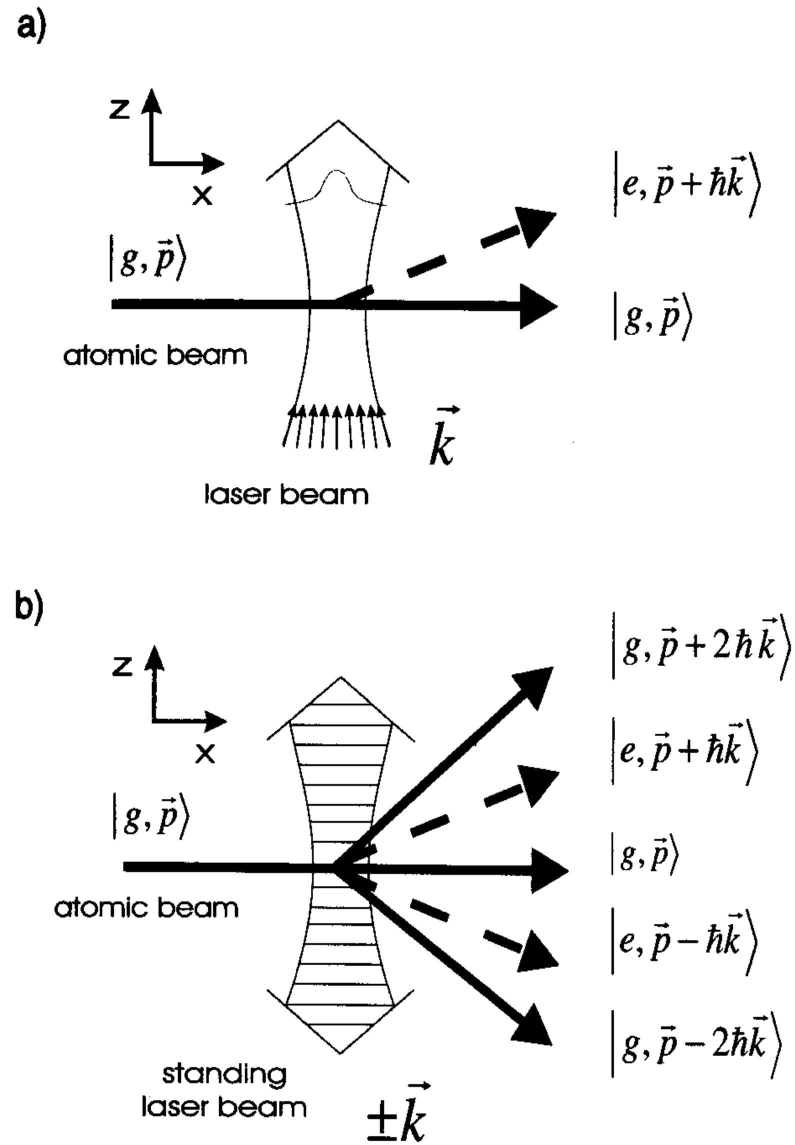
Atoms have internal states

Two level atom: g, e
Laser resonant on g e transition

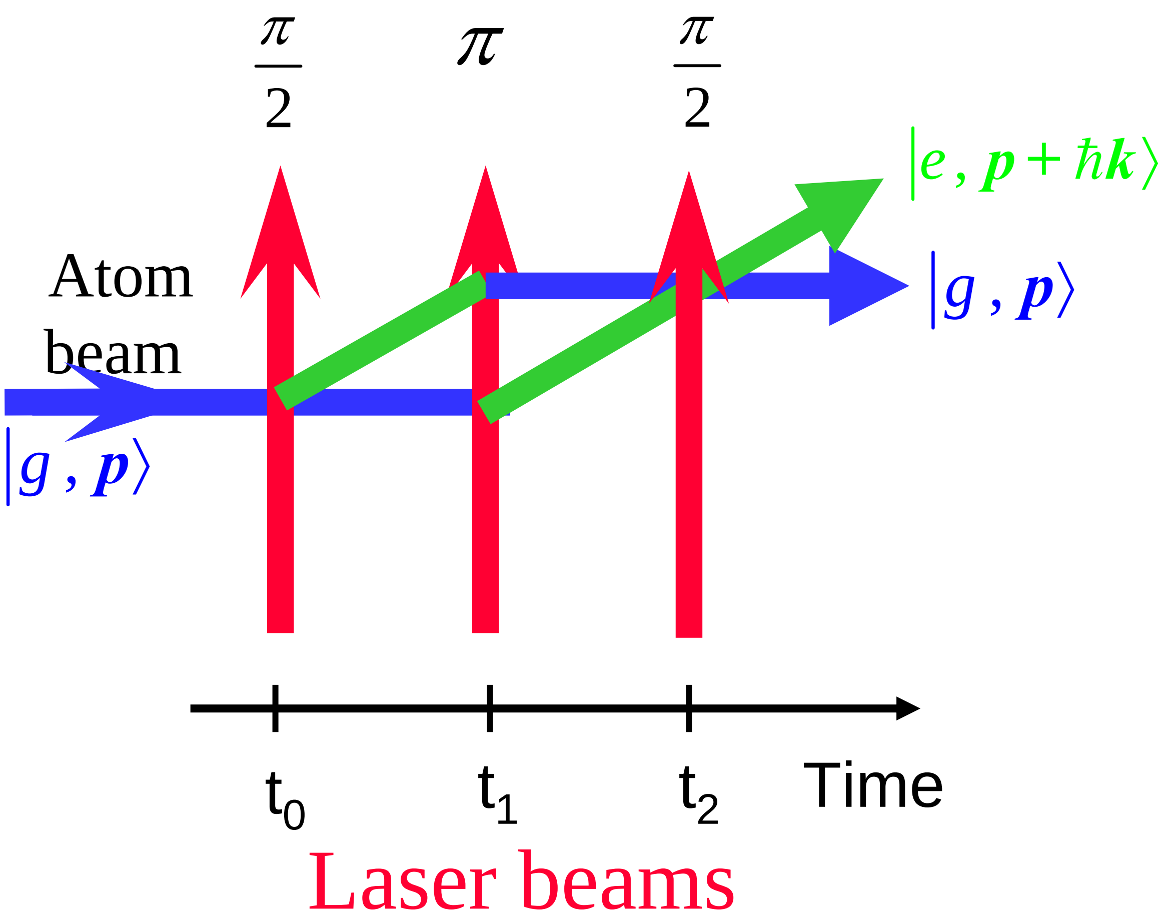
Neglect spontaneous emission
Use long lived upper states
Mg, Ca, Sr,…
or
Raman Transition between
hyperfine ground states
in alkalis for instance

Effective two-level system





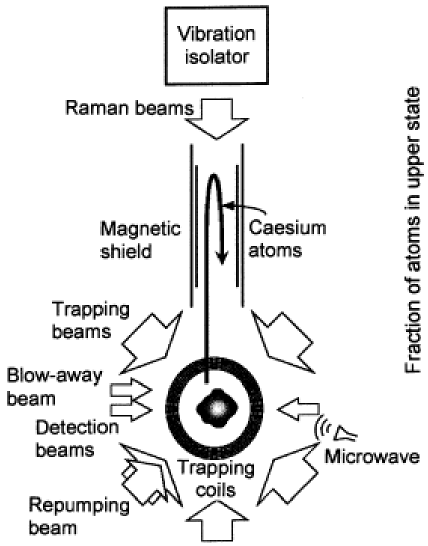
Mach-Zehnder interferometer with light beams

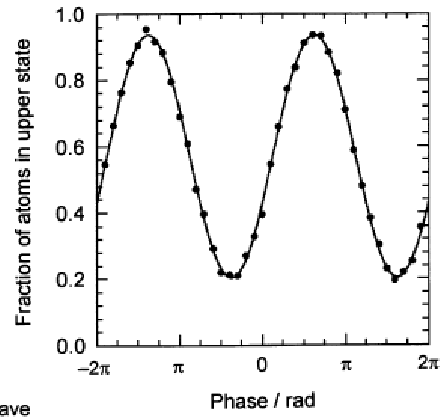


$$\delta\phi = \phi_1(t_0) - 2\phi_2(t_1) + \phi_3(t_2)$$

Sensitive to rotation and accelerations: gyrometers and gravimeters

Cold atom gravimeter

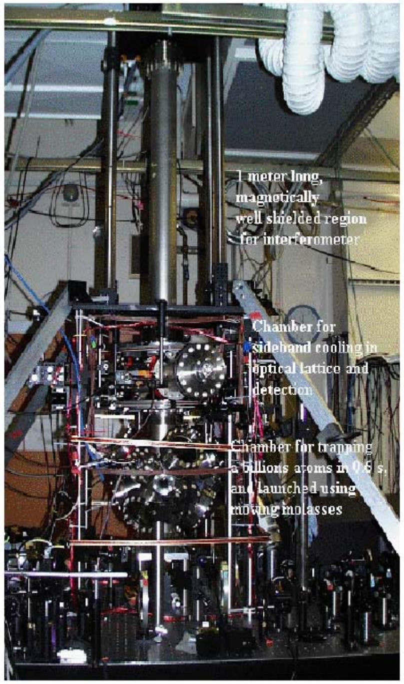


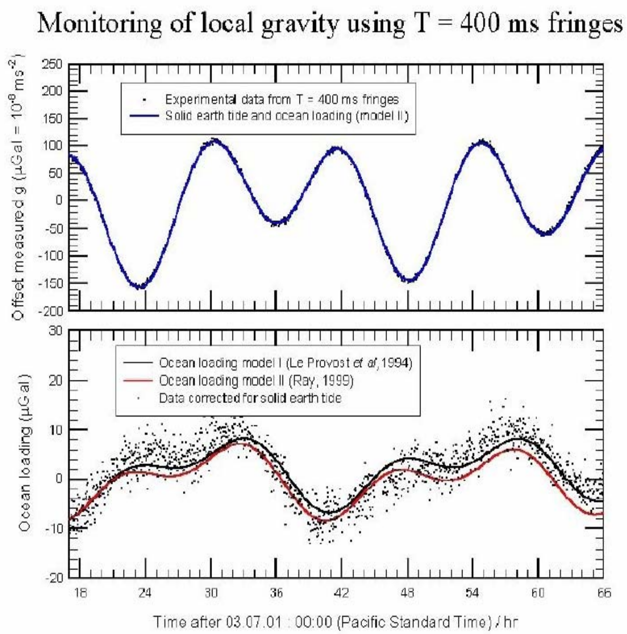


M. Kasevich and S. Chu, PRL 67, 1991

$$\delta\phi = -k_{eff}\, aT^2 = -k_{eff}\, gT^2 = -2k_L\, gT^2$$

$$a_{\min} = \delta\phi_{noise}\ \frac{\lambda_L}{4\pi T^2}\left[\frac{T_{cycle}}{\tau}\right]^{1/2}$$



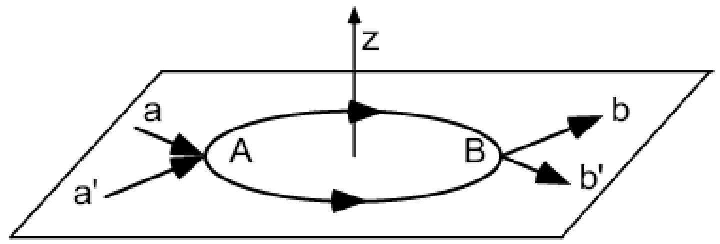


$$\delta\phi_{noise} = 0.01\, rad, \quad T = 0.4s$$

$$a_{\min} = 2\ 10^{-9}\, m/s^2\ at\ 1s$$

Detection of tides effects at ~10^{-7} g

Sagnac interferometer



Rotation with angular speed Ω around z

Interferometer area: A

Path length increase (decrease) for counter clockwise (clockwise) $\delta l = \Omega R T$

Travel time from A to B: $T = \pi R / v$

Phase difference $\delta\phi = 2k\delta l = 2k\Omega R T = 2k\Omega\pi R^2 / v = 2k\Omega A / v$

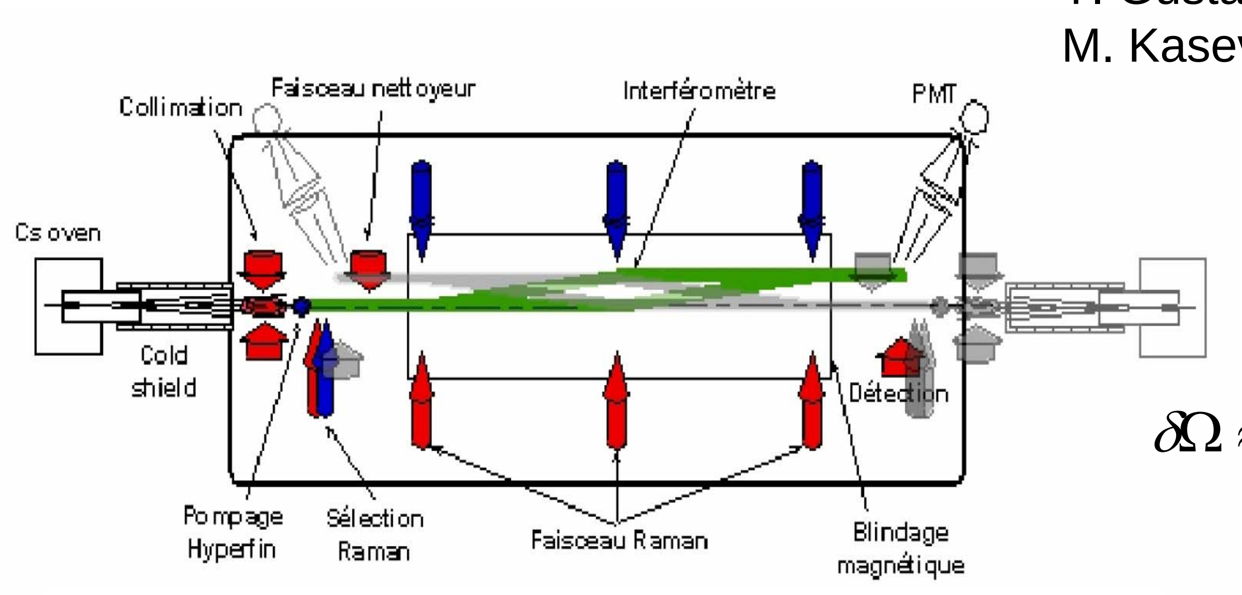
Photons: $v = c \quad k = \omega / c$ $\qquad \delta\phi_{photons} = 2\Omega A\omega / c^2$

Matter: $k = mv / \hbar$ $\qquad \delta\phi_{matter} = 2\Omega A m / \hbar$

Ratio of sensitivity: $$\frac{\delta\phi_{matter}}{\delta\phi_{photons}} = \frac{mc^2}{\hbar\omega} \sim \frac{100GeV}{1eV} \sim 10^{11}$$

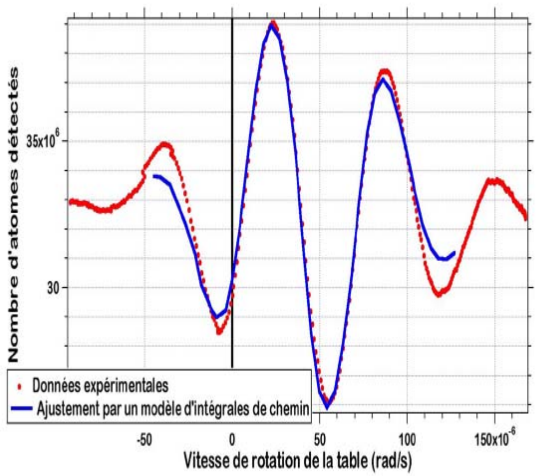
Stanford Gyroscope

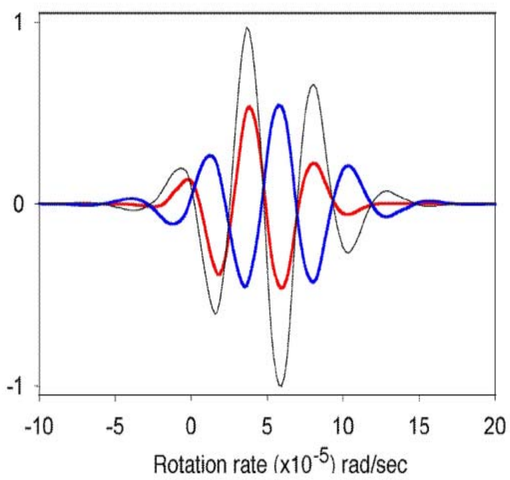
T. Gustavson, P. Bouyer and
M. Kasevich PRL 78, 2046, 1997



Current sensitivity

$$\delta\Omega \approx 6\ 10^{-10} rad\ s^{-1}\ Hz^{-1/2}$$

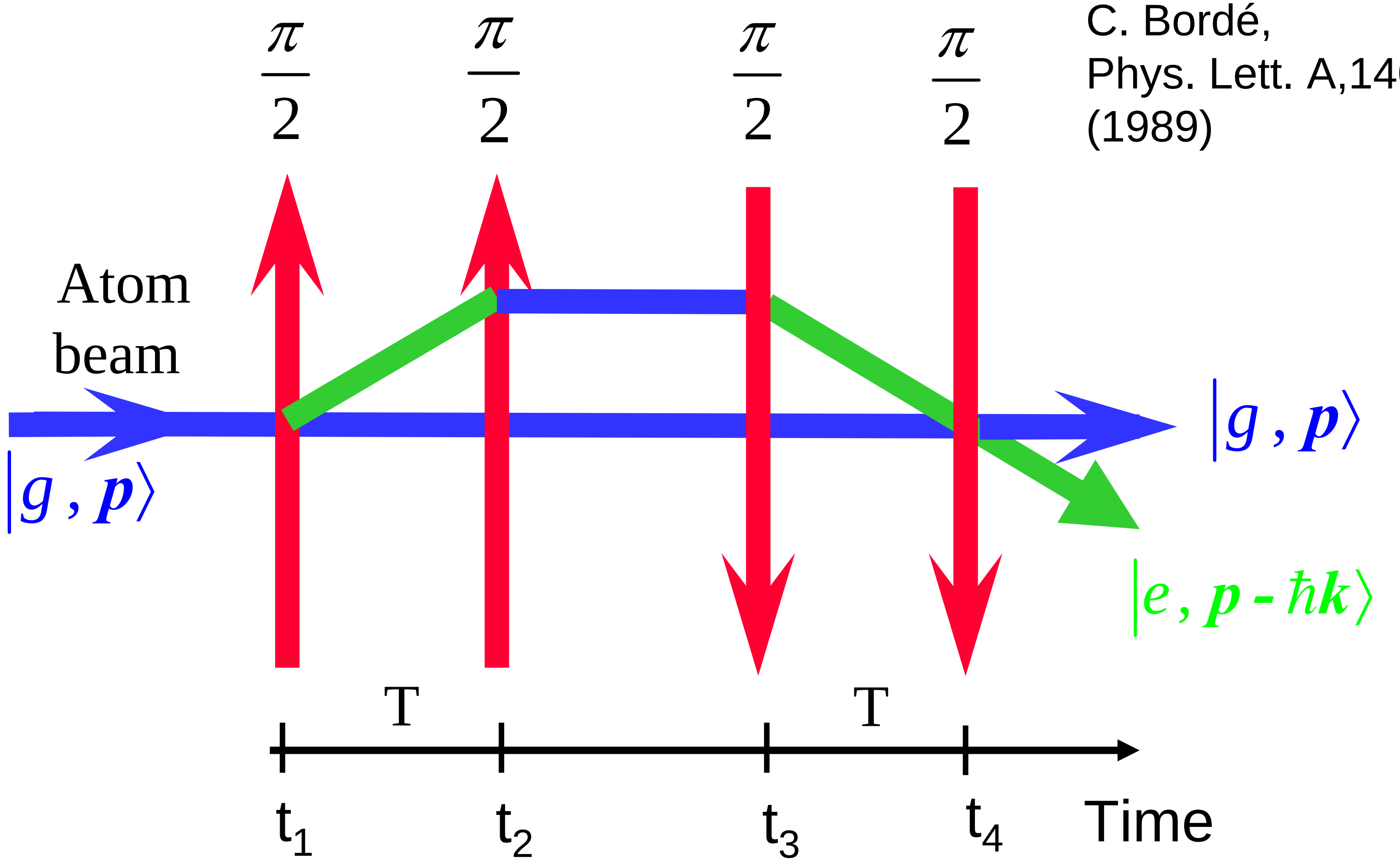




Measurement of
Earth rotation rate :
43 μrad/s

Ramsey-Bordé interferometer

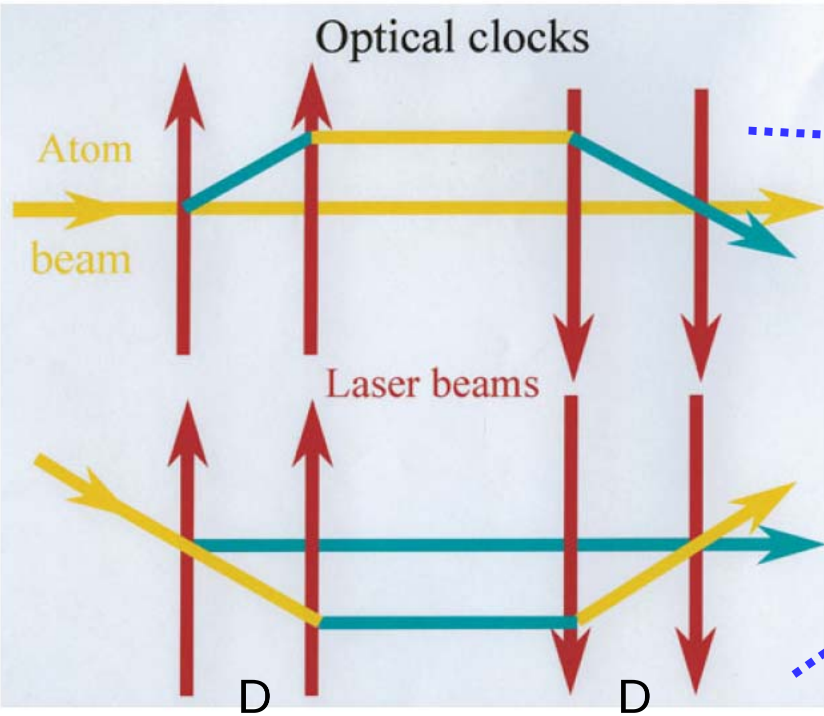
C. Bordé,
Phys. Lett. A,140
(1989)



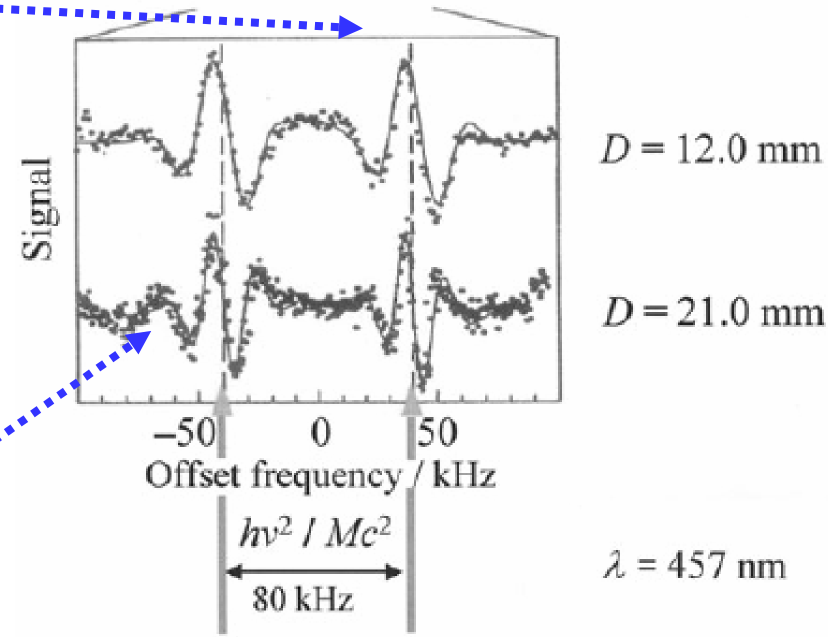
$$P_e \sim \cos\left(2T(\omega_L - \omega_0 + \delta) + \phi_L\right)$$

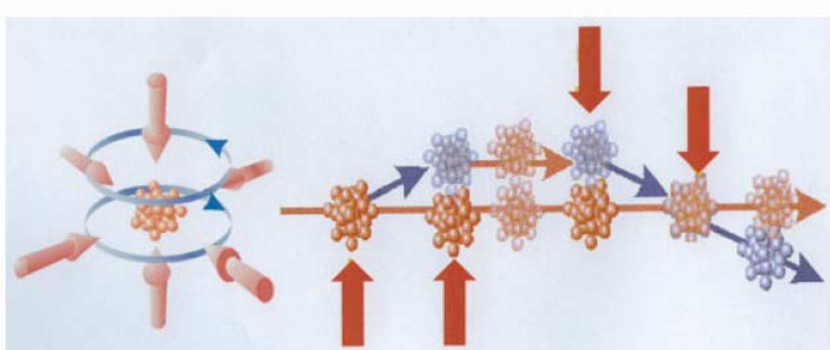
with $\delta = \hbar k^2 / 2m$ and $\phi_L = \phi_2 - \phi_1 + \phi_4 - \phi_3$

Recoil doublet in optical clocks



U. Sterr et al., atom interferometry,
P. Berman ed. 1997





Cold atoms frequency standards
Calcium: PTB, NIST… T increases
Lasers locked to Ca with
fractional instability
of 5 10^{-15} at 1 s and 2 10^{-16} at 2000 s
Accuracy: ~10^{-14}

h/m and fine structure constant α

Recoil splitting is: $\Delta f_{recoil} = h\nu^2 / m_{atom}c^2$

Measuring Δf_{recoil} is a measure of h / m_{atom}

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e} = \frac{2R_\infty}{c}\left(\frac{h}{m_{atom}}\right)\left(\frac{m_{atom}}{m_p}\right)\left(\frac{m_p}{m_e}\right)$$

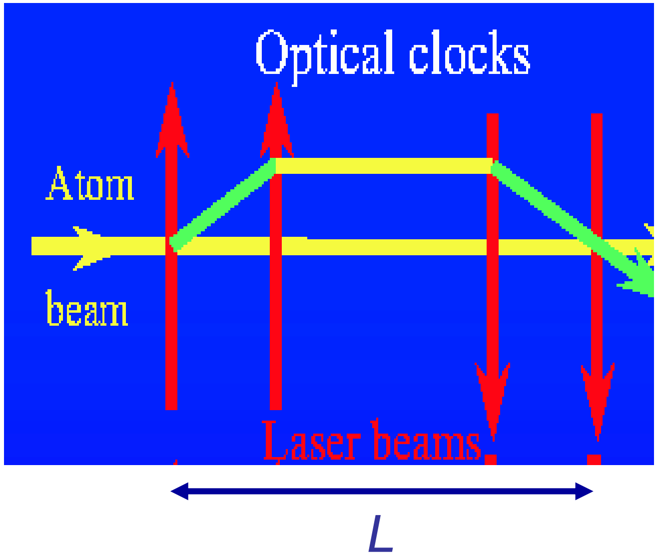
All other quantities can be measured at 10^{-9} or better
thus a photon recoil measurement at 10^{-9} can give α at 10^{-9}

Cesium atom interferometry: α at 7.7 10^{-9} — Wicht et al. Phys. Scripta (2002)
Bloch oscillations of Rb atoms : α at 6.7 10^{-9} — Cladé et al., PRL 96, (2006)
g-2 of electrons with QED calculations: α at 0.7 10^{-9}: Gabrielse et al, PRL, 97, (2006)

$$\alpha^{-1} = 137.035\,999\,710\,(96)\ [0.70\ \text{ppb}].$$

Cold atoms and precision measurements

Interferometers and clocks



T : interaction time with ELM field

Slow atoms: *T* large; atomic fountain
or microgravity of space
Interferometers on chips

Clocks: gain prop. to *T*

Inertial sensors:

Accelerometers: gain as T^2

Sagnac gyroscopes : gain as *L T*

Current sensitivity:

Acceleration: $\delta g/g = 3\ 10^{-9}$ in 1min

Rotation: $\Omega = 6\ 10^{-10}$ rad s^{-1} in 1 s

Summary

Atom interferometry has entered into high precision measurement phase
Fine structure constant and h/m
Towards a redefinition of the kilogram based on atomic masses

Earth rotation, g, g gradients, inertial base (GOM, CASI)
G: Magia (Firenze)
Tests of Newton law at short distances
Test of Equivalence principle

Prospects for ultra-high sensitivity inertial sensors in space
with long interrogation times: HYPER

Quantum gases sources and atom lasers with atom chips (ICE, Quantus)

See several talks at the workshop for most recent developments

Further reading

Atom Interferometry ed. Paul Berman, Academic Press, 1997

Atoms quanta and relativity, special Issue of J Phys B:
Atomic, Molecular and Optical physics , IoP (2005)
ed., T. Haensch, H. Schmidt-Boecking and H. Walther

C. Bordé, metrologia , 39, 435 (2002)
C. Cohen-Tannoudji, lectures at Collège de France 1992-1993
J. Dalibard, DEA lectures on cold atoms