Creation des paires e^+e^- dans un champ constant.

$$\mathcal{L} = -\bar\psi \not{D} \psi - m\bar\psi\psi = -\bar\psi \not{\partial}\psi - m\bar\psi\psi + i q \bar\psi \not{A} \psi$$

$$\mathcal{H}_I = -i q \bar\psi \not{A} \psi$$

$$S = T\exp\left(-i\int \mathcal{H}_I\right) = T\exp\left(-q\int \bar\psi \not{A}\psi \, d^3x\,dt\right)$$

Théorème de Wick:

$$\langle 0| T(\psi(x_1) \dots \psi(x_{2n}))|0\rangle = \sum_{\text{paires}} (-)^{\varepsilon(\sigma)} \langle 0|T(\psi(x_{\sigma(1)})\psi(x_{\sigma(2)})|0\rangle \times$$

$$\dots \langle 0|T\psi_{x_{\sigma(2n-1)}}\psi_{x_{\sigma(2n)}}|0\rangle$$

conservation de la charge: $\langle T\psi\psi\rangle = 0 = \langle T\bar\psi\bar\psi\rangle$

$$\langle T\psi_\alpha(x)\bar\psi_\beta(y)\rangle = S_{F\,\alpha\beta}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \left(\frac{-i}{i\not{p} + m - i\varepsilon}\right)_{\alpha\beta}$$

Permutations: $T(\bar\psi_1\psi_1 \dots \bar\psi_n\psi_n) \to \langle\psi_1\bar\psi_{p_1}\rangle \dots \langle\psi_n\bar\psi_{p_n}\rangle (-)^{\varepsilon(\sigma)}$

$$\sigma = \begin{pmatrix} \bar1 & 1 & \bar2 & 2 & \dots & \bar n & n \\ 1 & \bar p_1 & 2 & \bar p_2 & \dots & n & \bar p_n \end{pmatrix} = \begin{pmatrix} 1 & \bar1 & \dots & n & \bar n \\ \bar1 & 1 & \dots & \bar n & n\end{pmatrix} \circ \begin{pmatrix} \bar1 & \dots & \bar n \\ \bar p_1 & \dots & \bar p_n\end{pmatrix}$$

$$\varepsilon(\sigma) = (-)^n \varepsilon(P)$$

Γ operateur dans $V = L^2(\mathbb{R}^4) \otimes \{\text{spin space}\}$,

$$\langle x, \alpha | \Gamma | y, \beta \rangle = +q \, \cancel{A}_{\alpha\delta} \, S'_{\delta\beta}(x-y)$$

$$\langle 0|S|0\rangle = \sum_{n=0}^{\infty} \frac{(-q)^n}{n!} \int \langle 0 | T\left(\bar\psi(x_1)\gamma^{\mu_1}\psi(x_1) \dots \bar\psi(x_n)\gamma^{\mu_n}\psi(x_n)\right)|0\rangle$$
$$A_{\mu}(x_1) \dots A_{\mu}(x_n) \, dx_1 \dots dx_n =$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{p \in S_n} (-q)^n \varepsilon(p) \int \langle x_1, \beta_1 | \Gamma | x_{p_1}, \beta_{p_1} \rangle \dots \langle x_n, \beta_n | \Gamma | x_{p_n}, \beta_{p_n} \rangle$$
$$dx_1 \dots dx_n$$

on peut considerer $(x, \beta) = \alpha$ comme un indice d'une base pour V:

$$S_{00} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \sum_p \varepsilon(p) \langle \alpha_1 | \Gamma | \alpha_{p(1)} \rangle \dots \langle \alpha_n | \Gamma | \alpha_{p(n)} \rangle$$

$$\det A = \sum_{p \in S_n} \varepsilon(p) A_{1p_1} \dots A_{np_n} = \frac{1}{n!} \sum_{\{i\}} \sum_{p \in S_n} \varepsilon(p) A_{i_1 p(i_1)} \dots A_{i_n p(i_n)}$$

$$\det(1-A) = \sum_{p \in S_N} \varepsilon(p) \left(\delta_{1p_1} - A_{1p_1}\right) \dots \left(\delta_{Np_N} - A_{Np_N}\right) =$$

$$= \frac{1}{N!} \sum_{\{i_1 \dots i_N\}} \sum_{p \in S_N} \varepsilon(p) \left(\delta_{i_1 p(i_1)} - A_{i_1 p(i_1)}\right) \dots \left(\delta_{i_N p(i_N)} - A_{i_N p(i_N)}\right)$$

terme A^n: $\displaystyle \frac{(-)^n}{N!} \sum_{\{i\}, p} \varepsilon(p) \left(A_{i_1 p(i_1)} \dots A_{i_n p(i_n)} \delta_{i_{n+1}, p(i_{n+1})} \dots \delta_{i_N, p(i_N)} + \text{perm.}\right)$

$$= \frac{(-)^n}{N!} \sum_{\{i\}, p} \varepsilon(p) \binom{N}{n} A_{i_1 p(i_1)} \dots A_{i_n p(i_n)} \, \delta \dots \delta$$

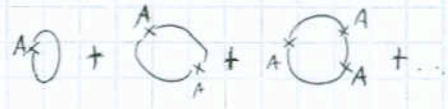
$$= \frac{(-)^n}{N!} \sum_{\{i_1 \dots i_n\}} \sum_{p \in S_n} \varepsilon(p) \frac{N!}{n!(N-n)!} A_{i_1 p(i_1)} \dots A_{i_n p(i_n)} (N-n)! = \frac{(-)^n}{n!} \sum_{\{i_n\}, p} \varepsilon(p) A \dots A;$$

$$\Gamma = q\not{A}\,\frac{-i}{i\not{\partial}+m-i\varepsilon} = -i q \not{A}\,\frac{1}{\not{\partial}+m-i\varepsilon} \;;$$

$$\det(1-\Gamma) = \det\left(\frac{\not{\partial} - iq\not{A} + m - i\varepsilon}{\not{\partial}+m-i\varepsilon}\right) = \frac{\det(\not{D}+m-i\varepsilon)}{\det(\not{\partial}+m-i\varepsilon)}$$

On peut obtenir le même résultat comme: $\dfrac{Z[A]}{Z[0]}$,

$$Z[A] = \int D\psi\, D\bar{\psi}\; e^{i\int \bar{\psi}(\not{D}+m)\psi} \;;\quad \frac{Z}{Z_0} \sim$$



Intégrale de Berezin: $\int d\psi = 0 \qquad \int \psi\, d\psi = 1$

$$\int d\psi\, f(\psi) = f'(0)$$

$$\int d\psi_i\, d\bar{\psi}_i\; e^{-\psi_i A_{ij}\bar{\psi}_j} = \frac{(-1)^n}{n!}\int d\psi_i\, d\bar{\psi}_i\, (\psi_{i_1} A_{i_1 j_1}\bar{\psi}_{j_1}) \ldots (\psi_{i_n} A_{i_n j_n} \bar{\psi}_{j_n})$$

$$= (-)^n \int d\psi\, d\bar{\psi}\, (\psi_1 A_{1 j_1} \bar{\psi}_{j_1}) \ldots (\psi_n A_{n j_n} \bar{\psi}_{j_n}) \to \{j_n\} \in S_n$$

$$= (-)^{\frac{n(n-1)}{2}} \det A$$

ψ et $\bar{\psi}$ sont considérés comme indépendants dans la théorie euclidienne.

Le signe est cancellé dans le rapport des déterminants.

On peut interpréter $Z[A]$ comme une action effective pour le champ A, qui tient compte des effets quantiques des fermions.

$$C\gamma^\mu C^{-1} = -(\gamma^\mu)^T$$

$$\det(\not{D}+m) = \det(\not{D}^T+m) = \det(-C\not{D}C^{-1}+m) = \det(\not{D}-m)$$

$$S_{00}^2 = \frac{\det(\not{D}+m-i\varepsilon)(\not{D}-(m-i\varepsilon))}{\det(\not{\partial} \quad)(\not{\partial} \quad)} = \frac{\det(\not{D}^2-m^2+i\varepsilon)}{\det(\not{\partial}^2-m^2+i\varepsilon)}$$

$$\not{\partial}^2 = \gamma^\mu\gamma^\nu\partial_\mu\partial_\nu = \Box;$$

$$\not{D}^2 = (\not{\partial}-iq\not{A})(\not{\partial}-iq\not{A}) = D_\mu D_\nu\left(\frac{1}{2}\{\gamma^\mu,\gamma^\nu\}+\frac{1}{2}[\gamma^\mu,\gamma^\nu]\right) =$$

$$= D^2 + \frac{1}{4}[\gamma^\mu,\gamma^\nu][D_\mu,D_\nu] =$$

$$= D^2 + \frac{1}{4}[\gamma^\mu,\gamma^\nu](-iq)F_{\mu\nu} = D^2 - \frac{q}{2}F_{\mu\nu}\sigma^{\mu\nu} \equiv H \equiv H_1+H_2;$$

En utilisant: $\log\frac{a}{b} = \int_0^\infty \frac{ds}{s}\left(e^{is(b+i\varepsilon)} - e^{is(a+i\varepsilon)}\right)$

$$\log S_{00} = \frac{1}{2}\int_0^\infty \frac{ds}{s}\, e^{is(-m^2+i\varepsilon)}\, \mathrm{Tr}\left(e^{is\Box} - e^{isH}\right)$$

Champ constant: $A_\mu = \frac{1}{2}F_{\mu\nu}x^\nu$ (choix de jauge)

$$H_1 = \left(\partial_\mu - \frac{iq}{2}F_{\mu\rho}x^\rho\right)\left(\partial^\mu - \frac{iq}{2}F^\mu{}_\nu x^\nu\right) = \partial^2 - \frac{q^2}{4}F_{\mu\rho}F^\mu{}_\nu x^\rho x^\nu$$

$\vec{E} = (0, 0, E)$

$F_{03} = E\ ; \quad A_3 = E\,x^0\ ;$

$$D^2 = (\partial_\mu - iqA_\mu)^2 = -\partial_0^2 + \partial_1^2 + \partial_2^2 + (\partial_3 - iqE\,x^0)^2 =$$

$$= -\partial_0^2 + \partial_1^2 + \partial_2^2 - q^2E^2\left(x^0 - \frac{1}{iqE}\partial_3\right)^2$$

$$\rightarrow P_0^2 - p_1^2 - p_2^2 - q^2E^2\left(x^0 - \frac{1}{qE}P_3\right)^2$$

$$[x^0, P_0] = +i \Rightarrow e^{-iaP_0}\,x^0\,e^{+iaP_0} = x^0 + i(ia) = x^0 - a$$

$$H_1 = D^2 = p_0^2 - p_1^2 - p_2^2 - q^2E^2\,e^{-iaP_0}(x^0)^2\,e^{iaP_0}\ , \quad a = \frac{P_3}{qE}$$

$$e^{isH_1} = e^{-is(p_1^2+p_2^2)}\,e^{-iaP_0}\,e^{is(p_0^2 - q^2E^2(x^0)^2)}\,e^{iaP_0} \qquad \left(\text{on utilise } e^{A^{-1}BA} = A^{-1}e^{B}A\right)$$

$$\mathrm{Tr}_{L^2}\,e^{isH_1} = \int d^4x\ \langle x|\,e^{isH_1}\,|x\rangle =$$

$$= \int d^4x \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\,e^{ix(p'-p)}\ \langle p,\omega|\,e^{isH_1}\,|p',\omega'\rangle =$$

$$= \int d^4x \int \frac{dp\,dp'}{(2\pi)^6}\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\,e^{ix(p'-p)}\,(2\pi)^3\,\delta(\vec{p}-\vec{p}')\ \langle|\ldots|\rangle =$$

$$= \int d^4x \int \frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\ e^{-is(p_1^2+p_2^2)}\,e^{\frac{ip_3(\omega-\omega')}{qE}}\ \langle p,\omega|\,e^{is(p_0^2 - q^2E^2(x^0)^2)}\,|p,\omega'\rangle$$

$$\int \frac{dp_1\,dp_2}{(2\pi)^2}\,e^{-is(p_1^2+p_2^2)} = \frac{1}{4\pi i s}\ ; \quad \int \frac{dp_3}{2\pi}\,e^{\frac{ip_3}{qE}(\omega-\omega')} = qE\ \delta(\omega-\omega')\ ;$$

$$\mathrm{Tr}_{c^2}\left(e^{isH_1}\right) = \int d^4x \, \frac{1}{4\pi i s} \, \frac{d\omega}{(2\pi)^2} \, qE \quad \langle \omega | \, e^{is\left(p_0^2 - q^2E^2(x^1)^2\right)} \, | \omega \rangle$$

Harmonic loscillator: $\hat{H} = \frac{p^2}{2M} + \frac{1}{2} M\Omega^2 q^2$

$$\mathrm{Tr}\left(e^{is\hat{H}}\right) = \sum_n e^{is\Omega(n+\frac{1}{2})} = \frac{e^{\frac{i\Omega}{2}s}}{1-e^{is\Omega}} = \frac{i}{2\sin\frac{s\Omega}{2}}$$

$$M = \frac{1}{2}, \quad \Omega = 2iqE \ : \quad \frac{i}{2\sin(isqE)} = \frac{1}{2\sinh(qEs)} \, ;$$

Trace sur les spineurs: $\mathrm{tr}\left(e^{isH_2}\right) = \mathrm{tr}\left(e^{-\frac{iqs}{2} F_{\mu\nu}\sigma^{\mu\nu}}\right)$

$$= \mathrm{tr} \sum_n \left(\frac{qs}{2}\right)^n \frac{E^n}{n!} \left[\gamma^0, \gamma^3\right]^n \neq$$

$$\left[\gamma^0, \gamma^3\right]^2 = \cancel{\ldots} \, 4\gamma^0\gamma^3\gamma^0\gamma^3 = 4\,\mathbb{1},$$

$$\Rightarrow \mathrm{tr} \sum_m \left(\frac{qs}{\cancel{2}}\right)^{2m} \frac{E^{2m}}{(2m)!} \cancel{4^m} \times \mathrm{tr}\,\mathbb{1} = 4\cosh(qEs),$$

Finalement,

$$S_{00} = -\frac{1}{2} \log \int_0^\infty \frac{ds}{s} \, e^{is(-m^2+i\varepsilon)} \int d^4x \, \frac{1}{4\pi^2} \, \frac{1}{is} \left[\frac{qE\,\mathrm{ch}(qEs)}{\mathrm{sh}(qEs)} - \frac{1}{s} \right]$$

Probabilité de ne pas produire de paires

$$|\langle 0|S|0\rangle|^2 = \exp\Big[\frac{VT}{4\pi^2}\int_0^\infty \frac{ds}{s^2}\Big(\frac{qE\,ch(qEs)}{sh(qEs)} - \frac{1}{s}\Big)\, \mathrm{Re}\Big(\underbrace{i\,e^{-ism^2 - s\varepsilon}}_{e^{-s\varepsilon}\sin(m^2 s)}\Big)$$

$$:= e^{-wVT}$$

w = taux de production pour unité de temps/volume

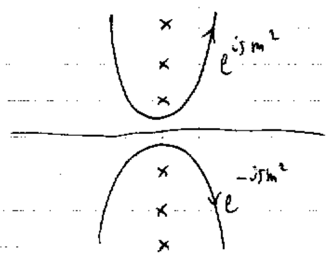
Convergence: $s \to \infty$: $e^{-s\varepsilon}$

$s \to 0$: $\frac{1}{s^2} \times s \times s \sim 1.$

Pôles: $sh(qEs) = 0$: $s_n = \frac{\pi i}{qE} n$, $n \in \mathbb{Z}\setminus\{0\}$

en étendant l'intégrale à $(-\infty, +\infty)$:

$$W = -\frac{1}{8\pi^2}\int_{-\infty}^{\infty} \frac{ds}{s^2}\Big(qE \coth(qEs) - \frac{1}{s}\Big)\frac{1}{2i}\Big(\underset{\substack{\downarrow \\ \mathrm{Im}\, s>0}}{e^{im^2 s}} - \underset{\substack{\downarrow \\ \mathrm{Im}\, s<0}}{e^{-im^2 s}}\Big)$$

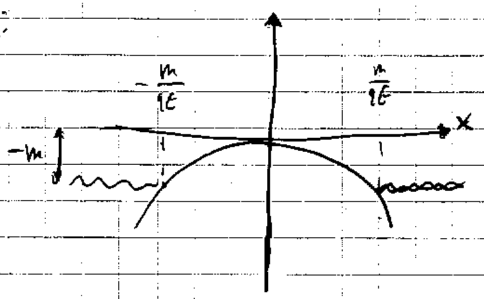


$$= -\frac{1}{16\pi^2 i} \times 2\pi i \sum_{n\neq 0} \mathrm{Res}_{s=s_n} \frac{1}{s^2}\, qE \coth(qEs)\, e^{m^2|s|}$$

$$= -\frac{1}{8\pi^2}\sum_n \frac{(qE)^2}{(-\pi^2 n^2)} e^{-\frac{m^2\pi}{qE} n} = \sum_{n=1}^{\infty} \frac{(qE)^2}{4\pi^3 n^2} e^{-\frac{m^2\pi}{qE} n}$$

Facteur $e^{-\frac{1}{qE}}$: effet non-perturbatif.

Effet tunnel:



$$V = -q^2 E^2 x^2$$

$$H = \frac{p^2}{2m} + V(x) = -m;$$

taux de probabilité de tunneling: $e^{-\int p\, dx}$ $= \int_{-\frac{m}{qE}}^{\frac{m}{qE}} dx \sqrt{(m - q^2E^2x^2)2m}$

$$= \sqrt{2m}\, \frac{m}{qE} \int_{-1}^{1} dx \sqrt{m - m x^2} = \sqrt{2}\, \frac{m^2}{qE} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\sin\theta \cos\theta = \frac{\pi m^2}{\sqrt{2}\, qE};$$

$$\sim e^{-\frac{\pi m^2}{qE}};$$

Champs electromagnetique: $A_3 = E x^0$ $\quad F_{03} = E$

$A_2 = B x_1$ $\quad F_{12} = B$

$$D^2 = (\partial_\mu - i q A_\mu)^2 = -\partial_0^2 + \partial_1^2 + (\partial_2 - i q B x_1)^2 + (\partial_3 - i q E x^0)^2$$

$$\to p_0^2 - p_1^2 - q^2 B^2 \left(x_1 - \frac{1}{qB} p_2\right)^2 - q^2 E^2 \left(x^0 - \frac{1}{qE} p_3\right)^2$$

$$= p_0^2 - p_1^2 - q^2 E^2 e^{-iap_0} (x_0)^2 e^{iap_0} - q^2 B^2 e^{-ibp_1} (x_1)^2 e^{ibp_1}$$

$$= e^{-iap_0} \left(p_0^2 - q^2 E^2 (x^0)^2\right) e^{iap_0} + e^{-ibp_1} \left(-p_1^2 - q^2 B^2 (x_1)^2\right) e^{ibp_1}$$

$$a = \frac{p_3}{qE}, \quad b = \frac{p_2}{qB}$$

$$\int d^4x \langle x| e^{-iap_0} e^{is(p_0^2 - q^2E^2(x^0)^2)} e^{iap_0} e^{-ibp_1} e^{is(-p_1^2 - q^2B^2(x_1)^2)} e^{ibp_1} |x\rangle =$$

$$= \int d^4x \int \frac{d\vec{p}}{(2\pi)^3} \frac{d\vec{p}'}{(2\pi)^3} \frac{d\omega\, d\omega'}{(2\pi)^2} e^{ix(p'-p)} (2\pi)^2 \delta(p_{2,3} - p'_{2,3}) e^{ia(\omega'-\omega)} e^{ib(p_1'-p_1)} =$$

$$= \int d^4x \int \frac{dp_2\, dp_3}{(2\pi)^2} \frac{dp_1\, dp_1'\, d\omega\, d\omega'}{(2\pi)^4} e^{i\frac{p_3}{qE}(\omega'-\omega)} e^{i\frac{p_2}{qB}(p_1'-p_1)} \langle \omega| e^{is(p_0^2 - q^2E^2(x_0)^2)} |\omega'\rangle \times$$

$$\times \langle p_1| e^{is(-p_1^2 - q^2B^2(x_1)^2)} |p_1'\rangle$$

$$= \int d^4x\, (qB)(qE) \int \frac{d\omega}{(2\pi)^2} \langle \omega| e^{is(\ )} |\omega\rangle \int \frac{dp}{(2\pi)} \langle p| e^{is(-p_1^2 - \dots)} |p\rangle =$$

$$= q^2 EB \frac{1}{(2\pi)^4} \frac{1}{2\,\mathrm{sh}(qEs)} \frac{(-i)}{2\sin(qBs)}$$

$$\mathrm{tr}\left(e^{isH_2}\right) = \mathrm{tr}\left(e^{-i\frac{qs}{2}F_{\mu\nu}\sigma^{\mu\nu}}\right) = *$$

$$F_{\mu\nu}\sigma^{\mu\nu} = i E\,[\gamma^0, \gamma^3] + i B\,[\gamma^1, \gamma^2]$$

$$* = \sum_n \left(\frac{qs}{2}\right)^n \frac{1}{n!}\,\mathrm{tr}\left(E\,[\gamma^0, \gamma^3] + B\,[\gamma^1, \gamma^2]\right)^n$$

Weyl basis: $\gamma^0, \gamma^3 = \sigma^0 \otimes 1, \sigma^3 \otimes 1$

$\gamma^1, \gamma^2 = \sigma^1 \otimes \sigma^2, \sigma^2 \otimes \sigma^2$

$$\Rightarrow \mathrm{tr}\left(e^{\frac{qs}{2}E[\gamma_0,\gamma_3]}\right)\mathrm{tr}\left(e^{\frac{qs}{2}B[\gamma^1,\gamma^2]}\right) = 4\,\mathrm{ch}(qEs)\cos(qBs)$$

$$\mathrm{Tr}\left(e^{isH} - e^{isH_0}\right) = \frac{(-i)q^2EB}{(2\pi)^4}\left(\frac{\mathrm{ch}(qEs)\cos(qBs)}{\mathrm{sh}(qEs)\,\mathrm{sen}(qBs)} - \frac{1}{q^2EBs^2}\right)$$

$$\log S_{00} = \frac{1}{2}\int dx \int \frac{ds}{s}\, i e^{is(-m^2+i\varepsilon)}\,\frac{1}{(2\pi)^4}\left(q^2EB\,\frac{\mathrm{ch}(qEs)\cos(qBs)}{\mathrm{sh}(qEs)\,\mathrm{sen}(qBs)} - \frac{1}{s^2}\right)$$

$$= \int dx\, \delta\mathcal{L},$$

Regardons la partie imaginaire:

$$\sim \int_0^\infty \frac{ds}{s}\cos(m^2 s) \quad \text{divergence logarithmique}$$

$$\cancel{\frac{\partial^2}{\partial s^2}(\;) = \frac{(qE\,\mathrm{sh}\cos - qB\,\mathrm{ch}\,\mathrm{sen})\,\mathrm{sh}\,\mathrm{sen} - \mathrm{ch}\cos(qE\,\mathrm{ch}\,\mathrm{sen} - qB\,\mathrm{sh}\cos)}{\mathrm{sh}^2\,\mathrm{sen}^2} = q^2E^2}$$

$$\frac{\operatorname{ch} x \cos y}{\operatorname{sh} x \operatorname{sen} y} = \frac{\left(1+\frac{x^2}{2}\right)\left(1-\frac{y^2}{2}\right)}{\left(x+\frac{x^3}{3}\right)\left(y-\frac{y^3}{3}\right)} = \frac{1}{xy}\left[1+\frac{x^2}{2}-\frac{y^2}{2}-\frac{x^2}{3}+\frac{y^2}{3}\right]$$

$$\delta\mathcal{L} = \log \mathcal{J}_{0e} = \frac{1}{8\pi^2}\int \frac{ds}{s} \cos(m^2 s) \frac{1}{6}\left(E^2-B^2\right) \to \text{renormalização de } e_{YM}$$

ou, se se produz $s \to -it$,

$$\delta\mathcal{L} = \frac{-1}{8\pi^2}\int_0^\infty \frac{dt}{t} e^{-tm^2} \frac{e^2}{6}\left(E^2-B^2\right)$$

$$A \to \frac{A}{e}, \quad \mathcal{L}^{(0)} = -\frac{1}{4e^2} F^2 = -\frac{1}{4e^2}\left(E^2-B^2\right)$$

$$\mathcal{L} = \mathcal{L}^0 + \delta\mathcal{L} = -\frac{1}{4e_R^2}\left(E^2-B^2\right), \qquad \frac{1}{e_R^2} = \frac{1}{e_0^2}\left(1+\frac{1}{12\pi^2} e_0^2 \int_0^\infty \frac{dt}{t} e^{-m^2 t}\right)$$

UV cut-off μ: $\displaystyle\int_{\mu^{-2}}^\infty \frac{dt}{t} e^{-m^2 t} = 2\log\frac{\mu}{m}$

$$\frac{1}{e_R^2} = \frac{1}{e^2(\mu)} + \frac{1}{6\pi^2}\log\frac{\mu}{m},$$