Theorie de Chern-Simons:

$$\mathcal{L} = \int d^3x \;\; \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

invariant de jauge mais non de la forme $f(F_{\mu\nu})$

E.o.m: $\quad \partial_{[\mu} A_{\nu]} = F_{\mu\nu} = 0 \qquad$ (du première ordre)

$$T^{\mu\nu} = -\frac{\delta \mathcal{L}}{\delta \partial_\mu A^\rho} \partial^\nu A^\rho + \eta^{\mu\nu} \mathcal{L}$$

$$\sim - \varepsilon^{\alpha\mu\rho} A_\alpha \partial^\nu A_\rho \qquad (\mathcal{L} = 0 \text{ on-shell})$$

$$\tilde{S}^{\rho\mu\nu} = -\frac{i}{2} \left(\varepsilon^{\alpha\rho\ell} A_\alpha \right) (-i) \left(\delta^\mu_\ell A^\nu - \delta^\nu_\ell A^\mu \right) =$$

$$= -\frac{1}{2} A_\alpha \left(\varepsilon^{\alpha\rho\mu} A^\nu - \varepsilon^{\alpha\rho\nu} A^\mu \right);$$

$$S^{\rho\mu\nu} = -\frac{1}{2} A_\alpha \Big(\varepsilon^{\alpha\rho\mu} A^\nu - \underset{\sim}{\varepsilon^{\alpha\rho\nu}} A^\mu - \varepsilon^{\alpha\mu\rho} A^\nu + \underline{\varepsilon^{\alpha\mu\nu}} A^\rho +$$
$$- \underline{\underline{\varepsilon^{\alpha\nu\rho}}} A^\mu + \underline{\varepsilon^{\alpha\nu\mu}} A^\rho \Big) =$$

$$= - A_\alpha \, \varepsilon^{\alpha\rho\mu} A^\nu ;$$

$$\Theta^{\mu\nu} = T^{\mu\nu} + \partial_\rho S^{\rho\mu\nu} = - \varepsilon^{\alpha\mu\rho} A_\alpha \partial^\nu A_\rho - \varepsilon^{\alpha\rho\mu} A_\alpha \partial_\rho A_\nu$$
$$= \varepsilon^{\alpha\rho\mu} A_\alpha \left(\partial^\nu A_\rho - \partial_\rho A^\nu \right) \sim 0.$$

Invariance sous difféomorphismes:

$$A'_\mu(x')\, dx'^\mu = A_\mu\, dx^\mu$$

$$x' = x - \xi(x) : \quad \delta A_\mu = \xi^\nu \partial_\nu A_\mu + \partial_\mu \xi^\nu A_\nu$$

$$\delta \mathcal{L} = \varepsilon^{\mu\nu\rho}\left(\delta A_\mu \partial_\nu A_\rho + A_\mu \partial_\nu \delta A_\rho\right) = \varepsilon^{\mu\nu\rho}\left(\delta A_\mu \partial_\nu A_\rho - \partial_\nu A_\mu \delta A_\rho\right) =$$

$$= \varepsilon^{\alpha\beta\gamma} \delta A_\alpha \left(\partial_\beta A_\gamma - \partial_\gamma A_\beta\right) = \varepsilon^{\alpha\beta\gamma} \delta A_\alpha F_{\beta\gamma} =$$

$$= \varepsilon^{\alpha\beta\gamma}\left(\xi^\sigma \partial_\sigma A_\alpha + \partial_\alpha \xi^\sigma A_\sigma\right) F_{\beta\gamma} = \varepsilon^{\alpha\beta\gamma} \xi^\sigma \left(\partial_\sigma A_\alpha - \partial_\alpha A_\sigma\right) F_{\beta\gamma} =$$

$$= \varepsilon^{\alpha\beta\gamma} \xi^\sigma F_{\sigma\alpha} F_{\beta\gamma} = 0.$$

$F_{\sigma\alpha} \sim \varepsilon_{\mu\sigma\alpha} V^\mu$, $\quad V_1 \otimes V_2 \otimes V_3$ le seul scalaire est ε_{ijk},

mais avec $V_2 = V_3 \Rightarrow 0$

$$\left([\tfrac{1}{2}] \otimes_s [\tfrac{1}{2}]\right) \otimes [1] = \left([0] \oplus [2]\right) \otimes [1] = [1] \oplus \left([1] \oplus [3]\right) \text{ no singlet}$$

explicitement, $F_{0[\alpha} F_{\beta\gamma]} = F_{01} F_{20} + F_{02} F_{01} = 0.$

La théorie a une invariance sous difféomorphismes avant le couplage à la gravité:

$$\Theta^{\mu\nu} = \frac{\delta S}{\delta g_{\mu\nu}} = 0.$$

Analyse canonique :

$$\mathcal{L}_{CS} = \int A_\mu \partial_\nu A_\rho \, \varepsilon^{\mu\nu\rho} \; ; \qquad \text{e.o.m.} \quad F_{\mu\nu} = 0$$

$$\Pi^\mu = \frac{\delta \mathcal{L}}{\delta \partial_0 A_\mu} = \varepsilon^{0\mu\nu} A_\nu = \varepsilon^{ij} A_j \; ; \qquad \begin{cases} \Pi_0 = 0 \; ; \\ \Pi^i - \varepsilon^{ij} A_j = 0 \end{cases}$$

$$\partial_i \Pi^i = \varepsilon^{ij} \partial_i A_j = F_{ij} = 0 \; ;$$

$$\begin{cases} \Pi^1 = A^2 \\ \Pi^2 = -A^1 \end{cases} \qquad \begin{matrix} p_1 = \Pi^1 - A^2 & q_2 = \Pi^2 + A^1 \\ q_1 = \frac{1}{2}(\Pi^2 + A^1) & p_2 = \frac{1}{2}(\Pi^1 + A^2) \; ; \end{matrix}$$

$$\{p_1, q_1\} = 2 \cdot \tfrac{1}{2} = 1 \; ;$$
$$\{p_1, q_2\} = \{\Pi^1, -A^1\} - \{A^2, \Pi^2\} = -1 - (-1) = 0 .$$

$(\Pi, A) \to (p, q)$ transformation canonique

on peut résoudre les contraintes : $A^1 \to \frac{q_2}{2} = \frac{1}{2} q$

$p_1, q_1 \to 0$: $A^2 \to p_2 = p$;

$$\mathcal{L} = A_0 (\partial_1 A_2 - \partial_2 A_1) + A_1 (\partial_0 A_2 - \partial_2 A_0) + A_2 (\partial_0 A_1 - \partial_1 A_0)$$
$$= \frac{1}{2}\Big[A_0 (\partial_1 p - \partial_2 q) - q (\partial_0 p - \partial_2 A_0) + p (\partial_0 q - \partial_1 A_0)\Big]$$

= après intégration par parties :

$$\frac{1}{2}\Big[A_0 (2\partial_1 p - 2\partial_2 q) + 2 p \dot{q}\Big] = p \dot{q} + A_0 (\partial_1 p - \partial_2 q)$$

$$H = p\dot{q} - L = -A_0 (\partial_1 p - \partial_2 q) \; ;$$

$$\Pi^0 = 0 \Rightarrow \frac{\partial}{\partial t} \Pi^0 = \{\Pi^0, H\} = \partial_1 p - \partial_2 q \equiv +\chi(x) \; ;$$

$$\{\chi(x), \chi(y)\} = \{\partial_1 p(x) - \partial_2 q(x), \; \partial_1 p(y) - \partial_2 q(y)\} =$$
$$= \left(-\frac{\partial}{\partial x_1}\frac{\partial}{\partial y_2} + \frac{\partial}{\partial y_1}\frac{\partial}{\partial x_2}\right) \delta(x - y) = 0 \Rightarrow \text{2 contraintes de 1ère classe !}$$

YM + couplage de Chern-Simons :

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{\lambda}{2}(\partial\cdot A)^2$$

e.o.m. : $$\frac{\partial}{\partial x^\rho}\left(\frac{\delta\mathcal{L}}{\delta\partial_\rho A_\mu}\right) - \frac{\delta\mathcal{L}}{\delta A_\mu} = \partial_\rho\left(-F^{\rho\mu} + \frac{m}{2}\epsilon^{\nu\rho\mu}A_\nu - \lambda\,\partial\cdot A\,\eta^{\rho\mu}\right)$$

$$- \frac{m}{2}\epsilon^{\mu\nu\rho}\partial_\nu A_\rho =$$

$$= -\partial_\rho F^{\rho\mu} + \frac{m}{2}\epsilon^{\mu\nu\rho}(\partial_\rho A_\nu - \partial_\nu A_\rho) - \lambda\,\partial^\mu\partial\cdot A$$

$$= -\partial_\rho F^{\rho\mu} - \frac{m}{2}\epsilon^{\mu\nu\rho}F_{\nu\rho} - \lambda\,\partial^\mu\,\partial\cdot A = 0$$

$$A_\mu = \int\frac{d^4p}{(2\pi)^4}\,e^{ipx}\,\varepsilon_\mu(p) \Rightarrow$$

$$p_\rho(p^\rho\varepsilon^\mu - p^\mu\varepsilon^\rho) - i\frac{m}{2}\epsilon^{\mu\nu\rho}(p_\nu\varepsilon_\rho - p_\rho\varepsilon_\nu) + \lambda p^\mu p^\rho\varepsilon_\rho =$$

$$= E^{\mu\rho}\varepsilon_\rho = 0\ ;$$

$$E^{\mu\rho} = (p^2\eta^{\mu\rho} - p^\mu p^\rho + \lambda p^\mu p^\rho - im\,\epsilon^{\mu\nu\rho}p_\nu)$$

$$= p^2\eta^{\mu\rho} + \xi p^\mu p^\rho + im\,\epsilon^{\mu\rho\nu}p_\nu \quad ; \quad \xi \doteq \lambda - 1.$$

inverse : $E^{\mu\rho}G_{\rho\sigma} = \delta^\mu{}_\sigma$

on peut le trouver avec un Ansatz :

$$G_{\rho\sigma} = A\,p^2\eta_{\rho\sigma} + B\,p_\rho p_\sigma + C\,\epsilon_{\rho\sigma\tau}p^\tau\ ;$$

$$E^{\mu\rho} G_{\rho\sigma} = p^2\left(A p^2 \delta^\mu_\sigma + B p^\mu p_\sigma + C \epsilon^\mu{}_{\sigma\tau} p^\tau\right) +$$

$$+ \xi p^\mu \left(A p^2 p_\sigma + B p^2 p_\sigma\right) +$$

$$+ i m \epsilon^{\mu\rho\nu} p_\nu \left(A p^2 \eta_{\rho\sigma} + B \cancel{p_\rho} p_\sigma + C \epsilon_{\rho\sigma\tau} p^\tau\right);$$

termes avec un ε: $\quad p^2\left(C \epsilon^\mu{}_{\sigma\tau} p^\tau + i A m \epsilon^\mu{}_{\sigma\tau} p^\tau\right) = 0$

$$C = - i m A;$$

$$p_\nu p^\tau \epsilon^{\mu\rho\nu} \varepsilon_{\rho\sigma\tau} = - p_\nu p^\tau \varepsilon^{\rho\mu\nu} \varepsilon_{\rho\sigma\tau} = - p_\nu p^\tau (-)\left(\delta^\mu_\sigma \delta^\nu_\tau - \delta^\mu_\tau \delta^\nu_\sigma\right)$$

$$= p^2 \delta^\mu_\sigma - p^\mu p_\sigma;$$

$$E^{\mu\rho} G_{\rho\sigma} = \delta^\mu_\sigma \left[A p^4 + m^2 A p^2\right] + p^\mu p_\sigma \left[B p^2 + \xi(A+B) p^2 - A m^2\right]$$

$$A = \frac{1}{p^2(p^2+m^2)}; \qquad B(1+\xi) p^2 = A(m^2 - \xi p^2)$$

$$(\xi = -1 : \text{ non inversible})$$

$$G_{\rho\sigma} = \frac{1}{p^2+m^2}\left[\eta_{\rho\sigma} + \frac{1}{1+\xi} \frac{p_\rho p_\sigma}{p^2}\left(\frac{m^2}{p^2} - \xi\right) - \frac{im}{p^2} \varepsilon_{\rho\sigma\tau} p^\tau\right]$$

$$= \frac{1}{p^2+m^2}\left[\eta_{\rho\sigma} + \frac{1}{\lambda}\left(\frac{p_\rho p_\sigma}{p^2}\right)\left(\frac{m^2}{p^2} + 1 - \lambda\right) - \frac{im}{p^2} \varepsilon_{\rho\sigma\tau} p^\tau\right]$$

$$= \frac{1}{p^2+m^2}\left[\eta_{\rho\sigma} - \frac{p_\rho p_\sigma}{p^2} - \frac{im}{p^2} \varepsilon_{\rho\sigma\tau} p^\tau\right] + \frac{1}{\lambda} \frac{p_\rho p_\sigma}{p^4}$$

Def. $B_\mu = \varepsilon_{\mu\alpha\beta} F^{\alpha\beta}$; $\partial^\mu B_\mu \equiv 0$

propagateur de B_μ: $\varepsilon_{\mu\alpha\beta}\, \varepsilon_{\nu\gamma\delta}\, p^\alpha p^\gamma g_{\beta\delta}$

$$p^\alpha p^\gamma p_\tau \varepsilon_{\mu\alpha\beta}\, \varepsilon_{\nu\gamma\delta}\, \varepsilon^{\beta\delta\tau} = p^\alpha p^\gamma p_\tau\, \varepsilon_{\nu\gamma\delta} \left(\delta_\mu^{\ \delta} \delta_\alpha^{\ \tau} - \delta_\alpha^{\ \delta} \delta_\mu^{\ \tau} \right) =$$

$$= p^2 p^\gamma \varepsilon_{\mu\nu\gamma} - p_\mu\, \cancel{\varepsilon_{\nu\gamma\delta} p^\gamma p^\delta} ;$$

$$p^\alpha p^\gamma \varepsilon_{\mu\alpha\beta}\, \varepsilon_{\nu\gamma\delta}\, \eta^{\beta\delta} = p^2 \eta_{\mu\nu} - p^\mu p^\nu ;$$

$$\langle B_\mu B_\nu \rangle \sim \frac{1}{p^2+m^2} \left(p^2 \eta_{\mu\nu} - p^\mu p^\nu - i m\, \varepsilon_{\mu\nu\gamma} p^\gamma \right)$$

Voir hep-th/9902115