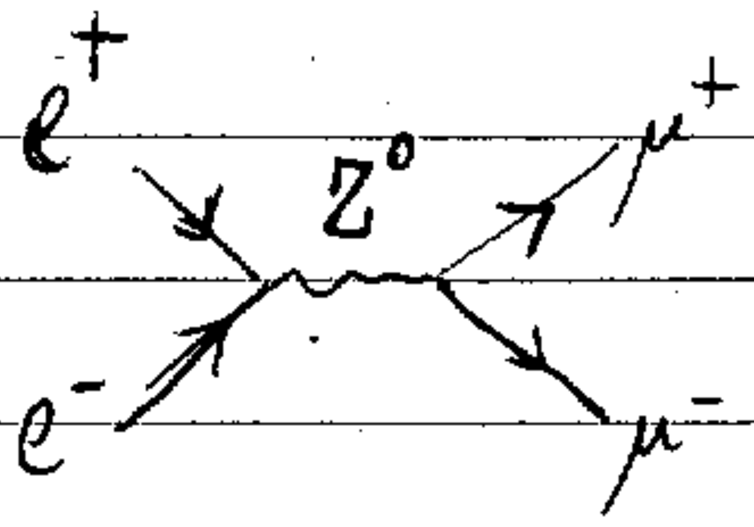
Diffusion $e^-\mu^- \to e^-\mu^-$ par un boson vectoriel massif

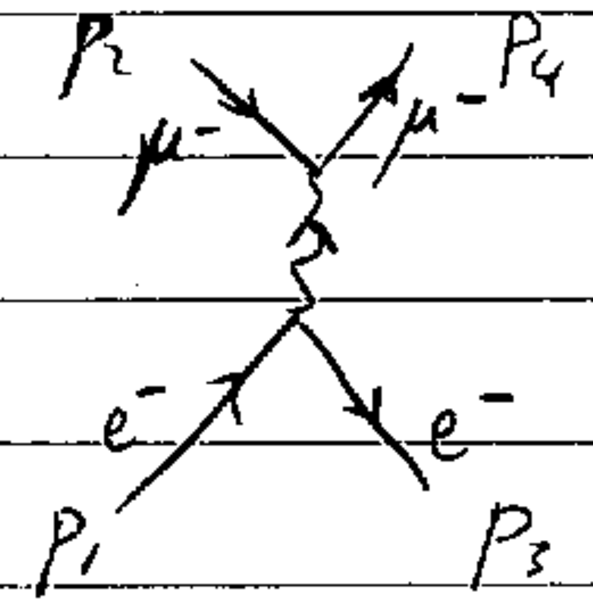


$$\frac{d\sigma}{d\Omega} : \text{formule } (7.44) \sim \frac{g^4}{32\pi^2}\,\frac{t^2+u^2}{s(s-m_Z^2)^2}$$

$$\sim \frac{g^4}{64\pi^2}\,\frac{s}{(s-m_Z^2)^2}\left(1+\cos^2\theta\right)$$

contributions de $\mathcal{H}_{int} = +ig\left(\overline{\psi}_e\,\gamma^\rho\,\psi_e\,Z_\mu + \overline{\psi}_\mu\,\gamma^\rho\,\psi_\mu\,Z_\mu\right)$

la même interaction cause la diffusion $e^-\mu^- \to e^-\mu^-$



$$\mathcal{M} \sim \overline{u}_e(p_3,\sigma_3,e)\,\gamma_\mu\,u(p_1,\sigma_1,e)\quad \overline{u}(p_4,\sigma_2,\mu)\,\gamma^\nu\,u(p_2,\sigma_4,\mu)\,\left(-i\,\Delta_{\mu\nu}(p_1-p_3)\right)$$

$$\Delta_{\mu\nu}(q) = \frac{1}{q^2+m^2-i\varepsilon}\left(\eta_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}\right)$$

$$(i\not{p}+m)\,u(p) = 0 \Rightarrow 0 = u^+\left(-i\,\not{p}^+ + m\right) = -\overline{u}\,\gamma^0\left(-i\,\not{p}^+ + m\right)\gamma^0 =$$

$$= \overline{u}\left(-i\,\gamma^0\gamma^{\mu+}\gamma^0 p_\mu - m\right) = \overline{u}\left(-i\,\not{p} - m\right);\quad 0 = \overline{u}\left(i\not{p}+m\right)$$

$$\overline{u}(p_3)\,\gamma_\mu\,u(p_1)\,i(p_1-p_3)_\mu = \overline{u}(p_3)\left(-i\not{p}_3 + i\not{p}_1\right)u(p_1) = \overline{u}\,(m-m)\,u = 0.$$

On peut remplacer $\Delta_{\mu\nu} \sim \dfrac{1}{m^2+q^2-i\varepsilon}\,\eta_{\mu\nu}$.

$$\mathcal{M} \sim \frac{1}{q^2+m_Z^2-i\varepsilon}\;\overline{u}_e(p_3,\sigma_3,e)\,\gamma_\mu\,u_e(p_1,\sigma_1,e)\quad \overline{u}(p_4,\sigma_4,\mu)\,\gamma^\mu\,u(p_2,\sigma_2,\mu)$$

$$\left(\bar{u}(1)\gamma^\mu u(2)\right)^\dagger = u^\dagger(2)(\gamma^\mu)^\dagger(\gamma^0)^\dagger u = -\bar{u}(2)\gamma^0(\gamma^\mu)^\dagger(-\gamma^0)u(1) =$$

$$= \bar{u}(2)\gamma^\mu u(1),$$

$$|M|^2 \sim \frac{1}{(q^2+m_Z^2+i\varepsilon)^2}\left[\bar{u}(p_3,\sigma_{3,e})\gamma_\mu u(p_1,\sigma_{1,e})\right]\left[\bar{u}(p_1,\sigma_{1,e})\gamma_\nu u(p_3,\sigma_{3,e})\right]\times$$

$$\times\left[\bar{u}(p_4,\sigma_{4,\mu})\gamma_\mu u(p_2,\sigma_{2,\mu})\right]\left[\bar{u}(p_2,\sigma_{2,\mu})\gamma^\nu u(p_4,\sigma_{4,\mu})\right]$$

$$\sum_\sigma u(p,\sigma)\,\bar{u}(p,\sigma) = \frac{1}{2p^0}(-i\not{p}+m)(i\gamma^0)(i\gamma^0) = \frac{1}{2p^0}(-i\not{p}+m)$$

$$\sum_{\sigma_1,\sigma_3}\bar{u}(3)_\alpha(\gamma_\mu)^{\alpha\beta}u(1)_\beta\quad\bar{u}(1)_\gamma(\gamma_\nu)^{\gamma\delta}u(3)_\delta =$$

$$=\left(-\frac{i}{2p_3^0}\right)(-i\not{p}_3+m_e)_{\delta\alpha}\left(-\frac{i}{2p_1^0}\right)(-i\not{p}_1+m_e)_{\beta\gamma}(\gamma_\mu)^{\alpha\beta}(\gamma_\nu)^{\gamma\delta} =$$

$$=\frac{1}{4p_1^0p_3^0}\mathrm{Tr}\left[(\not{p}_3+im_e)\,\gamma_\mu\,(\not{p}_1+im)\,\gamma_\nu\right]$$

$$\overline{|M|^2} = \frac{1}{p_1^0p_2^0p_3^0p_4^0}\;\frac{1}{(q^2+m_Z^2+i\varepsilon)^2}\,\mathrm{Tr}\left((\not{p}_3+im_e)\gamma_\mu(\not{p}_1+im_e)\gamma_\nu\right)\times$$

$$\mathrm{Tr}\left((\not{p}_4+im_\mu)\gamma^\mu(\not{p}_2+im_\mu)\gamma^\nu\right).$$

si on peut négliger la masse de m_e, m_μ ($<< m_Z$) :

$$p_3^\rho p_1^\sigma\;\mathrm{Tr}(\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\rho)\propto(p_3)_\mu(p_1)_\nu - p_1\cdot p_3\,\eta_{\mu\nu} + (p_1)_\mu(p_3)_\nu$$

$$\times\left((p_4)_\mu(p_2)_\nu + (p_2)_\mu(p_4)_\nu - p_2\cdot p_4\,\eta_{\mu\nu}\right) =$$

$$=(2p_{1\mu}p_{3\nu} - p_1\cdot p_3\,\eta_{\mu\nu})(p_2^\mu p_4^\nu + p_4^\mu p_2^\nu - p_2\cdot p_4\,\eta^{\mu\nu}) =$$

$$= \Big(2\, p_1\cdot p_2\, p_3\cdot p_4 + 2\, p_1\cdot p_4\, p_2\cdot p_3 - 2\, p_2\cdot p_4\, p_1\cdot p_3$$
$$- 2\, p_1\cdot p_3\, p_2\cdot p_4 + 4\, p_1\cdot p_3\, p_2\cdot p_4\Big)$$

Mandelstam variables: $s = -(p_1+p_2)^2 = -(p_3+p_4)^2 \quad (= E_{cm}^2)$

$$t = -(p_1-p_3)^2 = -(p_2-p_4)^2$$

$$u = -(p_1-p_4)^2 = -(p_2-p_3)^2$$

$$-(s+t+u) = p_1^2+p_2^2+2p_1\cdot p_2 + p_1^2+p_3^2-2\,p_1\cdot p_3 + p_1^2+p_4^2-2\,p_1\cdot p_4 =$$
$$= 3p_1^2+p_2^2+p_3^2+p_4^2+2p_1\cdot(p_2-p_3-p_4) = p_1^2+p_2^2+p_3^2+p_4^2 =$$
$$= -\sum m^2 \qquad \boxed{s+t+u = \sum m^2}$$

Si toutes les masses sont $=0$, $s+t+u=0$, et on a $p_1\cdot p_2 = p_3\cdot p_4 = -\frac{s}{2}$

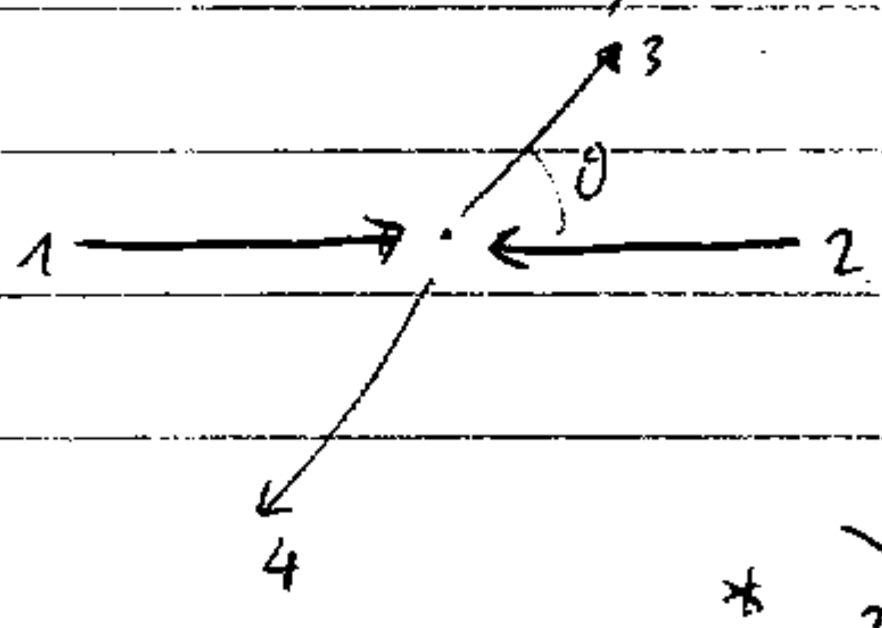
$$p_1\cdot p_3 = p_2\cdot p_4 = \frac{t}{2}$$

$$p_1\cdot p_4 = p_2\cdot p_3 = \frac{u}{2}$$

$$\overline{|M|^2} \sim \frac{1}{(-t+m_Z^2)^2}\,\frac{1}{p_1^0p_2^0p_3^0p_4^0}\left(2s^2+2u^2-2t^2-2t^2+4t^2\right) \sim \underbrace{\frac{1}{p_1^0p_2^0p_3^0p_4^0}\,\frac{s^2+u^2}{(-t+m_Z^2)^2}}_{\text{même que } e^+e^-\to\mu^+\mu^- \text{ avec } s\leftrightarrow t}$$

Crossing symmetry *

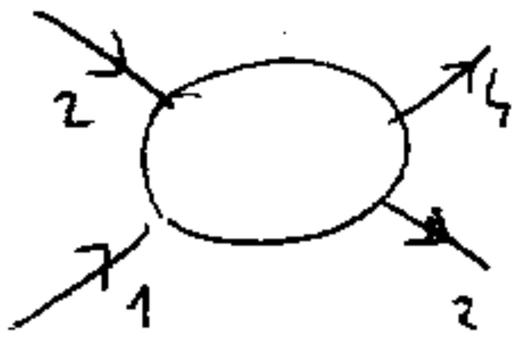
Dans le référentiel du c.m. de 1, 2



$$|p_1| = |p_2| = |p_3| = |p_4| = \frac{\sqrt{s}}{2}$$

$$p_1\cdot p_3 = -\frac{s}{4}(1-\cos\theta)\ ;\ t = -\frac{s}{2}(1-\cos\theta)$$

* $1+2\to3+4$



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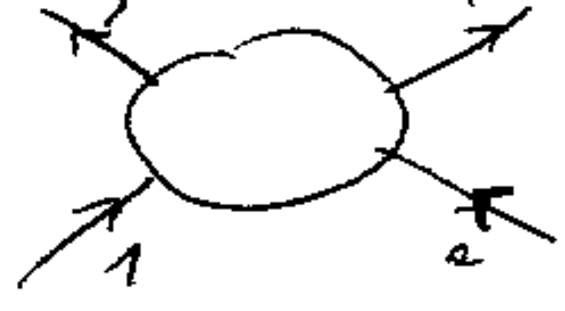
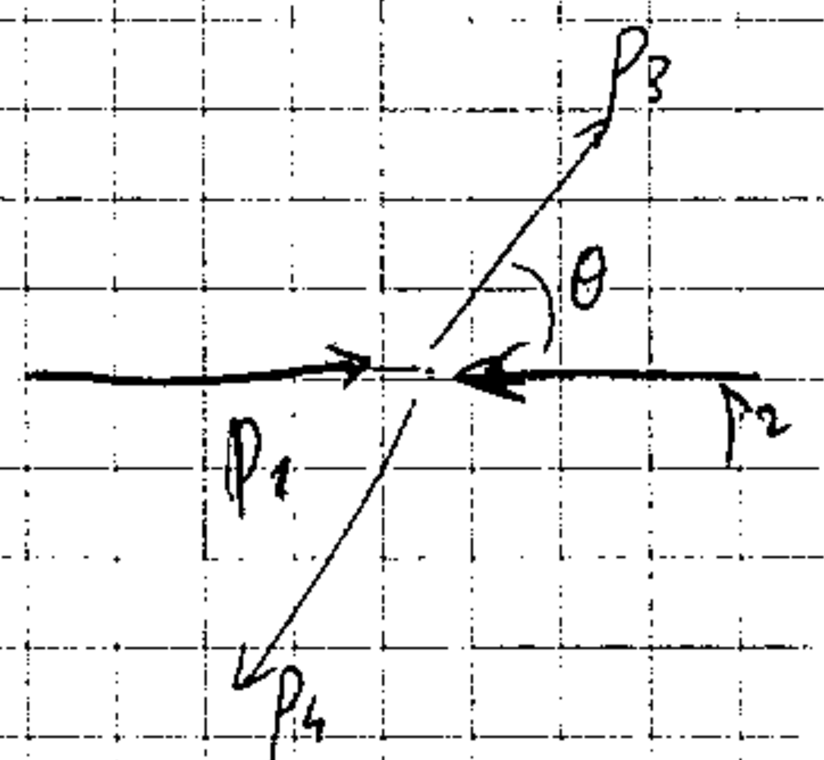


$= 1+\bar{3}\to\bar{2}+4$

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{1}{S}\, p_1^0 p_2^0 p_3^0 p_4^0\, |M|^2 = \frac{1}{s}\, \frac{s^2+u^2}{(t-m_Z^2)^2}$$



$$p_1 \cdot p_3 = \frac{t}{2} = -p_1^0 p_3^0 + \vec{p}_1 \cdot \vec{p}_3 =$$

$$= \frac{s}{4}\left(-1 + \cos\theta\right),$$

$$t = -\frac{s}{2}\,(1-\cos\theta);$$

$$u = -s - t = -\frac{s}{2}\left(2 - 1 + \cos\theta\right) = -\frac{s}{2}\left(1 + \cos\theta\right);$$

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{1}{S}\, \frac{\left(1 + \frac{u^2}{s^2}\right)}{\left(\frac{t}{s} - \frac{m_Z^2}{s}\right)^2} \underset{s \gg m_Z}{\sim} \frac{1}{s}\, \frac{1 + \frac{1}{4}\left(1+\cos\theta\right)^2}{\frac{1}{4}\,\left(1-\cos\theta\right)^2} =$$

$$= \frac{1}{S}\, \frac{1 + \frac{1}{4}\left(2\cos^2\frac{\theta}{2}\right)^2}{\frac{1}{4}\left(2\sin^2\frac{\theta}{2}\right)^2} = \frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}$$

divergence pour $\theta \to 0$ (la même que dans le scattering de Rutherford)

$$t = -(p_1 - p_3) = -q^2 = -\frac{s}{2}\left(1-\cos\theta\right) < 0,$$

$q^2 > 0$: la particule echangé est virtuelle (de type espace)

temps de vie $\tau \sim \frac{1}{\sqrt{q^2}}$, pour $\tau \to \infty$ on a echange de photons très soft (divergence infrarouge)

(Dans la limite $m_Z \to 0$, m_μ, $m_e \to 0$, c'est la même que le Bhabha scattering)

$$V(r) \sim \int d^3q\, \frac{e^{iqx}}{q^2} \sim \frac{1}{r} \quad \text{Coulomb potential}$$

Violation de la parité:

$$\mathcal{L} = g\, Z^\rho \left(\bar\psi_e\, \gamma_\rho (a-b\gamma^5)\, \psi_e + \bar\psi_\mu\, \gamma_\rho\, (a-b\gamma^5)\psi_\mu\right)$$

$$|p_e, \sigma_e\rangle\, |p_\mu, \sigma_\mu\rangle \longrightarrow |p'_e, \sigma'_e\rangle\, |p'_\mu, \sigma'_\mu\rangle$$

Parité: $h \to -h, \quad p \to Pp = (p^0, -\vec{p})$

Rotation de $180°$ dans le plan de diffusion: $h \to h, \quad p \to (p^0, -\vec{p})$

$$\Rightarrow S(\sigma_e, \sigma_\mu, \sigma'_e, \sigma'_\mu) = S(-\sigma_e, -\sigma_\mu, -\sigma'_e, -\sigma'_\mu)$$

sommant sur σ', moyennant sur σ_e on a $\underbrace{\sum |S|^2(\sigma_e)}_{\sigma_+} = \underbrace{\sum |S|^2(-\sigma_e)}_{\sigma_-}$

introduisant les composants d'hélicité donné: $\psi_L = \dfrac{1+\gamma_5}{2}\psi,$

$$\psi_R = \frac{1-\gamma_5}{2}\psi$$

$$\mathcal{L} = g\, Z^\rho \left(\bar\psi^e_L \gamma_\rho \psi^e_L (a+b) + \bar\psi^e_R \gamma_\rho \psi^e_R (a-b)\right) + (\text{muon})$$

M_L : muon initiale left:

$$\cancel{|M_L|^2 = \frac{1}{(q^2+m_Z^2)^2}\left(\bar u(p_3)\gamma_\mu u(p_1)\right)\left(\bar u(p_1)\gamma_\nu u(p_3)\right) \times}$$

$$\cancel{\times \left(\bar u(p_4)\gamma_\mu u_L(p_2)\right)\left(\bar u_L(p_2)\gamma_\nu u(p_4)\right) =}$$

$$\cancel{= \frac{1}{(q^2+m_Z^2)^2}\, \frac{1}{p_1^0 p_3^0 p_4^0}\, Tr\left(\not p_1 \gamma_\nu \not p_3 \gamma_\mu\right) Tr\left(u_L \bar u_L \gamma_\nu \not p_4 \gamma_\mu\right)}$$

$$|M_L|^2 = \frac{g^4(a+b)^2}{(q^2+m_Z^2)^2}\Big[(a+b)^2\left(\bar{u}_L(3)\gamma_\mu u_L(1)\right)\left(\bar{u}_L(1)\gamma_\nu u_L(3)\right) + (a-b)^2\left(\bar{u}_R(3)\gamma_\mu u_R(1)\right)\left(\bar{u}_R(1)\gamma_\nu u_R(3)\right)$$

$$\times\left[\bar{u}_L(4)\gamma_\mu u_L(2)\ \ \bar{u}_L(2)\gamma_\nu u_L(4)\right]$$

$$\sum_\sigma u_L(p)\bar{u}_L(p) = \frac{1+\gamma_5}{2}\left(\sum_\sigma u\bar{u}\right)\left(\frac{1-\gamma_5}{2}\right) = \left(\frac{1+\gamma_5}{2}\right)\frac{\not p}{2p_0}\left(\frac{1-\gamma_5}{2}\right) = \frac{1}{2p_0}\left(\frac{1+\gamma_5}{2}\right)\not p$$

$$|M_L|^2 = \frac{g^4(a+b)^2}{(q^2+m_Z^2)^2}\Big[(a+b)^2\,\mathrm{Tr}\left(\left(\frac{1+\gamma_5}{2}\right)\not p_1\gamma_\nu\left(\frac{1+\gamma_5}{2}\right)\not p_3\gamma_\mu\right) + \cancel{(a+b)}$$

$$+(a-b)^2\,\mathrm{Tr}\left(\left(\frac{1-\gamma_5}{2}\right)\not p_1\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)\not p_3\gamma_\mu\right)\Big]$$

$$\times\,\mathrm{Tr}\left(\left(\frac{1+\gamma_5}{2}\right)\not p_4\gamma^\nu\left(\frac{1+\gamma_5}{2}\right)\not p_2\gamma^\mu\right)$$

$$= \frac{g^4(a+b)^2}{(q^2+m_Z^2)^2}\Big[(a+b)^2\,\mathrm{Tr}\left(\not p_1\gamma_\nu\not p_3\gamma_\mu\right) + ab\,\mathrm{Tr}\left(\gamma_5\not p_1\gamma_\nu\not p_3\gamma_\mu\right)\Big]\times$$

$$\times\,\mathrm{Tr}\left(\left(\frac{1+\gamma_5}{2}\right)\not p_4\gamma^\nu\not p_2\gamma^\mu\right)\Big];$$

$$|M_R|^2 = \frac{g^4(a-b)^2}{(q^2+m_Z^2)^2}\Big[(a^2+b^2)\,\mathrm{Tr}(\quad) + ab\,\mathrm{Tr}(\gamma_5\quad)\Big]\,\mathrm{Tr}\left(\left(\frac{1-\gamma_5}{2}\right)\not p_4\gamma^\nu\not p_2\gamma_\mu\right)$$

$$S_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left(\not p_1\gamma_\nu\not p_3\gamma_\mu\right) = 2\left(p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - p_1\cdot p_3\, g^{\mu\nu}\right);$$ $S_{\mu\nu} =$ même pour [illegible]

$$A^{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left(\gamma_5\not p_1\gamma_\nu\not p_3\gamma_\mu\right) = 2i\,\varepsilon^{\mu\nu\rho\sigma}p_{1\rho}p_{3\sigma};$$ $a_{\mu\nu} = \dots;$

$$|M_L|^2 = \frac{g^4 (a+b)^2}{(t-M^2)^2} \left[(a+b)^2 (S_{\mu\nu} + a_{\mu\nu}) + (a-b)^2 (S_{\mu\nu} - a_{\mu\nu}) \right] (S^{\mu\nu} + A^{\mu\nu})$$

$$= \left[2(a^2+b^2) S_{\mu\nu} S^{\mu\nu} + 4ab\, a_{\mu\nu} A^{\mu\nu} \right]$$

$$|M_R|^2 = \frac{g^4 (a-b)^2}{(t-M^2)^2} \left[(a+b)^2 (S_{\mu\nu} + a_{\mu\nu}) + (a-b)^2 (S_{\mu\nu} - a_{\mu\nu}) \right] (S^{\mu\nu} - A^{\mu\nu})$$

$$= \frac{g^4 (a-b)^2}{(t-M^2)^2} \left[2(a^2+b^2) S_{\mu\nu} S^{\mu\nu} - 4ab\, a_{\mu\nu} A^{\mu\nu} \right]$$

$$\text{asymmetry} = \frac{|M_L|^2 - |M_R|^2}{|M_L|^2 + |M_R|^2} = \frac{(a+b)^2 \left[2(a^2+b^2)\, S\cdot S + 4ab\, a\cdot A \right] - (a-b)^2 \left[2(a^2+b^2)\, S\cdot S - 4ab\, a\cdot A \right]}{\text{''} \quad + \quad \text{''}}$$

$$= \frac{4ab(a^2+b^2)\left(S_{\mu\nu} S^{\mu\nu} + a_{\mu\nu} A^{\mu\nu} \right)}{2(a^2+b^2)^2 S_{\mu\nu} S^{\mu\nu} + 8a^2 b^2 a_{\mu\nu} A^{\mu\nu}} \neq 0$$ seulement si $a \neq 0$ et $b \neq 0$.

$S_{\mu\nu} S^{\mu\nu} = s^2 + u^2$;

$A_{\mu\nu} a^{\mu\nu}$ est antisymétrique en échangeant $1 \leftrightarrow 3$ ou $2 \leftrightarrow 4$: $s \leftrightarrow u$

$A \cdot a = s^2 - u^2$;