6.5.1 Boson vectoriel massif $\to$ deux scalaires

$(\omega \to \pi^+ \pi^-)$

a) $\mathcal{H}_{int} = i g V^\mu \phi^+ \partial_\mu \phi + h.c.$

$$= i g V^\mu \phi^+ \partial_\mu \phi - i g V^\mu \partial_\mu \phi^+ \phi \equiv i g V^\mu J_\mu$$

intégration par parts: $i g V^\mu \phi^+ \partial_\mu \phi + i g \cancel{\partial_\mu V^\mu} \phi^+ \phi + i g V^\mu \phi^+ \partial_\mu \phi$

$$\langle 0 | a(p_2, \pi^-) a(p_1, \pi^+) \int d^4x \, \mathcal{H}_{int} \; a^+(\vec{P}, \sigma=+1, \omega) | 0 \rangle =$$

$$= 2 i g \int d^4 x \frac{d^3 q}{\sqrt{(2\pi)^3 2 q^0}} \frac{d^3 q_1 d^3 q_2}{\sqrt{(2\pi)^3 2 q_1^0 (2\pi)^3 2 q_2^0}} e^{-iq_1 x - i q_2 x + i q x} (-i q_{2\mu}) \langle 0 | a(p_2, \pi^-) a(p_1, \pi^+) a^+(q_1, \pi^+) a^+(q_2, \pi^-) a(q, +1, \omega) a^+(\ldots) | 0 \rangle \, e^\mu(q, +1)$$

$$\left(\phi = \int \frac{dp}{\ldots} \left[e^{ipx} a(p, \pi^+) + e^{-ipx} a^+(p, \pi^-) \right], \; \phi^+ = \int \frac{dp}{\ldots} \left[e^{-ipx} a^+(p, \pi^+) + e^{ipx} a(p, \pi^-) \right] \right)$$

$$= \frac{2 g (2\pi)^4 \delta^{(4)}(P - p_1 - p_2)}{\sqrt{(2\pi)^9 8 P^0 p_1^0 p_2^0}} p_{2\mu} e^\mu(P, +1) = g \frac{(2\pi)^4 \delta^{(4)}(P - p_1 - p_2)}{\sqrt{(2\pi)^9 8 P^0 p_1^0 p_2^0}} (p_2 - p_1)_\mu e^\mu(P, +1)$$

$$M_{\alpha\beta} = \frac{g}{\sqrt{(2\pi)^3 8 P^0 p_1^0 p_2^0}} (p_2 - p_1)_\mu e^\mu(P, +1);$$

b) $$|M|^2 = \frac{g^2}{(2\pi)^3 8 P^0 p_1^0 p_2^0} e^\mu e^{\nu *} (p_2 - p_1)_\mu (p_2 - p_1)_\nu \qquad p_2 - p_1 = Q$$

$$e^\mu e^{\nu *} = \frac{1}{2} \begin{pmatrix} 0 \\ -1 \\ -i \\ 0 \end{pmatrix} (0, -1, i, 0) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & -i & 0 \\ 0 & +i & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad Q_\mu Q_\nu e^\mu e^{\nu *} = \frac{1}{2} \vec{Q}_\perp^2$$

(pour $\sigma = 0$, on aurait $e^\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $Q_\mu Q_\nu e^\mu e^{\nu *} = Q_z^2$)

Référentiel de ω: $p_1 = -p_2 = \frac{-Q}{2}$; $\quad \frac{1}{2}\vec{Q}_\perp^2 = \frac{4}{2}(\vec{p}_1^{\,\perp})^2 = 2\,|\vec{p}_1|^2 \sin^2\theta$

$$\frac{d\Gamma}{d\Omega_{cm}} = 2\pi |M|^2 \, d^3p_1 \, \delta(E - 2p_1^0) =$$

$$= \frac{2\pi\, |p_1|^2 d|p_1|\, d\cos\theta\, d\varphi}{|2E'(|p_1|)|}\,\delta(|p_1| - \kappa)\,|M|^2 = \frac{2\pi\,|\vec{p}_1|^2}{4|p_1|} m_\omega \, d\cos\theta\, d\varphi \, \frac{g^2}{(2\pi)^3 8 m_\omega \left(\frac{m_\omega^2}{4}\right)} 2|\vec{p}_1|^2 \sin^2\theta$$

$$= g^2 \frac{2\pi}{4} \underbrace{2\pi}_{d\varphi} \frac{1}{2(2\pi)^3} \frac{|p_1|^3}{m_\omega^2} d\cos\theta \,(1 - \cos^2\theta)$$

$$= \frac{g^2}{16\pi} \frac{1}{m_\omega^2} \left(\frac{m_\omega^2}{4} - m_\pi^2\right)^{3/2} 2\, d\cos\theta\,(1 - \cos^2\theta)$$

$$2\int d\cos\theta\,(1-\cos^2\theta) = 2\left[\cos\theta\Big|_{-1}^{1} - \frac{1}{3}\cos^3\theta\Big|_{-1}^{1}\right] = 2\left[2 - \frac{2}{3}\right] = \frac{8}{3}$$

$\sigma = 0$: $\quad Q_z^2 = 4\,p_{1z}^2 = 4\,|\vec{p}_1|^2 \cos^2\theta, \qquad 4\int d\cos\theta\, \cos^2\theta = \frac{8}{3}$

conséquence de l'invariance pour rotations.

$$\Gamma = \int \frac{d\Gamma}{d\Omega} d\Omega = \frac{g^2}{6\pi} \frac{1}{m_\omega^2} \left(\frac{m_\omega^2}{4} - m_\pi^2\right)^{3/2} = 0{,}14 \text{ MeV} \quad \Rightarrow g \simeq 0{,}18 \quad (\text{adimensionnel})$$

c) $P\, V^\mu(x) P^{-1} = -\eta\, P^\mu{}_\nu V^\nu(P x)$; pour $\eta = -1$, $\quad V^0 \to V^0, \quad \vec{V} \to -\vec{V}$

∂_μ est un vecteur: $\quad V^\mu \phi^+ \partial_\mu \phi$ invariant sous parité.

$\omega \to \pi^+\pi^-\pi^0$ à 89% $\quad$ contre $\omega \to \pi^+\pi^-$ ~1,7%.

$\begin{pmatrix}\pi^+\\ \pi^0 \\ \pi^-\end{pmatrix}$ triplet de $SU(2)_I$, ω singlet.

$\omega \to \pi^+\pi^-$

$\left.\begin{matrix} I = 0 \\ \ell = 1\end{matrix}\right\} \to$ symm. en isospin, antisymm. en space $\to$ fonction d'onde antisymétrique!

Si $SU(2)_I$ est une symétrie exacte, le processus est exclu.

6.5.3.

Matrices γ:

Algèbre de Clifford $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$

L'algèbre a une représentation sur les spineurs: $\gamma^\mu : V \to V$

Definissons $J^{\mu\nu} = -\frac{i}{2}\gamma^{\mu\nu}$; $[J^{\mu\nu}, J^{\rho\sigma}] = -i(\eta^{\nu\rho}J^{\mu\sigma} + \dots)$

V est une rep. de l'algèbre de Lorentz.

Une solution des relations de commutation est donné par

$$\gamma^0 = -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = -i\begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \qquad \text{(rep. de Weyl)}$$

$(\gamma^0)^2 = -1 \Rightarrow \gamma^0$ antihermitique $(\gamma^0)^\dagger = -\gamma^0 = \gamma^0\gamma^0\gamma^0$

$(\gamma^i)^2 = 1$ $\quad \gamma^i$ hermitique $\quad (\gamma^i)^\dagger = \gamma^i = \gamma^0\gamma^i\gamma^0 \Rightarrow \boxed{\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0}$

$$\left.\begin{aligned} J^{l0} &= \frac{i}{2}\begin{pmatrix} \sigma_l & \\ & -\sigma_l \end{pmatrix} \\ J^{kl} &= \frac{1}{2}\varepsilon_{klm}\begin{pmatrix} \sigma_m & \\ & \sigma_m \end{pmatrix} \end{aligned}\right\} \text{ repr. reductible: } \left(\tfrac{1}{2}, 0\right) \oplus \left(0, \tfrac{1}{2}\right)$$

$V = V_+ \oplus V_-$, $\quad \gamma^\mu : V_\pm \to V_\mp$

$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, $(\gamma_5)^2 = 1$, $\quad \gamma_5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$.

$P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ projecteurs sur $V_\pm$

$\{\gamma_5, \gamma^\mu\} = 0$; e.g. $\{\gamma^0\gamma^1\gamma^2\gamma^3, \gamma^0\} = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 + \gamma^0\gamma^0\gamma^1\gamma^2\gamma^3 =$

$$= -\gamma^0\gamma^0\gamma^1\gamma^2\gamma^3 + \gamma^0\gamma^0\gamma^1\gamma^2\gamma^3 = 0.$$

Traces: $\mathrm{Tr}(\mathbb{1}) = 4 = \dim V$

$\mathrm{Tr}(\gamma^\mu) = \mathrm{Tr}(\gamma^5) = 0$ vérifié dans la rep. de Weyl, mais la trace est indépendante de la représentation.

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = \frac{1}{2}\mathrm{Tr}(\{\gamma^\mu,\gamma^\nu\}) = 4\eta^{\mu\nu}, \quad \mathrm{Tr}(\gamma^{\mu\nu}) = 0$$

$$\mathrm{Tr}(\text{odd }\#) = 0$$

$$\mathrm{Tr}(\gamma^{\mu\nu\rho\sigma}) \propto \mathrm{Tr}(\gamma^5) = 0.$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = \mathrm{Tr}(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu) = 2\eta^{\sigma\mu}\,\mathrm{Tr}(\gamma^\nu\gamma^\rho) - \mathrm{Tr}(\gamma^\nu\gamma^\rho\gamma^\mu\gamma^\sigma) =$$

$$= 2\eta^{\sigma\mu}\eta^{\nu\rho}\times 4 - 2\eta^{\rho\mu}\eta^{\nu\sigma}\times 4 + \mathrm{Tr}(\gamma^\nu\gamma^\mu\gamma^\rho\gamma^\sigma)$$

$$= 2\eta^{\sigma\mu}\eta^{\nu\rho}4 - 2\eta^{\rho\mu}\eta^{\nu\sigma}4 + 2\eta^{\nu\mu}\eta^{\rho\sigma}4 - \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma)$$

$$\Rightarrow \mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 4\left(\eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\nu}\eta_{\rho\sigma}\right);$$

$$\mathrm{Tr}(\gamma^5\gamma^{\mu\nu\rho\sigma}) = c\,\varepsilon^{\mu\nu\rho\sigma};$$

$$c = c\,\varepsilon^{0123} = \mathrm{Tr}(\gamma^5\gamma^0\gamma^1\gamma^2\gamma^3) = \mathrm{Tr}(i(\gamma_5)^2) = 4i;$$