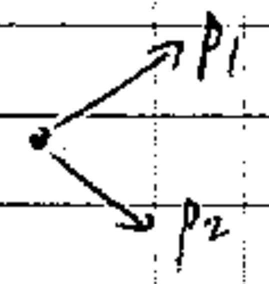
4.4.2

Γ intégrale sans spin, $M>0$;



$p_2 = -p_1$, m_1, m_2

rotation: $M_{p_1,\sigma_1;p_2,\sigma_2} = D(R)_{\sigma'_1,\sigma_1}\, D(R)_{\sigma'_2,\sigma_2}\, M_{Rp_1,\sigma'_1;Rp_2,\sigma'_2}$;

$$C \equiv \sum_{\sigma_1,\sigma_2} |M_{p_1 p_2}|^2 = \sum_{\sigma_1,\sigma_2} |M_{Rp_1,\sigma_1;Rp_2,\sigma_2}|^2$$ invariant sous rotation.

$$\frac{d\Gamma}{d\Omega_{cm}} = 2\pi |M_{p}|^2 \frac{E_1' E_2' p'}{M}$$ (Somme sur les polarisations finales)

$$E_1' = \sqrt{m_1^2 + p'^2} \qquad E_1' + E_2' = E_{cm} = M$$
$$E_2' = \sqrt{m_2^2 + p'^2}$$

$\pi^0 \to \gamma\gamma$; $m_1 = m_2 = 0$, $E_1' = E_2' = p' = \frac{M}{2}$; $\frac{d\Gamma}{d\Omega_{cm}} = 2\pi C \frac{1}{M}\left(\frac{M}{2}\right)^3 = \frac{\pi}{4} C M^2$;

$$\Gamma = \int d\Omega \frac{d\Gamma}{d\Omega} = 4\pi \times \frac{d\Gamma}{d\Omega} = \pi^2 C M^2$$

$$C = \frac{\Gamma}{\pi^2 M^2} = \frac{1}{\pi^2 M^2 \tau} \; ;$$

$[\hbar] = 1$: $\hbar = 6{,}6 \times 10^{-16}\, eV \cdot s$

$\tau = 8{,}5 \cdot 10^{-17} s \simeq 0{,}1\ eV^{-1}$;

$$C \sim 10^{-16}\, eV^{-1}$$

$$\langle \Phi_\beta | \Phi_\alpha \rangle \propto \delta^{(3)}(p_\beta - p_\alpha) \sim \frac{V^{n_\alpha}}{V} \qquad [\Phi_\alpha] = V^{\frac{n_\alpha}{2}} = L^{\frac{3}{2} n_\alpha}$$

$$[S_{\beta\alpha}] = L^{\frac{3}{2}(n_\alpha + n_\beta)} = [\delta^{(4)}(p)\, M_{\beta\alpha}] = L^4 [M_{\beta\alpha}]$$

$$[M_{\beta\alpha}] = L^{\frac{3}{2}(n_\alpha + n_\beta) - 4} \qquad [|M_{\beta\alpha}|^2] = L^{3(n_\alpha + n_\beta) - 8}$$

$n_\alpha = 1$, $n_\beta = 2$: $|M_{\beta\alpha}|^2 \propto L \sim M^{-1}$

$2+2$, $3+1$: $|M_{\beta\alpha}|^2 \propto M^{-4}$

Système du labos

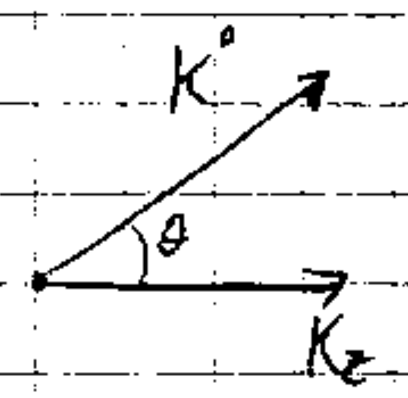
$d\Gamma(\alpha\to\beta)$ taux de désintégration $\sim \dfrac{dP(\alpha\to\beta)}{dt}$

$$d\Gamma_{lab} = d\Gamma_{cm}\,\frac{dt_{cm}}{dt_{lab}}, \qquad t_{lab} = \gamma\, t_{cm} \qquad \tau_{lab} = \gamma\,\tau_0; \quad \gamma = \frac{E}{M};$$

$$\vec{P}^2 = E^2 - M^2 = M^2(\gamma^2-1) = M^2\gamma^2\beta^2;$$

transformation de $d\Omega$; pour $m=0$:

$$K'^0 = \gamma(K^0 - \beta K^z)$$
$$K'^z = \gamma(K^z - \beta K^0)$$
$$K'_x = K_x$$



$$\cos\theta = \frac{K^z}{K^0},$$

$$\cos\theta' = \frac{K'^z}{K'^0} = \frac{K^z - \beta K^0}{K^0 - \beta K^z} = \frac{\cos\theta - \beta}{1-\beta\cos\theta};$$

$$d\Omega = d\cos\theta\, d\varphi; \qquad d\Omega_{lab} = d\cos\theta'\, d\varphi' = d\left(\frac{\cos\theta-\beta}{1-\beta\cos\theta}\right)d\varphi$$

$$= \frac{1-\beta^2}{(1-\beta\cos\theta)^2}\, d\cos\theta\, d\varphi = \frac{1}{\gamma^2(1-\beta\cos\theta)^2}\, d\cos\theta\, d\varphi,$$

$$\frac{d\Gamma_{lab}(\alpha\to\beta)}{d\Omega_{lab}} = \frac{\left(\gamma^2(1-\beta\cos\theta)^2\right)^{-1}}{\gamma}\frac{d\Gamma_{cm}}{d\Omega_{cm}} = \frac{1}{\gamma^3(1-\beta\cos\theta)^2}\,\frac{d\Gamma_{cm}}{d\Omega_{cm}};$$

directement; $d\Gamma(\alpha\to\beta) = 2\pi\,|M_{\alpha\beta}|^2\,\delta^{(4)}(\Sigma p' - P)\,d^3p_1' \ldots d^3p_N'$

$$= 2\pi\,\underbrace{|M_{P,p_1',p_2'}|^2\, E E_1' E_2'}_{\text{invariante}}\;\delta(E_1'+E_2'-E)\,\delta^{(3)}(p_1'+p_2'-P)\,\frac{1}{E}\,\frac{d^3p_1'}{E_1'}\,\frac{d^3p_2'}{E_2'}$$

$$= 2\pi\, C\,\frac{M^3}{4}\,\delta(E_1'+E_2'-E)\,\frac{d^3p'}{E E_1' E_2'} \qquad ; \quad p' \equiv p_1'; \; p_2' = P - p_1';$$

$$(E_2')^2 = (E-E_1')^2 = E^2 + |p'|^2 - 2E|p'|$$

$$= (\vec{P}-\vec{p}_1')^2 = \vec{P}^2 + |\vec{p}_1'|^2 - 2|\vec{P}||p'|\cos\theta = E^2 + |p'|^2 - M^2 - 2|P||p'|\cos\theta; \quad |\vec{p}'| = \frac{M}{2\gamma(1-\beta\cos\theta)}$$

$$\delta(E_1' + E_2' - E) = \delta\left(|p_1'| + E_2'(|p_1'|) - E\right) = \delta\left(f(|p_1'|) - E\right) = \frac{1}{f'(p_1)}\,\delta\left(|p_1'| - \frac{M}{2\gamma(1-\beta\cos\theta)}\right)$$

$$f(x) = x + \sqrt{x^2 - 2xM\beta\gamma\cos\theta + M^2\gamma^2\beta^2}$$

$$f'(x) = 1 + \frac{2x - 2M\gamma\beta\cos\theta}{2\sqrt{\ldots}} = 1 + \frac{E_1' - E\beta\cos\theta}{E_2'} = \frac{E(1-\beta\cos\theta)}{E_2'};$$

$$d\Gamma = \frac{\pi}{2} C M^3 \frac{1}{E E_1' \cancel{E_2'}} \frac{\cancel{E_2'}}{E(1-\beta\cos\theta)} |p'|^2 \, d\cos\theta \, d\varphi =$$

$$= \frac{\pi}{2} C M^3 \frac{1}{E^2 \, (1-\beta\cos\theta)} \frac{M}{2\gamma(1-\beta\cos\theta)} d\Omega = \frac{\pi}{4} C M^2 \frac{1}{\gamma^3 (1-\beta\cos\theta)^2} d\Omega .$$