Coherence and indistinguishability of single electron wavepackets emitted by independent sources

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Using two independent on-demand electron sources, the emission of two single-electron wavepackets is triggered at different inputs of an electronic beam splitter. Whereas classical particles would be randomly partitioned by the splitter, we observe two-particle interferences resulting from quantum exchange statistics. When both electrons are emitted in indistinguishable wavepackets with synchronized arrival time on the splitter, both particles exit in different outputs which can be recorded by the measurement of the low frequency noise of the output current. This two-electron interference experiment demonstrates the possibility to generate on-demand coherent and indistinguishable single electron wavepackets in a quantum conductor.

As for photons, the wave-particle duality plays a crucial role in the propagation of electrons in quantum conductors. The wave nature of electrons can be revealed in interference experiments [1, 2] probing the single-particle coherence of electron sources through the measurement of the average electrical current. The corpuscular nature of charge carriers shows up in the measurement of the fluctuations of the electrical current [3, 4]. Still, a few experiments cannot be understood within the wave nor the corpuscular description: this is the case when two particle interferences effects related to the exchange between two indistinguishable particles take place. These experiments have proven particularly interesting, first on a fundamental point of view as they require a full quantum treatment, and secondly, because the on-demand generation of indistinguishable partners is at the heart of many quantum information protocols [5]. Information coding in few electron states that propagate ballistically in quantum conductors [6] thus requires the production of coherent and indistinguishable single particle wavepackets emitted by several synchronized by otherwise independent emitters. The collision of two particles emitted at two different inputs of a beam splitter can be used to measure their degree of indistinguishability. In the case of bosons, indistinguishable partners always exit in the same output (see Fig.1). Fermionic statistics leads to the opposite behavior where particles exit in different outputs. The bunching of indistinguishable photons called photon coalescence has been observed by recording the coincidence counts by two detectors placed at the outputs of the splitter as a function of the time delay τ between the arrival times of the photons on the splitter. Bunching shows up in a dip in the coincidence counts (called the Hong-Ou-Mandel (HOM) dip in reference to the seminal experiment [7]), when τ is varied. The reduction of the coincidence counts directly measures the overlap between the single particle states at the input. It is maximum when the arrivals are synchronized and can be suppressed when the delay becomes larger than the wavepacket widths and thus provides a picture of the wavepacket envelope.

The production of indistinguishable partners is not an easy task and their generation by two independent sources has been only recently achieved in optics [8]. In one dimensional quantum conductors, the production of a continuous stream of undistinguishable electrons can be easily achieved by applying a dc voltage bias to two different electronic reservoirs. Due to the fermionic statistics, each source fills the electronic states up to the chemical potential $-eV$ and identical electron beams are generated. Using such sources, the π exchange phase of indistinguishable fermions has been measured in the above described collider geometry [9] and in a two particle interferometer based on a Mach-Zehnder geometry [10, 11]. However, as these sources generate a continuous beam of electrons, they do not reach the single particle resolution of their optical analog and two particle interferences cannot be interpreted as resulting from the overlap between two single particle wavepackets. The manipulation of single particle states thus requires to replace the dc emitters by triggered ac emitters that can generate in the circuit a single-electron wavepacket at a well defined time.

Dealing with electrons, one can benefit from the charge quantization of a small quantum dot enforced both by Coulomb interaction and fermionic statistics to trigger the emission of particles one by one [12–16]. Moreover, a ballistic one dimensional propagation in the conductor is required to guide the single electrons from the emitter to the electronic beam-splitter. Due to ballistic one dimensional chiral propagation, the edge channels of the quantum Hall effect provide an ideal test bench to implement optics like experiments with electron beams in condensed matter. We will consider here a mesoscopic capacitor [17, 18], which comprises small quantum dot capacitively coupled to a metallic top gate and tunnel coupled to a single edge channel by a quantum point contact of variable transmission. By applying a periodic rf excitation on the top gate which peak to peak amplitude matches the dot energy addition, $2eV_{exc} \approx \Delta$, a quantized current resulting from the emission of a single electron followed by a single hole is generated [12]. Beyond average current measurements, this emitter has been characterized through the study of current correlations on short times [19–22] as well as partition noise measurements in the Hanbury-Brown and Twiss geometry [23]. These measures have demonstrated that in the regime of escape times smaller than half the period of the excitation drive, exactly a single electron and a single hole excitations were successively emitted at each excitation period. Moreover, the tunnel emission of single particles from a discrete dot level should lead to electron and hole wavefunctions described by exponentially decaying wavepackets [24, 25]: $\phi(t) = \frac{1}{\sqrt{\tau_e}}\Theta(t - t_0)\ e^{i\frac{\Delta(t-t_0)}{2\hbar}}\ e^{-\frac{t-t_0}{2\tau_e}}$, where $\Theta(t)$ is the step function, $\Delta/2$ is the energy of emitted electrons and holes, and t_0 is the emission trigger that can be tuned with a few picoseconds accuracy. Measurements of the average current $\langle I(t)\rangle$ and short time correlations $\langle I(t)I(t+\tau)\rangle$ have confirmed that the probability of single particle detection (that is the envelope of the wavepacket) was following this exponential decay. However, these measurements are only sensitive to the modulus squared of the wavefunction, $|\phi(t)|^2$ and as such, do not probe the coherence of the electronic wavepacket related to the phase relationship between $\phi(t)$ and $\phi^*(t')$ ($t \neq t'$) and encoded in the off-diagonal components (coherences) of the density matrix $\rho(t,t') = \phi(t)\phi^*(t')$.

Using two such emitters at the two inputs of an electronic beamsplitter, the coherence and indistinguishability of two single electronic wavepackets can be probed by two electron interferences as suggested in Refs.[25–27]. Considering the electron emission sequence, each emitter generates an electronic wavepacket $|\phi_i\rangle$ ($i = 1, 2$) above the Fermi energy. These single electron wavepackets differ fundamentally from the states generated by applying a time dependent voltage $V_i(t)$ on each electronic reservoir connected to inputs $i = 1, 2$ and cannot be generated by such classical drive applied on a reservoir. This difference bears important consequences, as in the latter case, using the appropriate gauge transformation that shifts all the potentials by $V(t) = V_2(t)$, the two input problem can be made equivalent to a single input one where the potential $V_1(t) - V_2(t)$ is applied to input 1. Such reduction to a single input problem cannot be performed in our case as no gauge transformation can restore one input to its equilibrium state. Considering the two states $|\phi_i\rangle$ at each input of the splitter set at transmission $\mathcal{T} = 1/2$, the probability $P(1,1)$ that each particle exits in a different output is related to the overlap between wavepackets: $P(1,1) = \frac{1}{2}\left[1 + |\langle\phi_1|\phi_2\rangle|^2\right]$. An opposite sign occurs in the expression of the probability that both particles exit in the same output, $P(0,2) + P(2,0) = \frac{1}{2}\left[1 - |\langle\phi_1|\phi_2\rangle|^2\right]$. These signs are related to the exchange phase of π for fermions, they would be opposite for bosons. For fermions, the coincidence count for indistinguishable particles would thus be doubled compared to the classical case (see Fig.1). However, as single shot detection of ballistic electrons in condensed matter is not available, this antibunching is not probed by coincidence counts but rather by the low frequency fluctuations of the electrical current transmitted in the outputs. Indeed, while the average number of particles collected at the outputs $\langle N_3\rangle = \langle N_4\rangle = 1$ is not sensitive to the overlap $\langle\phi_1|\phi_2\rangle$, the fluctuations directly reflect the electronic antibunching: $\langle\delta N_3^2\rangle = \langle\delta N_4^2\rangle = \frac{1}{2}\left[1 - |\langle\phi_1|\phi_2\rangle|^2\right]$. Repeating this two electron collision at frequency f, and considering the successive emission of one electron and one hole in one period, the low frequency current noise at the output is then given by [25]:

$$S_{33} = S_{44} = e^2 f \times \left[1 - |\langle\phi_1|\phi_2\rangle|^2\right] \tag{1}$$

$$= e^2 f \times \left[1 - \left|\int dt\ \phi_1(t)\phi_2^*(t)\right|^2\right] \tag{2}$$

For perfect indistinguishability, $\phi_2(t) = \phi_1(t)$, a complete suppression of the output noise is obtained. By delaying by time τ the emission of one particle with respect to the other: $\phi_2(t) = \phi_1(t+\tau)$, the full random partitioning of classical particles $S_{33} = S_{44} = e^2 f$ can be recovered (see Fig.1). It is thus convenient to consider the noise normalized by the classical random partitioning $q = S_{33}/e^2 f$ which equals for exponentially decaying wavepackets:

$$q = 1 - e^{-|\tau|/\tau_e} \tag{3}$$

Note that Eq.(3) is valid at zero temperature, or when the Fourier components of the wavefunctions $\tilde{\phi}_i(\omega)$ have no overlap with the thermal excitations: $\tilde{\phi}_i(\omega) = 0$ for $\hbar\omega \approx k_B T$. Otherwise, as mentioned in Ref.[23], the random partitioning is also affected by antibunching with the thermal excitations, $S_{33} \leq e^2 f$. However, if one measures the normalized value of the excess noise Δq, between the situations where both sources are switched on and switched off, simulations describing the source in the Floquet scattering formalism [22, 28] show that Δq is accurately described by Eq.(3) for moderate temperatures $k_B T \ll \Delta$.

We now turn to the experimental realization of the two electron collision. The circuit (Fig.2), is realized in a 2D electron gas (2DEG) at a GaAsAl/GaAs heterojunction, of nominal density $n_s = 1.9 \times 10^{15}\,\mathrm{m}^{-2}$ and mobility $\mu = 2.4 \times 10^6\,\mathrm{cm}^{-2}\mathrm{V}^{-1}\mathrm{s}^{-1}$. A strong magnetic field $B = 2.68$ T is applied so as to work in the quantum Hall regime at filling factor $\nu = 3$ [29]. Two mesoscopic capacitors with identical addition energies $\Delta = 1.4$ K (much larger than the electronic temperature $T = 100$ mK) are used as electron/hole emitters and placed at a 3 μm distance from a quantum point contact used as an electronic beam-splitter. Single electron and hole emission in the outer edge channel are triggered with a square excitation at frequency $f = 2.1$ GHz with average emission times set to $\tau_{e,1} = \tau_{e,2} = 58 \pm 7$ ps corresponding to a transparency $D_1 = D_2 = 0.45 \pm 0.05$. The low frequency partition noise for a beam-splitter transmission set at $\mathcal{T} = \frac{1}{2}$ is measured on output number 3. Fig.3 presents the measurements of Δq as a function of the time delay τ between the two sources. We observe a dip in the noise measurements for zero time delay and a plateau for longer time delays. The noise values Δq are normalized by the value of the noise on the plateau. The sum of the partition noises for each source can also be measured by switching off each source alternately. This random partition noise is represented on Fig.2 by the blurry blue line, which extension represents the error bar. As expected, it agrees with Δq for large time delays.

The dip observed for short time delay is analogous to the HOM dip but is related here to the antibunching of indistinguishable single electrons (and single holes), we thus call it the Pauli dip. This dip reflects our ability to produce single particle states emitted by two different emitters with some degree of indistinguishability. The states are not perfectly identical as shown by the fact that the dip does not go to zero. Note that Coulomb repulsion between electrons and between holes on the splitter could also be responsible for a dip in the low frequency noise. However, this effect can be ruled out using the long time delay limit, $\tau \approx 240$ ps. In this limit, the arrival of one electron is synchronized with the arrival a hole in the other input. As can be seen on Fig.3, a random partitioning is observed while Coulomb attraction between electron and holes would also predict a dip in the low frequency noise. The dip around zero time delay can be well fitted by the expression $\Delta q = 1 - \gamma e^{-\frac{\tau - \tau_0}{\tau_c}}$ expected for two exponentially decaying wavepackets but with a non unit overlap γ. We find $\tau_e = 62 \pm 10$ ps, $\tau_0 = 11 \pm 6$ ps and $\gamma = 0.45 \pm 0.05$. As mentioned above, these results can be compared with numerical simulation in the Floquet scattering formalism. For identical emission parameters of both sources, $\Delta_1 = \Delta_2$, $D_1 = D_2$, Floquet theory predicts a unit overlap at zero time delay, $\Delta q_F(\tau = 0) = 0$. The red trace on Fig.3 represents $\Delta q = 1 - \gamma(1 - \Delta q_F(\tau))$ which imposes a non unit overlap γ in the Floquet scattering formalism. It reproduces well the shape of the dip using the following parameters: $\gamma = 0.5$, $D_1 = D_2 = 0.4$, $\Delta_1 = \Delta_2 = 1.4\mathrm{K}$ and $T = 100\mathrm{mK}$.

We believe that this non unit overlap can be attributed to two different origins. First, it could stem from some small differences in the emission energies related to small differences in the static potential of each dot. Using Eq.(2), a reduction to a 50% overlap can be obtained by shifting one level compare to the other by energy $\Delta/10$. The value of the static potential is fixed with a better accuracy but small variations could occur within the several hours of measurement time for each point. The second possibility is related to the decoherence of single electron wavepackets during propagation towards the splitter (that could arise from Coulomb interaction with the adjacent edge channel). In this case, the electron state cannot be represented by a pure state $\phi_1(t)$ but by a density matrix $\rho_1(t,t') = \phi_1(t)\phi_1^*(t')\mathcal{D}_1(t,t')$ where $\mathcal{D}_1(t,t')$ is a decoherence factor (see Ref. [12, 30]). We have $\mathcal{D}_1(t,t) = 0$, such that the average current $\langle I(t)\rangle$ is not affected, but $\mathcal{D}_1(t,t') \to 0$ for $|t-t'| \to \infty$, such that the off diagonal components (coherences) of the electronic wave-packet are suppressed. In that case, Eq.(2) becomes:

$$\Delta q = 1 - Tr[\rho_1\rho_2] \tag{4}$$

$$= 1 - \int dtdt'\phi_1(t)\phi_1^*(t')\mathcal{D}_1(t,t')\phi_2^*(t)\phi_2(t')\mathcal{D}_2(t,t') \tag{5}$$

Eq.() exemplifies that the noise suppression stems from a two particle interference effect encoded in the off diagonal, $t \neq t'$, components of the density matrices ρ_i, that is, on the coherence of the electronic wavepacket. Assuming $\mathcal{D}_1(t,t') = \mathcal{D}_2(t,t') = e^{-\frac{|t-t'|}{\tau_c}}$ to get an analytical expression of Eq.(5), we find that the overlap depends on the ratio between the intrinsic coherence time of the wavepacket τ_e and the coherence time τ_c associated with the propagation along the edge: $\gamma = \frac{\tau_c/(2\tau_e)}{1+\tau_c/(2\tau_e)}$. For $\tau_e \ll \tau_c$, the effects of decoherence can be neglected but in the opposite limit, $\tau_c \ll \tau_e$, the overlap is completely suppressed and the classical partitioning is recovered. In this case, electrons are rendered distinguishable through their interaction with the environment. Within this picture, our measurement of the overlap is compatible with $\tau_c \approx 100\mathrm{ps}$. Such decoherence effects underline the necessity to reach the subnanosecond timescale in electron emission to be able to generate indistinguishable electron wavepackets.

In conclusion, we have realized the collision on an electronic beam-splitter between two single electron wavepackets emitted by two independent sources. We observe a Pauli dip in the low frequency noise of the output current for short time delays between the arrival times of electrons on the splitter. This dip is related to two particle interferences caused by the π phase occurring in the exchange of two indistinguishable fermions. The random partitioning of classical particles is recovered for long time delays, when the overlap between incoming single particle wavepackets is suppressed. This two particle interference experiment demonstrates the possibility to generate coherent and indistinguishable single electron wavepackets in a quantum conductor with independent sources.

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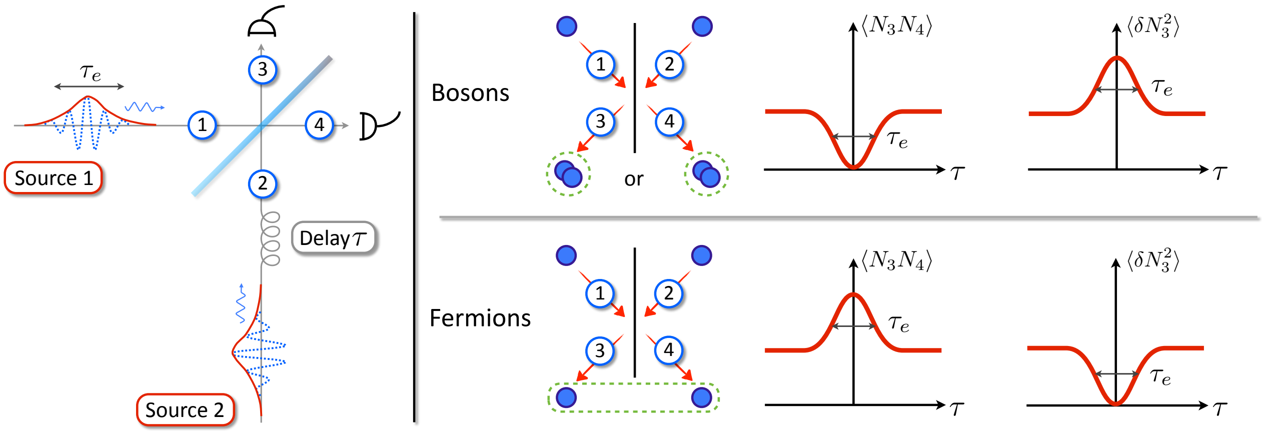

FIG. 1: Sketch of the experiment. Two single particle wavepackets of width τ_e are emitted at inputs 1 and 2 and partitioned on a splitter. Coincident counts $\langle N_3 N_4 \rangle$ and fluctuations $\langle \delta N_3^2 \rangle$ can be recorded at the outputs 3 and 4 as a function of the tuneable delay τ. Indistinguishable bosons always exit in the same output. This results in a suppression of the coincidence count and a doubling of the fluctuations at zero delay compared to the partitioning of classical particles obtained for $\tau \gg \tau_e$. An opposite behavior is expected for indistinguishable fermions (doubling of the coincidence counts and suppression of the fluctuations).

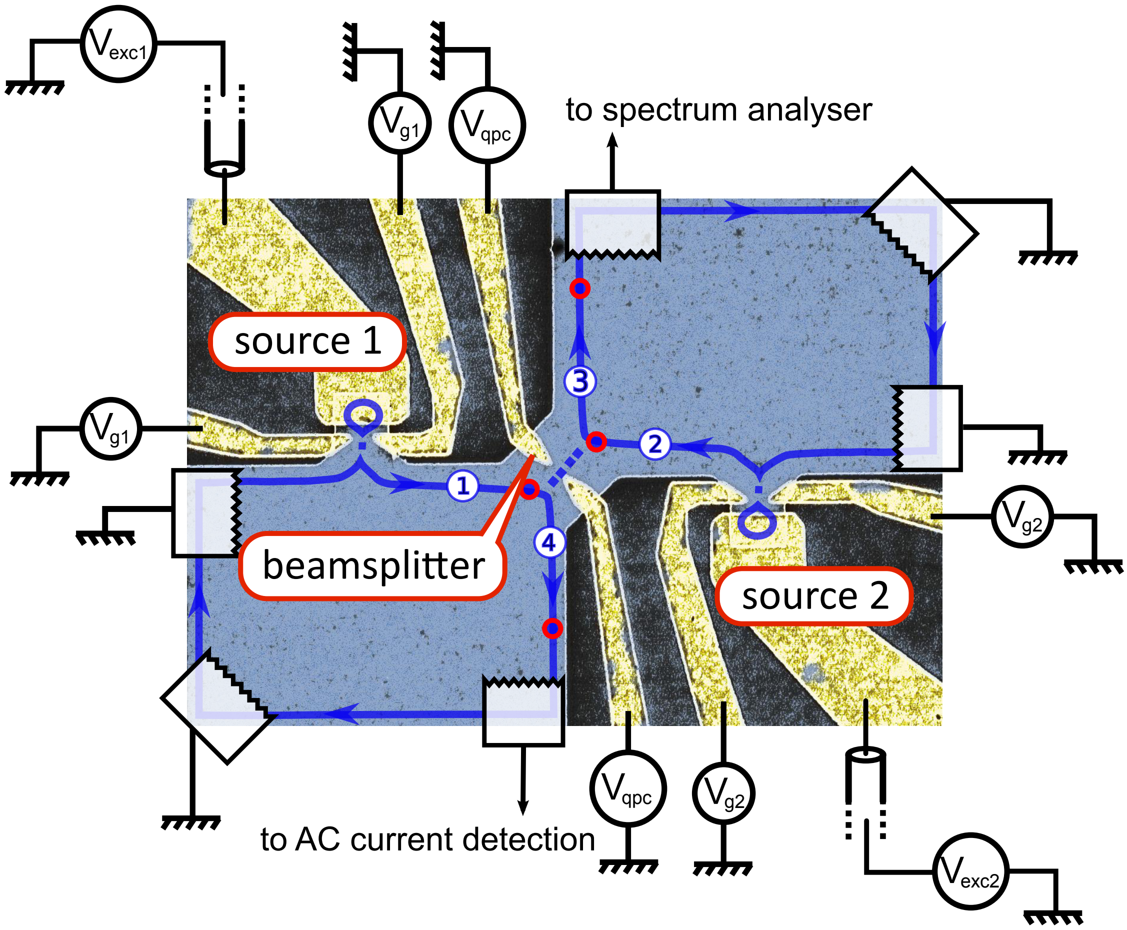

FIG. 2: Sketch of the sample based on a SEM picture. The electron gas is represented in blue. Two single electron emitters are located at inputs 1 and 2 of a quantum point contact used as a single electron beam-splitter. The transparencies D_1 and D_2 and static potentials of dots 1 and 2 are tuned by gate voltages $V_{g,1}$ and $V_{g,2}$. Electron/hole emissions are triggered by excitation drives $V_{exc,1}$ and $V_{exc,2}$ applied on the dot's metallic top gates. The transparency of the beam-splitter partitioning the inner edge channel (blue line) is tuned by gate voltage V_{qpc} and set at $\mathcal{T} = 1/2$. The average ac current generated by sources 1 and 2 are measured on output 4 while the low frequency output noise S_{33} is measured on output 3.

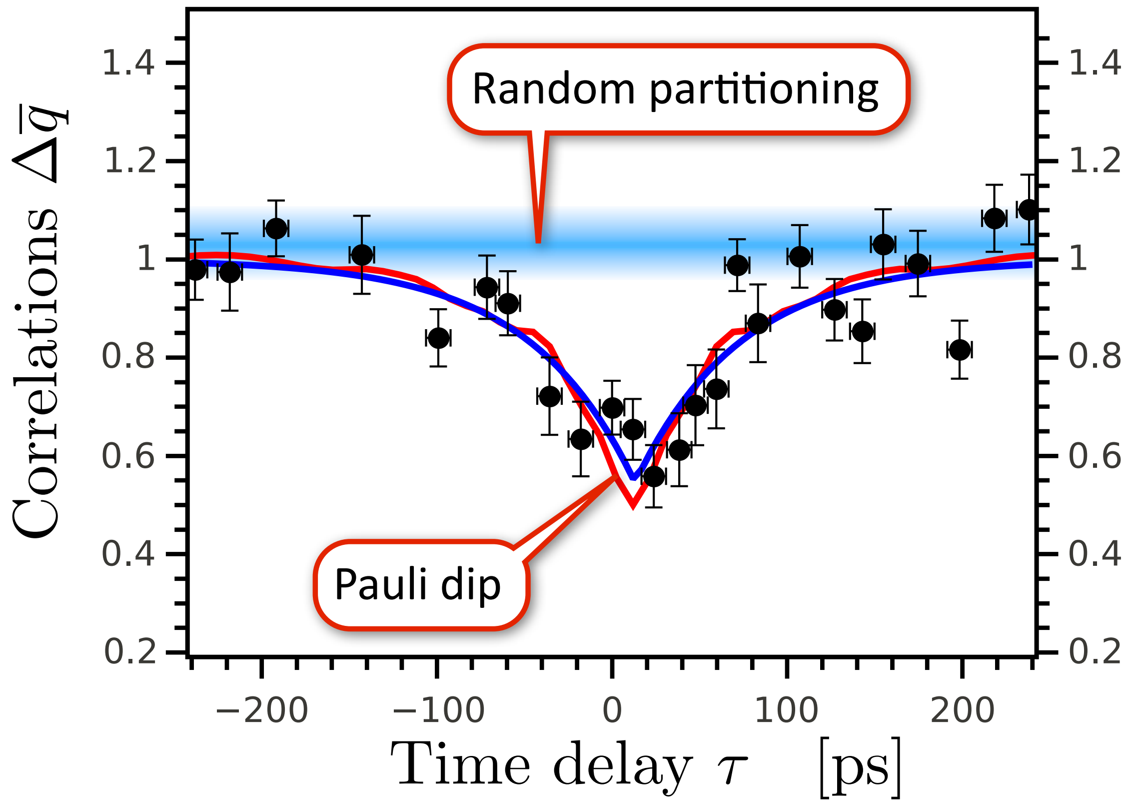

FIG. 3: Excess noise Δq between both sources switched on and both sources switched off as a function of the delay τ. The noise values are normalized by the value on the plateau observed for long delays. The blurry blue line represents the sum of the partition noise of both sources. The blue trace is an exponential fit by $\Delta q = 1 - \gamma e^{-|t-\tau_0|/\tau_e}$. The red trace is obtained using Floquet scattering theory.