Anomalous transverse voltages in the superconducting surface sheath

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The critical state and transport properties of the superconducting sheath are investigated, in slabs parallel to the applied magnetic field, when it makes an arbitrary angle \( \theta \) with the direction of the applied current. The observation of critical currents governed by surface defects, and linear current-voltage characteristics, corroborate the conclusion advanced by several authors in the past, that the surface sheath of a real rough surface is populated by quantized vortices or "flux spots," which should exhibit the same pinning and flux-flow properties as usual vortices. Nevertheless, transverse voltages measured above \( H_{c2} \) are at variance with the well-known general relation between macroscopic fields, \( \mathbf{E} = -\mathbf{v}_f \times \mathbf{B} \), contrary to those observed in the mixed state. Making allowance for the observed anisotropy of the surface critical current in parallel field, a model is proposed that accounts successfully for the unexpected distribution of electric currents and fields above \( H_{c2} \). We emphasize that experimental results cannot be reconciled with the existence of surface vortices, and our analysis would lead to serious difficulties in interpreting Joule dissipation, unless we rely on some unorthodox conclusions of a continuum theory of vortex motion developed recently by two of us. Measurements reported in this paper support the correctness of this theory.

I. INTRODUCTION

This paper is concerned with the critical state and transport properties of a slab in a parallel magnetic field, particularly above \( H_{c2} \). Figure 1 shows several current-voltage characteristics, such as commonly observed on increasing the magnetic field across \( H_{c2} \). The persistence of a critical current indeed confirms the capacity of the superconducting surface sheath for carrying a reasonably large current density, \( i_i \ A/m^2 \), without dissipation.

At first sight, the unchanging shape of the \( I-V \) curve suggests that some aspects or rules of the flow flux in the mixed state should extend beyond \( H_{c2} \). In particular, and as has long been noticed, the flux \( \Phi \) is governed by (surface) defects. Moreover, Hart and Swartz (HS) inferred from their experiments, that the superconducting layer of a rough (real) surface should be permeated by an array of quantized flux spots. As pointed out by several authors, many of the properties of the vortex lines in the mixed state, in connection with pinning and flux flow, may well pertain to these flux spots or surface vortices. While developing our own argument, we recall in Sec. II how this opinion prevailed at the end of the 1960s. Recently two of us have proposed a continuum theory of vortex motion in the mixed state, which we shall refer to below as the MS theory. As discussed in Sec. II, we believe that most of the ideas underlying the MS theory may be generalized to describe the critical state and vortex motion in the surface sheath, even though we are dealing with a quasi-two-dimensional (2D) situation.

In the mixed state, under steady-state conditions, a simple and well-known equation relates the electric field \( \mathbf{E} \) and the vortex line velocity \( \mathbf{v}_f \), viz. \( \mathbf{E} = -\mathbf{v}_f \times \mathbf{B} \). This equation, relating macroscopic fields, has been derived by Josephson in its stricter form, \( \mathbf{E}' = -\mathbf{v}_f \times \mathbf{B} \), where \( \mathbf{E}' = \mathbf{E} + \nabla \mu /\epsilon \) is the gradient of the electrochemical potential; the circulation of \( \mathbf{E}' \) is just what is measured by a voltmeter. Josephson’s method of proof is one of great generality. In the author’s own terms, it “is applicable to (any) systems which are inhomogeneous with respect to composition or flux line density.” \( \mathbf{E} = -\mathbf{v}_f \times \mathbf{B} \) implies the existence of strong transverse voltages in the mixed state, when \( \mathbf{B} \) makes an arbitrary angle \( \theta \) with the applied current (Fig. 2), in full agreement with experiment. We

![FIG. 1. Longitudinal voltage-current characteristics, \( \nu_c(I) \), of a Pb-In 17,5 at. % slab, for different values of the parallel applied field \( B_{||} \), and \( \theta = \pi/2 \) (see Fig. 2). The slab dimensions along the \( xyz \) directions shown in Fig. 2 are, respectively, \( L = 30 \) mm, \( W = 8 \) mm, and \( t = 1 \) mm. The distance between voltage probes is \( \Delta x = 13 \) mm. At \( T = 1.8 \) K, \( H_{c2} = 4700 \) Oe. The normal resistivity is \( \rho_n = 10.1 \) \( \mu \) cm. For \( B_{||} > B_{c2} \), the limiting slope of the \( V-I \) curve at large currents is the normal resistance \( R_n = \rho_n \Delta x/Wt \). At 5000 G, trailing of the curved part of the \( V-I \) curve reveals a relatively large dispersion or critical current along the sample, as explained in the text. By extrapolating at \( V = 0 \) the linear part of the characteristic at 5000 G (dashed line) we obtain the mean critical current \( I_c = 3.5 \) A. When \( B_{||} \) is aligned with the applied current (\( \theta = 0 \)), \( I_c \) is reduced to 2.8 A at 5000 G. On dividing these two values of \( I_c \) by 2\( W \), we get representative mean values of the extreme current densities \( a \) and \( b \) of the critical curve \( I_c(\varphi) \) of Fig. 5.
FIG. 2. The geometry of the sample. The magnetic field B is aligned with the $xy$ faces of the slab, and can be inclined at an arbitrary angle $\theta$ to the direction $x$ of the applied current. A 100–200 turns coil, wound directly on the slab as shown, presents mutual inductance with current loops along $yz$ planes. It is designed to confirm, through a low-frequency modulation experiment, the occurrence of circulating transverse currents above $H_{c2}$.

then expect that the transverse field suddenly falls to zero at $H_{c2}$, unless vortex motion takes place in the surface sheath. If so, and in so far as $E = -v_L \times B$ is a fundamental relationship pertaining to any kind of vortex motion, a similar effect ought to have been observed above $H_{c2}$, in spite of the fact that nearly the whole of the sample is in the normal state.

Transverse-voltage measurements are reported in Sec. III. We did observe transverse fields above $H_{c2}$, but their behavior is totally at variance with the equation $E = -v_L \times B$. Nevertheless, as explained in Sec. III, this is not inconsistent with the presence of vortices in the surface sheath, provided that the MS equation relating $E'$ and $v_L$ [Eq. (7) below] is used in the place of $E = -v_L \times B$. In most situations, in particular those regarding bulk flux flow, both equations are very much the same. Thus, the surface superconducting sheath appears, in this respect, as an unusual case well suited to support the correctness of the MS approach.

By relying on the observed anisotropy of surface critical currents, a model is proposed (Sec. IV) which well accounts for transverse voltages above $H_{c2}$. As a proof, a mutual inductance experiment, has been designed to probe the predicted current distribution. From these experiments it emerges that steady distributions of transverse electric fields and currents may occur, where a bulk normal Joule effect is balanced by a negative surface Joule effect. This rather surprising result again is well explained by the MS theory of flux-flow dissipation as discussed in Sec. IV.

II. THE CRITICAL STATE
OF THE SURFACE SHEATH:
A PRELIMINARY DISCUSSION

Earlier attempts to interpret $i_c$ as a fundamental thermodynamic property of the surface sheath have been highly unsatisfactory. Abrikosov$^9$ and Park$^{10}$ calculated the maximum current density $i_m$, which can be passed through the surface sheath, in accordance with the Ginzburg-Landau equations. But $i_c \ll i_m$ by 1 or 2 orders of magnitude. While $i_m$ certainly makes sense as a theoretical upper bound, it was clear that something else limited surface currents far below.

Fink and Barnes,$^{11}$ then Park,$^{12}$ defined the critical state of the surface sheath as that state for which the magnetic free-enthalpy difference $G - G_n$ between the superconducting state and the normal state is zero. Critical current densities obtained in this way are much lower than those given by Abrikosov$^9$ and Park,$^{10}$ and turn out to be the order of magnitude observed. However, besides the fact that minimum free-energy arguments are quite questionable when dealing with transport phenomena, severe objections have been made to this line of reasoning.$^{1,2}$ Though deriving unlike expressions for $i_c$, both models$^{11,12}$ make it to be dependent on the foil thickness and/or width; such a size dependence of $i_c$ has never been observed.$^2$ Another common prediction is worth mentioning, as it is inconsistent with our own results. In an experiment reported in Sec. IV, we have been led to destroy surface superconductivity on one face of the slab by nickel plating. Now, according to both models,$^{11,12}$ the measured critical current in this case should be reduced by a factor of $\sqrt{2}$. As far as both faces of the slab have been prepared alike, we instead observe that $i_c$ is smaller by a factor of 2 (Fig. 1), as clearly expected if $i_c$ is caused by surface defects. But the most direct criticism is still to argue that no transition to the normal state is observed,$^2$ as readily seen by mere inspection of the $I-V$ curve at large currents, where $V/I = R_n$, the normal resistance of the sample.

All four of the above-mentioned theories$^9-12$ suppose that the local magnetic field be everywhere parallel to a perfect planar surface on the scale of the coherence length $\xi(T)$. HS rightly noticed that such ideal conditions are unattainable, in practice, so that the field has a nonzero normal component over most of the surface. Therefore, in discussing their results, HS proposed that the magnetic field, locally inclined to the surface, crosses the superconducting sheath as an array of quantized flux spots. Not long after, Kulik,$^3$ following Abrikosov’s approach, showed that the superconducting layer, in an inclined field, actually has a vortex structure similar to the Abrikosov lattice. The mapping of the magnetic field sketched in Fig. 2 of Kulik’s paper illustrates and confirms the flux spot picture. Confining himself to rectangular unit cells, Kulik obtained a square vortex lattice at equilibrium, whose period $d$ is given by

$$d^2 B \sin \alpha = \phi_0,$$

where $\phi_0$ is the flux quantum. $B \sin \alpha$ is the normal component of the magnetic field. For small inclination angles $\alpha$, $d = (\phi_0 / B \alpha)^{1/2}$. For instance, taking $B = 5000 \text{G}$, and $\alpha = 1^\circ$, $d \approx 5000 \text{ A}$. However careful the alignment of the whole slab with the field, one cannot avoid large-scale roughness of the surface, and thereby penetration of magnetic flux through the sheath. So it is to be expected that a large number of Kulik’s vortices populate the surface. The generation of surface vortices, which may be pinned, enter a critical state, and move, provides an attractive explanation for all observations.

Let us return, for further comments, to the set of $I-V$ characteristics as shown in Fig. 1. The curved part at the foot of the characteristic, though of minor importance, is
a constant feature, which gives indirect evidence of the prominent part of defects, above and below \( H_{c2} \) alike. As the distribution of defects is hardly homogeneous in practice, each segment \( x \) of the sample has a different critical current, \( I_c(x) \), in a finite range, say from \( I'_c \) to \( I''_c \). Thus the curved region of the overall characteristic, between \( I'_c \) and \( I''_c \), results from the sum of individual linear characteristics. This fact is easily checked by using a series of close voltage probes.\(^{11}\) It is easily seen that the critical current \( I_c \), usually obtained by extrapolating the linear part of the \( I-V \) curve, represents the mean value of \( I_c(x) \) between the voltage probes. In discussing experimental results, it will be convenient to assume that the distribution of surface defects is homogenous and isotropic. Under such ideal conditions, the \( I-V \) curve should display the standard broken shape: \( V=0 \) for \( I < I_c \), and \( V \sim R(I-I_c) \) for \( I > I_c \). Below \( H_{c2} \), \( R = R_f < R_n \) is the field-dependent flux-flow resistance. Near \( H_{c2} \), the conductivity difference, \( \sigma_f - \sigma_n \), is an effect associated with the relaxation time of the order parameter.\(^{6,14}\) Vortex motion, if it occurs in the surface sheath, must entail similar time-relaxation effects. Because of the smallness of the volume involved, however, their contribution to the total dissipation is not significant, so that \( R \) cannot be distinguished, within experimental accuracy, from the normal resistance \( R_n \) (see Fig. 1).

The constant intercept of the linear part of the characteristic suggests that the surface sheath, above \( H_{c2} \), retains its ability to carry a constant nondissipative current \( I_1 \). That is its maximum value \( I_c \), as much as allowed by surface defects, while the voltage is proportional to the dissipative excess current \( I_2 = I-I_1 = V/R_n \).

The MS theory just leads to the same naive interpretation of the customary flux-flow regime in the mixed state.\(^{5,6}\) Yet there is an objection to be made: How does one explain the large part \( VI_1 = VI_c \) of the Joule power \( VI \), if \( I_1 \) itself does not contribute to dissipation? There is no inconsistency, however, as discussed at some length in Ref. 6. Whether below or above \( H_{c2} \), we are clearly faced with the same problems. We shall return to this point at the end of Sec. IV.

To begin with, we shall avoid arguing about the detailed nature of the pinning and dissipative mechanisms. First of all we wish to consider the previous and less complicated question of how electric fields and currents are distributed in the "vortex flow" regime above \( H_{c2} \), especially when the magnetic field is inclined at arbitrary angles \( \theta \). In whichever model of transport, a current \( I > I_c \) may be separated into a surface part \( I_1 = I_s \) (current density \( j_1 \)) and a bulk part \( I_2 \) (normal current density \( j_2 \)). We may accept as an experimental fact the existence of associated Joule effects: \( \mathbf{E} \cdot j_1 (W/m^2) \) and \( \mathbf{E} \cdot j_2 (W/m^3) \), but the very nature of dissipative mechanisms, however, will come into question inevitably, when we discuss our experimental results.

III. TRANSVERSE VOLTAGES: EXPERIMENTAL RESULTS

Longitudinal and transverse voltages, \( V_x \) and \( V_y \), have been measured in a series of 10–20 lead-indium slabs aligned with the applied magnetic field, for various values of the angle \( \theta \), as shown in Fig. 2. Typically, the dimensions \( L, \ W, \) and \( t \) of the slab, along the \( x, \ y, \) and \( z \) directions, respectively (Fig. 2), are \( L = 3 \ cm, \ W \leq 1 \ cm, \) and \( t = 0.5–1 \ mm \). Cast ingots of the solid solution Pb-In 17.5 at. \% were annealed for 2 weeks under purified argon (~\( 10^{-4} \) mm Hg) within about 10°C of the solidus point. Slabs were spark cut, either directly from the ingot, or from 0.5–1 mm sheets obtained by rolling, or by pressing a piece of ingot between glass microscope slides in a hydraulic press. Pressed samples exhibited a mirror-like finish, a good parallelism of the faces, and yet relatively large critical currents above \( H_{c2} \) (\( i \approx 1 \mu A/cm \)). Samples were used as rolled or as pressed, or after various surface treatments (mechanical or chemical polishing, electropolishing).

For the sake of the discussion, we shall refer to simple standard conditions, in which samples are assumed to exhibit a homogenous and isotropic distribution of (surface) defects, as stated in Sec. II, as also bulk homogeneity. Such conditions entail a translational symmetry along \( xy \) planes (Fig. 2). In the vortex state macroscopic space charges and Bernoulli effects are negligibly small, so that \( \mu = \text{const} \) in a homogeneous sample, and, in any case, \( \mathbf{E}' = 0 \). Moreover, translational symmetry and steady conditions \( (\nabla \times \mathbf{E} = 0, \ \nabla \cdot \mathbf{E} = 0) \) require \( \mathbf{E} \) to be uniform everywhere inside the sample.

In the mixed state, strong transverse voltages in an inclined field \( (\theta \neq \pi/2) \) have long been observed,\(^{8}\) and experimental results fully agree with theoretical predictions. Let us recall them briefly, for comparison with the unexpected and contrasted behavior of transverse fields above \( H_{c2} \). Except for the intrinsic situation where \( \mathbf{B} \) is aligned with the applied current \( (\theta \to 0) \), bulk flux flow in the mixed state is well understood. Vortex lines (density \( n \), direction \( \psi \)), defined as the lines on which the order parameter \( \psi \) vanishes, lie along the direction of the magnetic field \( \mathbf{B} \) so that

\[
\mathbf{B} = \alpha_0 \mathbf{n},
\]

and all theories of vortex motion state that, under stationary (and standard) conditions,\(^{5}\)

\[
\mathbf{E}' = -\mathbf{v}_L \times \mathbf{B},
\]

\[
\mathbf{E} = \rho_f (B) J_{11},
\]

where \( \rho_f \) is the flow-resistivity, and \( J_{11} \) is the component of the bulk current normal to vortex lines. In Eqs. (2) and (3), \( \mathbf{B} \) stands for the field inside the sample. Due to magnetization, \( \mathbf{B} \) differs from the applied external field \( \mathbf{B}_0 = \mu_0 \mathbf{H}_0 \). Note, however, that, in the parallel flat slab geometry, \( \mathbf{B} \) and \( \mathbf{B}_0 \) remain parallel, and \( B = \text{const}(\theta) \). Equation (3) alone prescribes a strong constraint on the direction of the electric field, since \( \mathbf{E} \) must be normal to \( \mathbf{B} \):

\[
E_y = -E_x / \tan \theta.
\]

If \( J_z \) is assumed to flow in the \( x \) direction, we have

\[
E_x = E \sin \theta = \rho J_z \sin^2 \theta.
\]
As \( \rho_2 = \text{const.}(\theta) \) in the geometry used, measurements of the apparent flux-flow resistivity, \( E_x/J_2 \) or \( dV_x/dI \), for \( 10^\circ < \theta < 90^\circ \), entirely confirmed the predicted \( \sin^2\theta \) dependence. As \( \theta \to 0 \), too large critical currents prevented us from observing a linear flux-flow regime.

It is noteworthy that the flux-flow constraint (5) must hold so long as some vortex structure exists. If, for instance, \( \theta = \pi/4 \), \( E_y = -E_x \), so that \( \mathbf{E} \) makes a constant \( \pi/4 \) angle with the applied current up to \( H_{c2} \), despite the fact that the sample is approaching the normal state continuously (\( \psi \to 0 \) and \( \mathbf{B} \approx \mathbf{B}_0 \)). Equation (3) breaks down at \( H_{c2} \), and \( \mathbf{E} \) is expected to line up suddenly with the direction of the applied current as it should be in the normal state, giving rise to jumps in longitudinal and transverse voltages. That is just what we observed. As shown in Fig. 3, when \( H_0 \) is increased at constant \( I \), \( |V_y| \) first increases (in the same ratio as \( V_x \)), and falls off abruptly at \( H_{c2} \) (over about 20–30 G). However, significant transverse voltages persist above \( H_{c2} \), according to a mechanism to be determined. Whatever it may be, it has to be ascribed to surface superconductivity: nickel plating both faces of the slab indeed makes \( V_y \) vanish above \( H_{c2} \) within experimental accuracy. It is to be noted that, because \( H_{c2} \) is very sensitive to the indium concentration, unannealed samples may exhibit a blurred transition over a few hundred G (see, for example, the data of Ref. 8). In spite of a narrow liquidus-solidus range, careful annealing is required to make the alloy composition uniform. In practice, we found that a sharp decrease of \( |V_y| \) at \( H_{c2} \) was the best way of testing bulk homogeneity, ensuring that the whole bulk sample has entered the normal state.

Let us now examine the outstanding features of transverse voltages above \( H_{c2} \), as compared with those observed below \( H_{c2} \). First of all, \( V_y \) changes sign (\( V_y > 0 \) in the case of Fig. 2, where \( I = I_x > 0 \)). Moreover, while the longitudinal characteristic \( V_x(I) \) has the common shape, \( V_x \) increasing linearly at large currents, the transverse characteristic \( V_y(I) \) shows a flat plateau (Fig. 4). This implies that, at given \( \theta \), the direction of the electric field is now current and field dependent: \( E_y/E_x \) may take any positive value, though rapidly decreasing with increasing \( I \) or \( B \) (\( I \to \infty \) or \( B \to B_{c2} \), \( E_y/E_x \to 0 \)).

Seeing that a vanishing vortex structure near \( H_{c2} \) still forces the orientation of \( \mathbf{E} \), we are not surprised that the superconducting sheath, thin though it may be, can affect the electric field direction. Moreover, referring to steady standard conditions, where \( \mathbf{E} \) is uniform throughout the sample, we are aware that any constraint on \( \mathbf{E} \) in the sheath must extend to the bulk. But the only reliable equation that presumably applies to any systems involving flux line motion (see Sec. I), \( \mathbf{E} = -\mathbf{v}_e \times \mathbf{B} \), happens to be inconsistent with our experimental results above \( H_{c2} \). In particular, \( E_y \) has the wrong sign. Whatever way we turn to interpret voltages along the surface sheath, we first have to remove this difficulty, unless we renounce surface vortices.

As far as its main ideas are readily extended to the surface vortex state, the phenomenological MS theory of the mixed state provides a satisfactory answer. The originality of the MS theory essentially lies in realizing from the outset that (i) at equilibrium, vortices must terminate perpendicular to the surface sample; (ii) Eq. (3) is not of general validity, and holds only for curl-free (in particular, in the absence of) supercurrents. In a thermodynamic treatment of the mixed state, regarded as a continuum, the vector \( \omega = n \phi \mathbf{v} \), which describes the local density and direction of vortex lines, and the macroscopic magnetic field \( \mathbf{B} \) must be considered as local independent variables: for a given value of \( \omega \) at some point \( M, \mathbf{B}(M) \) still may be varied by any change in the distribution of currents elsewhere. Most of equilibrium and transport problems may be reexamined from this point of view. Consider, for example, the equilibrium of a perfect cylinder or sphere in an external field \( \mathbf{B}_0 \) (see Fig. 2 in Ref. 6): it is found that vortex lines curve in over a small depth near the surface to end normal to the boundary, while field lines in this perturbed layer are bent in the opposite direction. A condition for the local equilibrium of vortices\(^{4,6}\) requires that supercurrents be associated with the local distortion of the vortex array, while \( J \approx 0 \) and \( \omega \approx \mathbf{B} \) in the bulk. These are nothing but the Meissner-like diamagnetic currents. Now, consider the

![FIG. 3. The transverse voltage \( V_y \) as a function of the magnetic field at \( T = 1.8 \, \text{K} \), and constant current \( I = 11 \, \text{A} \). Orientations of the applied magnetic field and current are those shown in Fig. 2. Voltage contacts are carefully aligned perpendicular to the direction of the applied current, or else a potentiometer setup is used as shown in the inset. Anyway, it is advisable to verify that \( V_y \), contrary to \( V_x \), is an odd function of \( \theta \). \( V_x \) and \( V_y \) are both even functions of \( B_0 \), and odd functions of \( I \). Negative transverse voltages below \( H_{c2} \) are fully explained by usual flux-flow equations. The unexpected observation of positive transverse voltages above \( H_{c2} \) are the subject of this paper. The abruptness of the voltage jump at \( H_{c2} \), over about 20–30 G, attests a good homogeneity in indium concentration after annealing.](image3)

![FIG. 4. Transverse voltage-current characteristics, \( V_y(I) \), above \( H_{c2} \), showing the voltage saturation at large currents. No transverse voltage is observed for \( \theta = 0^\circ \) and \( \theta = 90^\circ \). The effect has a maximum for \( \theta \approx 45^\circ \).](image4)
case of a rough slab in perpendicular field: there are no
diamagnetic currents. But in the presence of surface
roughness on a scale comparable to or smaller than the
vortex spacing, there are many ways for the vortices to
end normal to the actual surface, allowing for a large
number of metastable or nondissipative solutions.5,6
Associated supercurrents in this case may well appear as
nondissipative transport currents $I_1$. Again, across a thin
surface layer, vortex lines (i.e., the singular lines $\psi=0$)
strongly deviate from magnetic field lines, and $\omega \neq \mathbf{B}$ (see
Fig. 3 of Ref. 6).

Using a standard rigorous method, MS derived a com-
plete set of transport equations. In particular, under sta-
nionary conditions, the equation [Eq. (39) of Ref. 6]

$$E' \approx \mathbf{E} = -\mathbf{v}_L \times \omega$$  \hspace{2cm} (7)

is obtained as a straightforward consequence of conserva-
tion laws.5,6 Equation (7) should be substituted into Eq. (3)
of more general applications.

It should be noted that the normal ending of vortices at
the sample surface is consistent with (required by) the
Ginzburg-Landau boundary condition, $\partial \psi / \partial n = 0$,
where $\partial / \partial n$ is the normal derivative at the surface.
Kulik’s vortices do not make an exception to this rule,
though this is not pointed out by the author. By simple
inspection of Kulik’s solution for the order parameter in
the surface sheath, it is seen that the lines $\psi=0$ are
indeed normal to the surface, whatever the angle $\alpha$
between surface and field may be. Since surface vortices are
very short and melt away rapidly in the normal bulk,
they cannot change direction through the surface sheath.
According to Eq. (1) the vortex density is $n = a^{-2}$, and thus
$\omega$, and $\alpha$ dependent, so that $\omega$ should be a highly variable
function of position, in particular $\omega$, changes sign, along
a rough $xy$ face of the slab. The vector $\omega$, however, keeps
close to the $z$ direction. Therefore, it is clear that any
direction of the electric field in the $xy$ plane becomes
compatible with the new transport equation (7).

IV. ANISOTROPY

OF SURFACE CRITICAL CURRENTS:
A MODEL OF TRANSVERSE VOLTAGES ABOVE $H_{c2}$

Due to Ohm’s law, $J_x = \sigma_e E_x$, the existence of a trans-
verse field $E_x > 0$ at large currents implies that of a trans-
verse normal current $J_{xy} = \sigma_n E_x > 0$. This is a marked
difference with bulk currents $J_x$ in the mixed state, which
are flowing in the $x$ direction. Since no net current can flow in
the $y$ direction ($I_y = 0$), surface currents must flow
on both faces in the negative $y$ direction so that

$$2i_{1y} + J_{2y} = 0$$  \hspace{2cm} (8)

while the applied current $I > I_c$ is

$I = I_x = W(2i_{1x} + J_{2x})$  \hspace{2cm} (9)

where $I_x = 2W_{1x} > 0$ and $J_{2x} > 0$. Therefore, for large
currents, the surface current density $i_1$ makes an angle
with the $x$ direction, except for $\theta = 0$ and $\pi / 2$. If we
succeed in explaining the clockwise rotation of $i_1$, we
should state, conversely, that a normal current must re-

sult from the backflow of the transverse surface currents,
giving rise to transverse voltages:

$$V_y = -2W_{1y} / \tau \sigma_n.$$

(10)

For $\theta = 0$ and $\pi / 2$, no transverse voltages are observed,
as expected from symmetry considerations. This fact
warrants that no preferred direction exists on the surface
except for that of the magnetic field itself. For $I > I_c$,

$$i_1 = i_{1x} = W_{1x} / 2W.$$  \hspace{2cm} (11)

As discussed in Sec. II, we only have access, in actual
samples, to average values of critical currents between
two probes, and all current densities $i_1$ and $J_x$ in
the above standard equations should be replaced by appro-
priate mean values. Nevertheless, as we systematically
observed that $I_c (\theta = \pi / 2)$ were 20–30% larger than $I_c
(\theta = 0)$, we may reasonably conclude that surface critical
currents are locally anisotropic: the maximum current density $i_1 = i_{1x}$, that the surface is capable of carrying
without dissipation, depends on the angle $\varphi$ between $i_1
and $B$. Thus, an ideal standard sample would be charac-
terized, for given values of $T$ and $B$, by a critical curve
$i_1(\varphi)$ such as sketched in Fig. 5; extreme values
$i_1(\pi / 2) = a > b = i_1(0)$ correspond to $I_c (\theta = \pi / 2) = 2W a$
and $I_c (\theta = 0) = 2W b$, respectively.

Once we have accepted the anisotropy of surface critical
currents as an experimental evidence, it is a simple
matter to establish the connection between this anisot-
ropy and the observed behavior of the transverse voltage-
characteristics above $H_{c2}$. For this purpose we
still refer to simple standard conditions, where the criti-
cal state of the surface sheath, at any point of the surface,
is uniformly described by the same curve $i_1(\varphi)$. Then we
make the reasonable assumption that transport currents are
so distributed as to minimize the total power input
$V_x I_x = V_y I_y$; at $I$ constant, this means $V_y$ minimum.
In particular, $V_y = 0$ as far as allowed by critical properties
of the surface. Low nondissipative surface currents can
flow in the $x$ direction up to $i_{1x} = i_1(\varphi) = OM$ (Fig. 5).
The transport current $I = 2W_{1x}$ can be further in-

FIG. 5. Graphical construction of the critical state of the
surface sheath in inclined fields. As shown by experiment,
the critical surface current density $i_1 (A / m)$ is anisotropic, and de-
deps on the angle $\varphi$ between the direction of the magnetic field
and that of the surface current density $i_1$. Assuming homogene-
ous surface conditions (referred to as standard conditions in the
context), any point of the $xy$ faces should be characterized by the
same theoretical curve $i_1(\varphi)$. $i_1 = OC$ represents the current
density in the critical state, defined as that state achieving
the maximum transport current $i_{1x}$ for zero longitudinal voltage.
creased, while maintaining $V_x = 0$, provided that the surface
current $i_1$ rotates clockwise along the critical curve. When $i_1 = 0$ (Fig. 5), the critical current is attained:
$I = I_c(\theta) = 2W$ OH. Any excess current $I - I_c$ will appear
as bulk normal currents $J_{2\perp} > 0$. As $i_1$ is rotating, a
transverse voltage $V_y$ arises, which is associated to the
normal backlight in accordance with Eq. (10). $V_y$ comes to saturation for $i_{1y} = HC$ (Fig. 5). The curve traced out
by the vector OH, from A to B, is the so-called pole curve of the curve $i_1(\theta)$. If the critical curve is approximated
by an ellipse with semiaxes $a$ and $b$, it is easily shown that $HC$ has a maximum $a - b$ near $\theta = \pi/4 \tan \theta = (b/a)^{1/2}$. Hence, Eq. (10) yields the
maximum transverse voltage to be expected:

$$V_{y\text{max}} = (2W/\text{HCmax})/\sigma_n$$

$$= 2W(a - b)/\sigma_n = \Delta I_c/\tau \sigma_n,$$  \hspace{1cm} (12)

where $\Delta I_c = I_c(\pi/2) - I_c(0)$. $V_{y\text{max}}$ can be estimated by
using the experimental mean values of critical currents;
taking, for instance, data from Figs. 1 and 4 at 5000G, we
find $\Delta I_c = 0.7$ A and, from Eq. (12), $V_{y\text{max}} = 71 \mu V$, which is close to the measured saturation value for $\theta = \pi/4$ (Fig. 4).
In view of the dispersion of critical currents in actual samples, perhaps such a quantitative agreement is fortuitous, but the predicted order of magnitude remains significant.

This model well accounts for the main features of transverse voltages above $Hc_2$: sign, amplitudes, and saturation at large currents. This could have been our conclusion, but we cannot evade a last difficulty, in connection with the transverse current distribution implied by our interpretation. Before coming to this point, let us report the result of an experiment designed to confirm the presence of the transverse superconducting-normal counterflow.

A pickup coil consisting of $n$ turns of wire was wound close to the slab surface as shown in Fig. 2. Surface currents $i_{1y}$ on both faces, and return normal currents $J_{2\perp}$, form double current loops in the $yz$ planes. By symmetry, the magnetic flux through the coil as a result of these transverse currents is zero. By nickeling one face of the slab, the superconducting sheath is destroyed on this face, so that transverse currents $i_{1y}$ and $J_{2\perp}$ now form a single loop: $I_y$ and $V_y$ are reduced by a factor of 2, but a magnetic flux $(\sim n \mu_0 J_{1y} Wt)$ links the coil. If then the sample is driven by a modulated current $I = I^* e^{i\omega t}$ along the normal component of the $V_y(I)$ characteristic, $V_y$ and $i_{1y}$ are modulated accordingly, and we may expect an induced emf in the coil, which we estimate roughly as

$$(\text{pickup signal}) \sim \omega n \mu_0 J_{1y} Wt - \omega n \mu_0 V_y I^* t^2 \sigma_n.$$  \hspace{1cm} (13)

The frequency $\omega$ is low enough to ensure a quasistatic
modulation of the characteristics, as checked experimentally.
With $I^* = 0.4$ A, $\omega/2\pi = 130$ Hz, $n = 200$ turns, and data of Figs. 1 and 4, Eq. (13) predicts $\omega \sim 1 - 10 \mu V$, a signal readily measured by a lock-in amplifier. We did observe such a signal under the required conditions. That is, at $I^* = \text{const}$, the signal vanishes, as expected, in the normal state, in the mixed state, and, immediately
above $Hc_2$, for either subcritical or large currents (as $V_y$ is saturated). It vanishes also at any current and field, when either no face or both faces are nickel plated.

Let us return to the graphic construction of the transverse
current in a standard sample (Fig. 5). As $i_1$ rotates from $\text{OM}$ to $\text{OC}$, $V_y$ increases, while $V_x = 0$. Then, as $I$ is
further increased, $V_y = \text{const}$, while $V_x$ starts rising.
Indeed, we are prevented from observing such an ideal behavior, for the critical-current dispersion makes both $V_x(I)$ and $V_y(I)$ characteristics spread out over a large current interval (Figs. 1 and 4). Nevertheless, we are entitled to consider the standard case as being physically possible. Thus, within a short current interval $(OM < I < 2W < OH)$, $E$ may be a purely transverse field ($E_z = 0$). Under these conditions, the bulk normal Joule effect $\mathbf{E} \cdot J$ turns out to be exactly balanced by a negative surface Joule effect, $\mathbf{E} \cdot i_1 = E_{ix} i_x < 0$, associated with vortex flow in the surface sheath. Note that, according to Eq. (7), electric field and vortex motion are inseparable. This is a most unusual situation. In dc experiments we are not accustomed to obtain circulating steady currents in an imperfect conductor without needing the emf of some generator to keep them going. Note, however, that no physical law is violated. As pointed out at the end of Sec. II, the difficulty lies elsewhere, in connection with the usual interpretation of terms such as $\mathbf{E} \cdot i_1$ in the Joule effect.

Consider an operating point $(I, V)$ in the flux-flow regime along any longitudinal characteristic, either below or above $Hc_2$. $VI$ represents the electrical power input to the sample. In steady conditions, energy is transferred at the same rate to the heat reservoir, as required by the law of energy conservation. It is also generally stated that $VI$ is the rate at which energy is dissipated within the portion of circuit considered. For instance, the part

$$(V / (I - I_y)) = V^2 / R_f$$

of the Joule effect, in the mixed state, is clearly associated with the viscous vortex flow: the dissipation rate by unit length of vortex can be written as

$$\eta \nu^2,$$

where $\eta = B \phi_0 / \rho_f$ is a viscosity coefficient. The
remaining part of the Joule effect, $V L = V L$, is the integral of sum of terms such as $E i_1$, is not so easily accounted
for (including above $Hc_2$), but it is usually thought of as resulting from the same damping mechanism as $\eta \nu^2$: elastic instabilities at the pinning sites should give large local fluctuations of $\nu_L$, so that $\eta \nu^2 > (\nu_L)^2$, where $\nu_L = E / B$ stands for the mean velocity of the vortex array. Thus, $\eta (\nu_L)^2$ should correspond to $(V / (I - I_y))$, while $\eta \nu^2$ should be responsible for the extra dissipation $V L$. Though very ingenious, this model is not entirely satisfactory, but that is not the point.

As argued in Sec. II, there is no reason to believe that the mechanism underlying the Joule effect $V L$ is different above and below $Hc_2$. Now, if $E i_1$ has to be associated with some dissipative mechanism, it must be positive, a condition which is inconsistent with the intermediate regime described above. The model for critical currents and surface Joule effect in soft materials, that proceeds from the MS theory of transport, avoids this further paradox.

In conclusion, we briefly recall the relevant arguments
of the MS theory with regard to pinning and dissipation in soft samples. The current density $J$ is separated into two parts, as $J_1 + J_2$, where $J_1 = -
abla \psi$ is the nondissipative part of the supercurrent. Here $\psi(x)$ is a “vortex potential” closely related to the reversible magnetization curve. $J_1$ is defined at any point and time as the supercurrent (including diamagnetic currents) that would come into equilibrium with the vortex array in its instantaneous configuration. $J_1$ does not enter into the dissipative function, but it may contribute to transport currents. As stated in Sec. III, in the presence of surface irregularities, there are many vortex configurations allowing nondissipative transport currents $J_1$ to flow close to the surface. On the scale of the sample these currents can be regarded as surface currents $I_1$. The instantaneous critical current is defined as the flux $I_1$ of $J_1$ through a cross-section averaged over the measured length of the sample. Small fluctuations of $I_1$ ($\sim 10^{-3} - 10^{-4}$), resulting from stronger local fluctuations, are responsible for the flux-flow noise. Critical-current data, as also a number of old or more recent experiments, have confirmed this point of view. Now, an important convective term $\psi(x)\omega L$ appears in the general expression of the energy flux. In usual flux-flow regimes, because of the mean steady bending of the vortex lines near the surface, the surface normal component of $\psi(x)\omega L$ is pointing outwards, and contributes to the heat ejected to the surrounding medium (see Fig. 4 of Ref. 6). In any circumstances, the net outward flux of energy due to the term $\psi(x)\omega L$ is just $\beta I_1$, but clearly, nothing in this description requires it to be necessarily positive.