Abstract
The effect of disorder on interacting fermions of both spinless and spinful two coupled chains has been examined by applying the renormalization group method to the bosonized Hamiltonian. A phase diagram is calculated as a function of disorder, interaction and interchain hopping to find localization induced by impurities and confinement of single particle.

Keywords: disorder, coupled chains, localization, confinement, renomalization group.

1. Introduction
The interplay of disorder and interactions has been studied extensively in low dimensional system, where bosonization and renormalization group (RG) techniques can be used. In one dimension, attractive interactions of the order of the bandwidth were needed to suppress Anderson localization and produce a metallic state while, in two coupled fermions chain, localization is reduced even for weak interactions due to the opening of gaps in excitations [1,2,3]. In the presence of interchain hopping, a competition between interaction and disorder becomes noticeable for the case close to the non-interacting point where the gaps vanish. In
the present paper, we show such an effect of interchain hopping based on two coupled chains of spinless and spinful fermions [4,5].

2. Model

Two coupled chains of spinless fermions are described by a Hamiltonian [4],

\[
H_{\text{spinless}} = -t \sum_{i,p} \left( c_{i,p}^+ c_{i+1,p} + H.c. \right) - t_\perp \sum_{i} \left( c_{i,1}^+ c_{i,2} + c_{i,2}^+ c_{i,1} \right) + V \sum_{i,p} n_{i,p} n_{i+1,p} + \sum_{i} \left( \xi_{i,1} c_{i,1}^+ c_{i,1} + \xi_{i,2} c_{i,2}^+ c_{i,2} \right),
\]

(1)

where \(c_{i,p}(p = 1, 2)\) denotes a fermion operator in the \(p\)-th chain and \(n_{i,p} = c_{i,p}^+ c_{i,p}\). The case of \(t > t_\perp\) is examined for intrachain hopping energy \(t\) and interchain hopping energy \(t_\perp\) where \(V\) is the interaction between nearest neighbor sites. The impurity potentials, \(\xi_{i,1}\) and \(\xi_{i,2}\), acting on the chain 1 and the chain 2 satisfy \(\xi_{i,p}^\prime \xi_{i,p'} = W_\delta_{i,p} \delta_{p,p'}\). The parameter for interaction is taken as \(K_\rho = 1 - V(1 - \cos(2k_F a))/\pi v_F\), where \(v_F = 2ta\sin(k_F a)\) and \(a\) is a lattice spacing. For the spinful case, we consider the two-chain Hubbard model (spin \(\sigma\)) [5],

\[
H_{\text{spinful}} = -t \sum_{i,p,\sigma} \left( c_{i,p,\sigma}^+ c_{i+1,p,\sigma} + H.c. \right) - t_\perp \sum_{i,\sigma} \left( c_{i,1,\sigma}^+ c_{i,2,\sigma} + H.c. \right) + U \sum_{i,p,\uparrow} n_{i,p,\uparrow} n_{i,p,\downarrow} + \sum_{i,p,\sigma} \xi_{i,p} c_{i,p,\sigma}^+ c_{i,p,\sigma} \ ,
\]

(2)

where \(K_\rho = 1 - Ua^2/2\pi v_F\) for interaction. Equations (1) and (2) are examined by diagonalizing the \(t_\perp\) term and making use of the bosonization. We consider only the backward scattering for impurity, which leads to localization by disorder. The backward scattering for the spinful case is expressed as [5],

\[
H_{\text{imp}} = \frac{2}{\pi a} \int dx \left[ \xi_a(x) O_{\text{CDW}}^\dagger(x) + \xi_x(x) O_{\text{CDW}}(x) + H.c. \right],
\]

(3)
where \( O_{\text{CDW}^a}(x) \) and \( O_{\text{CDW}^b}(x) \) denote operators of \( 2k_F \) CDW (charge density wave) with
in phase and out of phase between two chains (\( k_F \) being Fermi wave number). They are
defined by
\[
O_{\text{CDW}^a}(x) = e^{i\phi_{\rho-}} \{ \sin \phi_{\rho-} \cos \phi_{\sigma^+} \cos \phi_{\sigma^-} - i \cos \phi_{\rho-} \sin \phi_{\sigma^+} \sin \phi_{\sigma^-} \},
\]
\[
O_{\text{CDW}^b}(x) = e^{i\phi_{\rho-}} \{ \cos \theta_{\rho-} \cos \theta_{\sigma^+} \cos \phi_{\sigma^+} + i \sin \theta_{\rho-} \sin \theta_{\sigma^-} \sin \phi_{\sigma^+} \}.
\]
The quantity \( \phi_{\rho\pm}(\phi_{\sigma\pm}) \) denotes the
phase for charge (spin) fluctuation and + (-) corresponds to symmetric (antisymmetric) one. In
Eqn. (3), \( [\phi_{\rho\pm}(x), \theta_{\rho\pm}(x')] = i\pi \text{sgn}(x-x') \) and \( \xi_{\rho}(x)\xi^*_{\rho}(x') = 4Wa\delta(x-x')\delta_{\tau\tau'} \ (r = a, s) \).

3. Phase Diagram

The parameter for disorder is taken as \( D = 2Wa^2 / \pi v_{f}^2 \). The conditions for the relevance and
the irrelevance of interaction, disorder (\( D(l) \)) and interchain hopping (\( t_{\perp}(l) \)) are examined by
using the RG method where the scale invariance is assumed under the transformation of
\( a(l+dl) \rightarrow a' \). The localization given by the pinned CDW is obtained for \( D^* \rightarrow \infty \) and the
confinement of single particle is obtained for \( t_{\perp}^* \rightarrow 0 \) where \( D^* \) and \( t_{\perp}^* \) denotes the fixed
point values for \( D(l) \) and \( t_{\perp}(l) \) with increasing \( l \) in RG equations.

![Fig. 1. Phase diagram on the plane of disorder (D) and interactions for \( t_{\perp}/t = 10^{-1} \)
(solitary curve) and \( 10^{-2} \) (dotted curve). Interactions are given by \( K_{\rho} = 1 - Va \sin k_F a / (\pi t) \)
for spinless case (a) and \( K_{\rho} = 1 - Ua / (4\pi \sin k_F) \) for spinful case (b).]
The phase diagram for the spinless case is shown in Fig. 1(a). One obtains $D^* = 0$ in the region A of the superconducting (SC) phase while disorder is relevant, i.e., $D^* = \infty$ in the regions of B and C of the $2k_F$ pinned CDW phase. The region C denotes the confinement, which leads to the irrelevant $t_\perp$ (i.e., $t_\perp^* = 0$). The global phase diagram for the spinful case is shown in Fig. 1(b). The region I denotes SCd (d-wave SC) phase with $D^* = 0$. The pinned $2k_F$ CDW is obtained in the regions III and IV where the confinement occurs in the region IV. Regions IIa and IIb lead to the SCs (s-wave SC) phases with $D^* = 0$. In the region IIa (IIb), $t_\perp^* = 0(\infty)$ and then the confinement is found in the region IIa leading to an interchain Josephson coupling. Note that pinning of $4k_F$ CDW is expected in the regions I, IIa and IIb although the explicit calculation is beyond the present scheme of RG. The boundary between confinement and deconfinement in the pinned CDW region moves continuously to that in the SCs state. The former is determined by disorder while the latter is determined by attractive interaction. With decreasing $t_\perp$ (dotted curve), the confinement and the pinned CDW are enhanced. We note that the spinful case shows the $2k_F$ CDW induced localization-delocalization transition for both repulsive and attractive interaction.

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4. References


