Formation interuniversitaire de physique

Evolution and Measurement of Quantum States

Exercise class n°3: Describing a real photodetector

Sylvain Nascimbène

A perfect photodetector is a measurement device that would record n 'clicks' for n incoming photons. In practice photodetectors exhibit imperfections that one should understand and model quantitatively in order to interpret measurements accurately.

1 Probability operator measure of a photodetector

The probability $P(n)$ that n photocounts are registered is related to the a priori probabilities $p(m)$ that m photons were present

$$P(n) = \sum_m P(n|m)p(m),$$

where $P(n|m)$ is the conditional probability that there are n photocounts if m photons were present.

1. Show that in the absence of information on $p(m)$ the probability that m photons were present given that n photocounts are recorded reads

$$p(m|n) = \frac{P(n|m)}{\sum_m' P(n|m')}.$$

2. Show that the quantum state associated with the detection of n photons is described by the density matrix

$$\hat{\rho}_n = \frac{\sum_m P(n|m) |m\rangle \langle m|}{\sum_m' P(n|m')}.$$

3. Show that this density matrix can be written as

$$\hat{\rho}_n = \frac{\hat{\pi}_n}{\mathrm{Tr}\,(\hat{\pi}_n)},$$

where the operators

$$\hat{\pi}_n = \sum_m P(n|m) |m\rangle \langle m|$$

are the elements of a probability operator measure.

2 Detector tomography of an avalanche photodiode

Avalanche photodiodes are photodiodes that use the avalanche effect (i.e. impact ionization leading to the exponential multiplication of carriers) to achieve sensitivity of single photons. These detectors have the remarkable ability to detect a single photon. Unfortunately, if more than one photon is detected this information is lost in the avalanche. Hence, this detector acts as a binary detector; the two possible detection outcomes are detection of at least one photon ('click') and the detection of no photon (no click). For each photon impinging on the detector there is an intrinsic efficiency of the detector η that the photon will cause an avalanche.

4. Show that detector can be modeled using the probability operators

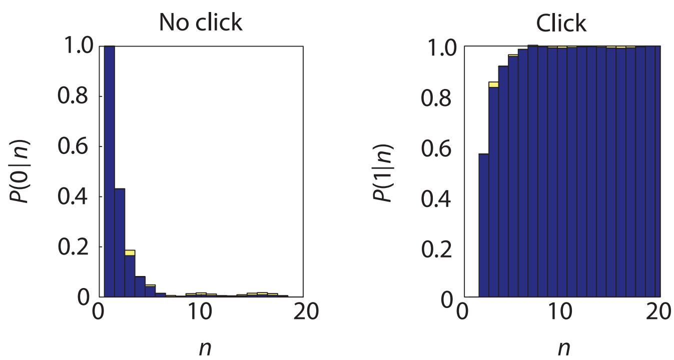
$$\hat{\pi}_0 = \sum_{n=0}^{\infty} (1-\eta)^n |n\rangle \langle n| , \quad \hat{\pi}_1 = \hat{\mathbb{1}} - \hat{\pi}_0.$$

5. Background photons and electronic noise may induce spurious dark counts in the detection. Explain why one expects the dark count probability distribution to be poissonian, given by $\nu^n e^{-\nu}/n!$, where ν is the mean dark count number.

6. Show that a noisy avalanche photodiode can be described by the POM

$$\hat{\pi}_0 = \sum_{n=0}^{\infty} e^{-\nu}(1-\eta)^n |n\rangle \langle n| , \quad \hat{\pi}_1 = \hat{\mathbb{1}} - \hat{\pi}_0.$$

7. Plot the conditional probabilities $P(0|n)$ and $P(1|n)$ for a detector of quantum efficiency $\eta = 90\%$ and a dark count probability $\nu = 20\%$. How does it compare with the experimental measurements in the graph below?



Let us discuss how to measure the probability operator measure associated with a detector. We will see that it can be characterized by measuring its action on a set of quantum states that can be prepared with good accuracy. Coherent states

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

of arbitrary amplitude α can easily be prepared.

8. How to prepare a coherent state $|\alpha\rangle$ of large amplitude?

9. Assume that one has a coherent state $|\alpha\rangle$ of large amplitude at hand. Show that one can create an arbitrary coherent state $|\alpha'\rangle$ of small amplitude (i.e. $\alpha' = \sqrt{\epsilon}\,\alpha$ with $0 < \epsilon < 1$) using a density filter of attenuation ϵ.

10. The detector response to coherent states is given by the so-called Husimi Q_n functions

$$Q_n(\alpha) = \langle\alpha|\,\hat{\pi}_n\,|\alpha\rangle\,.$$

Calculate the functions $Q_0(\alpha)$ and $Q_1(\alpha)$ for an avalanche photodiode.

11. Imagine how to reconstruct the POM of an avalanche photodiode from the measurement of Husimi functions. One should introduce a photon number cutoff N to consider a finite Hilbert space. How many measurements should be performed to reconstruct the conditional probabilities $P(n|m)$ ($n = 0, 1$, $0 \le m \le N$)?

References:

Tomography of quantum detectors, J. S. Lundeen, A. Feito, H. Coldenstrodt-Ronge, K. L. Pregnell, Ch. Silberhorn, T. C. Ralph, J. Eisert, M. B. Plenio, and I. A. Walmsley , Nature Phys. 5, 27 (2009)

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