Master ICFP

Ultracold Atoms

Exercise class n°9: Bloch oscillations in an optical lattice

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Understanding transport phenomena in electronic systems requires to model the dynamics of quantum particles in a lattice, in the presence of an external force. Zener predicted in 1934 that in clean systems electronic motion under electric field should be periodic, contrary to the common intuition of conduction. These so-called Bloch oscillations cannot be observed in usual metals, due to the predominance of electron scattering against impurities. On the contrary, ultracold atoms can be trapped into optical lattices with almost no defect, making it possible to observe Bloch oscillations.

We consider in the following the 1D problem of an atom in an optical lattice $V(x) = U\cos^2(kx)$.

1 General description of Bloch oscillations

1. Show that the eigenstates of the single-particle Hamiltonian can be written as
$$\psi_{n,q}(x) = e^{iqx}u_{n,q}(x),$$
where $u_{n,q}(x)$ is a periodic function of x, of period π/k. Give the allowed values for n and q.

2. Plot the dispersion relation $E_{n,q}$ expected for a depth $U = 10\,E_r$.

3. Starting from a BEC at rest, we slowly increase the lattice depth U up to the final value $U = 10\,E_r$. Give a criterion on the loading timescale leading to the preparation of the ground state in the lattice with high fidelity.

Assume that one prepares an atomic wavepacket that can be described as the ground state of the optical lattice, i.e. $\psi(x, t = 0) = u_{0,0}(x)$. We suddenly apply a force F to the atom.

4. Explain why the system can be described by the Hamiltonian
$$H = \frac{p^2}{2m} + U\cos^2(kx) - Fx.$$

5. Does one expects that the atom remains in a Bloch state under the action of this Hamiltonian?

6. We suggest to perform the gauge transform defined as
$$|\psi'\rangle = U(t)\,|\psi\rangle$$
$$U(t) = e^{-iFtx/\hbar}.$$
Give the expression of the novel Hamiltonian $H' = U(t)HU^\dagger(t) + i\hbar[\partial_t U(t)]U^\dagger(t)$.

7. Show that under the action of H' the atom remains described as a Bloch state of quasi-momentum $q' = 0$.

8. Explain that this implies that the evolution under the actual Hamiltonian H leads to a state
$$\psi(x,t) = e^{iq(t)x}u(x,t), \quad q(t) = Ft/\hbar,$$
where $u(x,t)$ is a function of spatial period $\lambda/2$.

9. Show that the evolution of the quasi-momentum is periodic, and give the period t_B of these oscillations.

10. Explain why for weak forces F we expect $u(x,t) \simeq u_{n,q(t)}(x)$. What is the maximum timescale for such an adiabatic evolution? Explain why a particle held into a very weak lattice does not undergo Bloch oscillations.

11. Using the definition of the group velocity

$$v_g = \frac{1}{\hbar} \left. \frac{\mathrm{d}E_{n,q}}{\mathrm{d}q} \right|_{q=q_0}$$

of a wavepacket centered around q_0, find the time evolution of the center of the wavepacket $x_c(t)$. What is the spatial amplitude of Bloch oscillations?

2 Bloch oscillations and Bragg diffraction

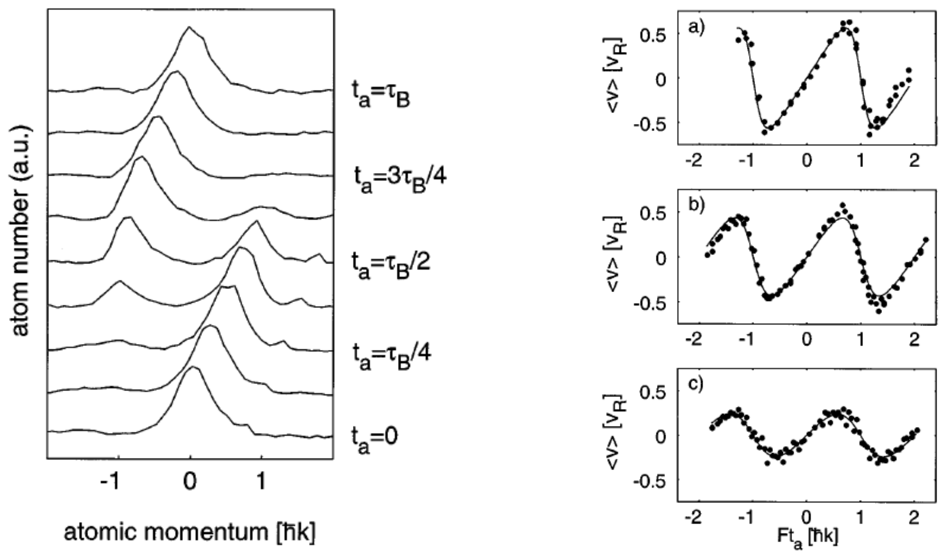
A nice picture of Bloch oscillations can be provided using Bragg diffraction arguments.

12. Recall which momentum states are resonant for Bragg diffraction.

13. Explain the interpretation of Bloch oscillations as periodic Bragg diffraction events occuring at the edges of the Brillouin zone.

3 Experimental realization

We show below a measurement of the velocity distribution of an atomic gas undergoing Bloch oscillations (left panel). The right panel shows the evolution of the mean velocity of the wavepacket, for lattice depths $U/E_r = 1.4$, 2.3 and 4.4 (from PRL **76**, 4508 (1996)).



14. Instead of applying a force to the atoms, one uses an accelerated optical lattice. Explain the principle of this method and the order of magnitude of the required acceleration.

15. The velocity distributions shown above correspond to the lattice frame of reference. What would be the time evolution of velocity in the lab frame?

16. Explain the shape of the time evolution of the mean velocity. Give an analytic expression for the velocity evolution in the limits $U \ll E_r$ and $U \gg E_r$.