Master ICFP

Ultracold Atoms

Exercise class n°8: Hanbury Brown and Twiss effect

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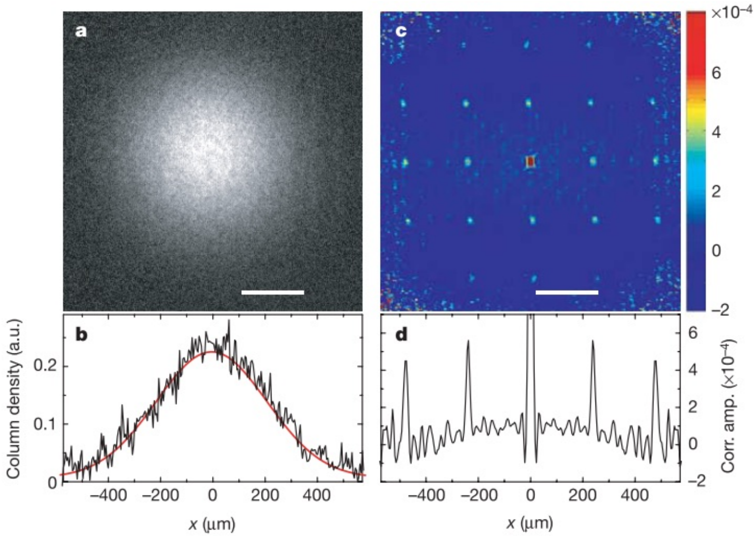
We consider cold atoms confined in an optical lattice. Observing the density noise of the atoms during time of flight it is possible to demonstrate the Hanbury Brown and Twiss effect which states that independent particles have a tendency to bunch or antibunch depending on their quantum statistics.

We show below such a measurement for bosons in the Mott insulating regime (from Nature 434:481 (2005)). The autocorrelation of time of flight pictures shows a regular pattern revealing the lattice order of the atoms in the trap. The fact that these peaks have positive amplitude comes from the quantum statistics of the atoms.



In order to quantify this effect, we estimate in the following the second order correlation function defined as:

$$g^{(2)}(\mathbf{x}_1, \mathbf{x}_2; t) = \frac{\langle : \hat{n}(\mathbf{x}_1, t)\hat{n}(\mathbf{x}_2, t) : \rangle}{\langle \hat{n}(\mathbf{x}_1, t)\rangle\langle \hat{n}(\mathbf{x}_2, t)\rangle}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ corresponds to different positions, t is the time of flight and $: \bullet :$ indicates the normal ordering. Experimentally, this is done by calculating the spatially averaged density-density correlation function

$$C(\mathbf{d}) = \frac{\int \langle n(\mathbf{x} + \mathbf{d}/2) \cdot n(\mathbf{x} + \mathbf{d}/2)\rangle d^2\mathbf{x}}{\int \langle n(\mathbf{x} + \mathbf{d}/2)\rangle\langle n(\mathbf{x} + \mathbf{d}/2)\rangle d^2\mathbf{x}}$$

which can be deduced from time of flight pictures. In this equation $n(\mathbf{x})$ is the column density obtained from single pictures and the brackets $\langle\rangle$ denote an averaging over an ensemble of independently acquired images.

Bosons in the Mott insulating regime

We consider a system of N bosons confined in an optical lattice. We assume that the lattice sites are independent, *i.e.* the system is in the Mott insulating regime.

1. The density-density correlations can be expressed as $\langle \hat{n}(\mathbf{x}_1,t)\hat{n}(\mathbf{x}_2,t)\rangle = \langle \hat{a}^\dagger(\mathbf{x}_1,t)\hat{a}(\mathbf{x}_1,t)\hat{a}^\dagger(\mathbf{x}_2,t)\hat{a}(\mathbf{x}_2,t)\rangle$, where $\hat{a}$ is the field operator. We assume that $\mathbf{x}_1 = \mathbf{x} - \frac{1}{2}\mathbf{d}$ and $\mathbf{x}_2 = \mathbf{x} + \frac{1}{2}\mathbf{d}$ where $\mathbf{d}$ corresponds to the distance between the two detectors. Taking into account the free propagation of the atoms during the time of flight t, show that the operator $\hat{a}(\mathbf{x},t)$ is related to the on-site operators $\hat{a}(\mathbf{r}_j)$ for the lattice sites j at positions $\mathbf{r}_j$ as

$$\hat{a}(\mathbf{x},t) = \sum_j w(\mathbf{x}-\mathbf{r}_j,t)e^{i(m/2\hbar t)(\mathbf{x}-\mathbf{r}_j)^2}\hat{a}(\mathbf{r}_j)$$

where w is the expanding wavefunction originally localized to the Wannier function at the site.

2. In the Mott insulating regime the state of the system corresponds to a product of Fock states with site occupation n_i at site i. Deduce that $\langle \hat{a}^\dagger(\mathbf{r}_k)\hat{a}^\dagger(\mathbf{r}_m)\hat{a}(\mathbf{r}_l)\hat{a}(\mathbf{r}_n)\rangle$ is the sum of two terms.

3. Show that the correlation function $C_{3D}(\mathbf{d}) = C(\mathbf{x}_1 - \mathbf{x}_2)$ can be written

$$C_{3D}(\mathbf{d}) = 1 + \frac{1}{N^2}\sum_{k,l} e^{i(m/2\hbar t)(\mathbf{x}_1-\mathbf{x}_2)\cdot(\mathbf{r}_k-\mathbf{r}_l)}n_k n_l$$

4. For a regular one-dimensional lattice with unity filling and spacing a_{lat} show that the sum simplifies to $C_{1D}(d) = 1 + \sin^2(\pi N d/l)/[N^2\sin^2(\pi d/l)]$ with $d = x_2 - x_1$ and $l = \hbar t/(m a_{\text{lat}})$.

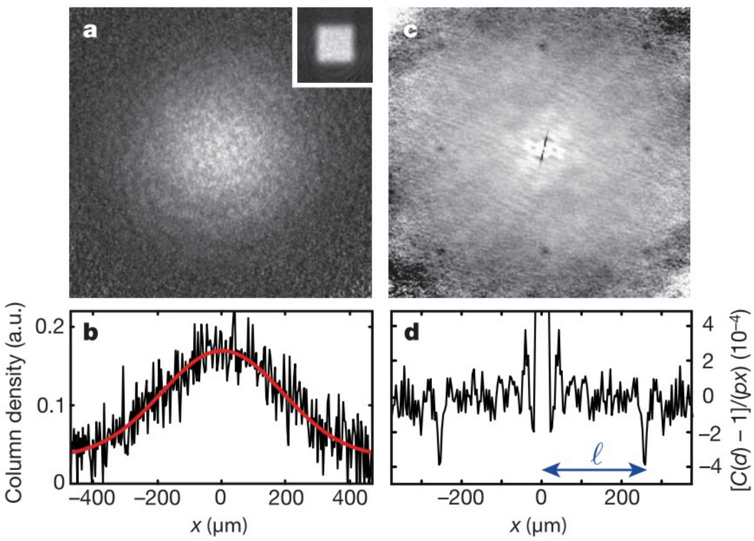
5. Express C_{1D} in the large N limit. How does it translate to the 3D case ? It is useful to introduce here the reciprocal three-dimensional lattice momenta $\mathbf{p}_j$.

6. We now take into account the finite resolution of the detector. The operators $\hat{n}(\mathbf{x}_{1,2})$ have to be convolved by the inverse point spread function (approximated by a gaussian of r.m.s. width σ). For unity filling, show that

$$C(\mathbf{d}) = 1 + \frac{1}{4\pi N}\left(\frac{l}{\sigma}\right)^2\sum_j e^{-\{[\mathbf{d}-\mathbf{p}_j t/m]^2/4\sigma^2\}}$$

Free fermions in a degenerate Fermi gas

An analog of the previous experiment has been performed with fermions (see Nature 444:733 (2006)).



1. How does the quantum statistics of the free fermions affect the calculation of $C_{3D}(\mathbf{d})$?
2. Following the same reasoning than for the boson lattice, find an expression for $C_{3D}(\mathbf{d})$. Note that here the lattice sites occupation is given by the Fermi-Dirac distribution.