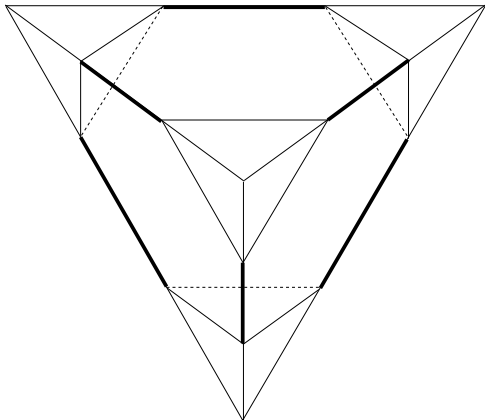

Analytical theories for effective Hamiltonians in frustrated magnets

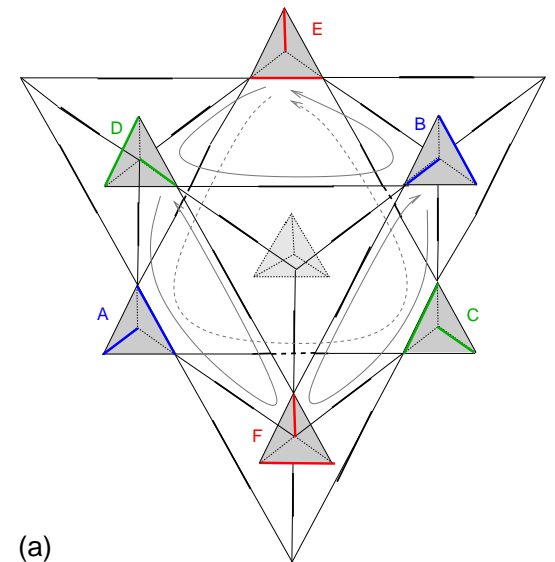


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cond-mat/0502146+0508504



(a)

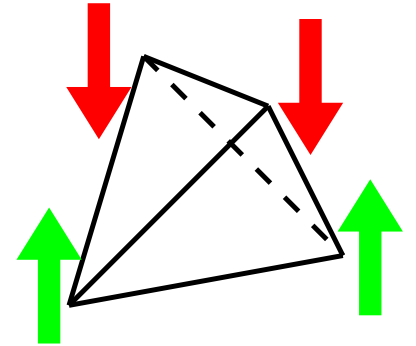
Overview

- Interplay analytics-numerics
 - **problem: no exact information (in either case)**
- Strong-coupling pictures
 - ***qualitative* understanding from analytics**
- Downwards from high energies to effective Hamiltonians: the pyrochlore $\text{Sp}(N)$ antiferromagnet – expansion in $1/N$
 - $1/N^0$: **dimer limit**
 - $1/N^1$: **exact solution of quantum dimer model symmetry**
 - $1/N^2$ and beyond: **effective dimer Hamiltonians**
- Custom-tailoring phases via Klein models
 - ⇒ **$\text{SU}(2)$ invariant spin liquids and valence-bond phases**
Anderson 1972, 1987.

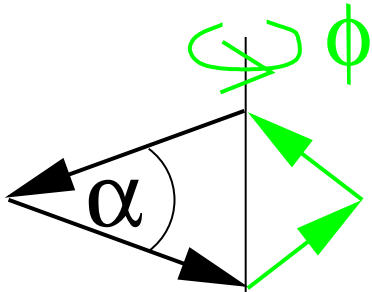
Frustration from a tetrahedron onwards

Consider four Heisenberg spins $\mathbf{S}^2 = S(S + 1)$ with $J > 0$:

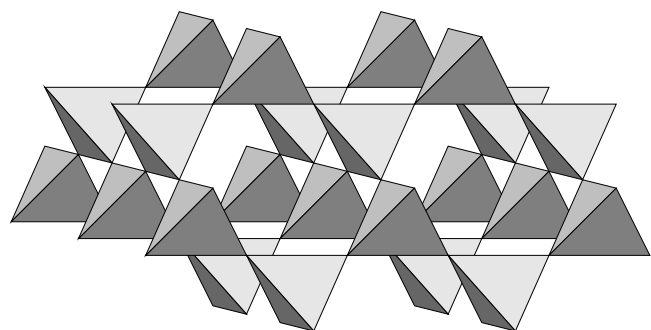
$$\mathcal{H}_{tet} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \simeq \frac{J}{2} \left(\sum_{i=1}^4 \mathbf{S}_i \right)^2 \equiv \frac{J}{2} \mathbf{L}^2$$



- $\mathbf{L} = 0$ in ground states \Rightarrow degeneracy
- Classical: two continuous **internal** d.o.f. (α, ϕ)
- Quantum: $2S + 1$ ground states
- $S = 1/2$:
 - two ground states
 - three dimerisations (linearly dependent)



The full lattice



$$\mathcal{H} = \sum_{tet} \mathcal{H}_{tet} = \frac{J}{2} \sum_{tet} \left(\sum_{i \in tet} \mathbf{S}_i \right)^2 = \sum_{tet} \mathbf{L}_{tet}^2$$

- Operators \mathbf{L}_{tet} do not commute on different tetrahedra
 $\implies ???$
- Two paths for analytical progress:
 - Construct model in which operators (almost) commute
 \implies large- N
 - Not in fact necessary for terms in \mathcal{H} to commute
 \implies Klein models
- previous work on $S = 1/2$: Harris *et al.*, Canals *et al.*, Tsunetsugu, Berg *et al.*,
...

The $Sp(N)$ Hamiltonian: from high energies down

- Basic strategy for $S = 1/2$: Schwinger Bosons Arovas+Auerbach
⇒ introduce many Boson flavours N
⇒ controlled theory in $1/N$ Read+Sachdev

- Schwinger Bosons $\{b_\uparrow, b_\downarrow\}$ represent the spin operators:

$$S^z = (b_\uparrow^\dagger b_\uparrow - b_\downarrow^\dagger b_\downarrow)/2, \quad S^+ = b_\uparrow^\dagger b_\downarrow,$$

- Spin length constraint: $n_b \equiv b_\uparrow^\dagger b_\uparrow + b_\downarrow^\dagger b_\downarrow = 2S + 1 \equiv 1$
- $\mathbf{S}_i \cdot \mathbf{S}_j = (\epsilon^{\sigma\tau} b_{i\sigma}^\dagger b_{j\tau}^\dagger / \sqrt{2})(\epsilon^{\mu\nu} b_{i\mu} b_{j\nu} / \sqrt{2}) - 1/4 \equiv s_{ij}^\dagger s_{ij} - 1/4$

- Generalise singlet operator to many flavours:

$$s_{ij} \equiv \epsilon^{\sigma\tau} b_{i\sigma} b_{j\tau} / \sqrt{2} \rightarrow \mathcal{J}_{AB}^{\sigma\tau} b_{i\sigma}^A b_{j\tau}^B / \sqrt{2N} \text{ with } \mathcal{J}_{AB}^{\sigma\tau} \equiv \epsilon^{\sigma\tau} \delta_{AB}$$

$$\Rightarrow \mathcal{H} = -J \sum_{\langle ij \rangle} \left\{ s_{ij}^\dagger s_{ij} - \frac{1}{4N^2} \right\} \text{ 'counts' the number of singlets}$$

The large- N limit of $\mathcal{H} = -J \sum_{\langle ij \rangle} \left\{ s_{ij}^\dagger s_{ij} - \frac{1}{4N^2} \right\}$

- Many $N \rightarrow \infty$ limits possible
 - at fixed n_b/N , get Schwinger-Boson mean-field theory
 - at fixed $n_b = 1$, get dimer model Sachdev \rightarrow this talk
- (classical) dimer model at $O(1/N^0)$: unlike SU(2)
 - all dimerisations are orthogonal
 - all dimerisations are ground states
 - the manifold of dimer coverings may or may not incorporate non-trivial correlations
- Corrections at $O(1/N^1)$ lead to degenerate perturbation theory \Rightarrow quantum dimer model

The quantum dimer model at $O(1/N^1) = O(1/N)$

- Dimer coverings non-orthogonal at subleading order
- Leading correction occurs for dimer coverings differing by one pair of dimers only

⇒ effective dimer Hamiltonian flips a pair of dimers

⇒ pictorial representation as resonance term:

$$\mathcal{H}_{QDM} = \frac{2J}{N} |\begin{array}{c} \bullet \text{---} \bullet \\ \bullet \text{---} \bullet \end{array}\rangle \langle \begin{array}{c} \bullet \\ \bullet \end{array} \bullet \bullet|$$

- Solution of quantum dimer model in general difficult
- Pyrochlore lattice: exact solution possible

The pyrochlore quantum dimer model

- Single tetrahedron: three dimerisations
- Effective dimer model: $\mathcal{H}_{tet} = 0$ if 0 or 1 dimer on tet, else:

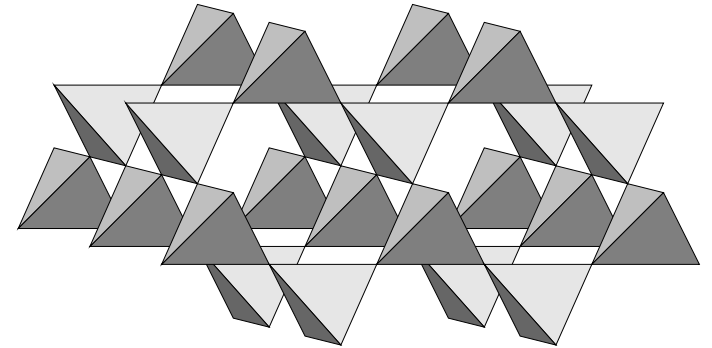
$$\mathcal{H}_{tet} = \frac{2J}{N} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

- \mathcal{H}_{tet} has two ground states with $E = -2J/N$
- ⇒ Ground states of $\mathcal{H} = \sum_{tet} \mathcal{H}_{tet}$ maximise number of tetrahedra occupied by two dimers

Q: What do these states look like?

The ground states of the pyrochlore QDM

- All dimers occupy one sublattice of tetrahedra
 - (# dimers) = (# tet)
 - doubly occupied tet cannot be neighbours
- breaks inversion symmetry (and nothing else)
- displays residual entropy: $\mathcal{S} = (1/4) \ln 2$
- formally identical to starting point of mean-field and CORE calculations: Harris+Berlinsky+Bruder, Tsunetsugu, Berg+Altman+Auerbach
 - effective d.o.f. on one sublattice of tetrahedron only
 - effective d.o.f. form **E**-representation of T_d symmetry



Beyond $O(1/N)$

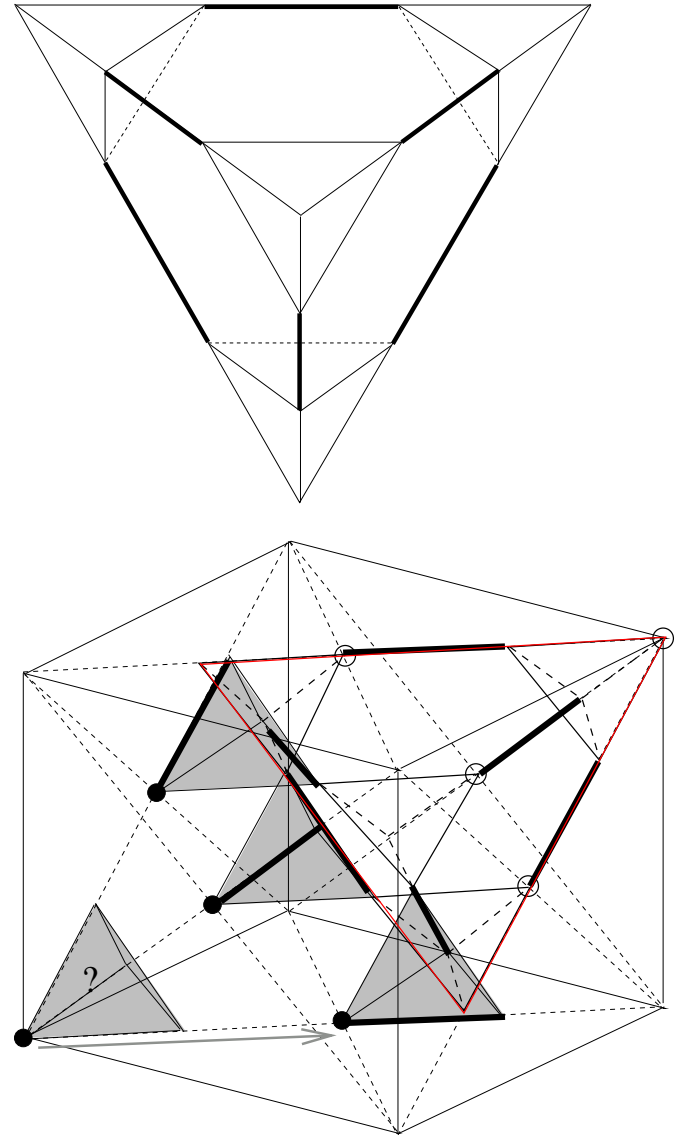
- No exact solution known beyond $O(1/N^1)$
- Can we use effective d.o.f at $O(1/N^1)$ to guess at H_{eff} ?
- Dimer *kinetic* terms (longer loops, e.g. hexagonal): impossible as they shift dimers onto wrong sublattice
- Dimer *potential* terms still possible, e.g.:

$$H_3 = - \sum_{\text{hex}} |'_{-}\rangle \langle'_{-}| ; \quad H_2 = - \sum_{\text{hex}} |' \setminus \rangle \langle' \setminus |$$

- Written out explicitly, they have *exactly* the same form as Hamiltonians of **HBB-T**; **EEA**!
- Large- N : simple picture for effective Hamiltonians

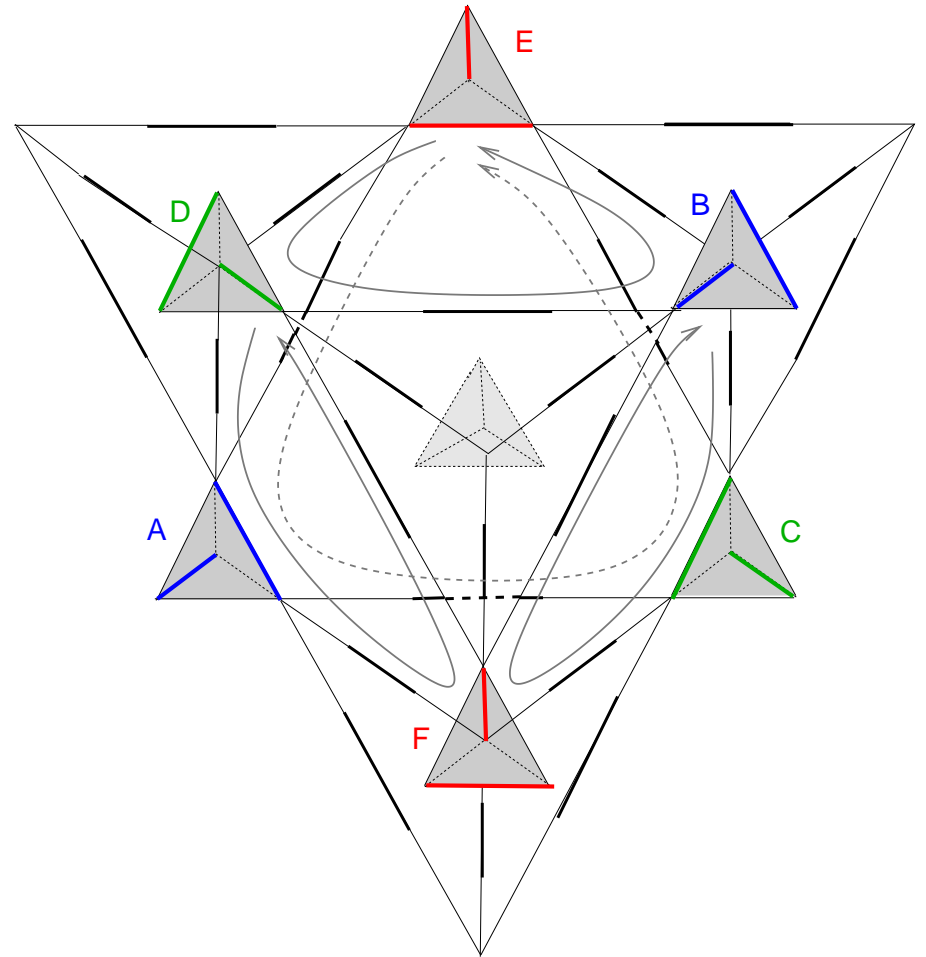
The *HBB-T* state as maximally flippable dimer state

- Minimising $H_3 \Leftrightarrow$ maximising # of flippable hexagonal loops
- This is done by the densest tiling of ‘flippable supertetrahedra’
 - $1/4$ of all hex loops flippable
 - $1/4$ of all dimers unusued
- Mean-field theory of *HBB-T* predicts four-sublattice tetrahedral ordering with one disordered sublattice
 - This is maximally flippable!
 - Different focus: tetrahedra vs. supertetrahedra



Fate of disordered sublattice

- Our treatment no longer controlled \Rightarrow no reason for going further
- Can nonetheless ask questions about longer loops
- One resolution: flippable supertetrahedra of the second generation



Implications of the large- N theory

- Large- N scenario: hierarchy of symmetry breakings
 - $O(1/N^0)$: dimer manifold with $\mathcal{S} = \mathcal{S}_{dimer}$
 - $O(1/N^1)$: inversion symmetry (definitely) broken; $\mathcal{S} = (1/4) \ln 2$
 - beyond $O(1/N^1)$: full ordering
- multi-stage quantum order-by-disorder
 - Maximal flippability at $O(1/N^1)$ and in HBB-T state
- hierarchical structure experimentally observable?
- provides simple ('strong-coupling') physical picture
 - different effective H at each level
- presumably this approach shares strengths+weaknesses with other RVB-type approaches (see planar pyrochlore, ...)

From effective Hamiltonians up

- Basic question: knowing effective H , how do we generate appropriate d.o.f and interactions between them?

Q: How to generate dimerised phases for $SU(2)$ Heisenberg?

- Strategy: find a set of non-commuting but simultaneously satisfiable (by dimerisation) projectors.
- Write \mathcal{H} as sum of projectors onto maximal spin state (triplet: $L = 1$) of each bond: $\mathcal{H} = J \sum_{\langle ij \rangle} \mathcal{P}_{L=1}$, where $L = \mathbf{S}_i + \mathbf{S}_j$.
- Different \mathcal{P} do not commute in H'berg case.

Q: Can they be simultaneously be satisfied?

A: In general: **No!** See e.g. square lattice H'berg $S=1/2$ afm

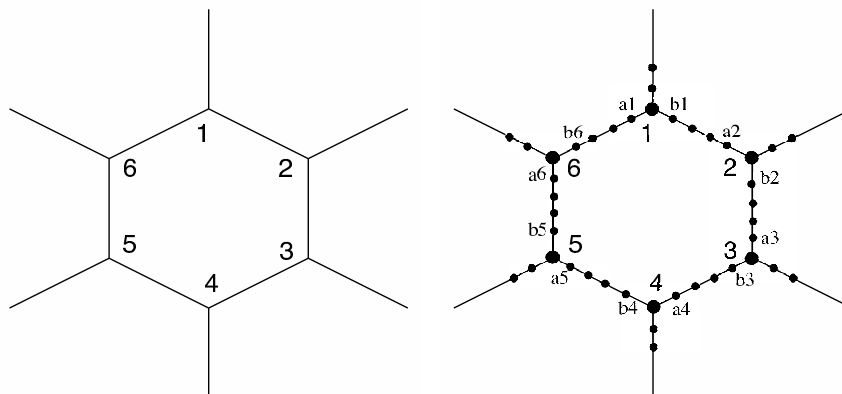
Q: Are there spin models where this is nonetheless possible?

A: **Yes!** Ferromagnets; Majumdar-Ghosh chain, ...

The Klein model route to quantum dimer models

- H'berg: project bond onto maximal spin state $L = 1$
 - Klein: project spin and its z n.n. onto $L = (z + 1)/2$
- ⇒ Klein $\mathcal{H} = \sum_i \mathcal{P}_{L=(z+1)/2}$ minimised by n.n. dimer coverings
- dimer coverings non-orthogonal
 - dimer coverings not the only ground states

- decoration makes dimer states orthogonal and gaps spinons



- add small perturbations to gapped system to generate QDM
- can thus realise SU(2) invariant phases:
topological+deconfined; Coulomb; Cantor deconfined; VBS;

Summary

- Aim: describe systems+models exhibiting new phases
- Analytical contributions:
 - $1/N$ expansion
 - custom-tailored controlled \mathcal{H} via Klein models
- Generates scenarios
- Identifies ‘good’ (and bad) effective variables
 - hierarchical quantum order by disorder
 - multiple-step entropy loss
 - partially ordered phases
- Supplements numerics
- Can make ‘in-principle’ statements