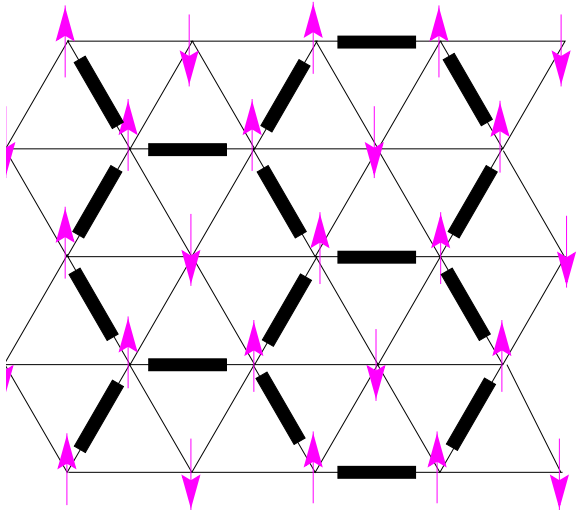

A Fermionic magneto-supersolid on the triangular lattice



Roderich Moessner

CNRS and ENS Paris

Shivaji Sondhi

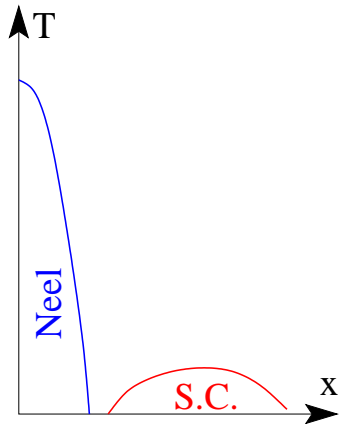
Princeton

Outline

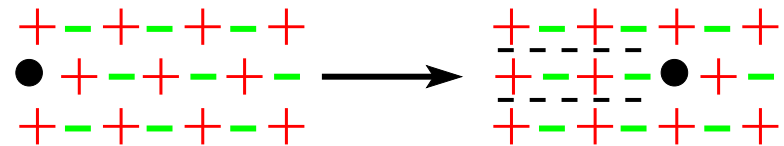
- Motivation
 - correlated electrons
 - itinerant frustration
- Single hole: a frustrated Nagaoka theorem
 - translational and Ising symmetry breaking
 - 'nano-phase separation'
- Many holes: an effective attractive interaction
 - $U(1)$ charge symmetry breaking

⇒ magneto-supersolid
- Heavy doping: an incompressible state
- Finite temperatures: stability of order by disorder
- Outlook

Motivation



‘frustrated kinetic energy’:
hopping holes \Rightarrow domain walls



\Rightarrow new physics?

- Itinerant electrons on frustrated lattice are interesting
 - heavy Fermions without d -electrons: LiV_2O_4
 - cobaltate superconductors
 - triangular RVB liquid
 - ...

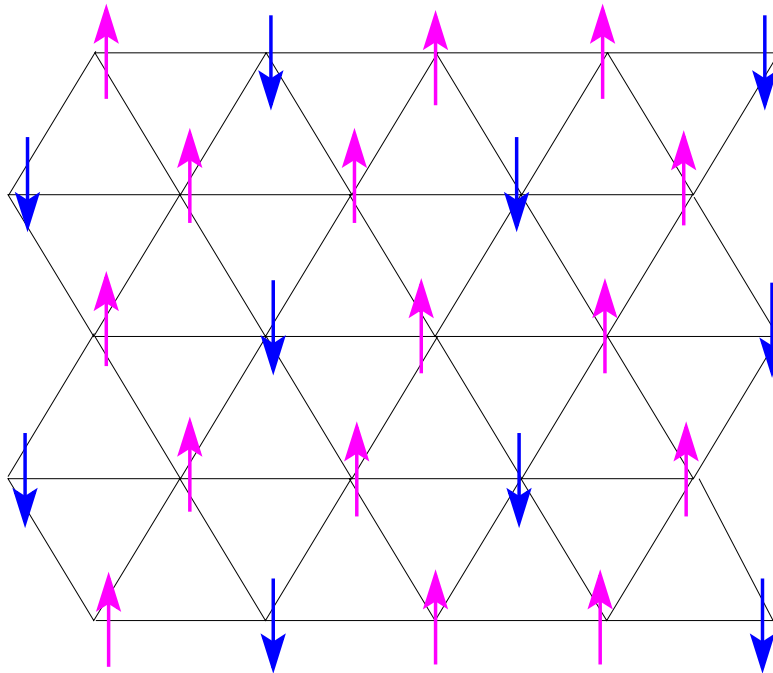
The model: electrons on a triangular lattice

- Treat exchange as leading term ('strong correlations')
- Ising exchange
 - undoped model is classical
 - undoped model is extensively degenerate
- No double occupancy preserved by hopping term
- Additional density interaction ($\eta = 1/4$ for Hubbard model)

$$H = J^z \sum_{\langle ij \rangle} S_i^z S_j^z - t \sum_{\langle ij \rangle} \mathcal{P} c_{i\sigma}^\dagger c_{j\sigma} \mathcal{P} - \eta J^z / 4 \sum_{\langle ij \rangle} n_i n_j$$

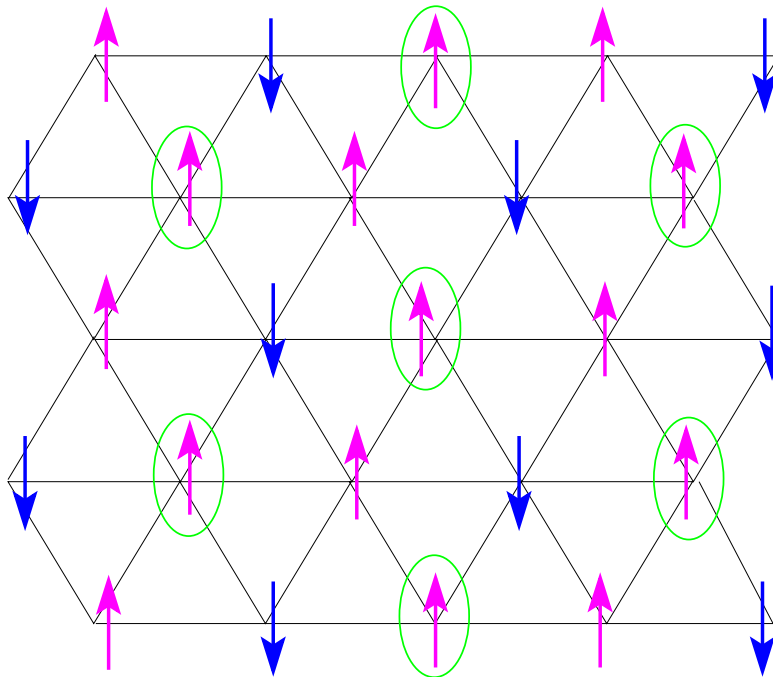
The leading term: triangular Ising AFM *Wannier; Houtappel*

- ground-state condition: $|\sum_{i \in \mathcal{S}_i} S_i| = 1/2$
 - finite entropy in ground state: $\mathcal{S} = 0.323k_B$
 - ‘flippable spins’ experience vanishing exchange field
- ⇒ lower bound on entropy $\mathcal{S} = (k_B/3) \ln 2$ *Anderson*



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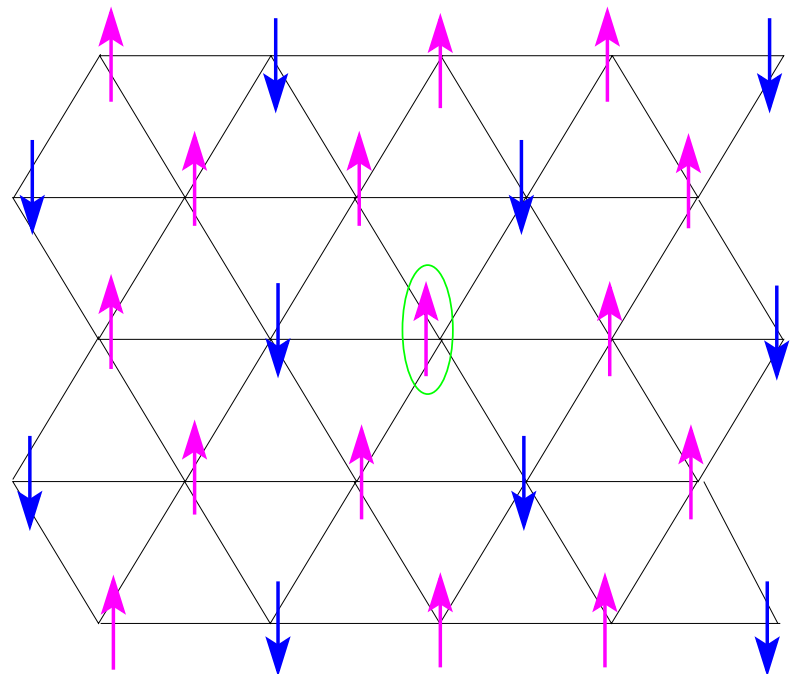


Doping a single hole: frustrated Nagaoka theorem

- Removing spin costs no magnetic energy iff it's flippable
- Degenerate perturbation theory between one-hole states
- max. local coordination of flippable spins is 3 $\Rightarrow E_0 \geq -3t$
- unique state with coordination three everywhere

\Rightarrow flippable spins form hexagonal backbone

- Result: one hole breaks
 - Ising symmetry ($M = 1/3$)
 - translational symmetry ($Q \rightarrow \sqrt{3} \times \sqrt{3}$)

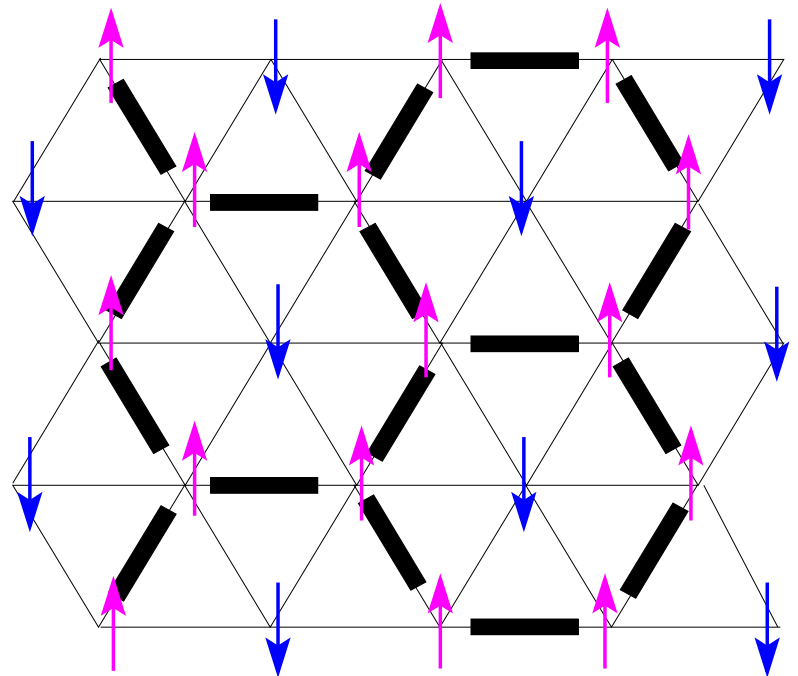


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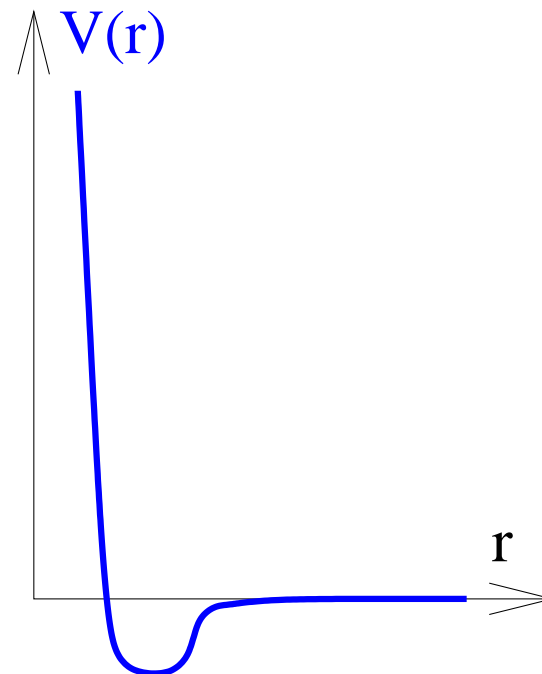
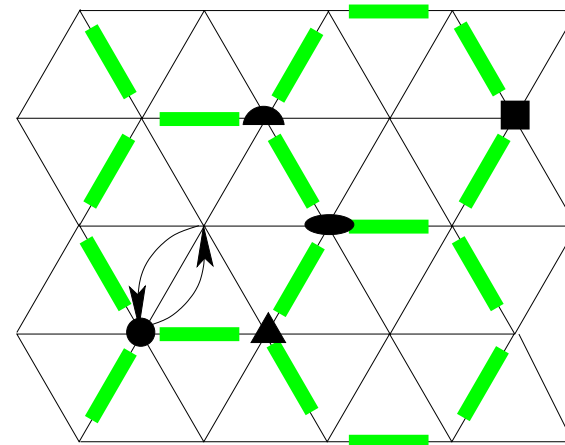
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Two-hole interaction (low density)

- Assume one-hole physics (maximise local coordination) is robust
- ⇒ all holes hop on hexagonal backbone: **2d Fermi liquid**
- Hopping off backbone gives a two-body potential V , perturbative in t/J^z
 - Attractive tail: **superconducting instability** in *odd* (possibly high) relative angular momentum channel
- ⇒ **magnetic supersolid**



Stability to thermal fluctuations

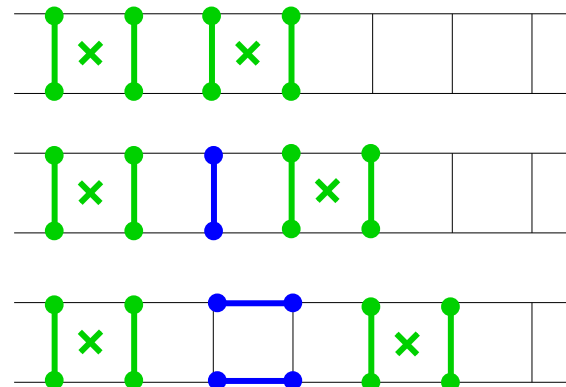
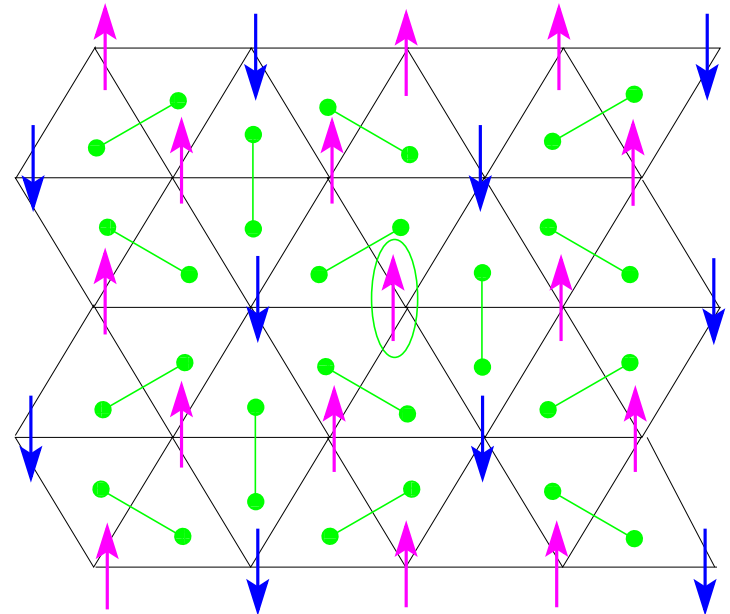
- calculation of classical two-hole potential
- ⇒ soluble in dimer language
- effective potential:

$$\beta v(r) \sim -\frac{\Xi}{\Upsilon^2} \frac{\cos(4\pi x/3)}{x^2 + y^2}$$

$$\Xi = \frac{729 - 324\sqrt{3}\pi - 324\pi^2 + 96\sqrt{3}\pi^3 + 64\pi^4}{288\pi^6}$$

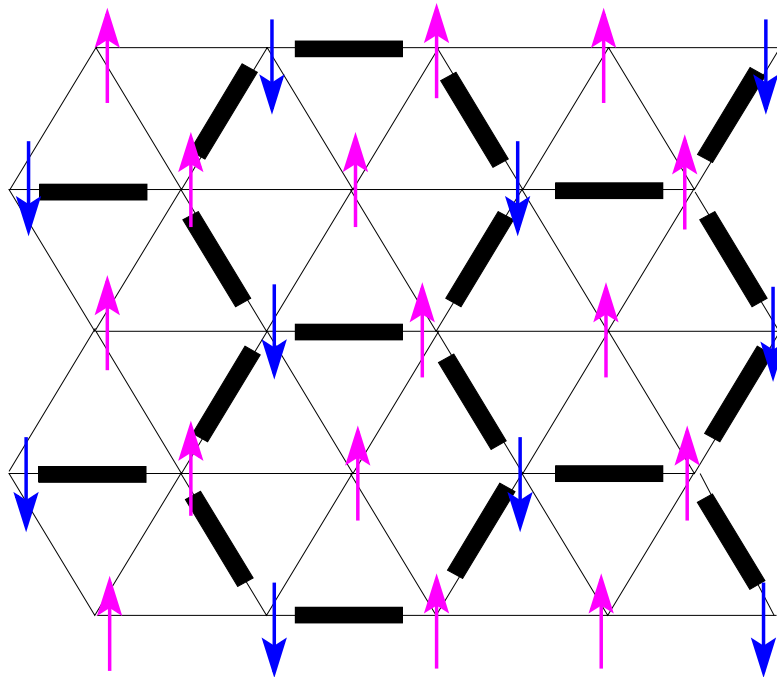
$$\Upsilon = \frac{2}{27} - \frac{3\sqrt{3}}{8\pi^3} + \frac{1}{2\sqrt{3}\pi}$$

- also favours $\sqrt{3} \times \sqrt{3}$ pattern
- cf. ladder result



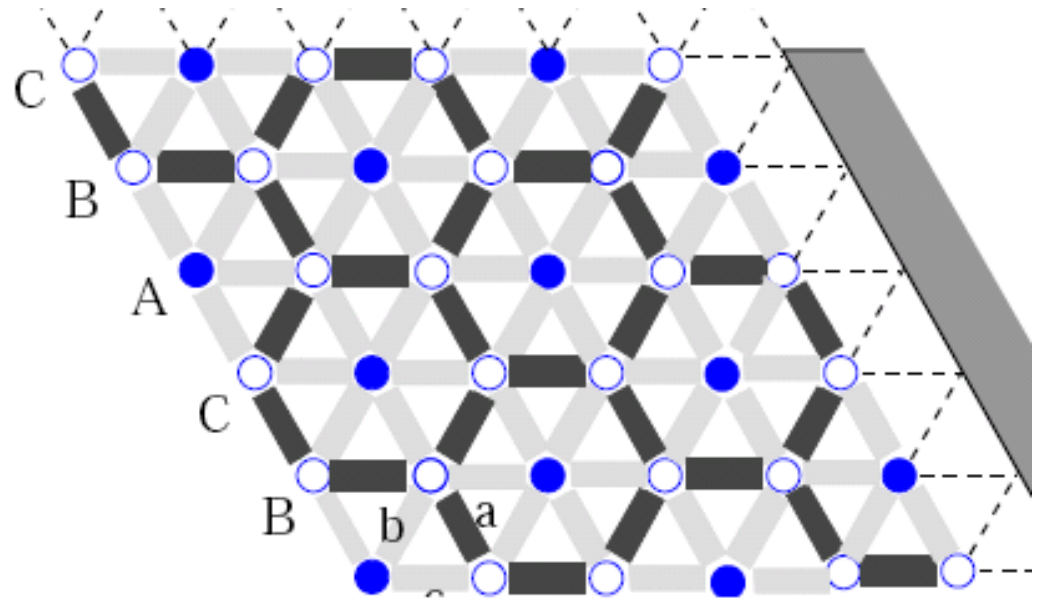
Larger hole densities

- doping upto hole density $1/3$ costs no magnetic energy
- ⇒ fully occupied (bipartite) honeycomb lattice
- doping one additional hole costs $E \sim O(J^z)$
 - discontinuity of $\mu \Rightarrow$ incompressible state



Related work: bosons

- hardcore bosons hopping on triangular lattice
- presence of boson \Leftrightarrow Ising spin
- n.n. repulsion \Leftrightarrow Ising afm
- hopping \Leftrightarrow ferromagnetic XY exchange
- weak hopping limit \Rightarrow supersolid



dark bonds: high k.E.
dark sites: high density

Melko et al., [Heidarian+Damle](#), [Wessel+Troyer](#), [Boninsegni+Prokofev PRL 2005](#)

also work on 'long-range' supersolids

[Melko et al. 2006](#)

Summary

- spectacular symmetry breaking by single hole
- magneto-supersolid for low doping
- easy for hole to hop through frustrated magnet
 - rearrange, rather than create debris
- connections to supersolids
- also: 'Ising models of quantum frustration'
- relevance to Heisenberg? Shastry: kinetic antiferromagnetism