On bipartite Rokhsar-Kivelson points and Cantor deconfinement

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References and collaborators

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E. Fradkin, D. Huse, R. Moessner, V. Oganesyan and S. Sondhi, *On bipartite Rokhsar-Kivelson points and Cantor deconfinement*, Phys. Rev. B **69**, 224415 (2004); arXiv:cond-mat/0311353.

R. Moessner, S. L. Sondhi and E. Fradkin, *Short-ranged RVB physics, quantum dimer models and Ising gauge theories*, Phys. Rev. B **65**, 024504 (2002); arXiv:cond-mat/0103396.

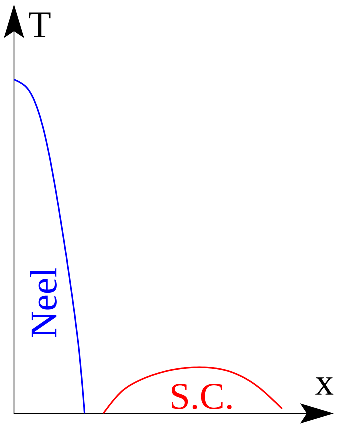
see also A. Vishwanath, L. Balents, T. Senthil, *Quantum Criticality and Deconfinement in Phase Transitions Between Valence Bond Solids*, Phys. Rev. B **69**, 224416 (2004); arXiv:cond-mat/0311085.

Outline

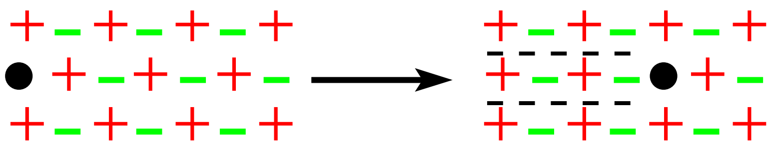
- Origin and usefulness of quantum dimer models
- Phase diagram of the Rokhsar-Kivelson QDM:
  valence-bond crystals and the deconfined RK critical point
- Phase diagram of the perturbed quantum dimer models:
  tilted phases, incommensurate states and devil's staircase
- Field theory for the RK points
  - Duality to quantum height models
  - Quantum Lifshitz action for the RK point
  - Generic critical behavior of perturbed quantum dimer models on the honeycomb lattice
  - Behaviour on other lattices
- Conclusions

Short-range RVB physics

Basic problem of high-T_c: how do holes hop through an antiferromagnetic Mott insulator?



Hole motion is frustrated:
hopping creates domain walls

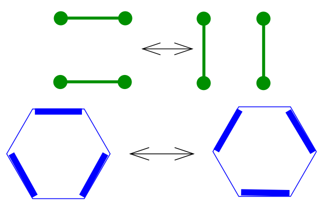


Possible resolution: magnet enters a different phase
resonating valence bond liquid phase
which breaks no symmetries.

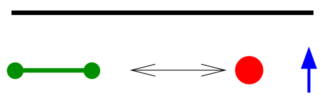
Neighbouring electrons form a singlet ("valence") bond
$\rightarrow$denoted by a dimer.

The basic RVB scenario - electron fractionalisation

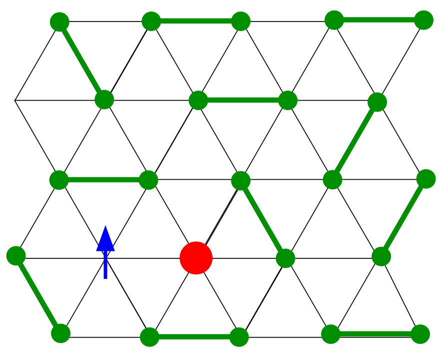
Energetics	RVB	Neel
single pair	valence bond optimal	
higher coordination	energy from resonance	... each neighbour
hole doping	motion unimpeded	motion frustrated

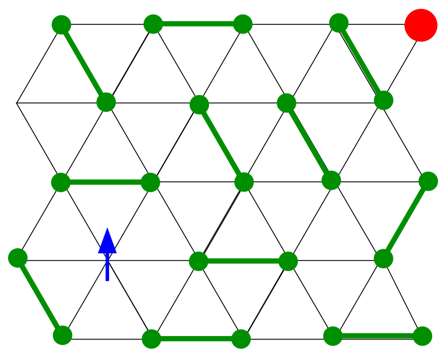


- Basic resonance move is that of benzene



- Removing an electron $\rightarrow$ holon + spinon



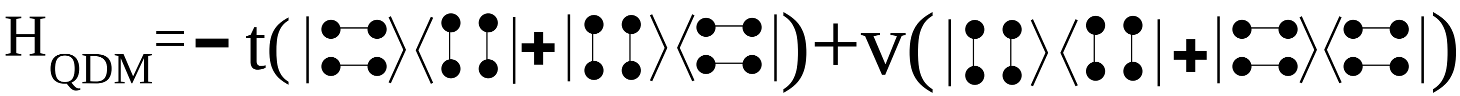


spinon and holon are deconfined

$\downarrow$

(bosonic) holons can condense

The Rokhsar-Kivelson quantum dimer model



- Resonance (t) and potential (v) term from uncontrolled approximation – one parameter: v/t
- Simple model for quantum frustration (cf. equivalences):

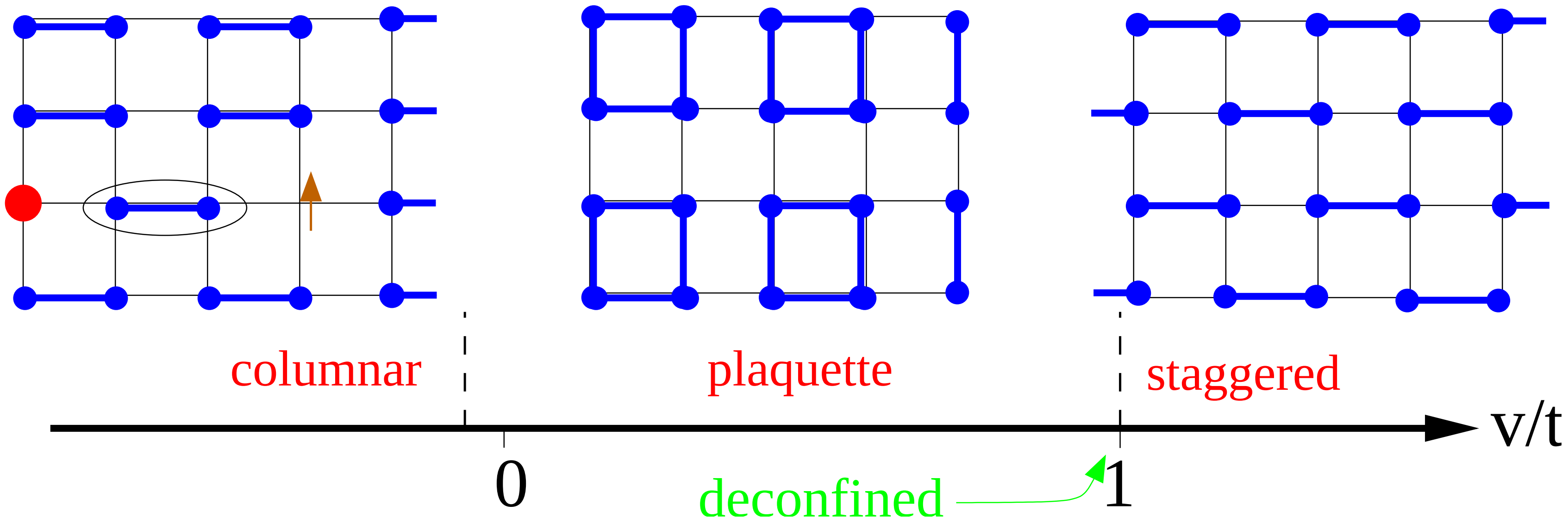
  degeneracy + quantum fluctuations

- $v/t > 1$ and limits of $v/t \rightarrow -\infty$ give solid (staggered and columnar, respectively) phases
- RK point $v/t = 1$ is exactly soluble in $d = 2$ at $T = 0$:

$$|0\rangle = \frac{1}{\sqrt{N_c}} \sum_c |c\rangle \rightarrow \langle \hat{P} \rangle = \frac{1}{N_c} \sum_{c,c'} \langle c|\hat{P}|c'\rangle = \frac{1}{N_c} \sum_c p_c$$

$\rightarrow$ classical calculation for diagonal operators

Phase diagram for the square lattice



- all phases confining (break translational symmetry) RK, Fradkin; Read, Sachdev, Jalabert; Ioffe, Larkin; Leung; ...
- RK point deconfined RM+Sondhi, see recent work on deconfined QCPs
- RK point highly degenerate RK

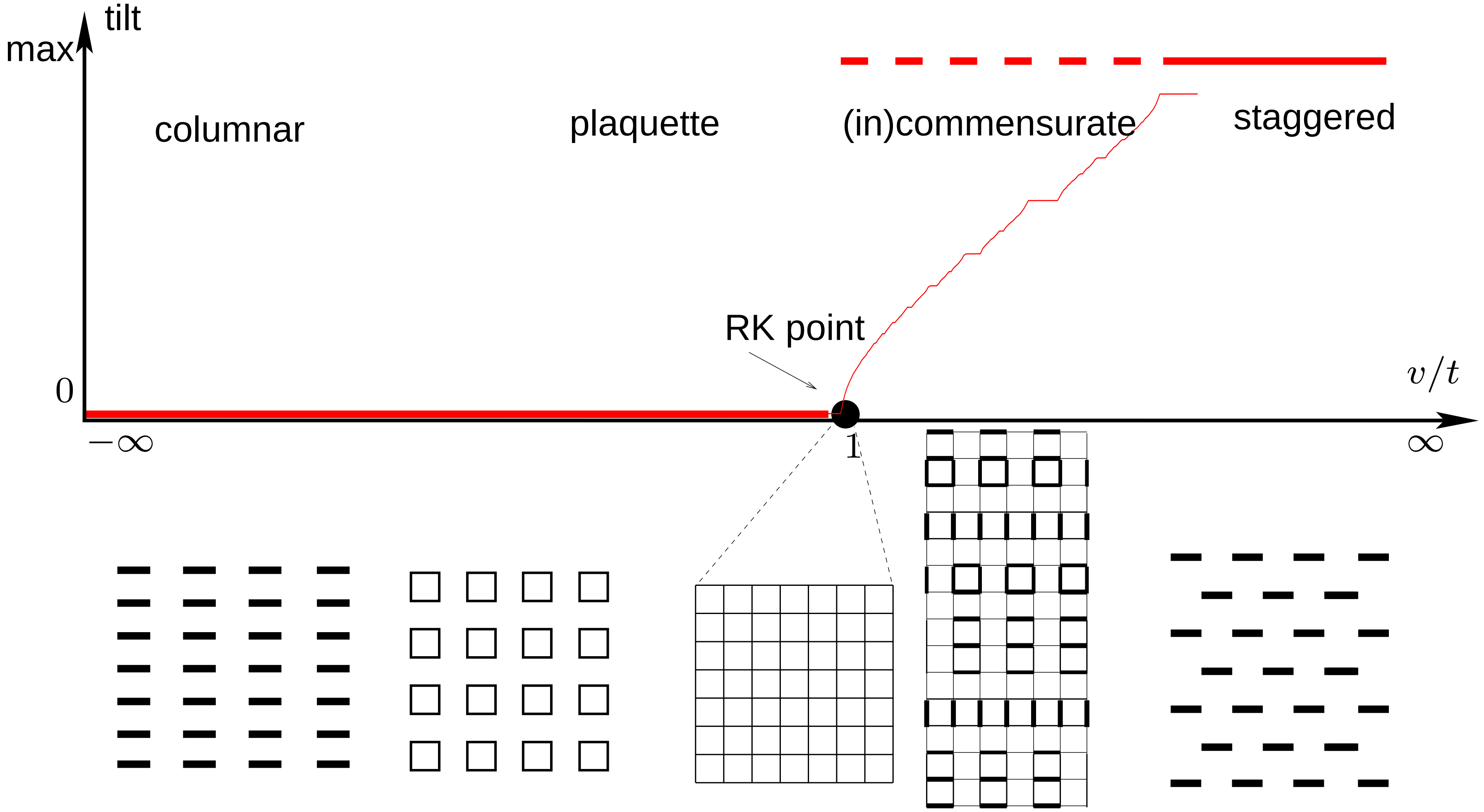
Questions about the RK point phase transition

- Why is there a deconfined critical point separating two confining phases?
- Is this transition generic – is it robust to local perturbations? Why this is not a first order transition (as naively expected)?
- QDM models are known to have columnar, plaquette and staggered phases – these are all commensurate. Are there also incommensurate ones? Global phase diagram?
- What is the effective field theory of this quantum phase transition? Is this a new universality class?

Some answers

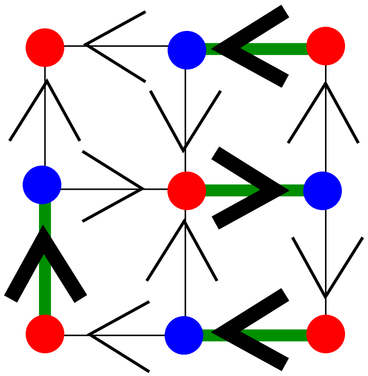
- Ordering (and confinement) is via dangerously irrelevant operator ('invisible' at deconfined critical point)
- RK point is non-generic:
  - Generic phase transition is first order or a continuous one into a different (not the staggered) phase.
  - perturbations yield a devil's staircase of interleaved commensurate and incommensurate phases
- Field theory is defined in terms of dual variables, not the obvious dimer order parameter

Schematic phase diagram of generic QDMs



Height/gauge mapping of square lattice dimer model

Orientation of dimers (from red to blue sublattice) is possible.



Magnetic analogy: dimer = magnetic flux $\vec{B}$

- Link with dimer $\rightarrow$ flux $\vec{B} = +3$
- Unoccupied link $\rightarrow$ flux $\vec{B} = -1$
- $\nabla \cdot \vec{B} = 0 \rightarrow \vec{B} = \nabla \times \vec{A} = \nabla \times h$

- 'Vector potential' $\vec{A}$ in $d = 2$ is simple scalar height function h
  Youngblood et al.
- Columnar phase: $\langle|\nabla h|\rangle = 0$ Staggered phase: $\langle|\nabla h|\rangle \neq 0$
- Mapping to height takes care of hardcore constraint
  $\Rightarrow$ we can coarse-grain safely to get effective long-wavelength theory Henley

Mapping for dimer density operators

- Microscopically: dimers (on links) vs. heights (on sites of dual)

- Upon coarse graining, dimer density operators pick up an extra term Blöte, Hilhorst, Nienhuis, Nightingale, ...:

$\square$: $$n_x - \frac{1}{4} = \frac{1}{4}(-1)^{x+y}\partial_y h + \frac{1}{2}[(-1)^x \exp(\pi i h/2) + \text{h.c.}]$$

⬡ : $$n_1 - \frac{1}{3} = \frac{1}{3}\partial_x h + \frac{1}{2}[\exp(4\pi i x/3)\exp(2\pi i h/3) + \text{h.c.}]$$

Effective field theory of the RK QCP Henley; RM, Sondhi+Fradkin

$$\mathcal{L} = \frac{1}{2}(\partial_\tau h)^2 + \frac{1}{2}\rho_2(\nabla h)^2 + \frac{1}{2}\rho_4(\nabla^2 h)^2 + \lambda\cos(2\pi h)$$

- $\mathcal{L}$ only contains gradients and periodic functions of h
- Lock-in potential, λ, keeps track of the discreteness of the microscopic heights
- Properties of RK point as desired:
  - ρ_2 vanishes and changes sign at the RK point $\Rightarrow z = 2$.
  - non universal $\rho_4 = (\pi/32)^2$;$(\pi/18)^2$ (square;honeycomb)
  - lock-in potential is irrelevant
  - reproduces correct critical classical correlations.
  - Same action as smectic layers in $d = 3$ at Lifshitz transition: ‘quantum Lifshitz model’, Grinstein; Ardonne et al.

The vicinity of the RK Point

- On bipartite lattices only highly commensurate ordered states (and critical points) are allowed in the QDM see Read+Sachdev
- The transition at the RK point between the plaquette $(\rho_2 > 0)$ and staggered phases $(\rho_2 < 0)$ is unusual.
  - The plaquette order parameter vanishes continuously but the staggered order parameter appears in full strength: ‘3/2 order phase transition’

The generic field theory on the honeycomb

- $\mathcal{L}$ is rotationally invariant and does not depend on the symmetries of the underlying lattice
- A generic effective Lagrangian must include the relevant cubic perturbation

$$\mathcal{L}_3 = g_3(\partial_x h)(\frac{1}{2}\partial_x h - \frac{\sqrt{3}}{2}\partial_y h)(\frac{1}{2}\partial_x h + \frac{\sqrt{3}}{2}\partial_y h)$$

and the marginal quartic perturbation,

$$\mathcal{L}_4 = g_4 \left[\nabla h \cdot \nabla h\right]^2 .$$

- If $g_3 \neq 0$, the relevant perturbation takes over, drives the transition first order and the resulting staggered state has maximal tilt – in accordance with Landau expectation.

Stability of the RK Point at $g_3 = 0$

- RG flow for $g_3 = 0$ (fine-tune or consider ice model):

$$\frac{d\lambda}{dt} = -\left(\frac{\pi}{2\rho_4^{1/2}} - 2\right)\lambda$$
$$\frac{dg_4}{dt} = -\frac{9}{4\pi\rho_4^{3/2}} g_4^2 \,,$$

- $g_4 > 0 \quad \Rightarrow \quad \lambda$ irrelevant and g_4 marginally irrelevant
- logarithmic corrections to scaling
- finite renormalization of ρ_4
- The multicritical RK point is stable on a surface of codimension 2 ($\rho_4^{\text{eff}} \leq \pi^2/16$).

The tilted phase ($v/t > 1$)

- For g_4 large enough, the tilt is not immediately maximal
- Small fluctuations in the weakly tilted state

$$h(\mathbf{r}, \tau) = \mathbf{C} \cdot \mathbf{r} + \delta h(\mathbf{r}, \tau), \qquad \mathbf{C} = C\mathbf{e}_x$$

$$\delta\mathcal{L} = \frac{1}{2}(\partial_\tau \delta h)^2 + \frac{\rho_4}{2}(\nabla^2 \delta h)^2 + \frac{v_l^2}{2}(\partial_x \delta h)^2 + \frac{v_t^2}{2}(\partial_y \delta h)^2$$

- New Bragg peaks in the dimer density structure factor
  - *commensurate* Bragg peak at the wavevector of the maximally “staggered” state located at the origin
  - *incommensurate* peaks displaced from the wavevector of the columnar/plaquette pattern, $(4\pi/3, 0)$, by an amount proportional to C.

Lock-in of crystals

- We have so far ignored the possibility of the height lock-in into tilted configurations.
- The height and the lattice points together define a 3D lattice $\Gamma = \{(h, \mathbf{x})\}$, the simple cubic lattice with the $[111]$ direction measuring height $\Rightarrow$ General lock-in potential:

$$V_{\rm lock}(h, \mathbf{x}) = \sum_{\{\mathbf{G}\}} V_{\mathbf{G}} \; \exp[i(G_h h + \mathbf{G}_{\mathbf{x}} \cdot \mathbf{x})], \quad \mathbf{G} \in \mathbf{\Gamma}^*$$

- Tilt commensurate with any of the $\{\mathrm{G}\}$: Lock-in potential is relevant – the ground state is a gapped valence bond solid.
- Incommensurate tilts: lock-in depends upon the strength of $V_{\rm lock}$ relative to the remaining quantum fluctuations.

Strong fluctuations or weak lock-in

Near the RK point:

- Higher order commensuration to lock-in $|\mathbf{G}| \sim 1/C$
  $\Rightarrow$ lock-in potentials get weak and increasingly irrelevant
- $V_{\text{lock}}^{\text{eff}} \to 0$ and the gap $\Delta \to 0$ as $C \to 0$: even the commensurate phases are essentially gapless!

$$\Delta \sim C^{a/(\rho_4 C^2)}, \qquad \xi \sim 1/C \;, \qquad \xi_c \sim \xi^{a'\xi^2/\rho_4}$$

- Ground states are incommensurate dimer crystals (measure $\to 1$ for $C \to 0$) with a Bragg peak at the incommensurate wavevector but with a gapless (phason) spectrum

$\Rightarrow$ 'Cantor deconfinement'

Weak fluctuations or strong lock-in

Further away from RK point:

- Ground states form a complete devil’s staircase
- Aubry’s “breaking of analyticity” transition beyond which
  - the incommensurate ground states are pinned,
  - their low lying excitations are localized,
  - the incommensurate ground states occupy a set of measure zero of the phase diagram.

Effective Theories for other lattices

- Square lattice
  - two additional quartic terms allowed by symmetry, which lead to fluctuation-driven first-order transition
  - RK point requires fine-tuning of two parameters.
- Triangular ice (20-vertex) model
  - same as hexagonal QDM but *without* relevant perturbation g_3.
  - RK point is genuine critical point

Conclusions

- The simple RK model displays very rich behaviour
- RK points are deconfined multicritical points
- Effective field theory in keeping with Landau theory, but only in terms of dual variables
- The effective field theories near the RK points admit additional operators, which
  - make the transition first order or
  - give continuously growing tilt leading to a complex sequence of commensurate and incommensurate phases (“Cantor deconfinement”).
  - (almost) a deconfined phase for a U(1) gauge theory in $d = 2 + 1$

Generic phase diagram